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ROCKING AND OVERTURNING RESPONSE OF RIGID BODIES TO EARTHQUAKE MOTIONS

A Report of an Analytical and Experimental Study on the Rocking and Overturning Response of Rigid Blocks to Simultaneous Horizontal and Vertical Accelerations

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LIST OF SYMBOLS

a	Amplitude of acceleration
B	Total width of the block
H	Total height of the block
b	$B/2$
h	$H/2$
R	$\sqrt{b^2 + h^2}$
K	Stiffness
M	Mass of the block
W	Weight of the block
$\ddot{u}$	Horizontal ground acceleration
$\ddot{v}$	Vertical ground acceleration
g	Acceleration of gravity
T	Period of vibration
ω	Angular frequency
θ	Angular displacement
$\dot{\theta}$	Angular velocity
$\ddot{\theta}$	Angular acceleration
ν	Coefficient of restitution
μ	Coefficient of friction

ABSTRACT

This report is a fundamental study of the rocking and overturning response of massive concrete blocks (with relatively high aspect ratio) to simultaneous horizontal and vertical earthquake ground motions. This problem occurs when large concrete blocks are used as radiation shields in particle accelerators or similar nuclear installations. The results of this study also offer insight into the response of the rigid bodies (such as are approximated by some electrical machinery and mechanical equipment) which are not anchored to the ground.

The mathematical model used was based on the assumption of a constant coefficient of restitution. A computer program was written to predict the rocking and overturning behavior of rigid rectangular blocks under simultaneously applied horizontal and vertical ground accelerations.

To check the accuracy of the computer model, tests were carried out on a 20 ft x 20 ft shaking table at the University of California, Berkeley. Free vibration tests, as well as forced vibration tests, were conducted under the effect of simultaneously applied horizontal and vertical accelerations. A comparison was made between the test and theoretical results and a satisfactory agreement was found between the two.

Using the computer program, the response of rigid blocks of various aspect ratios and sizes was studied under the accelerograms of various strong motion earthquakes. The coefficient of restitution was varied to see its effect on the rocking response. Results regarding

the angular displacement, velocity, and acceleration response of the blocks to various accelerograms were plotted on the Calcomp plotter. The consequences of prestressing the massive concrete blocks (with relatively higher aspect ratios) to the floor was also studied.

In light of this report and the danger of overturning if rocking occurs, it is recommended that the radiation shielding systems be prevented from rocking either by prestressing to the ground or by reducing the coefficient of friction between the block and the floor, thus allowing the block to slide. The sliding of rigid bodies in earthquakes has been discussed in an earlier report.[1]

KEY WORDS

Radiation shielding systems, Rocking of blocks, Overturning of blocks, Coefficient of restitution, Coefficient of friction, Sliding of blocks, Earthquake motions.

SUMMARY AND CONCLUSIONS

1. Response Modes

Systems comprised of solid blocks (such as radiation shielding blocks or heavy electrical machinery and mechanical equipment approximating rigid bodies) can be designed to respond to earthquakes in the following three modes: (a) sliding within predetermined limits, (b) rocking without overturning, or (c) moving integrally with the ground. For heavy masses, costs for support structures are least where sliding can be tolerated and greatest where no movement can be permitted relative to the ground. The structural engineer must choose which of the three support structure types (corresponding to response modes a, b, or c) will best meet the system and cost requirements.

For rigid bodies which are not firmly anchored to the ground, the two response modes to earthquakes are (a) sliding or (b) rocking. This report describes theoretical and experimental results developed during the course of this investigation to determine the dynamic response of solid blocks for those cases where the rocking-mode of response is initiated under simultaneous vertical and horizontal ground accelerations. A separate report[1] treats the sliding-mode of response.

2. Boundary Between Sliding- and Rocking-Modes

The boundary between the sliding- and rocking-modes of solid blocks depends on μ_s, the static coefficient of friction between the block and the floor, and on H/B, the height-to-width ratio of the block. For an unrestrained block with perfectly plane interface with the floor, an earthquake can induce (a) sliding if $H/B < 1/\mu_s$ or (b) rocking if $H/B > 1/\mu_s$. If the interface surfaces are not plane, rocking can start at lower H/B ratios.

3. Computer Program BLOKROC

A computer program, BLOKROC, has been developed to give the time history response (angular displacement, angular velocity, and angular acceleration) of an unrestrained block responding in the rocking-mode to simultaneous vertical and horizontal earthquake accelerations as a function of the block's coefficient of restitution, size, and shape.

BLOKROC was developed as a result of the present investigation. The basic assumptions for the mathematical model solved numerically by BLOKROC are listed in Section 2.7 of this report. This model represents the equations of motion of a rocking block with a constant coefficient of restitution driven by simultaneously applied vertical and horizontal ground accelerations. Computer and test results on the angular displacement of a block within the region of stability were found to agree within 10 percent. The acceleration time histories in both the vertical and horizontal planes of any real or postulated earthquake, the block's coefficient of restitution, the block's aspect ratio, and the block size (for a given aspect ratio) can be used as input data to the BLOKROC program.

BLOKROC will become available from the Earthquake Engineering Research Center of the University of California of at Berkeley.

4. Detailed-Results Using BLOKROC

Detailed results on the rocking response of blocks to earthquakes are given in Section 4 of this report. Five different strong motion earthquake accelerograms were used. Blocks with different aspect ratios, and of different sizes (for a given aspect ratio), and with different coefficient of restitution values were investigated.

In general, it was found that once the rocking-mode response has started, the possibility of overturning is quite real for blocks with aspect ratios (height-to-width) as low as 2.0 for the strong earthquakes considered. This emphasizes the desirability of preventing, where possible, the initiation of the rocking response.

5. Conclusions

(1) A mathematical model of the equations-of-motion has been developed to predict the rocking response of solid blocks (or rigid bodies approximated by electrical machinery or mechanical equipment) driven by simultaneous vertical and horizontal ground motions during an earthquake. This theoretical model assumes a constant coefficient of restitution (i.e., one independent of impact velocity) and has been validated by experiment. There was good agreement (within 10 percent) between theoretical predictions and experimental results.

(2) The rocking response is very sensitive to small changes in ground acceleration, coefficient of restitution, or any external forces acting on the block. This sensitivity is due to the dependence of the natural period of vibration on the amplitude of vibration of a rocking block.

(3) Rocking response is independent of the density of the material of the block, but is dependent on the overall dimensions; i.e., the block with the same aspect ratio but different dimensions will have a different response. In general, the stability of blocks against overturning increases as the size of the block increases for a fixed aspect ratio.

(4) A decrease in the value of the coefficient of restitution does not always decrease the maximum response of a block under a given ground accelerogram.

(5) If the rocking-mode of response is initiated, there is a high probability that a rigid block with an aspect ratio of 3 or more and a height of less than 15 ft will overturn during a severe earthquake. There is a low probability that a block will overturn if it has an aspect ratio of 3 or less and a height of more than 15 feet. However, comparatively small-sized blocks with aspect ratios of even less than 3 can overturn easily under rocking conditions.

(6) Whenever there is a danger of rocking and overturning, free-standing blocks could be allowed to slide by reducing the coefficient of friction between the ground and the block. Prestressing of massive concrete blocks with comparatively high aspect ratios should be done after careful study assuring that the foundation can withstand the dynamic forces. Allowing free-standing blocks to slide may be the best solution to overcome the undesirable rocking response during earthquakes. In this case the bottom surface of the block should be made slightly concave so that it rests on its outermost edges thus avoiding any premature initiation of rocking.

SECTION 1

INTRODUCTION

A general investigation has been undertaken to determine the seismic response of large free-standing concrete blocks. Such blocks, stacked in various configurations, are used to provide radiation shielding in particle accelerator laboratories. While the investigation is directed to large concrete blocks, any massive equipment presents a similar problem to the structural engineer. In the present state of the art, there is a lack of fundamental data and detailed analysis for selecting practical alternative solutions to the basic seismic problem for supporting massive equipment. Alternative approaches to the solution are as follows:

(1) To design foundations or floor structure of sufficient strength to prevent any relative motion between the support and the block system. The problems here relate to cost and the adequacy of the foundation to withstand the resulting forces, the determination of a credible design-basis earthquake, and the costs associated with tailor-made designs to resist the particular design-basis earthquake. In cases where the foundation strength is in question, any attempt to prevent relative motion may aggravate the earthquake damage and safety hazard.

(2) To provide a safer or lower cost design which uses some decoupling of the earthquake motions from the block system. The problem here is that a better understanding of the nature of the seismic response is needed to furnish a rational basis for such designs.

It is hoped that the present investigation will indicate solutions to the immediate problem of shielding blocks as well as contribute to the state of the art for seismic safety of massive equipment in general.

At the Lawrence Berkeley Laboratory and other such laboratories, massive shielding blocks are often stacked as much as 20 feet high and 15 feet deep to shield high energy physics experiments. Some of these blocks are provided with a vertical keying system that prevents relative horizontal movement between them, but does not prevent rocking. While rocking of the blocks and their possible overturning would be extremely destructive in an earthquake, a reasonable amount of sliding between the blocks and the floor might be tolerated as being the least destructive means of accommodating earthquake forces. This raises the questions of (1) how much sliding-displacement of a rigid block--or of a system of such blocks--could be expected in an earthquake, (2) how can the sliding-mode response be made to dominate the more hazardous rocking-mode by selecting proper design parameters, and (3) how much angular displacement occurs during the rocking-mode; what conditions induce a rocking block to overturn, etc. Little work had been reported on these questions prior to the initiation of this general investigation.

A free-standing rigid block can either slide or rock, or a combination of both, under the simultaneous vertical and horizontal ground excitation that occurs during an earthquake. The sliding-mode of response was fully investigated and the results presented in an earlier report[1] which discusses both experimental and theoretical studies, showing a good agreement between the two. The present report deals with the rocking behavior of rigid blocks, which may overturn if the blocks are not properly designed.

It was observed in some preliminary tests that the stability against rocking is very sensitive to the boundary conditions. If the block is not resting on its edges, rocking may start at accelerations lower than the theory would predict. Hence the block or ground surfaces that are not perfectly plane aggravate the rocking problem. This may be overcome by making the lower block surfaces slightly concave.

Experimental evidence also shows that it is reasonable to assume a constant coefficient of restitution (i.e., independent of impacting velocity), provided there is no serious damage to the edges of the block during rocking.

Using a constant coefficient of restitution, a general computer program was developed to solve numerically the equation of motion of a rocking block under horizontal and vertical ground accelerations. The computer program can take into account any prestressing forces acting on the block. Horizontal and vertical ground accelerations may, in general, be given at different points in time. Details of this and other theoretical treatment of the rocking problem are given in Chapter 2.

Tests were carried out on a 20 ft x 20 ft shaking table recently completed at the University of California, Berkeley. Tests were made under simultaneously applied horizontal and vertical ground accelerations. These tests were conducted on two concrete blocks, having height and width dimensions of 30 in. x 6 in. and 36 in. x 9 in., respectively. Free-rocking tests were conducted to determine the coefficient of restitution and the data were digitized and plotted for comparison with the analytical results.

Tests were conducted under harmonic as well as earthquake accelerograms. The angular displacement of the block was measured by means of a displacement meter. The data that were digitized and plotted included the applied horizontal and vertical accelerations of the table and the displacement of the block. Using the digitized accelerograms which were used in the tests and the coefficient of restitution determined from the free-rocking tests, computer analysis was carried out. The test and predicted results were compared and a satisfactory agreement was found between the two. The details of experimental studies and comparison with the computer results are described in Chapter 3.

After establishing the reliability of the computer model, some parametric studies were made under various strong motion earthquake accelerograms to investigate the rocking and overturning behavior of rigid blocks. The aspect ratios of the blocks, as well as the size of the blocks, were varied and the response was plotted on the Calcomp plotter. The effect of the coefficient of restitution on the rocking and overturning response of the blocks was also studied. Details are given in Chapter 4.

From the test and analytical data, some important observations have been made regarding the behavior of rocking blocks under ground motions. The report ends with a set of conclusions.

SECTION 2

THEORY OF ROCKING BLOCK PROBLEM

2.1 Conditions for Sliding and Rocking

Consider the block shown in Fig 2.1 with width and height equal to B and H, respectively, and subjected to horizontal and vertical ground accelerations $\ddot{u}$ and $\ddot{v}$, respectively. The block will be on the verge of rocking when the moment of the horizontal inertial force around one edge is equal to the restoring moment, i.e.,

$$M\ddot{u}(H/2) = W(1 + \ddot{v}/g)B/2$$

$$\ddot{u} = (B/H)\ g(1 + \ddot{v}/g) \qquad (2\text{-}1)$$

where M and W are the mass and weight of the block, respectively, and g is the acceleration of gravity. Rocking will, therefore, start only if

$$\ddot{u} > (B/H)\ g(1 + \ddot{v}/g)\ .$$

Now suppose that the static coefficient of friction μ is such that the block would start sliding instead of rocking under the ground accelerations. The block will be on the verge of sliding where the horizontal inertial force equals the frictional force, i.e.,

$$M\ddot{u} = \mu W(1 + \ddot{v}/g)$$

$$\ddot{u} = \mu g(1 + \ddot{v}/g)\ . \qquad (2\text{-}2)$$

Therefore the block will slide if $\ddot{u} > \mu g(1 + \ddot{v}/g)$.

A comparison of equations (2-1) and (2-2) shows that if

$\mu > (B/H)$, a smaller value of $\ddot{u}$ can set the block into rocking than the $\ddot{u}$ required for the block to slide, and vice versa. Therefore a block will rock only if

(1) $\mu > B/H$

(2) $\ddot{u} > (B/H)\, g(1 + \ddot{v}/g)$,

2.2 Free Vibrations

The rigid block shown in Fig. 2.2 will oscillate about the centers of rotation O and O° when it is given an initial displacement θ_o and then released. Let h be the distance of the centroid from the base of block, and b the distance from edge to the centroid. The radial distance from the center of rotation O to the centroid is R . I_o is the mass moment of inertia about O . α is the angle of the block as shown in Fig. 2.2. The tilting of the block is measured by the angle θ and $R = \sqrt{b^2+h^2}$.

When the block is rotated through an angle θ , the weight of the block will exert a restoring moment and the equation of motion is

$$I_o \frac{d^2\theta}{dt^2} = - WR \sin(\alpha - \theta) \qquad (2\text{-}3)$$

For tall slender blocks, Eq. (2-3) may be approximated by

$$I_o\ddot{\theta} - WR\theta = - WR\alpha$$

Let

$$p = \sqrt{\frac{WR}{I_o}} = \sqrt{\frac{3gR}{4R^2}} = \sqrt{\frac{3g}{4R}}$$

then Eq. (2-3) becomes

$$\ddot{\theta} - p^2\theta = -p^2\alpha \tag{2-4}$$

Equation (2-4) is independent of the weight of the block and is only dependent upon the dimensions of the block and not the density of the material. This equation is subject to the condition $\theta = \theta_0$ and $\dot{\theta} = 0$ at $t = 0$, which represents the block released from rest and has the solution[2]

$$\theta = \alpha - (\alpha - \theta_0) \cosh pt \tag{2-4a}$$

The block will fall from $\theta = \theta_0$ to $\theta = 0$ in a time $t = T/4$, where T = natural period, and at this instant, Eq. (2-4a) becomes

$$0 = \alpha - (\alpha - \theta_0) \cosh p \frac{T}{4}$$

$$T = \frac{4}{p} \cosh^{-1}\left(\frac{1}{1-\theta_0/\alpha}\right) . \tag{2-5}$$

Equation (2-5) gives the period T in terms of R and θ_0/α. A graph of this equation is shown in Fig. 2.3. It will be seen that the period is strongly dependent upon the amplitude ratio θ_0/α and is highly a non-linear problem. When θ_0/α is close to unity the period is long, and when θ_0/α is close to zero the period is short.[2]

During the rocking of a real block, there would be a dissipation of energy at each impact during each half cycle, and the period of each half cycle would be longer than that which will follow it. If the impact is without bouncing, the coefficient of restitution ν is defined as

$$\nu = \sqrt{\frac{1}{2} I_0 \dot{\theta}_{i+1}^2 / \frac{1}{2} I_0 \dot{\theta}_i^2} = \frac{\dot{\theta}_{i+1}}{\dot{\theta}_i} \tag{2-6}$$

where

$$\dot{\theta}_i = \text{angular velocity before impact}$$

and

$$\theta_{i+1} = \text{angular velocity after impact.}$$

2.3 Overturning by Constant Acceleration

If the block is resting on a base which is suddenly given a constant acceleration a of duration t_1, the block may or may not overturn, depending on the magnitude of a and the duration t_1.

A necessary condition for motion to be initiated is that $a/g > \alpha$. For a given value of a and for small angles of α (i.e., slender blocks), a good approximate value of t_1 required to overturn the block can be found by the following equation (the proof of which is given in Reference 2):

$$\cosh \left(\sqrt{\frac{3g}{4R}}\, t_1\right) = 1 + 1/\left[\frac{2a}{g\alpha}\left(\frac{a}{g\alpha} - 1\right)\right] \qquad (2\text{-}7)$$

Fig. 2.4 is a graph of this equation giving the duration t_1 of constant ground acceleration a required to overturn the block. Fig. 2-5 shows a plot of time t at which blocks of various dimension would overturn under a constant acceleration a lasting 0.1 sec.

The foregoing analysis is not realistic for earthquake ground motions since it assumes constant ground accelerations of finite duration followed by a constant velocity of the ground. This type of ground motion does not occur during earthquakes and hence it is not meaningful to discuss the overturning of blocks in this context.

2.4 Overturning by Sinisoidal Acceleration

The accelerograms recorded during earthquakes show maximum peaks which can be approximated by half sine waves. Now suppose that a half sine wave acceleration pulse of period T_s and amplitude a is applied to the base of the block. For slender blocks (i.e., $\sin \alpha \approx \alpha$) it can be shown (2) that the following equation specifies the value of T_s (for a given value of amplitude a) at which the block will overturn.

$$\frac{a}{g\alpha} = \sqrt{1 + (\omega/p)^2} = \sqrt{1 + \frac{4R}{3g}\left(\frac{2\pi}{T_s}\right)^2} \qquad (2\text{-}8)$$

where

$$\omega = 2\pi/T_s \ .$$

Equation (2-8) is a minimum condition to overturn the block as it satisfies the condition $\dot{\theta} = 0$ when $\theta = \alpha$. A plot of Eq. (2-8) is presented in Fig. 2.6 for small values of (ω/p) . For large values of (ω/p), i.e., $\omega/p > 3$, Eq. (2-8) can be approximated by

or

$$\frac{a}{g\alpha} = \omega/p \qquad (2\text{-}9)$$

or

$$\frac{a}{g\alpha} = \omega/\sqrt{\frac{3g}{4R}}$$

or

$$aT_s = 4\pi\alpha\sqrt{\frac{Rg}{3}}$$

$$\frac{aT_s}{2} = 2\pi\alpha\sqrt{\frac{Rg}{3}} \ . \qquad (2\text{-}10)$$

Equation (2-10) determines whether the block will overturn or not depending upon whether or not the left-hand side of Eq. (2-10) is larger than the right-hand side. Note that the left-hand side of Eq. (2-10) is simply the product of amplitude of the sinusoidal pulse and its duration. Two observations can be made from Eq. (2-10).

(1) For a given value of α (i.e., for geometrically similar blocks) the product of amplitude of the pulse with its duration increases proportionally with $\sqrt{R}$ to overturn the block. In other words, a larger block would be more stable than a smaller block. This indicates that larger blocks may be more stable than the smaller blocks for a given angle α under earthquake motions. It will subsequently be seen that although this is true in general, it is not always the case because an earthquake accelerogram is much more complicated than a simple sinisoidal pulse.

(2) For a given value of $\sqrt{R}$, the product of the amplitude with the pulse duration varies proportionally with the angle α to overturn the block.

Note that the analyses shown in Articles 2.2, 2.3, and 2.4 are applicable only when $\sin \alpha$ can be approximated by α and when the angle of rotation is also small.

2.5 General Rocking Problem

Unfortunately, the acceleration pulses during an earthquake are randomly distributed with varying amplitude and there is no simple way to treat the problem of the rocking of a block. As a rigid block starts rocking under an earthquake, there is an energy build-up into the system as the block is subjected to successive acceleration pulses.

The block can overturn at much smaller accelerations than those predicted by a single pulse of certain duration. Therefore, the overturning of a block under a single pulse, as described in the previous articles, is of an academic interest, but does not give much useful information on the rocking and overturning behavior of rigid blocks under earthquake ground motions.

To determine the rocking response of a rigid block under an earthquake, the equation of motion has to be solved numerically on the digital computer. The general equation of motion of a rocking block under simultaneously applied horizontal and vertical ground accelerations, basic assumptions of the mathematical model, and the numerical scheme used in the computer program to solve the basic equation of motion are described in articles that follow. The coefficient of restitution was determined experimentally and the validity of the computer program was checked against test results.

2.6 Equation of Motion of a Rocking Block

Consider the block shown in Fig. 2.7 with height H and width B. b and h are the distances of the centroid of the block from the edge and the base, respectively, as shown in Fig. 2.7. Suppose the block starts rocking about the edge under the action of horizontal $(\ddot{u})$ and vertical $(\ddot{v})$ ground accelerations. R and α are the same as defined in article 2.3 and shown in Fig. 2.7. θ and $\ddot{\theta}$ are the angle of rotation and the angular acceleration of the block, respectively, at any moment and are defined as positive as shown in Fig. 2.7.

If $\ddot{\theta}$ is the angular acceleration of the centroid of the block, then

$R\dot{\theta}^2$ = radial acceleration of G (along OG line)

and

$R\ddot{\theta}$ = tangential acceleration of G ($\perp$ to OG) .

Let

$\ddot{u}_G$ = horizontal acceleration of G (resultant)

and

$\ddot{v}_G$ = vertical acceleration of G (resultant) .

Then

$$\ddot{u}_G = \ddot{u} - R\ddot{\theta}\cos\beta - R\dot{\theta}^2 \sin\beta$$

and

$$\ddot{v}_G = \ddot{v} + R\ddot{\theta}\sin\beta - R\dot{\theta}^2\cos\beta$$

where

$$\beta = \alpha - \theta .$$

Let

I = mass moment of inertial of the block about $G = \frac{M}{3} R^2$.

Taking moment about O $\Sigma M_0 = o$ and making $K = o$

$$I\ddot{\theta} + Wx + M\ddot{v}_G x - M\ddot{u}_G y = 0 \qquad (2\text{-}11)$$

where

$$x = b\cos\theta - h\sin\theta$$

and

$$y = b\sin\theta + h\cos\theta$$

Note that K is the stiffness of the prestressing rods that may be present in order to tie down the block to the floor. If $K \neq 0$, the effect of the forces due to prestressing or the extension of the bars can be included in Eq. (2-11).

Substituting the values of $\ddot{u}_G$ and $\ddot{v}_G$ in equation (2-11) and replacing β in terms of $(\alpha - \theta)$, i.e.,

$$\sin\beta = \sin(\alpha - \theta) = \sin\alpha\cos\theta - \cos\alpha\sin\theta$$
$$= \frac{b}{R}\cos\theta - \frac{h}{R}\sin\theta$$

$$\cos\beta = \cos(\alpha - \theta) = \cos\alpha\cos\theta + \sin\alpha\sin\theta$$
$$= \frac{b}{R}\cos\theta + \frac{b}{R}\sin\theta$$

Equation (2-11) reduces to

$$I\ddot{\theta} + MR^2\ddot{\theta} - M\ddot{u}y + M(\ddot{v}+g)\,x = 0$$

$$\frac{M}{3}R^2\ddot{\theta} + MR^2\ddot{\theta} - M\ddot{u}y + M(\ddot{v}+g)\,x = 0$$

$$\frac{4}{3}MR^2\ddot{\theta} - M\ddot{u}y + M(\ddot{v}+g)\,x = 0\,. \qquad (2\text{-}12)$$

Note that $\frac{4}{3}MR^2 = I_0$, the mass moment of inertia of the block about the edge 0. The mass of the block M being a common factor in Eq. (2-12) can be cancelled, and by replacing x and y, Eq. (2-12) will become

$$\frac{4}{3}R^2\ddot{\theta} - \ddot{u}(b\sin\theta + h\cos\theta)$$
$$+ (\ddot{v}+g)\,(b\cos\theta - h\sin\theta) = 0\,. \qquad (2\text{-}13)$$

Equation (2-13) is independent of the mass of the block and is only a function of the dimensions of the block. Thus, the rocking behavior of a block does not depend on the density of the material. As the block rotates about 0 in the clockwise direction and hits the ground, there will be a change in velocity after impact (θ_{i+1}). This is related to the velocity before impact (θ_i) by the coefficient of restitution ν, which is a function of the elastic properties of the material of the block and the base, and has to be determined by tests.

2.7 Basic Assumptions for the Mathematical Model

(1) The coefficient of friction between the contact surfaces

of the block and the ground and the dimensions of the block are such that the block rocks without any sliding.

(2) The block is rocking on its edges without any bouncing at the time of impact with the ground.

(3) The blocks have a uniform density so that the geometric center coincides with the center of gravity.

(4) The surface of the block and the ground are perfectly plane so that the block will rock only on its edges.

It is not easy to get perfectly plane surfaces. Therefore, for convenience, the bottom surface of the block should be made slightly concave so that the block will touch the ground only on its edges.

(5) The loss in velocity at the time of impact, representing the energy loss (at impact), is defined by the coefficient of restitution ν defined by the relationship

$$\dot{\theta}_{i+1} = - \nu \, \dot{\theta}_i$$

where

$\dot{\theta}_i$ = angular velocity before impact

$\dot{\theta}_{i+1}$ = angular velocity after impact.

(6) There is no spalling of the edges of the block to cause any change in the value of ν for a particular material during rocking of the blocks.

(7) ν will be assumed constant for a given material.

2.8 Brief Description of the Computer Program and Numerical Integration Procedure

A computer program was written to solve numerically the equation of motion of a rocking block under simultaneously applied horizontal and vertical ground accelerations. The conditions for initiation of rocking as described earlier were incorporated in the computer program, and the change in velocity at the time of impact was calculated using the coefficient of restitution ν .

The program was written to read in the digitized horizontal and vertical ground acceleration records, the values of which could be specified at different time intervals. To increase the accuracy of numerical integration, the consecutive values of digitized accelerograms could be further subdivided into any equal number of parts and the distribution of acceleration between the two consecutively given points was taken as a straight line. Numerical integration of Eq. (2-13) was carried out in two steps using the predictor-corrector approach.

A provision was made in the computer program to include the effect of any prestressing and elastic forces as a result of tying the block to the floor. It is assumed that the prestressing rods are hinged to the ground and remain elastic.

Let the stiffness of the prestressing rods be equal to K (Fig. 2.7) and let each of the prestressing rods have a prestressing force equal to F_o . S is the distance of each rod from the edge of the block. If δ_1 and δ_2 are the extensions in the rods, the prestressing rods will exert, respectively, restoring moments RM1 and RM2 about O . For the values of θ encountered in most practical cases, RM1 and RM2 can, with sufficient accuracy, be written as:

$$RM1 = (Fo + K.\delta_1)\ S \cos\theta$$

$$RM2 = (Fo + K.\delta_2)(B - S) \cos\theta$$

And when K is not equal to zero, Eq. (2-12) becomes:

$$\frac{4}{3}MR^2\ddot{\theta} - M\ddot{u}(B \sin\theta + H\cos\theta) + M(\ddot{v}+g)(B\cos\theta - H\sin\theta) + RM1 + RM2 = 0 \qquad (2\text{-}14)$$

The integration procedure used to get the angular displacements was the same as before, except that angular acceleration was calculated from the general Eq. (2-14) instead of Eq. (2-13).

A Calcomp plotting subroutine was added in the computer program to plot the time history of the applied ground motion and the response of the rocking block in terms of angular acceleration, velocity, and displacement. A typical Calcomp plot of the response of a rigid block 2 ft. wide and 8 ft. tall has been shown in Fig. 2.8. The coefficient of restitution (COR) is 0.95; the total stiffness of the rods and the total prestressing force are 0.4W/in. and 0.4W, respectively. The horizontal and vertical ground accelerations, angular acceleration, velocity, and displacements of the block are plotted from top to bottom in Fig. 2.8. The parallel lines shown in the displacement time plot of the block envelope the stable position of the block. A displacement outside this envelope indicates an unstable position.

Figures 2.9 and 2.10 show the response of a freely rocking block given an initial displacement θ. In Fig. 2.9 the value of the coefficient of restitution ν has been assigned to be 1.0 which represents no energy loss. In Fig. 2.10 the value of ν is 0.95 and this shows a decay of rocking amplitude with time.

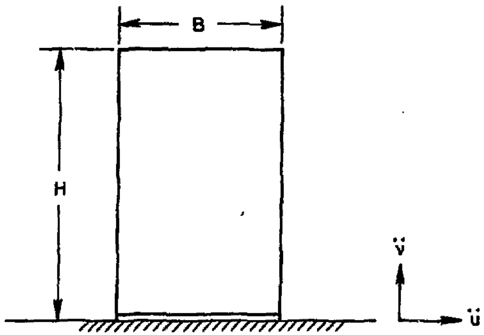

FIG. 2.1 RIGID BLOCK UNDER GROUND ACCELERATIONS

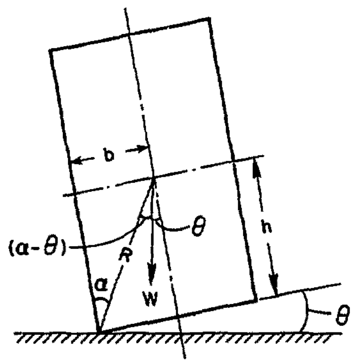

FIG. 2.2 A FREELY ROCKING BLOCK

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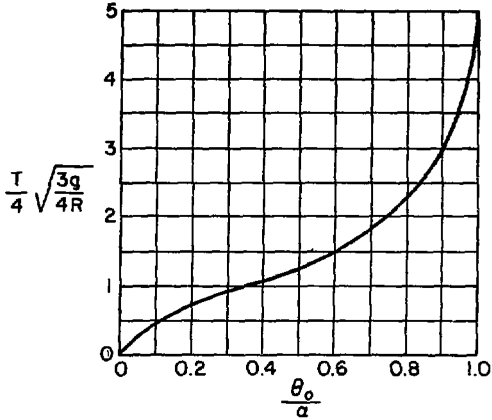

FIG. 2.3 PERIOD T OF BLOCK ROCKING WITH AMPLITUDE θ_0

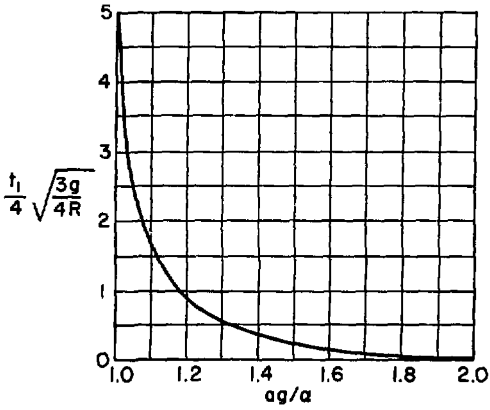

FIG. 2.4 CONSTANT ACCELERATION a OF DURATION t_1 REQUIRED FOR OVERTURNING

XBL 784-8348

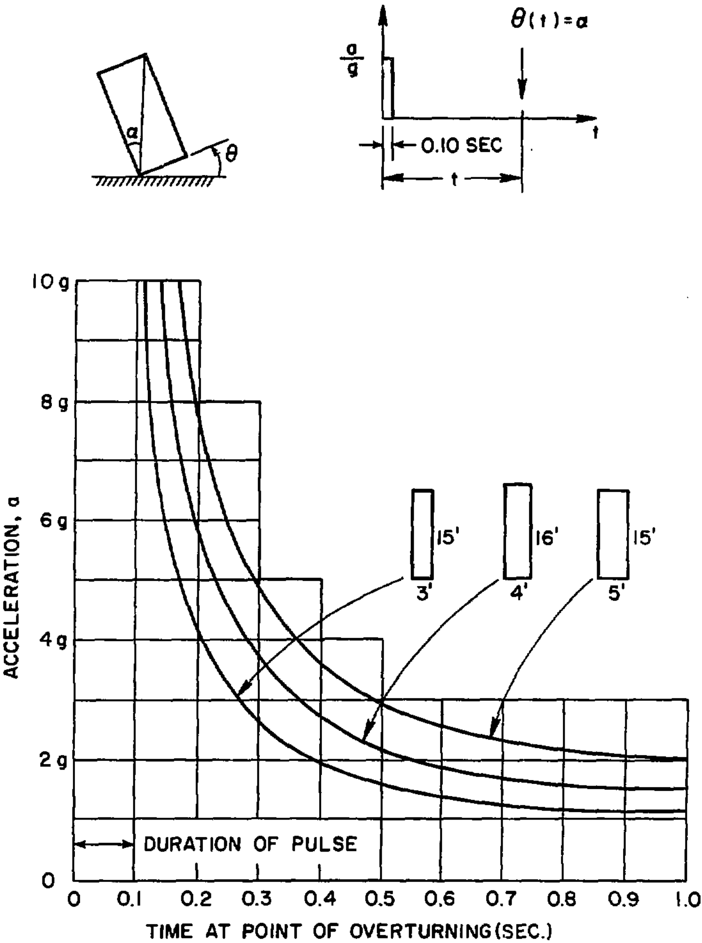

FIG. 2.5 RESPONSE TO SQUARE ACCELERATION PULSE

XBL 784-8352

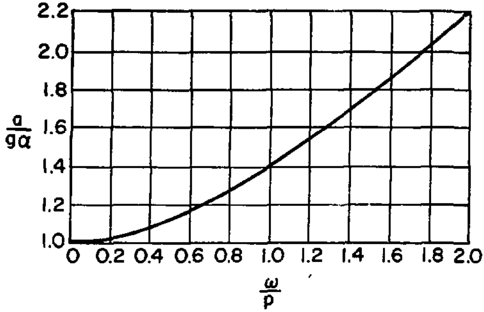

FIG. 2.6 SINUSOIDAL ACCELERATION PULSE a SINwt REQUIRED FOR OVERTURNING

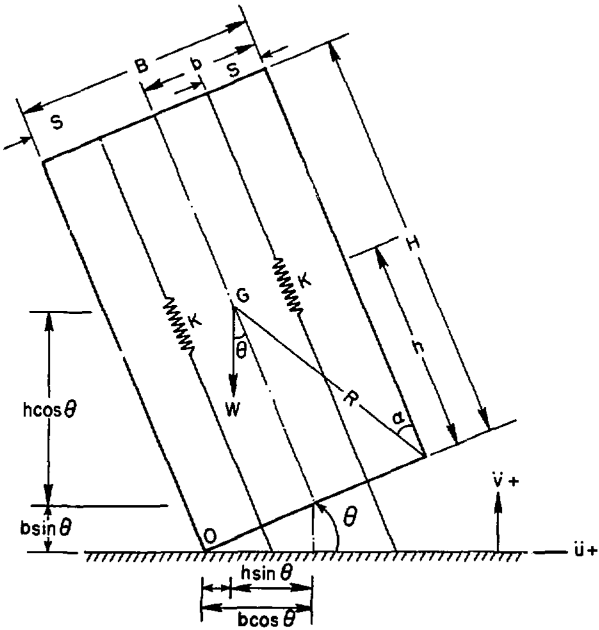

FIG. 2.7 ROCKING OF A BLOCK UNDER GROUND ACCELERATIONS

XBL 784-8353

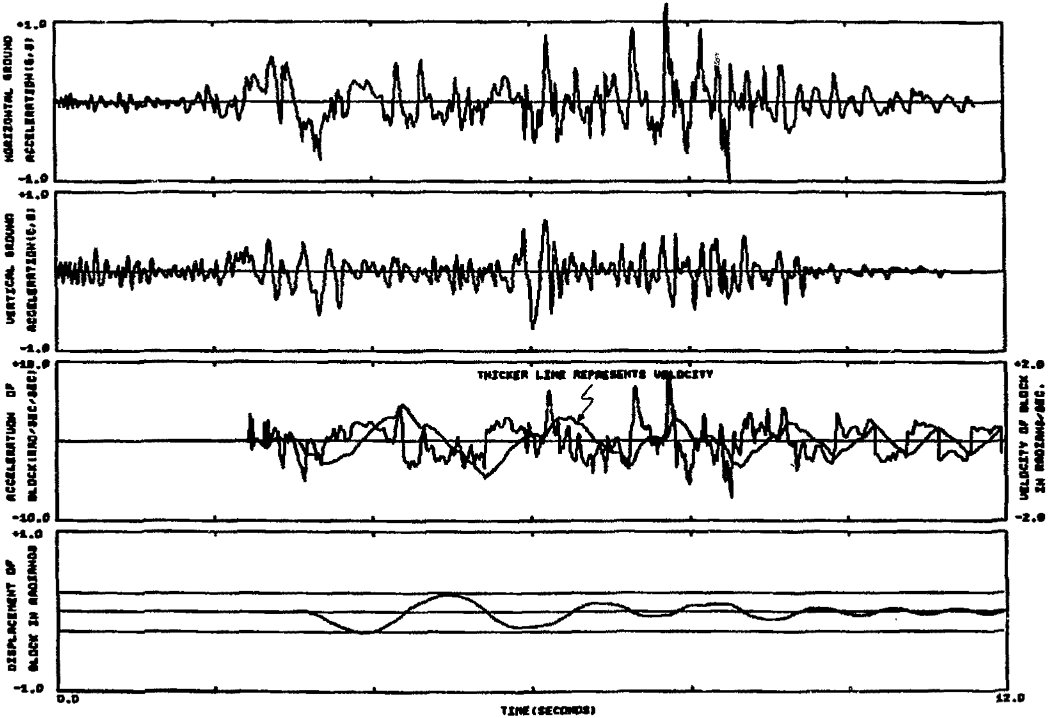

FIG. 2.8 ROCKING OF A BLOCK SUBJECTED TO SAN FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S16E)B=24IN.,H=96IN.,COR=0.95 K=0.2W/IN,PRESTRESSING FORCE=0.2W

XBL 784-8355

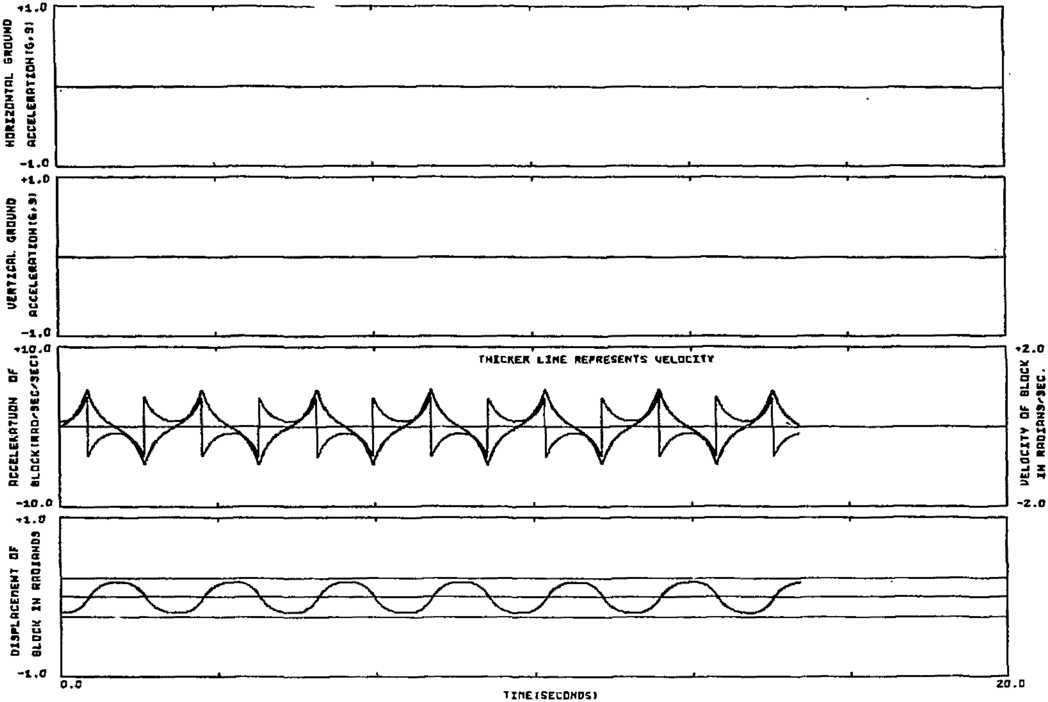

FIG. 2.9 FREE ROCKING OF A BLOCK, B=9.0IN, H=36IN, COR=1.00

XBL 784-8345

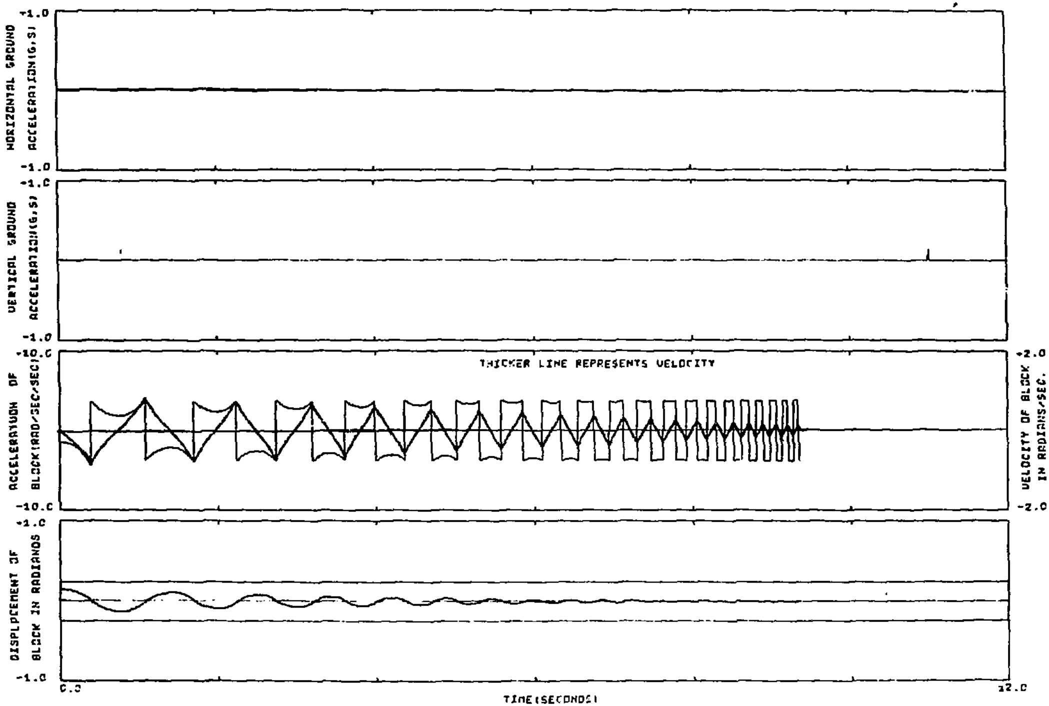

FIG. 2.10 FREE ROCKING OF A BLOCK, B=9 IN, H=36IN, COR=0.95 $\theta_0 = 0.15$

XBL 784-8356

SECTION 3

EXPERIMENTAL STUDIES AND COMPARISON WITH COMPUTER RESULTS

3.1 General

To validate the computer model, tests were carried out on the rocking response of rigid blocks under sinusoidal and actual earthquake ground motions. Free-rocking tests were conducted to determine the coefficient of restitution ν and to find any significant dependency of ν on the angular velocity with which the block impacts the ground.

The test data regarding the shaking table acceleration and block displacement were digitized and plotted for comparison with the computer results. Details of these tests, and a comparison of the tests and corresponding computer results, are presented in the following articles.

One of the most important things that was learned from these tests was the importance of the boundary condition at the base of the block, as explained in the next article.

3.2 Considerations in Model Design

Preliminary tests on the shaking table showed that the response of a block in the rocking mode is very sensitive to the boundary condition between block and ground. If the lower surface of the block or of the ground is even slightly convex or irregular, rocking may start at horizontal ground acceleration lower than given by Eq. (2-1). Once a block starts rocking, there is a build up of energy into the system. Thus, the overall response of the block could be entirely different than that expected if the initiation of rocking were governed by the condition that the horizontal ground acceleration $\ddot{u}$ must be greater than $(B/H)g(1 + \ddot{v}/g)$ in order to start the block rocking. The theoretical boundary conditions can be ensured if the surface of the block touching the ground is made concave and the ground is flat, making the block touch the ground only on its edges. Such a block, in general, would be more stable under ground accelerations than a block with an irregular or convex boundary.

To ensure the correct boundary condition for test purposes, a 3/8 in. thick aluminum plate was cemented to the bottom surface of the block. The lower surface of the plate was made concave so that the block would be resting on the ground only on its edges. This plate also prevented possible spalling or damage to the concrete edges which could have changed the coefficient of restitution ν during the test.

Since the surface of the shaking table was rough and uneven, a plane surface for rocking was provided by a 1 in. thick steel plate which was hydrostoned and prestressed to the shaking table. The steel plate was 40 in. long and 15 in. wide and the top was machined to provide a plane surface so that rocking would start when the condition $\ddot{u} > (B/H)g(1 + \ddot{v}/g)$ was met.

The top surface of the steel plate was sand blasted to increase the coefficient of friction μ so that μ was significantly greater than the width to height (B/H) ratio of the block. This ensured that the block would rock without sliding on the steel plate.

Since the purpose of these tests was to check the accuracy of the computer model, no specific consideration was given to precise physical modeling or similitude. Tests were carried out on two blocks having a height to width ratio of 4.0 and 5.0, respectively. They were chosen to produce pure rocking without any sliding movements between the block and the table. Most of the tests were carried out on a block 30 in. high and 6 in. wide. The third dimension of the block, which is perpendicular to the table motion, does not affect the response of the block. Some free-rocking tests were also carried out on the other block, which was 36 in. high and 9 in. wide.

3.3 Model Instrumentation and Test Set-up

Figure 3.1 shows the test set-up of the concrete block on the shaking table. The height and width dimensions of the block are 30 in. and 6 in., respectively. The block can be seen standing on the steel plate, which is prestressed to the shaking table.

The displacement at the top of the block was measured relative to the table with a spring-loaded potentiometer. The potentiometer wire exerted a horizontal force of 20 ounces at the top of the block, and as this would have affected the response of the block. A second potentiometer was connected on the other side, as shown in Fig. 3.1, to cancel this effect.

The potentiometers were mounted on steel-I posts, each prestressed to the shaking table with 1 1/4 in. diameter steel rods. The prestressing force in each rod was 10,000 lbs. Steps were taken to ensure that the steel posts carrying the potentiometers would not rock or have significant elastic vibrations. This rigidity was necessary for measuring the displacement of the top of the block relative to the shaking table. A close-up view of a potentiometer mounted on the post is shown in Fig. 3.2.

Two horizontal cantilever beams, one on each side of the block, were fixed to the steel posts (Fig. 3.1). These were used to stop the block on the table from overturning and prevented damage to the potentiometers, which had a range of ± 15 in. Sufficient clearance was left, however, between the block and the cantilever arms to allow the block to go into the unstable range so that it could be considered as overturned.

The horizontal acceleration of the table was measured with an accelerometer fixed near the base of block, as shown in Fig. 3.1. The vertical acceleration of the table was measured with two accelerometers, and an average of the two was taken.

The potentiometer and accelerometers were connected to the data acquisition system of the table, and data regarding the table accelerations were digitized to carry out the computer analysis for comparison purposes.

3.4 Shaking Table

All tests were carried out on the 20 ft x 20 ft shaking table located at the University of California's Richmond Field Station. Figure 3.3 shows the 20 ft x 20 ft shaking table during the preliminary testing with the 30 in. x 6 in. concrete block. This shaking table can reproduce prescribed independent horizontal and vertical ground motions. The table is driven horizontally by three

50-kip hydraulic actuators and vertically by four hydraulic actuators. The weight of the table and the structure it supports is balanced by pressurized air when the table is in operation.

Electronic control for the shaking table was supplied by the MTS System Corporation, Minneapolis, Minnesota. The shaking table command signals are in the form of displacement-time histories. A mini-computer (NOVA) is used to derive this from the acceleration records. After the displacement-time histories are available, they are fed via a digital-to-analog converter to an analog tape recorder and then to the MTS Control Console.

During the test, NOVA collects the data, which can be sampled at the rate of 100 samples per second and stored on a disc or a magnetic tape. These data can be punched on a paper tape and plotted on a Versatic printer/plotter. For details regarding the shaking table and its associated system, see Reference (1).

3.5 Determination of Coefficient of Restitution ν

The value of the coefficient of restitution ν was determined by free-rocking tests at the beginning and at the end of the test series. The block was given an initial angular displacement θ_0 less than the block angle α and then allowed to rock freely on its edges. The shaking table was kept stationary and care was taken to release the block from initial angle θ_0 with zero angular velocity. As the block continued to rock on its edges, the amplitude of the angular displacement decreased until the block came to rest.

As the block started rocking, a continuous record of the angular displacement versus time was taken and digitized at the rate of 50 samples per second. These test data were plotted on the Versatec plotter and the initial angular displacement was read from the digitized record.

Now, using the computer program BLOKROC, a theoretical analysis of the same block, rocking freely, was done for the same initial angular displacement θ_0 for various values of ν and the displacement versus time curve was plotted for each. These theoretical plots were superimposed on the experimental plots, and the plot which showed the best fit gave the value of the coefficient of restitution. Figure 3.4 shows the experimental and theoretical plots of angular displacements for the 6 in. x 30 in. concrete block. The value of the coefficient of restitution used in the computer analysis was 0.925. This value of $\nu = 0.925$ was later used in the computer analysis to determine the theoretical response of this test block under table motions for comparison with the test results.

Free-rocking tests were also done on a 36 in. x 9 in. concrete block and a comparison of these results is given in Article 3.7.

3.6 Procedure for Ground Motion Tests

Tests were conducted under harmonic as well as simulated earthquake ground motions. The test set-up is s[illegible] 3.1. Before each test, the surface of the steel plate on which the block is resting was checked and levelled by adjusting the vertical actuators of the shaking table.

After carefully levelling, the shaking table was given horizontal and vertical motions. At the same time, the data acquisition system associated with the table was started to gather the displacement response of the block as well as the horizontal and vertical table accelerations. These data were digitized at a rate of 50 samples per second and were kept on a magnetic disc.

The data regarding the table accelerations and angular block displacements, (obtained from the top displacements of the block), were plotted on the Versatec plotter which is a part of the shaking table data acquisition system. A typical plot of the test data is shown in Fig. 3.5. The plot shows the response of a rigid block in the rocking mode of vibration under an artificially generated earthquake having the characteristics of the San Fernando earthquake of 1971. The maximum horizontal and vertical ground accelerations are approximately 0.5 g and 0.2 g, respectively. The block was 30 in. high and 6 in. wide, and it overturned after 4 seconds after the start of ground motion, as shown in Fig. 3.5.

The measured shaking table accelerations were transferred to tape from the magnetic disc in order to carry out the computer analysis for comparison with the test results. All the tests under ground accelerations were performed on a 30 in. x 6 in. block. The value of the coefficient of restitution ν for this block, as determined from free rocking tests, was 0.925, and this value was used in the computer analysis.

3.7 Comparison of Test and Theoretical Results

(1) Free-Rocking

Comparison between the test and computer results of a freely rocking block is shown in Fig. 3.4. The block was 30 in. high and 6 in. wide. The value of the coefficient of restitution used in the computer program was constant and equal to 0.925. It can be seen in Fig. 3.4 that the agreement between the test and computer results is excellent, and that the two curves are identical, for all practical purposes. In both cases, the block came to rest in approximately 6 seconds, after about 10 vibrations.

The agreement between the two curves also shows that the coefficient of restitution is independent of the velocity of impact in this test. Hence it is reasonable to consider it as a constant. It should be recognized, however, that for a block of the size used in this test, the stresses remain elastic throughout and this might not apply to large specimens.

Free-rocking tests were also conducted on another block having a height of 36 in. and a width of 9 in. The third dimension (i.e., the length of the edges about which the block rocks) was 18 in.; however, this dimension does not affect the response in any way. A comparison between the natural periods, as obtained from the test, and computer results for this block is shown in Table 3.1 and Fig. 3.6.

The period of vibration for the test was determined from the digitized record of a freely rocking block by calculating the time that the block takes from maximum angular displacement to zero displacement and multiplying this by a factor of 4. This procedure was necessary because the amplitude of vibration decreases after every impact and the period of vibration is highly dependent on amplitude.

Figure 3.6 shows a good agreement between the test and computer results for decreasing amplitudes of vibration. In the tabulated values of Table 3.1, the difference between the test and theory is within 2 percent.

(2) Rocking Under Ground Accelerations

Comparisons were made of the test and theoretical responses when both horizontal and vertical table accelerations were applied to the test specimen. The block for these tests was 30 in. high and 6 in. wide, giving a height to width ratio of 5.0. This represents an upper bound on the shielding blocks according to the present design criteria.

Measured ground accelerations and angular displacements of the block were taken directly from the Versatec plots of the digitized test data. The theoretical displacements were obtained by using the same measured table accelerations as input ground motions.

In Fig. 3.7 the horizontal component of ground motion was harmonic with a frequency of 2 Hz and an amplitude of 0.5 g, and the vertical component was zero. It can be seen that the agreement between the test and theoretical results is good and that the differences are small. In both cases, the block becomes unstable at about the same point and overturns in the same direction.

Figures 3.8 and 3.9 show the test and computer results under simultaneously applied horizontal and vertical ground acceleration. Ground motion for both components was harmonic with a frequency of 2 Hz.

In Fig. 3.8 the intensities were 0.38 g and 0.33 g for the horizontal and vertical components respectively, and in Fig. 3.9 the intensities were 0.45 g and 0.26 g. It can be seen in these figures that the agreement between the test and theoretically predicted angular displacements of the block is quite good. The block becomes unstable at about the same points in both test and computer results, and overturning is in the same direction. The value of the coefficient of restitution ν used for the computer analysis was 0.925, as determined experimentally.

It can be seen in Figs. 3.7 through 3.9 that the agreement between the test and theory, though good, is not as precise as in the case of the free-rocking tests shown in Figs. 3.4 and 3.6. The reason for this may have been the rotation (pitch) component of the table, which is always present to some degree when the table is in operation and has a small effect on both the horizontal and vertical components of table motion at the base of the block. The frequency of the pitch of the table was found to be approximately 12 Hz. and the angular acceleration of the table was found to be between 0.2 to 0.25 radians/sec^2.

The effect of this rotation component of the shaking table was not taken into account in the analytical result and clearly it would have some effect on the response of the block. It was found, however, that when the block was under harmonic motions of the table at a low frequency of 2 Hz and at comparatively high amplitudes of vibration, the effect of the pitch was relatively small and thus it was possible to get the good agreement between the test and computer results shown in Figs. 3.7, 3.8 and 3.9.

It was not possible, however, to get the same level of agreement between the test and theoretical response of the block under an earthquake accelerogram because in this case the pitch of the table had a larger effect. This was confirmed by repeating a test with the same earthquake accelerogram. It was found that the response of the block was not repeatable using the same horizontal and vertical command displacements because the pitch of the shaking table was different in each case. Also, when earthquake records were used the frequency components of the horizontal and vertical table accelerations were higher than in the harmonic frequency tests, and therefore were closer to the pitch frequency of the table. When the block starts rocking under earthquake motions, the initial amplitude of rocking due to the horizontal ground acceleration is relatively small, and hence, the effect of the rotation component of the table motion becomes more important. Also, as the period of vibration is very dependent on the rocking amplitude, a change in period could cause a phase shift, the new displacements (which include the effect of pitching), being in a different phase for the same applied horizontal ground motion.

The factor that makes the rocking problem so sensitive to many parameters, including the boundary conditions, coefficient of restitution, the applied accelerations, and any horizontal or vertical forces applied to the block, is the dependency of the period of vibration on the displacement amplitude. A small change in any of these factors can cause a drastic change in the rocking response of the block. Therefore, any test conducted on the rocking response of a rigid structure for the purposes of validating theory has to be extremely precise.

From the good agreement that was found between the test and theoretical results on free-rocking and under harmonic table accelerations (both with and without vertical table motion) where the effect of pitching of the table was either zero or small, it is concluded that the computer program can adequately predict the dynamic response of a rigid structure vibrating in a rocking mode under any simultaneous horizontal and vertical ground motions. Comparisons of test and theoretical results for earthquake-type ground motions are not given, as these did not show the same level of agreement for the reasons discussed above.

TABLE 3.1 TEST AND COMPUTED VALUES OF NATURAL PERIOD OF A BLOCK ROCKING WITH AMPLITUDE θ. HEIGHT AND WIDTH OF THE BLOCK ARE 36 IN. AND 6 IN. RESPECTIVELY

DISPLACEMENT θ IN DEGREES	PERIOD OF VIBRATION IN SECONDS		TEST/THEORY
	TEST	THEORY	
(1)	(2)	(3)	(4)
9.57	1.88	1.84	1.02
7.96	1.48	1.51	0.98
6.76	1.29	1.30	0.99
5.90	1.14	1.15	0.99
3.15	0.77	0.75	1.03

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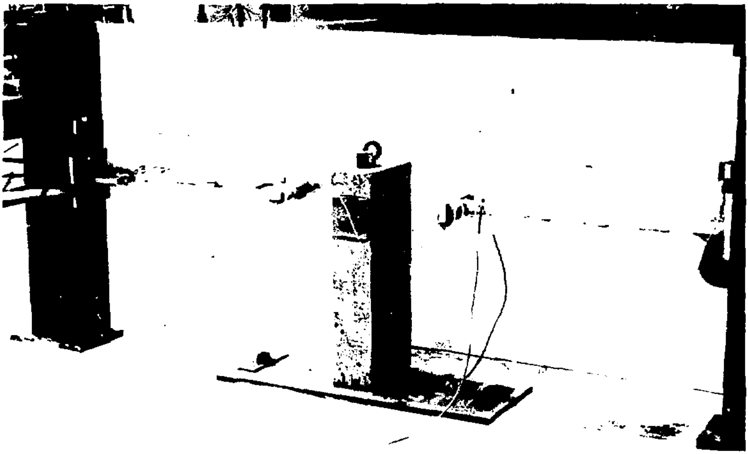


FIG. 3.1 TEST SET UP OF A 30 IN. × 6 IN. CONCRETE BLOCK SHOWING INSTRUMENTATION.

XBB 785-5069

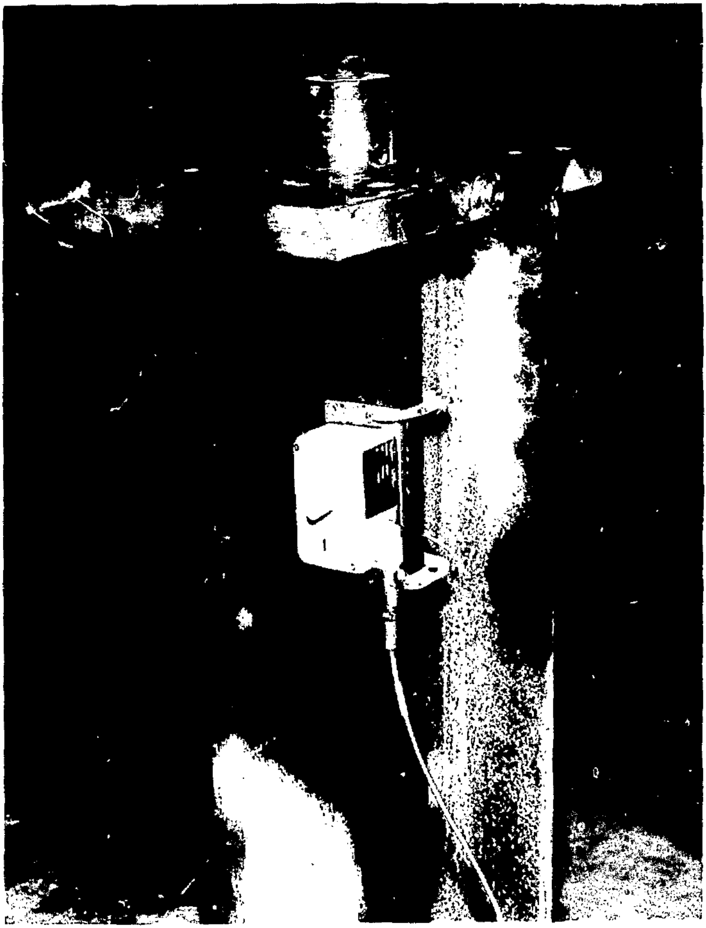


FIG. 3.2 CLOSE UP OF POTENTIOMETER MOUNTED ON STEEL POST.

XBB 785-5071

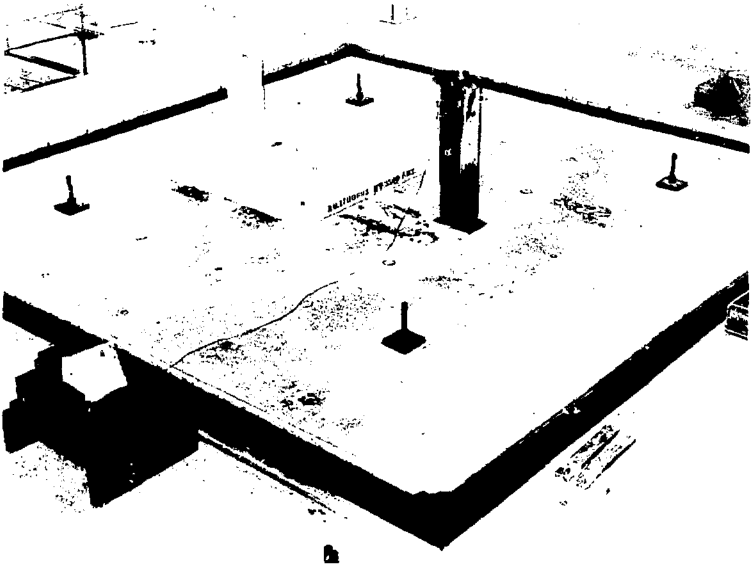


FIG. 3.3 SHAKING TABLE WITH CONCRETE BLOCK.

XBB 785-5070

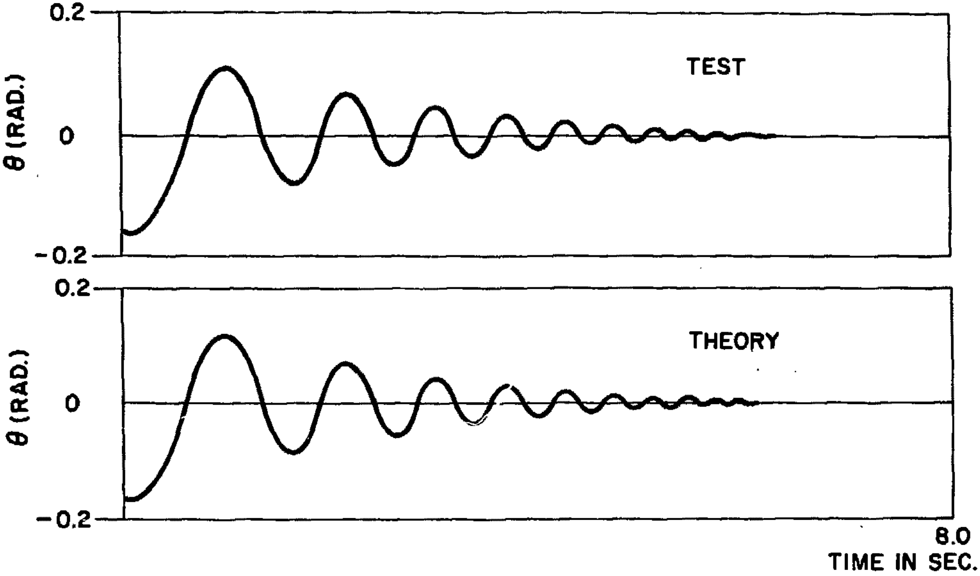


FIG. 3.4 COMPARISON OF ANGULAR DISPLACEMENTS OF A FREELY ROCKING 30 x 6 IN. BLOCK ($\nu = 0.925$)

XBL 784-8358

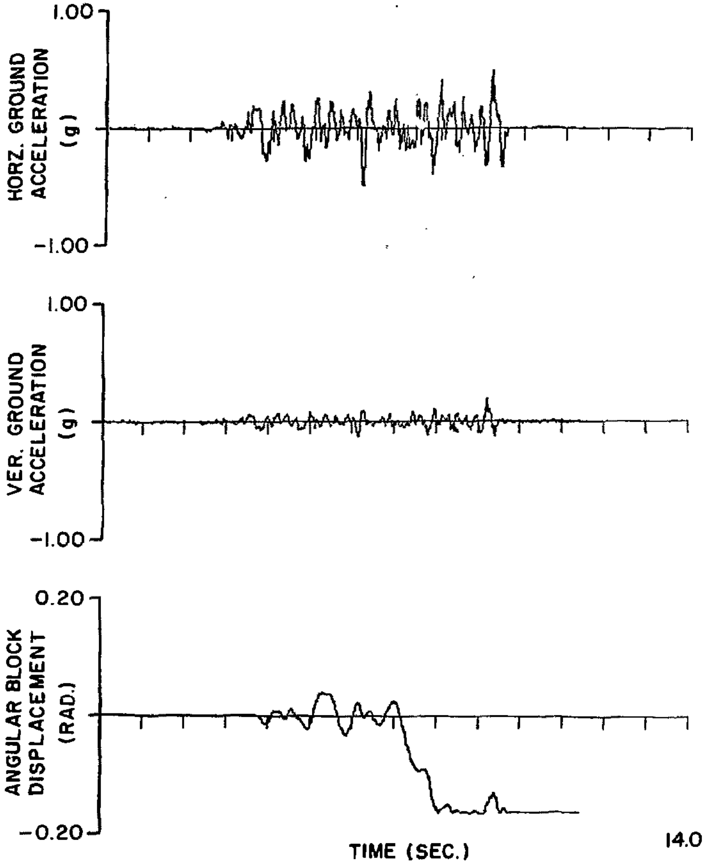


FIG. 3.5 RESPONSE OF A 30x6 IN. BLOCK IN THE ROCKING MODE UNDER AN EARTHQUAKE ACCELEROGRAM (TEST DATA)

XBL 784-8347

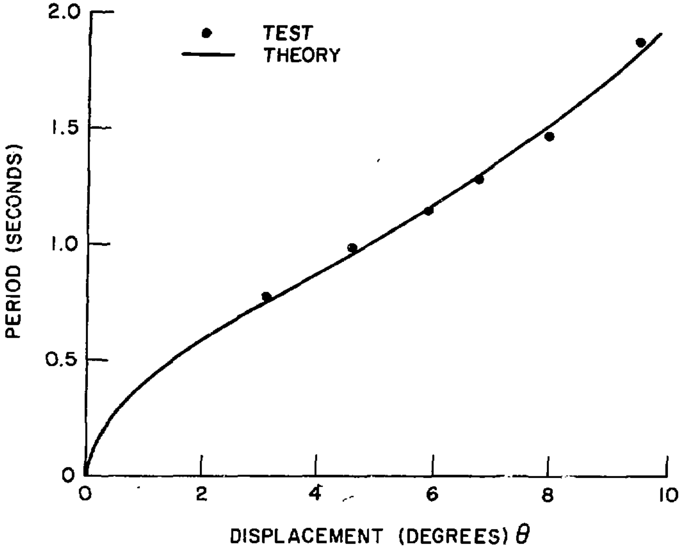


FIG. 3.6 TEST AND COMPUTED VALUES OF NATURAL PERIOD OF A BLOCK ROCKING WITH AMPLITUDE θ, HEIGHT AND WIDTH OF THE BLOCK ARE 36 IN. AND 9 IN. RESPECTIVELY

XBL 784-83ↄ1

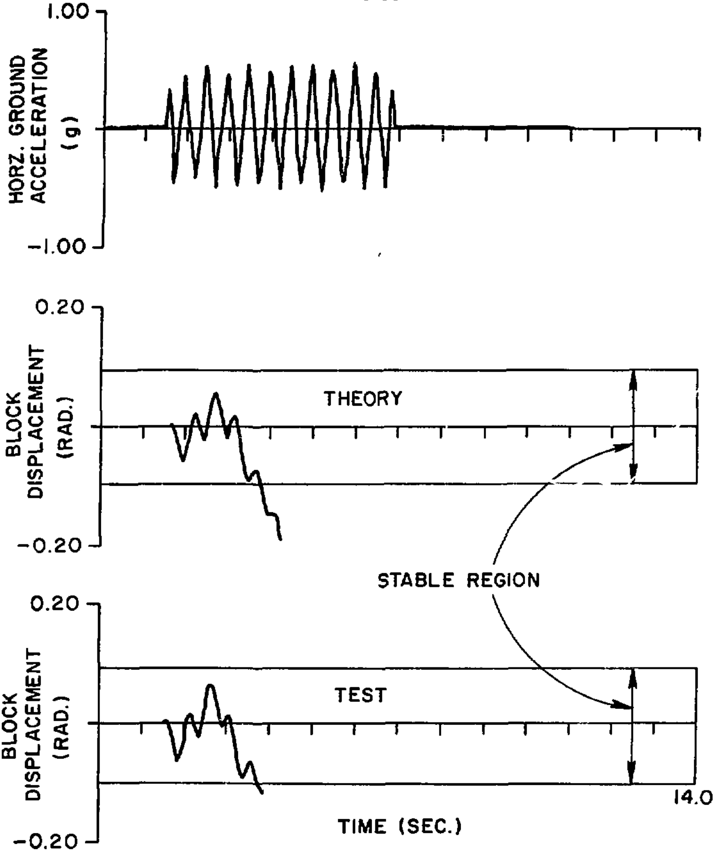


FIG. 3.7 COMPARISON OF TEST AND THEORETICAL DISPLACEMENTS OF A 30x6 IN. ROCKING BLOCK UNDER HORIZONTAL GROUND ACCELERATION

XBL 784-8349

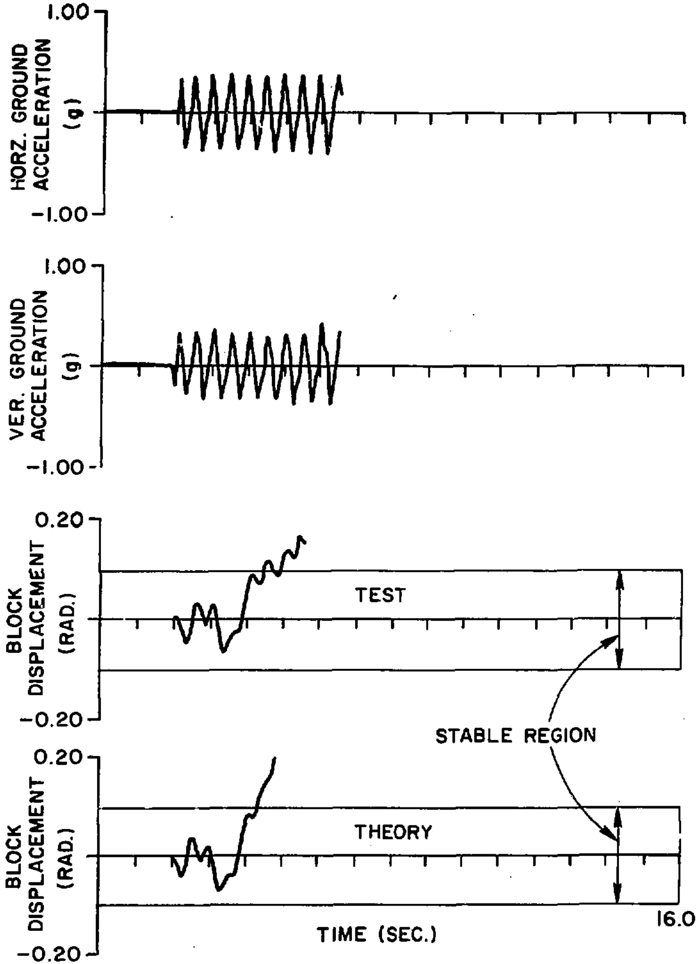


FIG. 3.8 COMPARISON OF TEST AND THEORETICAL ANGULAR DISPLACEMENTS OF A 30 x 6 IN. BLOCK UNDER HORIZONTAL AND VERTICAL GROUND ACCELERATIONS

XBL 784-8346

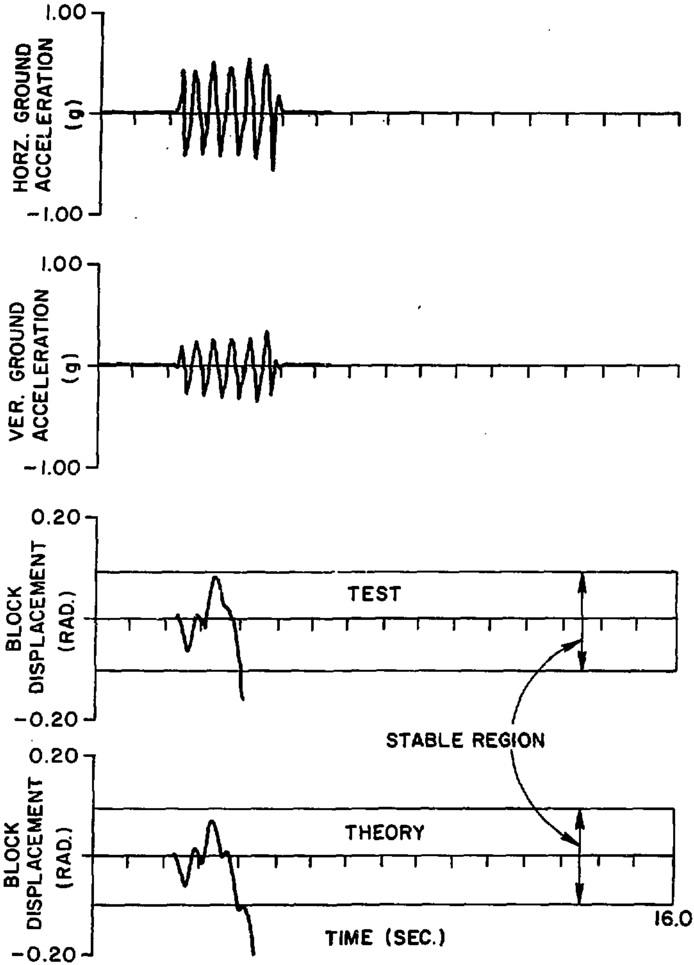


FIG. 3.9 COMPARISON OF TEST AND THEORETICAL ANGULAR DISPLACEMENTS OF A 30 x 6 IN. BLOCK UNDER HORIZONTAL AND VERTICAL GROUND ACCELERATIONS

XBL 784-8354

SECTION 4

ROCKING RESPONSE OF RIGID BLOCKS TO VARIOUS EARTHQUAKE ACCELEROGRAMS

4.1 General

Having established the reliability of the computer model (Chapter 3), a study was undertaken to determine the dynamic response of rigid blocks under pure rocking conditions (without any sliding movements between the block and the ground) to various earthquake records. The results of rigid blocks with the sliding mode of response (no rocking) have been presented in an earlier report (1). The purpose of the present study is to determine the stability of rocking blocks against overturning in typical earthquakes.

The response of the blocks was studied under five different strong motion earthquake accelerograms. These included: S16°E and S74°W components of the horizontal accelerations recorded at Pacoima Dam during the San Fernando earthquake of 1971, an artificially generated accelerogram representing the characteristics of the San Fernando earthquake at Olive View Hospital in Los Angeles with maximum acceleration of 0.5 g, and two artificially generated accelerograms A-1 and B-1 (3). The artificial accelerograms A-1 and B-1 have the maximum accelerations on the order of 0.5 g and 0.4 g, respectively. At the time these records were produced, these values were thought to represent the maximum accelerations in earthquakes of magnitudes 8 and 7, respectively, on the Richter scale. The accelerations recorded at Pacoima Dam during the San Fernando earthquake of 1971 were up to 1.25 g, even though the magnitude of the earthquake was only 6.5 on the Richter scale. However, the length of strong motion of the accelerograms A-1 and B-1 was 60 seconds and 30 seconds, respectively, while for the San Fernando earthquake it was about 12 seconds.

Detailed results of these studies for blocks of various dimensions and with different coefficient of restitution values are presented in the following subsections. Based on these results, some observations are made regarding the rocking behavior of rigid blocks.

4.2 Rocking and Overturning of Rigid Blocks under Various Earthquake Accelerograms

A summary of the rocking motions of blocks of different aspect ratios and of different sizes (for a given aspect ratio) is presented in this section. The analysis of the rocking response was carried out by the computer program and the results were plotted on the Calcomp plotter. Two typical examples of the Calcomp plots for each of the earthquake records used in this study are shown in Figs. 4.1 through 4.10. These figures show the response of a block that is 192 in. (16 ft) high and 48 in. (4 ft) wide, giving an aspect ratio of height to width equal to four.

In each of these figures, the horizontal ground accelerations, the vertical ground accelerations, a superimposition of the angular acceleration and velocity of the block, and the angular displacement of the block are plotted from top to bottom. Angular acceleration, velocity, and displacements have been expressed in radians and second units. The ground accelerations are plotted in g's where g is the acceleration due to gravity. Two horizontal lines drawn parallel to the x-axis in the displacement plot of the block in each of these figures bound the stable region. As soon as the angular displacement of the block goes outside this region, the block becomes unstable. Each of these horizontal lines represents an angular displacement of the block equal to the block angle α , as defined in Fig. 2.7.

Figures 4.1 and 4.2 show the rocking motion of a 16 ft high by 4 ft wide rigid block under the San Fernando earthquake of 1971 (Pacoima Dam Record S16E). The values of the coefficient of restitution in Figs. 4.1 and 4.2 are 1.0 and 0.95, respectively. (The abbreviation COR is used for coefficient of restitution in these figures). The maximum value of the angular displacement of the block divided by the block angle α is 0.78 in Fig. 4.2, and the block does not overturn. However, in Fig. 4.1 the block becomes unstable and overturns. In these Calcomp plots, K is the stiffness of any prestressing rod which may be used to tie the block to the floor. When the blocks are not tied to the ground, the prestressing force and the stiffness of the prestressing rod would be zero. However, if the prestressing force and K are not zero, they can be in included in the analysis.

Figures 4.3 and 4.4 show the rocking response of the same (16 ft x 4 ft) block when subjected to the San Fernando earthquake of 1971 (Pacoima Dam record S74°W). The coefficients of restitution were 1.0 and .95 in Figs. 4.3 and 4.4, respectively. The maximum angular displacement expressed as a ratio of block angle α are 0.59 and 0.33, respectively, in Figs. 4.3 and 4.4. The effect of the actual recorded vertical accelerogram at Pacoima dam was included in the analysis of

...s. 4.1 through 4.4.

Figures 4.5 and 4.6 are two typical examples of the response of the same 16 ft x 4 ft block under the San Fernando earthquake record at Olive View Hospital. Figures 4.5 and 4.6 show the response for a ν value of 1.0 and 0.95, respectively. The vertical ground accelerations for this record were not generated and are assumed to be zero in this case. In both cases the block remains stable.

Figures 4.7 and 4.8 are typical Calcomp plots showing the response of the block under the artificial earthquake A-1. Only the horizontal ground accelerations records were available. Therefore, the vertical ground acceleration is zero in Figs. 4.7 and 4.8. The block overturns in the case when ν is equal to 1.00.

Figures 4.9 and 4.10 show typical examples of the same block rocking under the artificial earthquake B-1. Vertical ground accelerations are again zero in this case. The block remains stable in both cases and does not overturn.

A summary of the results of the rocking response of rigid blocks for various aspect ratios under various earthquake accelerograms is given in Tables 4.1, 4.2 and 4.3. The results show the maximum angular displacement of the block and are given as a ratio of the block angle α. Obviously a ratio greater than unity means that the block becomes unstable. A value of F in these tables means that the block overturns.

Table 4.1 shows the results on blocks which have approximately the same height, i.e., 15 ft, 16 ft, and 15 ft, respectively, but vary in width, being 5 ft, 4 ft, and 3 ft, respectively, giving an aspect ratio of 3, 4, and 5. An aspect ratio of 5 represents an upper limit allowed by the present design practice, and a height as high as 15 ft is quite common in radiation shielding blocks. The results are given for values of the coefficient of restitution equal to 1.0 and 0.95. A value of 0.95 for ν would be reasonably conservative, provided that the bottom surface of the block is flat, or preferably, concave, so that the block is resting on its edges. The values of the response for ν equal to 1.0 (no energy loss at impact) have been included here for a comparison with the response at ν equal to 0.95. The results are tabulated for the accelerograms S16°E and S74°W recorded at Pacoima Dam during the San Fernando Earthquake of 1971, the Olive View Hospital accelerogram, and the artificial earthquake accelerograms A-1 and B-1. Only the Pacoima Dam accelerograms included the vertical ground acceleration because in the other cases the vertical accelerograms were not available.

Tables 4.2 and 4.3 give the maximum response of the rocking blocks over a wider range of aspect ratios and sizes of blocks. The aspect ratio (height/width) was varied from 2 to 5. The height of the blocks was varied from 2 ft to 15 ft, and the width of the blocks was varied from 1 ft. to 3 ft. Table 4.2 gives the values of maximum angular displacements normalized with respect to the block angle α for a ν value of 1.0, while the same results are repeated in Table 4.3 for a ν value of 0.90. The results in Tables 4.2 and 4.3 are tabulated for accelerograms S16°E and S74°W recorded at Pacoima Dam and for the artificial accelerograms A-1 and B-1. For the Pacoima Dam record, the vertical ground acceleration was also included, but no vertical component of ground acceleration was included in the accelerograms A-1 and B-1.

4.3 General Discussion of Results Regarding the Rocking and Overturning of Blocks

From the data presented in Tables 4.1, 4.2, and 4.3, some important observations can be made regarding the general behavior of the rigid blocks rocking under ground motions. It will be seen that because of the dependency of the natural period of vibration on the amplitude of the rocking motion, the problem of rocking is very different from the normal elastic vibration problem. This is the property that makes rocking, and consequently overturning, of blocks very sensitive to many factors.

The following observations can be made from the data presented here.

(1) It can be seen that the possibility that the shielding blocks will overturn under strong ground motions is quite real for aspect ratios (height to width ratio) as low as 2.0, depending upon the overall dimensions of the block and level of ground accelerations. It was shown in Chapter 2 that under a single sinusoidal pulse of acceleration the stability of the block increases with $\sqrt{R}$ for a given angle α. It can be seen in Tables 4.2 and 4.3 that it is generally true that under a given earthquake ground acceleration, and for a given α (i.e., a constant aspect ratio), a block with higher dimension is more stable. For example, by comparing the response of a 10 ft x 2 ft block to that of a 15 ft x 3 ft block, it can be seen in Tables 4.2 and 4.3 that the response of the 15 ft x 3 ft block is always smaller than the response of the 10 ft x 2 ft block, however, the exact mathematical relationship does not hold under arbitrary ground motion.

It will be seen in Tables 4.1, 4.2, and 4.3 that the only block that survives the most severe earthquake accelerogram ever recorded (Pacoima Dam record S16°E) is the 15 ft x 5 ft block when the value of the coefficient of restitution used was 0.95. This means that the probability of overturning a block with an aspect ratio of 3.0 or smaller and a height of more than 15 ft in any future earthquake would be rather small (provided the boundary conditions are right at the base).

(2) In general, a block with a higher value of R and α should be more stable, but this may not be always true as can be seen in Table 4.2 by noting the displacements of 16 x 4 ft and 15 x 3 ft blocks. Comparing the response of these blocks under the Olive View Hospital record and a ν value of 1.0, the corresponding normalized maximum displacements for the 16 ft x 4 ft and 15 ft x 3 ft blocks are 0.32 and 0.30. (Also, compare the same results in Figs. 4.5 and 4.12.) It should be noticed, however, that the 16 ft x 4 ft block with a smaller aspect ratio and larger height should, in general, have a smaller displacement than the 15 ft x 3 ft block.

(3) It can be seen in Table 4.1 and by comparing Tables 4.2 and 4.3 that, generally speaking, the maximum response of a given block under a given accelerogram decreases when the coefficient of restitution ν is decreased. However, this is not always true, and the response of a rocking block actually may increase with a reduction in ν (which is equivalent to an increase in damping). This may be seen by comparing the maximum response of a 15 ft x 3 ft block at ν values of 1.0 and 0.95 under the Olive View Hospital and B-1 accelerograms. The time

history response of this phenomenon can be seen in another example by comparing Figs. 4.13 and 4.14 where a 6 ft high and 2 ft wide block has a higher maximum response at ν = 0.90 than the maximum response at ν = 1.0 under the San Fernando Earthquake of 1971 (Pacoima Dam record S74°W). The maximum values of the response of this block, as listed in Tables 4.2 and 4.3, are 0.38 and 0.58, respectively, for ν values of 1.0 and 0.90.

This rocking behavior in which an increased response results from a decreased ν (or, in other words, at increased damping) can be explained by the dependence of the natural period on the displacement amplitude. This is very different from an elastic vibration problem in which the damped period of vibration is affected very little by a small change in the value of damping and is not a function of the amplitude. In a rocking problem the natural period is very sensitive to the displacement amplitude, and this may result in an appreciable phase shift between the displacement of the block and the applied ground motion. Hence a situation may arise where the displacement of the block, at a smaller value of ν , becomes in phase with a certain pulse of the ground acceleration thus giving a higher response.

Because of this dependency of the period of vibration on the amplitude of vibration, the response of a rocking block may become very sensitive to the coefficient of restitution, as can be seen in Fig. 4.15. It can be noted that the response of a 30 in. x 6 in. block under the same ground motion is entirely different when the value of ν is changed from 0.92 to 0.90. Although, in the beginning, displacements remain small, the block suddenly assumes higher amplitude of motion in the second case (i.e., $\nu = 0.90$), becomes unstable after approximately 6.7 seconds, and overturns in the negative direction. With ν value of 0.92, however, the block becomes unstable after 7.5 seconds and then overturns in the positive direction.

For the same reason mentioned above (namely, the variation of period with θ) the response of a rocking block would be very sensitive to even very small changes in the ground acceleration or other biasing factors, such as addition of external forces.

(4) It may be interesting to note here how the maximum acceleration of a single half sine wave required to overturn a block compares with the maximum acceleration of an earthquake record that will also overturn the same block. Let us assume that a single sine pulse of duration 0.1 second (i.e., of period 0.2 second), which is a good estimate of typical earthquake pulses, is applied to a block 16 ft high and 4 ft wide. Using Eq. (2-9) in Chapter 2, it can be calculated that the amplitude of acceleration of a single sine wave of duration 0.1 second has to be at least 4.6 g in order to overturn the 16 ft x 4 ft block. However, it can be seen in Table 4.1 that the same block overturns under the earthquakes which only have maximum accelerations of 0.5 g. From this comparison it can be seen that using single pulse response it is not possible to derive useful information about the stability of a block under earthquake ground motions.

4.4 Prestressing of Blocks to the Ground

There may be two ways to control the dynamic rocking response and stop the overturning of rigid blocks under earthquake ground motions.

(1) Increase damping.

(2) Prestressing the blocks to the ground.

As far as the effectiveness of increasing the damping is concerned (i.e., reducing ν), it has been shown in the previous subsection that this does not always result in a lower response of a rocking block under siesmic accelerations and, moreover, it may not be easy to reduce the coefficient of restitution. Figure 4.16 shows that an 8 ft x 2 ft block, which overturned at ν values of 1.0 and 0.90 (Tables 4.2 and 4.3) under the San Fernando Earthquake of 1971 (Pacoima Dam accelerogram S16°E), also overturns at a ν value as low as 0.70, which may not even be possible to achieve. Therefore, this alternative does not seem to be attractive.

Excessive rocking can be prevented by properly prestressing the blocks to the foundation. In this case, special consideration should be given to the design of the foundation with respect to the dynamic forces of the design earthquake. This type of design may be more expensive than that used in a sliding system.

Figure 4.17 shows the rocking response of the 8 ft x 2 ft block under the same accelerograms as shown in Fig. 4.16. The value of the coefficient of restitution used in this analysis was 0.95. The block was assumed to be prestressed to the ground in the center, and the end condition for the prestressing rod was assumed to be hinged at the ground. It was also assumed that the prestressing rod remains elastic and the stiffness of the rod was taken to be 0.4W/in., where W is the total weight of the block. The prestressing rod initially had a prestressing force of 0.4W. The analysis was carried out using the computer program and the results can be seen in Fig. 4.17. It should be noticed that the block which overturns without any prestressing in Fig. 4.16 becomes stable with a maximum normalized displacement (θ/α) equal to 0.3 (Fig. 4.17).

The maximum force a foundation must withstand occurs when the block has a maximum angular displacement. The total concentrated maximum tensile force that the foundation must take, in this case, would be = $0.4W+.4W\,\Delta$ where Δ is the extension of the prestressing rods. It is quite obvious that when the concrete blocks are very large the foundations may gave to be specially designed to withstand the maximum force in the ties.

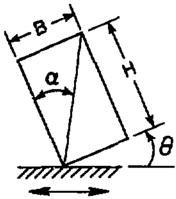
4.5 Rocking Versus Sliding

It is apparent from the studies on the rocking response of free-standing blocks under earthquake ground motions that overturning under severe earthquakes is a very real possibility. The probability of overturning a block with an aspect ratio of 3 together with a height of more than 15 ft is rather low and increases with an increase in aspect ratio or for a fixed aspect ratio with a decrease in height (see Tables 4.1, 4.2, and 4.3).

It has also been shown in the previous article that prestressing these massive tall concrete blocks to the ground would result in very large forces on the foundation in the event of a strong earthquake. Therefore, it is possible that in some cases these shielding blocks should be allowed to slide by a reduced and carefully controlled coefficient of friction between the block and the ground, thus changing the response from the rocking mode to sliding.

As a guide to the expected sliding movements of rigid blocks under earthquake ground motions, Table 4.4 can be used. This table shows the maximum relative displacement of a block with respect to the ground for values of the coefficient of dynamic friction (μ_d) equal to 0.1, 0.2, and 0.3. The values of relative displacments in Table 4.4 are given for the accelerograms of the San Fernando Earthquake of 1971 (Pacoima Dam records S16°E and S74°W) and for the artificially generated accelerograms A-1, A-2, and B-1 (3). Details of these studies on the sliding movements of rigid blocks are presented in Reference (1). It can be seen that the maximum sliding movements always occur under Pacoima Dam records because of the unusually high accelerations recorded at this site. The probability of exceeding the maximum values given in Table 4.4 in a future earthquake would be rather low. Considering the results presented in Table 4.4, an estimate can be made regarding the expected sliding movements during earthquakes for a given value of the dynamic coefficient of friction. Having estimated the sliding movements, enough clearance should be provided between the blocks and surrounding equipment to allow free movement and to prevent any damage due to collision.

It should be noted here that, whereas the value of the dynamic coefficient of friction should be used in Table 4.4 to estimate the maximum expected sliding movements, it is the static coefficient of friction that should be used to determine the boundary between rocking and sliding, as explained in Article 2.1.



H/B (ft)	COR ν	MAXIMUM θ/α VALUES UNDER EARTHQUAKES: SAN FERNANDO EARTHQUAKE, PACOIMA DAM S16°E	PACOIMA DAM S74°W	OLIVE VIEW HOSPITAL RECORD	ARTIFICIAL EARTHQUAKE A-1	ARTIFICIAL EARTHQUAKE B-1
15/5	1.00	F	0.13	0.15	0.35	0.08
	0.95	0.55	0.10	0.11	0.01	0.01
16/4	1.00	F	0.59	0.32	F	0.67
	0.95	0.82	0.33	0.24	0.60	0.54
15/3	1.00	F	F	0.30	F	F
	0.95	F	0.99	0.34	F	0.41

TABLE 4.1 ROCKING RESPONSE OF A RIGID BLOCK UNDER VARIOUS STRONG MOTION ACCELEROGRAMS

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TABLE 4.2 ROCKING RESPONSE OF A RIGID BLOCK UNDER VARIOUS STRONG MOTION ACCELEROGRAMS (ν = 1.0)

HEIGHT/WIDTH (H/B) (ft/ft)	MAXIMUM θ/α VALUES UNDER EARTHQUAKES			
	SAN FERNANDO EARTHQUAKE		ARTIFICIAL EARTHQUAKE	
	PACOIMA DAM RECORD		A-1	B-1
	S16°E	S74°W		
2/1	F	0.63	0.0	0.0
3/1	F	F	F	F
4/1	F	F	F	F
5/1	F	F	F	F
4/2	F	0.42	0.0	0.0
6/2	F	0.38	F	0.40
8/2	F	0.73	F	F
10/2	F	F	F	F
6/3	F	0.20	0.0	0.0
9/3	F	0.29	F	0.16
12/3	F	0.65	F	0.72
15/3	F	F	F	F

XBL 784-8376

TABLE 4.3 ROCKING RESPONSE OF A RIGID BLOCK UNDER VARIOUS STRONG MOTION ACCELEROGRAMS (ν = 0.90)

HEIGHT/WIDTH (ft/ft)	MAXIMUM θ/α VALUES UNDER EARTHQUAKES			
	SAN FERNANDO EARTHQUAKE		ARTIFICIAL EARTHQUAKE	
	PACOIMA DAM RECORD		A-1	B-1
	S16°E	S74°W		
2/1	F	F	0.00	.000
3/1	F	F	0.005	.002
4/1	F	F	F	F
5/1	F	F	F	F
4/2	F	0.30	0.000	0.000
6/2	F	0.58	0.003	0.001
8/2	F	0.43	0.33	0.62
10/2	F	0.75	F	0.66
6/3	0.38	0.23	0.00	0.00
9/3	0.75	0.22	0.002	0.001
12/3	F	0.28	0.22	0.56
15/3	F	0.43	F	0.37

XBL 784-8377

TABLE 4.4 MAXIMUM DISPLACEMENT OF A SLIDING BLOCK RELATIVE TO GROUND UNDER VARIOUS ACCELEROGRAMS

DYNAMIC COEFICIENT OF FRICTION	MAXIMUM RELATIVE DISPLACEMENT (INCHES)				
	SAN FERNANDO EARTHQUAKE AT PACOIMA DAM (1971)		ARTIFICIAL EARTHQUAKES		
	S16°E	S74°W	A-1	A-2	B-1
0.1	17.1	5.6	12.0	7.0	5.8
0.2	4.4	8.8	2.4	1.8	0.4
0.3	2.1	5.1	0.1	0.3	0.03

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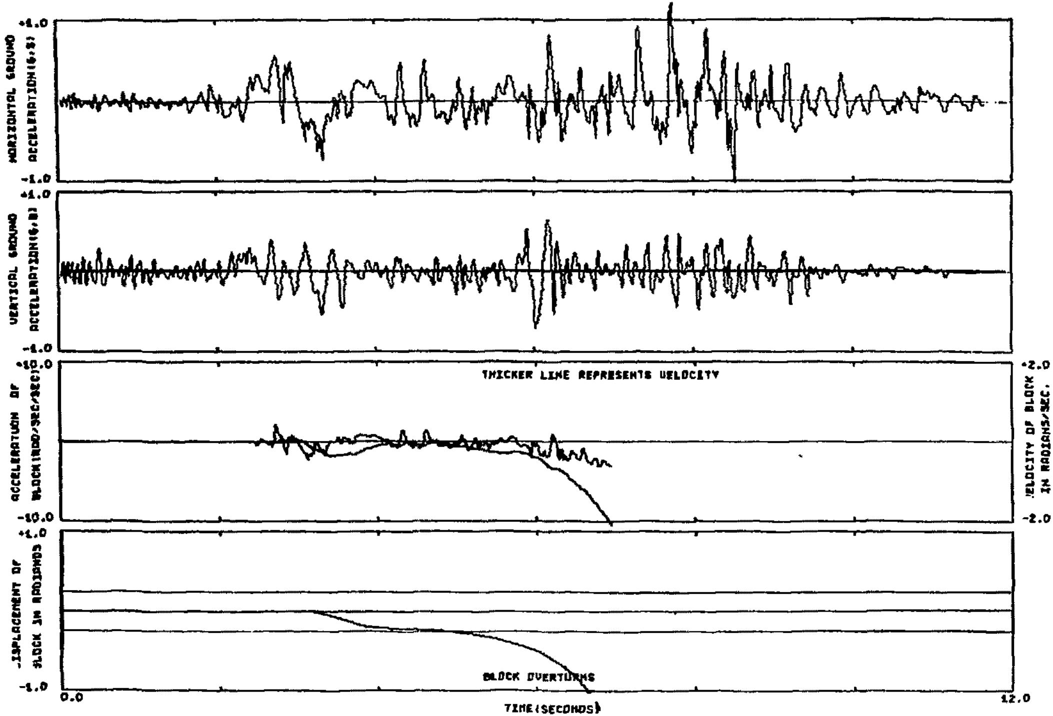

FIG. 4.1 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S16E)B=48IN.,H=192IN,COR=1.00 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 7810-12069

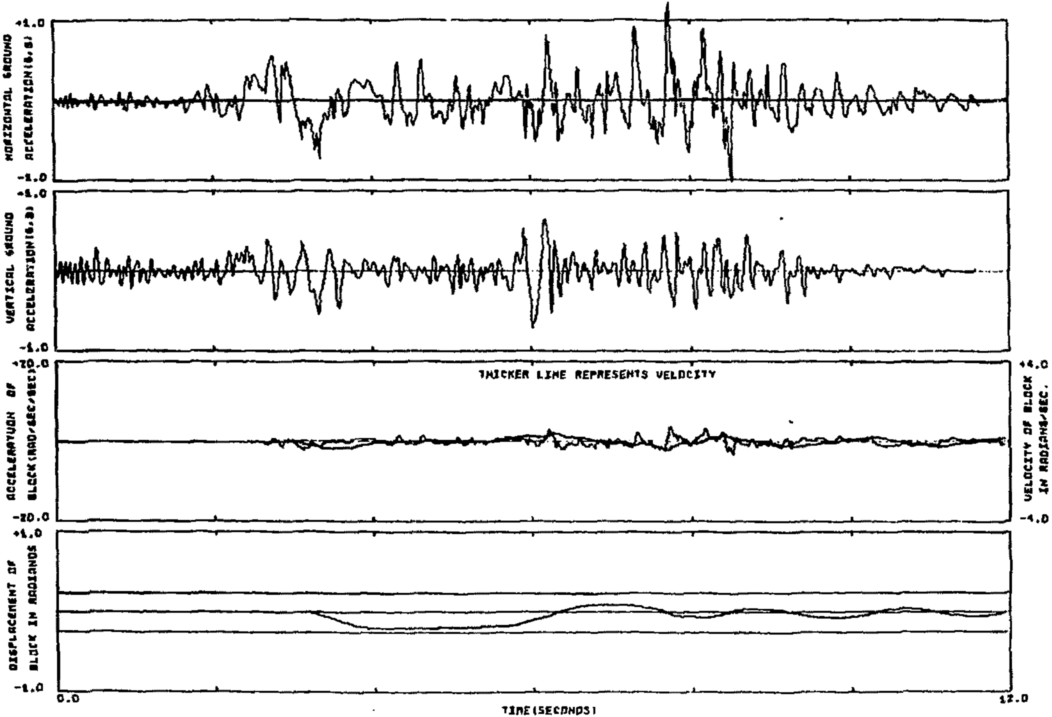

FIG. 4.2 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S16E)B=48IN.,H=192IN,COR=0.95 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 7810-12056

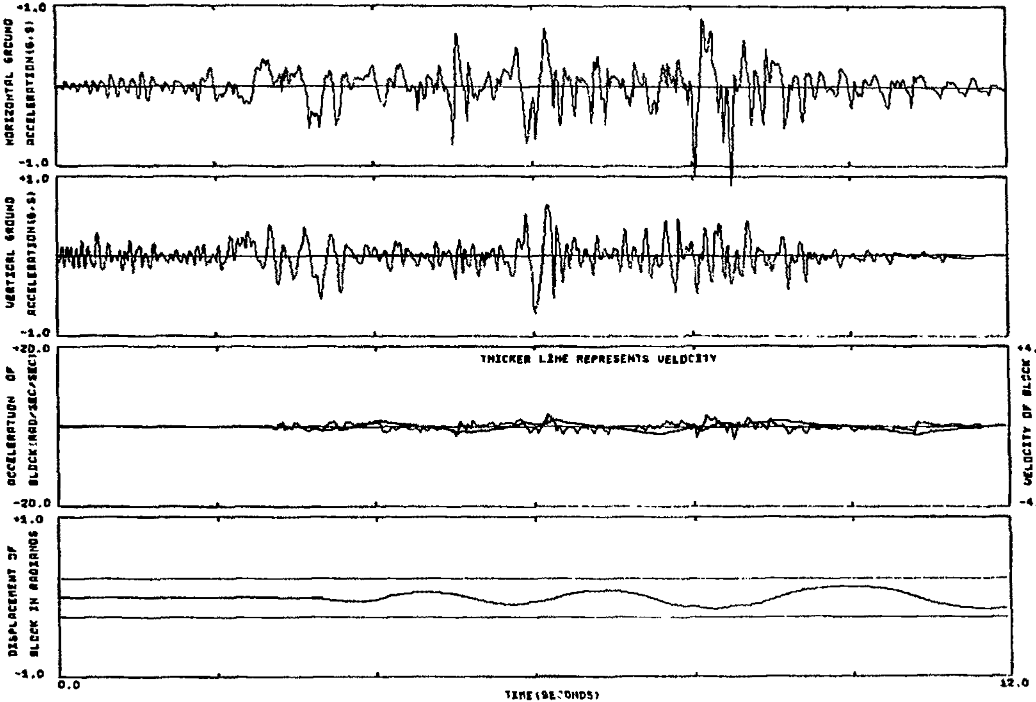

FIG. 4.3 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S74W)B=48IN.,H=192IN,COR=1.00 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 7810-12057

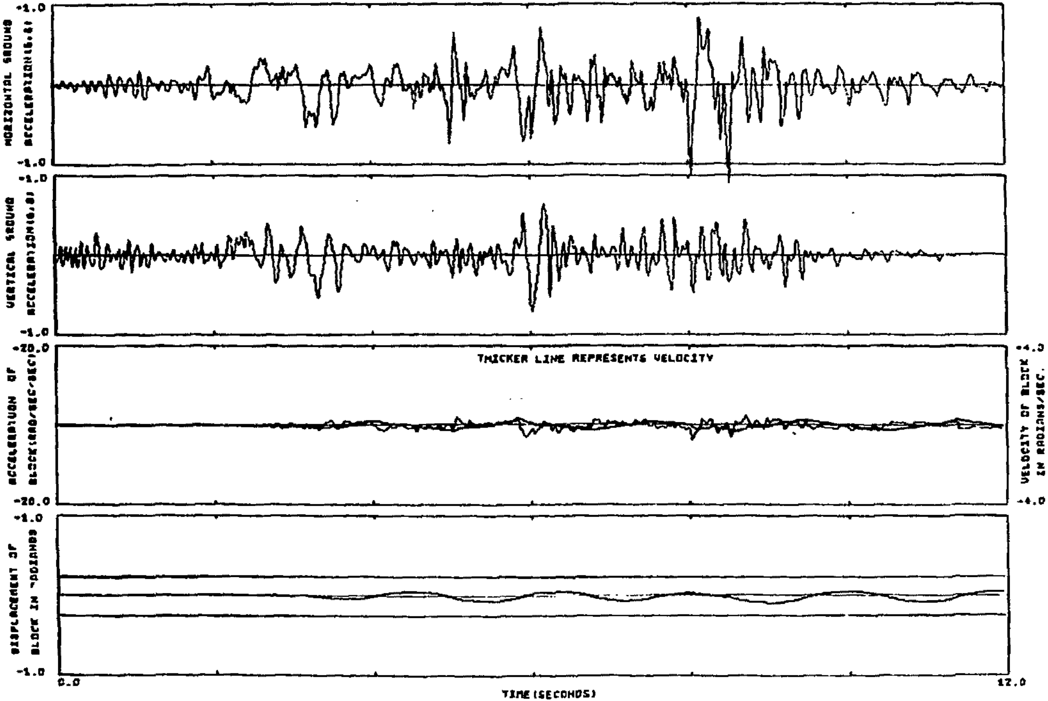

FIG. 4.4 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S74W)B=48IN.,H=192IN,COR=0.95 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 7810-12058

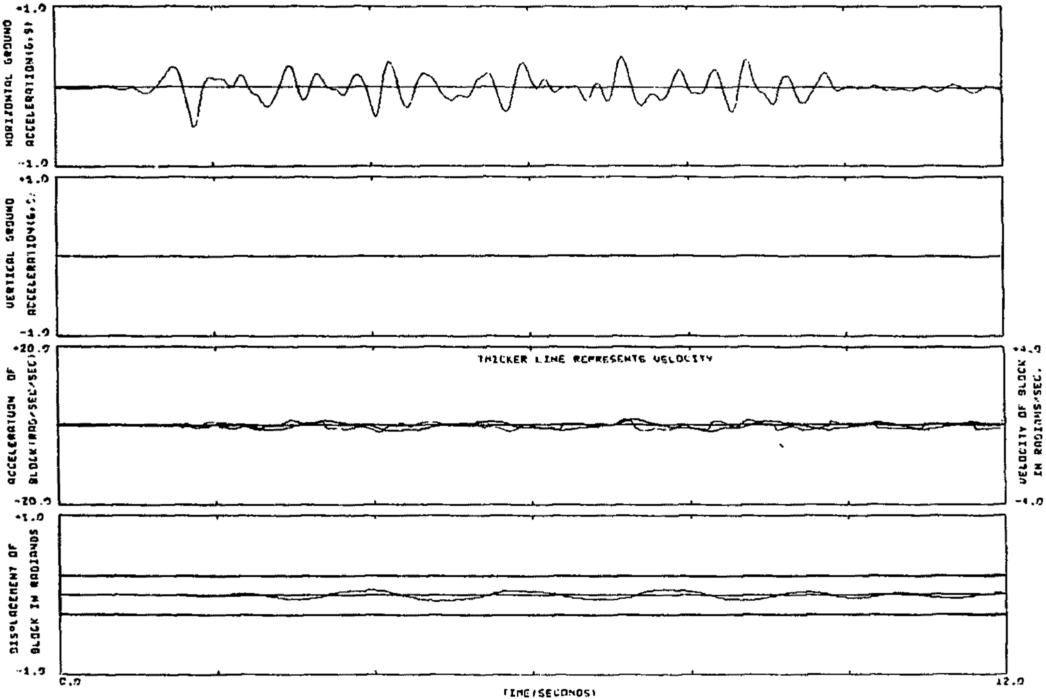

FIG. 4.5 ROCKING OF A BLOCK SUBJECTED TO SAN FERNANDO EARTHQUAKE 1971(OLIVE VIEW HOSP. RECORD)B=48IN.,H=192IN,COR=1.00 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 784-8361

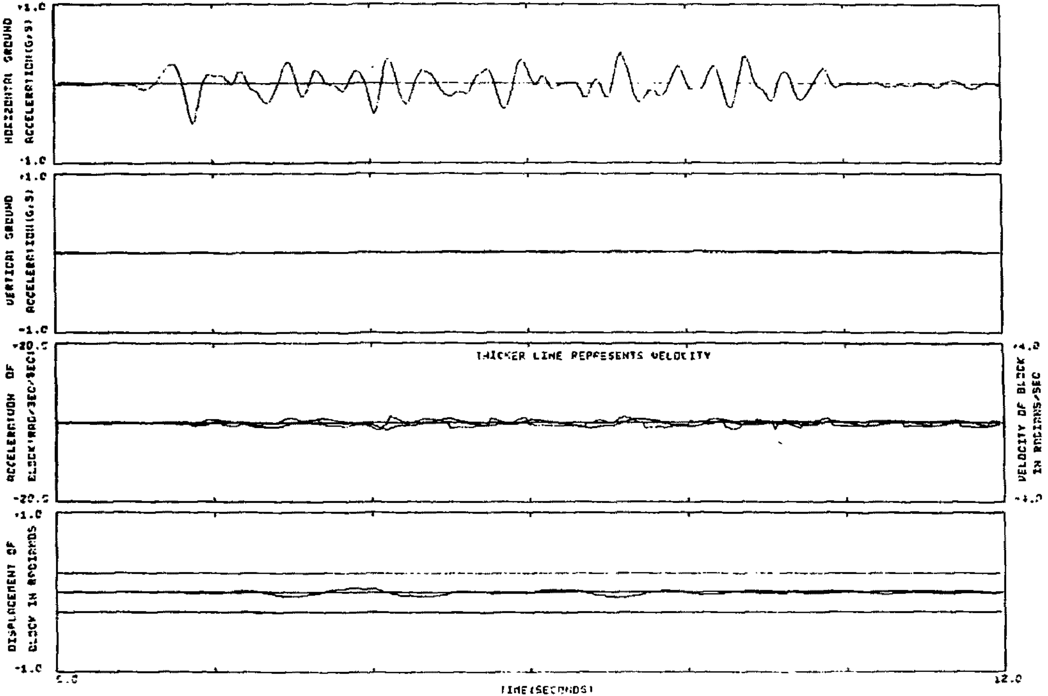

FIG. 4.6 ROCKING OF A BLOCK SUBJECTED TO SAN FERNANDO EARTHQUAKE 1971(OLIVE VIEW HOSP. RECORD)B=48IN.,H=192IN,COR=0.95 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 784-8372

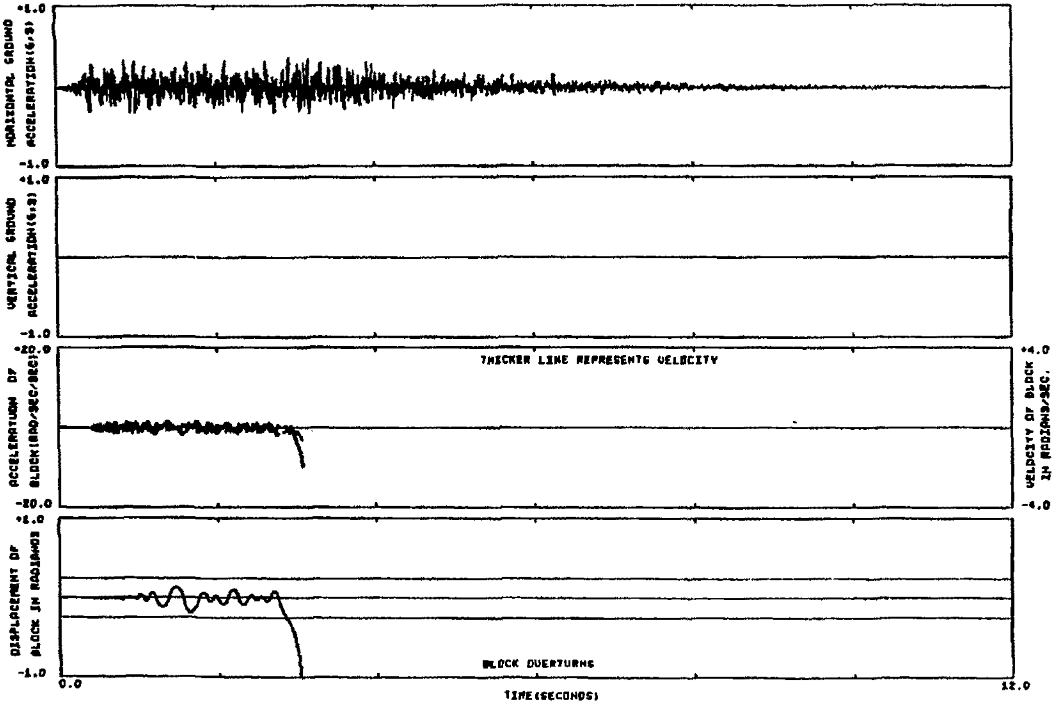

FIG. 4.7 ROCKING OF A BLOCK SUBJECTED TO ARTIFICIAL EARTHQUAKE A-1
B=48IN, H=192IN, COR=1.0, K=0.0W/IN, PRESTRESSING FORCE=0.0W

XBL 7810-12059

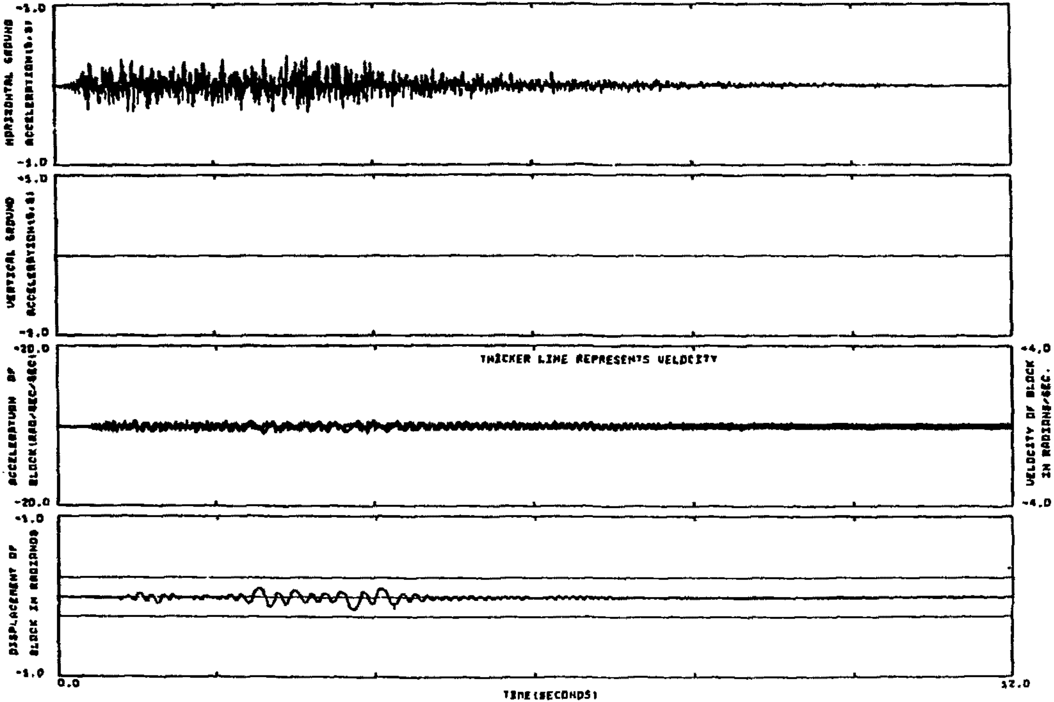

FIG. 4.8 ROCKING OF A BLOCK SUBJECTED TO ARTIFICIAL EARTHQUAKE A-1
B=48IN, H=192IN, COR=.95, K=0.0W/IN, PRESTRESSING FORCE=0.0W

XBL 7810-12060

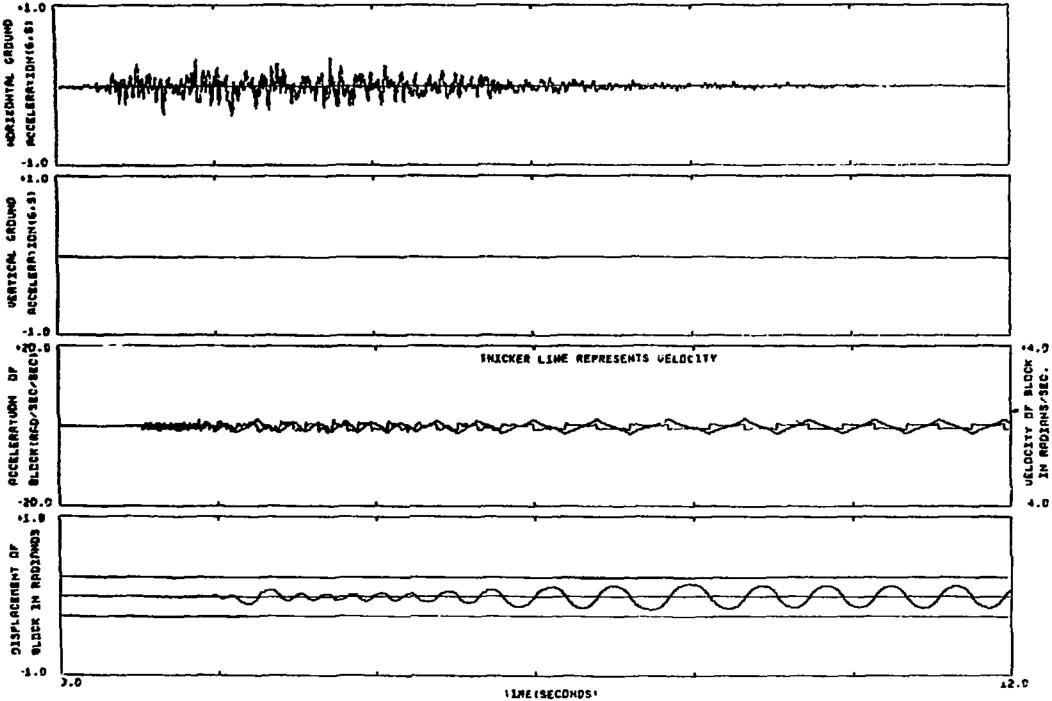

FIG. 4.9 ROCKING OF A BLOCK SUBJECTED TO ARTIFICIAL EARTHQUAKE B-1
B=48IN, H=192IN, COR=1.0, K=0.0W/IN, PRESTRESSING FORCE=0.0W

XBL 7810-12061

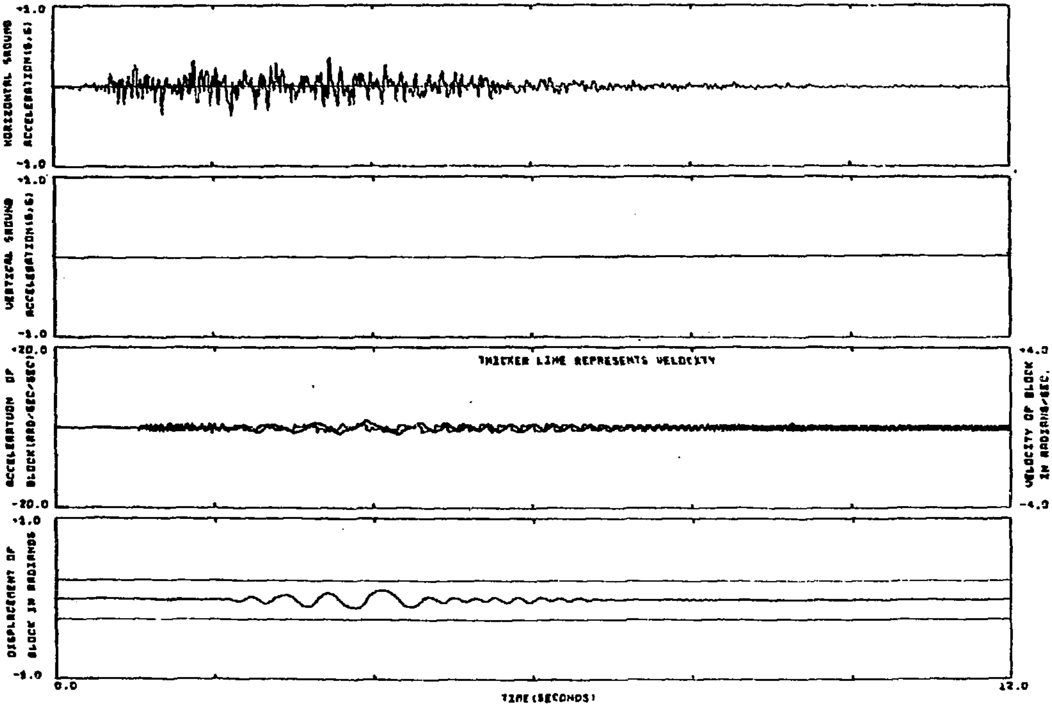

FIG. 4.10 ROCKING OF A BLOCK SUBJECTED TO ARTIFICIAL EARTHQUAKE B-1
B=48IN, H=192IN, COR=.95, K=0.0W/IN, PRESTRESSING FORCE=0.0W

XBL 7810-12062

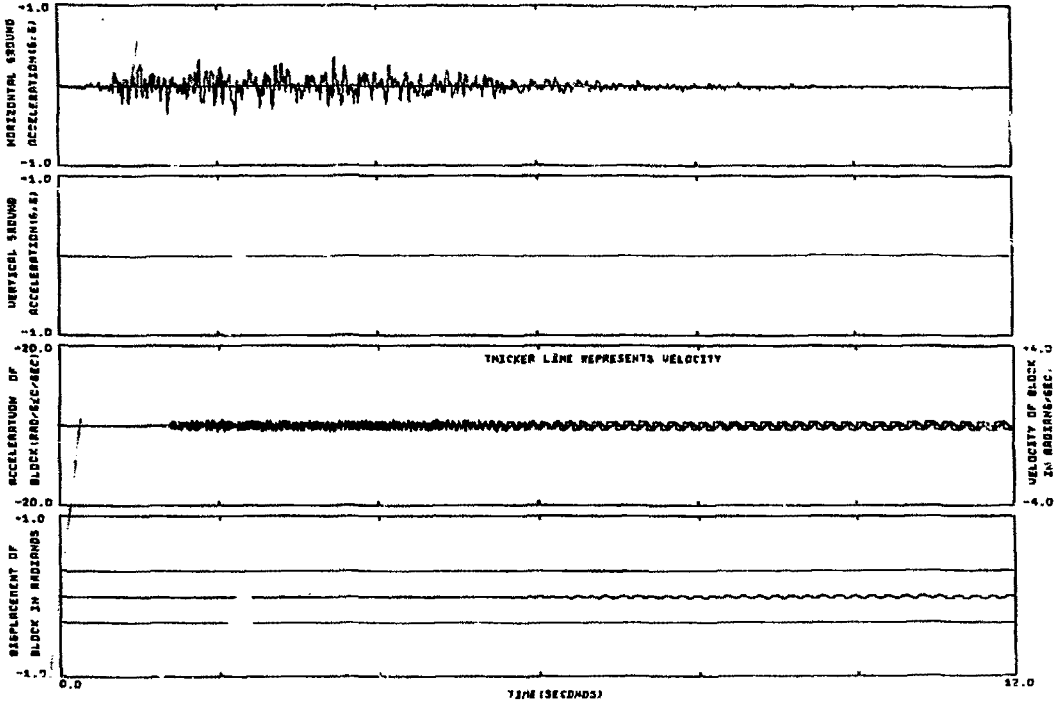

FIG. 4.11 ROCKING OF A BLOCK SUBJECTED TO ARTIFICIAL EARTHQUAKE B-1
B=60IN, H=180IN, COR=1.0, K=0.0W/IN, PRESTRESSING FORCE=0.0W

XBL 7810-12063

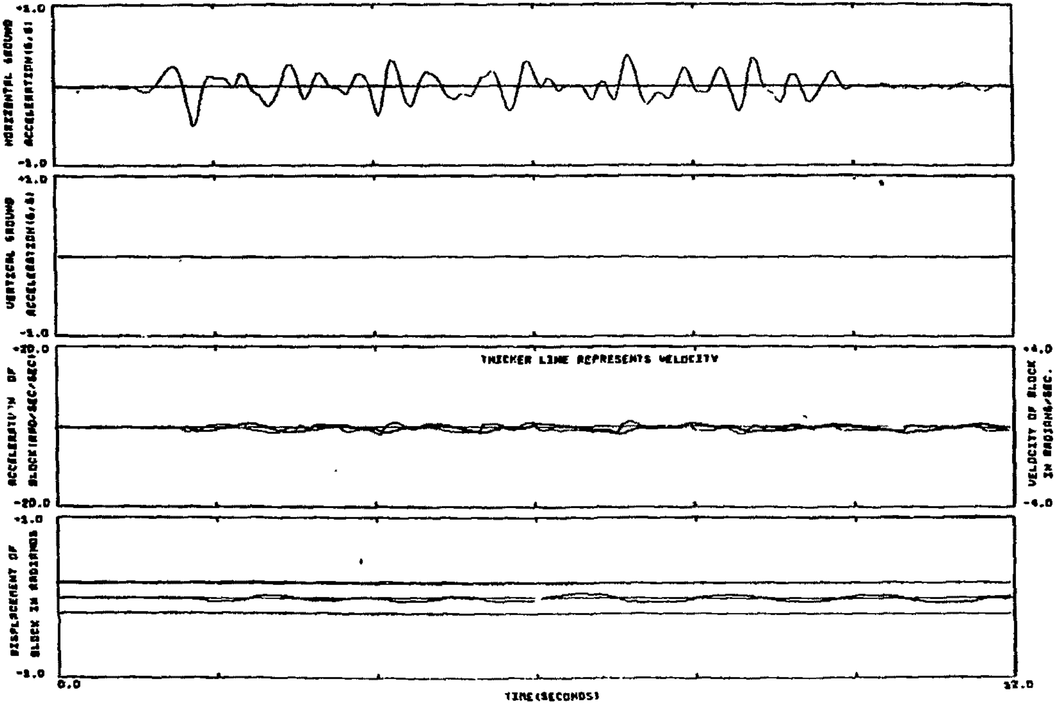

FIG. 4.12 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(OLIVE VIEW HOSP. RECORD) B=36IN., H=180IN, COR=0.95 K=0.0W/IN, PRESTRESSING FORCE=0.0W

XBL 7810-12064

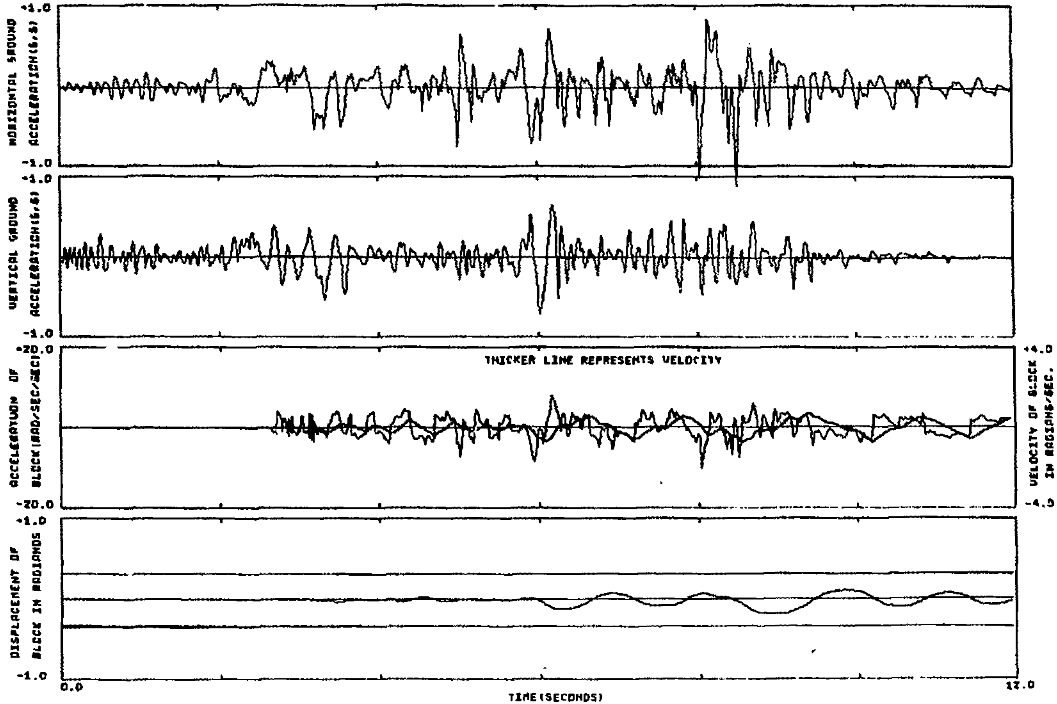

FIG. 4.13 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S74W)B=24IN.,H=72 IN,COR=0.90 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 7810-12065

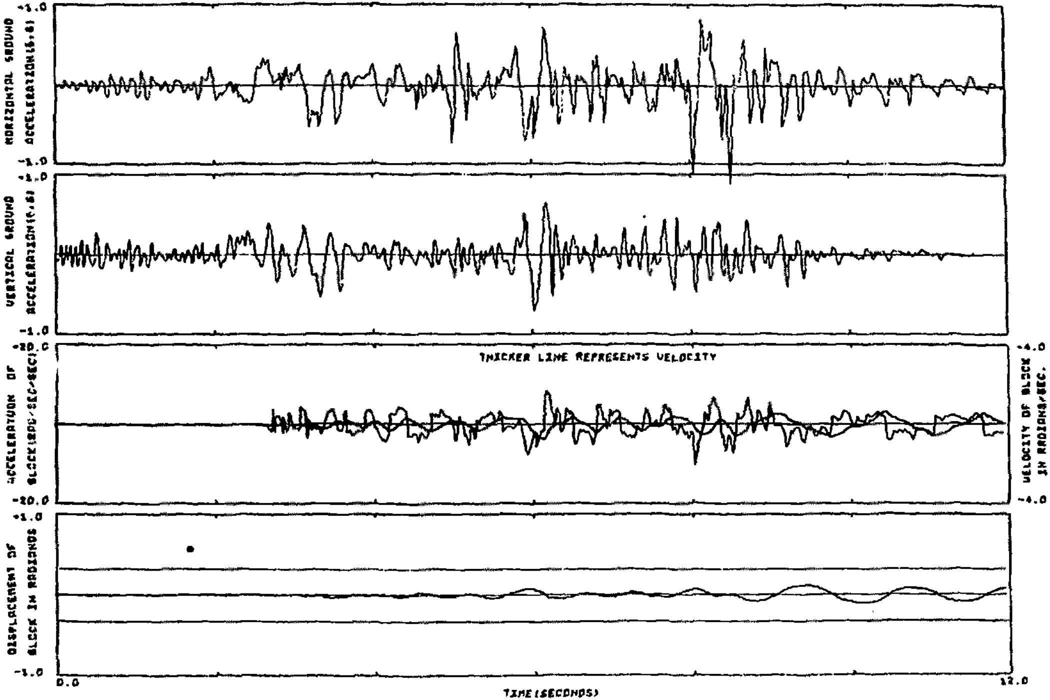

FIG. 4.14 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S74W)B=24IN.,H=72 IN,COR=1.0 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 7810-12066

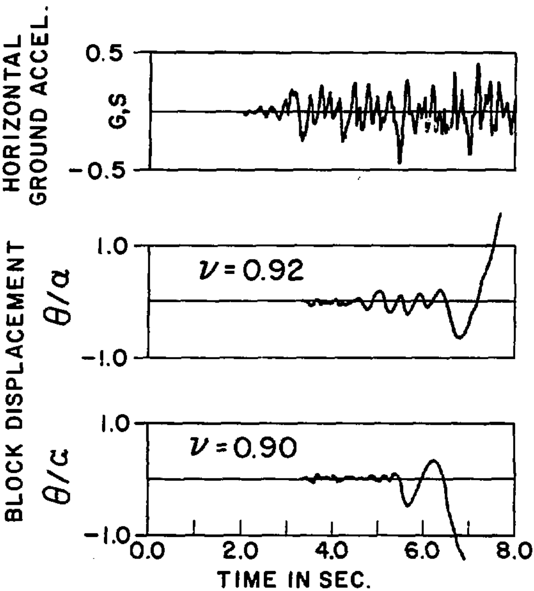

FIG. 4.15 ROCKING RESPONSE OF A 30x6 IN. BLOCK TO THE OLIVE VIEW HOSPITAL GROUND MOTION SHOWING SENSITIVITY TO COEFFICIENT OF RESTITUTION.

XBL 784-8369 A

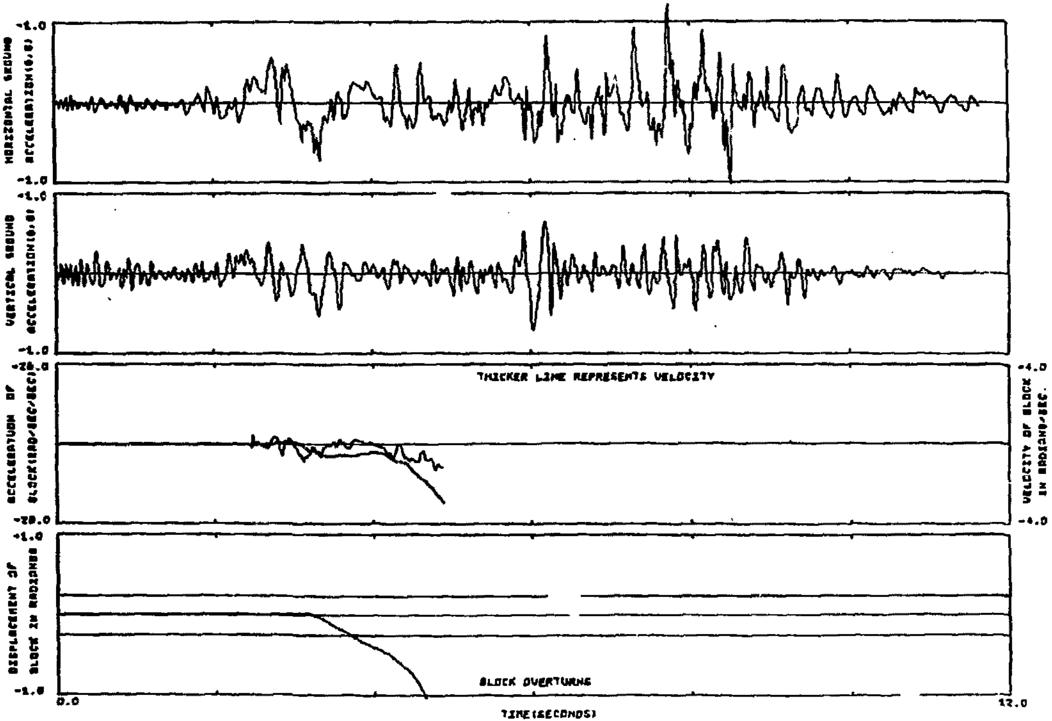

FIG. 4.16 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971(PACOIMA DAM RECORD S16E)B=24IN.,H=96 IN,COR=0.70 K=0.0W/IN,PRESTRESSING FORCE=0.0W

XBL 7810-12067

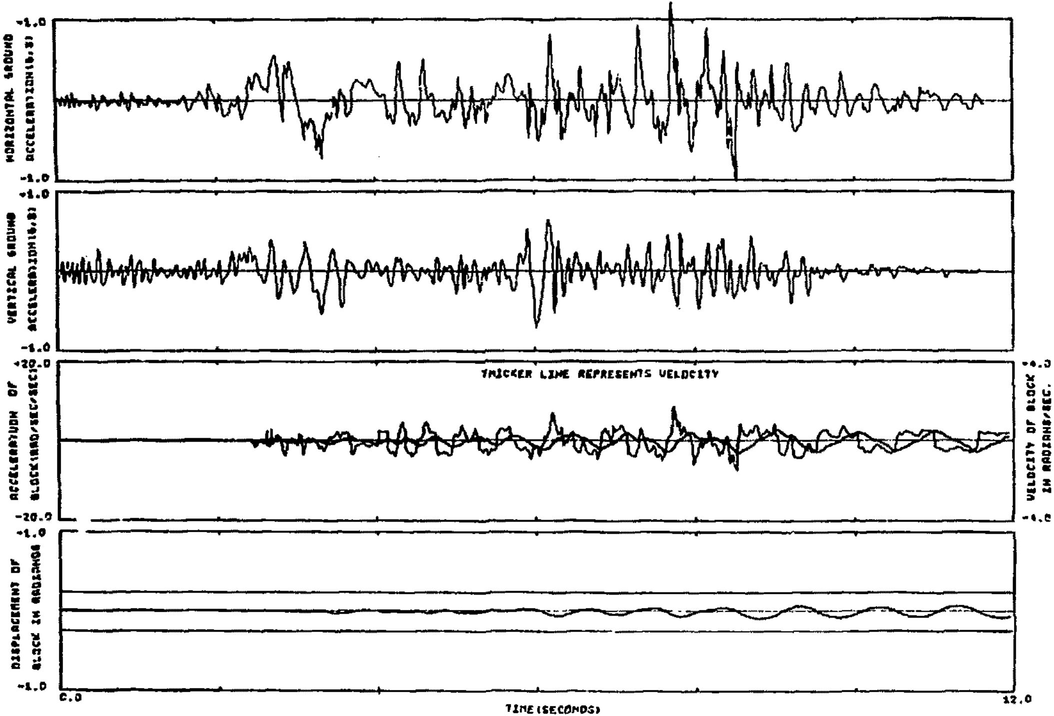

FIG. 4.17 ROCKING OF A BLOCK SUBJECTED TO SAN-FERNANDO EARTHQUAKE 1971 (PACOIMA DAM RECORD S16E) B=24IN., H=96 IN, COR=0.95 K=0.4W/IN, PRESTRESSING FORCE=0.4W

XBL 7810-12068

REFERENCES

1. Aslam, M., Godden, W.G., and Scalise, D.T., "Sliding Response of Rigid Blocks to Earthquake Motions," LBL-3868, Lawrence Berkeley Laboratory, September 1975.
2. Housner, G.W., "The Behavior of Inverted Pendulum Structures During Earthquakes," Bulletin of the Seismological Society of America, Vol. 53, no. 2, pp. 403, February 1963.
3. Jennings, P.C., Housner, G.W., and Tsai, N.C., "Simulated Earthquake Motions," California Institute of Technology, Pasadena California, April 1968.

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