Title
How Teaching Fractions to Elementary Students Looks in the Common Core Era

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Publication Date
2020

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

How Teaching Fractions to Elementary Students Looks in the Common Core Era

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Education

by

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2020

ABSTRACT OF THE DISSERTATION

How Teaching Fractions to Elementary Students Looks in the Common Core Era

by

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Doctor of Education

University of California, Los Angeles, 2020

Professor Megan Loef Franke, Chair

In 2019, forty-five percent of fourth grade students scored proficient on the California Assessment of Student Performance and Progress. Previous research indicates that fractions and algebra are two domains that cause confusion for students (NMAP, 2008). This study examined the pedagogical decisions regarding fraction instruction for third and fourth grade students. Twenty third and fourth grade teachers within Los Angeles County participated in semi-structured interviews. I used a qualitative research design in order to determine how and why teachers make particular instructional choices and their challenges related to fraction instruction. Once data were collected, I analyzed patterns of reasoning to illuminate similarities and differences within the sample. My findings showed teachers are seeking and open to more effective fraction instruction and consider their school and student dynamics to guide pedagogical decisions. This suggests an opening for professional development in the area of elementary fraction instruction.

The dissertation of Opal Chambers is approved.

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2020

DEDICATION PAGE

I dedicate this dissertation to Nick, my best friend and husband, who has been my greatest source of encouragement. Your passion for math has always been contagious. Thank you for being the rock that I needed and always supporting me in pursuing my goals.

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ACKNOWLEDGEMENTS

I would like to thank Dr. Franke, my committee members, Dr. Cindy Kratzer, Rosanne, my cohort, my friends, and my family. Dr. Franke, your continued support and feedback pushed me to new levels of understanding. To my committee, thank you all for your insight and improving education in countless ways.

Cindy, you are such an amazing educator and I am so thankful that our cohort were able to be part of your extremely thoughtful lessons and guidance. You are the instructor we will always remember, and we cannot thank you enough for the way you worked tirelessly to help us conquer each task. Thank you for using your classes to show us how real professional development looks and seeking the answers to our many, many questions all while forging strong relationships with every one of us.

Terrific 25, it is crazy for me to think that three years ago we were all strangers! Thank you for all the lessons learned. It has been a privilege to share this experience with all of you.

Rosanne! Something brought us together like magnets and I am forever grateful! Thank you for being my study buddy, my sounding board, my cause for laughter in this journey, and my dear friend. I cannot wait to see how your future unfolds!

To my family, thank you for teaching me the value of education and the power of perseverance. To my friends, thank you so much for supporting me!

Finally, I would like to thank my husband, who politely nudged me to keep going, who somehow figured out what I needed before I did, and who indulged in my many late-night discussions surrounding the world of education.

VITA

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CHAPTER ONE: INTRODUCTION

Background

According to the 2019 National Assessment of Educational Progress, the majority of fourth grade students are not proficient in math. In fact, only forty-one percent of students in 4th grade in our country demonstrated proficiency on the 2019 National Assessment of Educational Progress (National Center for Education Statistics [NCES], 2019). Furthermore, when disaggregating the data, only 20% of Black students were proficient in math. NAEP defines proficient as being able to "demonstrate solid academic performance and competency over challenging subject matter" (NCES, 2019). In Los Angeles County, fourth grade proficiency scores on the 2019 California Assessment of Student Performance and Progress (CAASPP) mirrors this level of understanding, with just under forty-five percent of students meeting or exceeding standards (California Department of Education, 2019).

Fractions and algebra are two domains within the math trajectory that cause the most confusion (National Mathematics Advisory Panel, 2008). Over the past few decades, researchers have repeatedly linked fraction fluency to algebra success (Bailey et al., 2012; Jordan et al., 2013; Siegler et al., 2013; Siegler et al., 2011; Siegler et al., 2012). In addition, algebra proficiency is linked to college graduation rates and higher post-graduate earnings (Gaertner et al., 2014).

Elementary teachers must find ways to better address fraction concepts to support students' success in future math courses. Drawing on the research that highlights the need for conceptual knowledge of fractions and its correlation with future success, I studied how third and fourth grade teachers introduce fraction concepts by conducting teacher interviews.

Why Students Struggle to Understand Fractions

Students have struggled to understand fractions for many years. In fact, according to the National Council of Teachers of Mathematics, in 2007, half of middle and high school students struggled to understand fractional concepts (National Mathematics Advisory Panel, 2008). In recent decades, research has illuminated many reasons why students struggle with fractions. Research suggests students have difficulties assessing fraction magnitude, or the value of a fraction. Research also indicates that students need multiple opportunities throughout the academic year to practice, discuss, and revise their knowledge of fractions, rather than one confined unit on the topic. Finally, textbook alignment and new nation-wide standards have posed challenges for teachers to learn how to best teach their students.

Teachers who teach fractions as a single unit may be adding to their students' confusion. Multiple studies have suggested that some students who struggle with fraction concepts incorrectly apply a whole number bias, also referred to as a natural number bias (Ni & Zhou, 2005; Kainulainen et al., 2017). This bias means students apply the properties of whole numbers to work with fractions. For example, students might incorrectly assume that numbers that are greater in the denominator means the magnitude or value of that fraction is also greater. Students might assume that multiplying two fractions results in a greater product. However, when educators teach fraction concepts simultaneously with whole numbers, students are able to understand that the principles applied to whole numbers are different than those applied to fractions.

Introduction of the Common Core State Standards

In an attempt to improve student outcomes in math, the Common Core State Standards for Mathematics (CCSSM) revamped the fraction standards to focus more on

conceptual understanding (Common Core State Standards Initiative, 2010). In 2009, the National Governor's Association for Best Practices and the Council of Chief State School Officers (CCSSO) created the Common Core State Standards with the intention of designing a more focused, coherent, and rigorous set of standards. In math, the content was deliberately cumulative to help students draw connections between old and new concepts. With the hope of becoming more competitive internationally, and aligning the national curricula, the CCSSM has been adopted by 41 states and the District of Columbia (CCSSO, 2010).

The Number and Operations-Fractions domain first appears in the Common Core State Standards for Mathematics (CCSSM) in the third grade. The standards specify that by the end of third grade, students will demonstrate how to write a fraction based on their knowledge of partitioning whole shapes, create equivalent fractions, and represent fractions on a number line. Students in third grade are only expected to work with fractions containing the denominators of 2, 3, 4, 6, and 8. Prior to the Common Core State Standards, there was no explicit mention of the number line to plot fractions (California Department of Education, 2013). The CCSSM did, however, incorporate the importance of equal partitioning, a skill many students are familiar with prior to the academic setting (Carpenter et al., 1993; Kouba, 1989).

Constraints to Implementing the Common Core State Standards

Now that the Common Core State Standards for Mathematics have directly incorporated the plethora of research on fraction magnitude, partitioning, and plotting fractions on the number line alongside whole numbers, one must consider the constraints for teachers to implement this set of standards.

To start, we know that all teachers needed to become familiar with the Common Core State Standards for Math. At the time of CCSSM implementation, a survey conducted by the 2012 National Survey of Science and Mathematics Education found that 82% of elementary teachers were over 40, which means that those teachers needed to adapt to, or learn about, the Common Core State Standards (Banilower et al., 2013). This large percentage of teachers all required significant training in the differences between the old California state math standards and the Common Core State Standards for Mathematics. Research shows that pre-service teachers have used the Common Core State Standards for Mathematics in their teacher preparation programs (Murphy & Marshall, 2015). We do not know, however, how veteran teachers, or those who taught under the prior standards, have rolled out the CCSSM standards and how beneficial the teachers found the training.

Textbook alignment is another challenge that teachers face. Walters et al. (2014) found that 46% of teachers needed textbooks aligned to the CCSSM. Lee and Wu (2017) added to Walters' study, finding that school-adopted textbooks did not match the rigor or content of the Common Core State Standards. According to their website, "The standards will not prescribe *how* they are taught and learned but will allow teachers flexibility to teach and students to learn in various instructionally relevant contexts." (Common Core Standards Initiative, 2010). Because of this flexibility and lack of aligned materials, teachers may resort to creating or searching for instructional resources. Research has not addressed how teachers make pedagogical decisions on how to teach the standards.

Project Statement

My study added to the research by explicating the pedagogical decisions that third and fourth grade teachers make while teaching the Number and Operations-Fractions

Domain of the CCSSM. Knowing how teachers make pedagogical decisions about the introductory lessons for fractions allows leaders of professional development, math coaches, and school administrators to provide more specific professional development. The objective of my research was to discover how teachers introduce fractions concepts with third and fourth grade students.

Research Questions

1. How do third and fourth grade teachers choose what and how to teach fractions in the context of the 2013 California Math Framework?
    a. Why do they make those pedagogical choices?
    b. What teaching and learning experiences have influenced third and fourth grade teachers' pedagogical choices when teaching fractions?
    c. What do third and fourth grade teachers perceive as challenges when teaching fractions?

Research Design

In order to address my research questions about how teachers are introducing fractions to their third and fourth grade students, I focused on 20 third and fourth grade teachers in Los Angeles County who vary in their use of school curriculum.

Although fourth grade nationwide math scores are readily available, fraction concepts are first addressed as a domain in the Common Core State Standards for Mathematics in third grade and continue to be explored in fourth grade. I included both grades in this study in order to obtain a more wholistic idea of teaching pedagogy with fractions. Some third-grade teachers might provide the most useful information as research has demonstrated that the introduction of fractions is the most difficult time for students to correctly identify fraction

magnitudes as they often utilize the whole number bias (McMullen et al., 2015). Fourth grade teachers will be able to explain what their students understood at the beginning of the year and go more in depth on what they base their teaching decisions. Utilizing both grades allowed for trends to be identified.

The population for this study was third and fourth grade teachers within Los Angeles County. Working in various school sites allowed me to discover ways that teachers have adapted to the standards in distinct settings.

Research Methods

My research utilized interviews. As part of the interview process, teachers completed a short screening questionnaire. This questionnaire asked teachers their email address, how long they had been teaching, how long they had been teaching third or fourth grade, their dependence on the school-adopted math curriculum, the name of the math curricula at their school, and the name of their school site.

Teacher interviews were divided into two parts. The initial interview questions identified the decision-making process teachers use when deciding the introductory activity for fraction concepts to their students. Although some of this information could have been gathered in a survey, a semi-structured interview allowed for follow-up questions and in-depth analysis of the teachers' pedagogical processes.

The second part either took place at the same time as the first or within a few days of the initial interview, depending upon teacher availability. This part introduced five activities and asked the participants to explain why they would or would not use the activities with their students as well as what they found to be the advantages and disadvantages to each proposed activity.

Significance of the Research

The findings of this study will help teachers and curriculum developers align lessons to the research-based standards and offer ways to improve student understanding of fractions. Furthermore, administrators and instructional coaches could use the data from this research to design professional development for their veteran teachers to further assist them in their conceptual understanding of the fraction's standards. This study could also be helpful for surrounding grade level teachers, such as second and fifth, as the findings might provide them with ways in which to design their own curricula to best meet the needs of their students. Most importantly, teachers, administrators, teacher preparation programs, and instructional coaches from underperforming schools could be given the roadmap necessary for their teachers to ultimately improve student performance in math. Finally, learning about the ways teachers make decisions about fraction activities and teacher background knowledge could lead to changes in math instruction and begin to positively address the achievement gap.

CHAPTER TWO: LITERATURE REVIEW

Introduction

Proficiency with fractions has been a struggle for students for many years. To understand the factors affecting student achievement with fractions, I summarize the dismal student achievement scores relating to fractions and its significance to algebra proficiency which is the foundation for many mathematical concepts. Then, I explain the effective and ineffective teaching strategies teachers use to teach fraction concepts. Later, I introduce the Common Core State Standards for Mathematics and exemplify how its authors integrated those best practices for teaching fractions into the standards.

History of Student Challenges in Understanding Fraction Concepts

Students in the United States have struggled to understand fractions for years (NMAP, 2008). In 2007, according to the National Council of Teachers of Mathematics, half of middle and high school students scored proficient with problems about fraction concepts (NMAP, 2008). When asked to order three fractions from least to greatest when using different denominators, only half of eighth grade students correctly solved the problem. Furthermore, in 2008, the National Mathematics Advisory Panel (NMAP) surveyed 1,000 algebra teachers within the United States who rated weak fraction knowledge as the second largest issue facing their students understanding for math. NMAP went on to conclude that proficiency with fractions was the most important foundational skill that would lead to improved student outcomes with algebra (NMAP, 2008).

Within the past few decades, research has not only linked proficiency with fractions to success in algebra but has also illuminated best practices with fractions. Magnitude, or the size of a fraction, has been identified as one common student misunderstanding (Siegler et

al., 2011). Before introducing fractions, students can improve their understanding of magnitude by practicing the value of whole numbers. In fact, whole number arithmetic skills, such as adding four plus five in first grade, are correlated to a strong understanding of fraction magnitude (Bailey, Siegler, & Geary, 2014; Booth & Siegler, 2008). The goal of student success in algebra starts with whole number proficiency, but rests upon strong fraction concepts.

In 2010, the Common Core State Standards for Math (CCSSM), a nation-wide set of standards, integrated the research-based practices to improve student understanding of fractions. The CCSSM begins with partitioning whole amounts and slowly adds concepts like comparing and operating on fractions. Although the CCSSM incorporates best practices regarding fraction instruction, we do not know how teachers and school administrators have implemented the CCSSM and what constraints still exist. My research elicited the ways in which teachers currently instruct fraction concepts in the third and fourth grades and the pedagogical decisions they make as they implement lessons.

Best Practices to Aid Student Understanding of Fractions

Integrated Theory of Numerical Development

A common way to teach fractions is by using the part-whole relationship of fractions. Focusing on these vocabulary terms, teachers would identify the numerator and denominator to focus on counting the parts of a whole. They might then draw a rectangle where three of the four parts are shaded. Teachers would then mention that the three is called the numerator and the four is called the denominator. Although this approach focuses on vocabulary, it affects the students' ability to realize that a fraction is one value, not two separate numbers. For example, students may believe that seven-fourteenths is larger than two-thirds because

both the seven and fourteen are larger whole numbers than two and three. When students continue to analyze a fraction by separating the numerator from the denominator, they apply a whole number bias.

When applying a whole number bias, or natural number bias, students overgeneralize the rules they have learned about whole numbers to rational numbers (Ni & Zhou, 2005). While whole number values state that multiplication increases the value while division decreases the value, fraction properties have the opposite outcomes (Bailey, Siegler, & Geary, 2014). For example, students might assume that $\frac{1}{2}$ times $\frac{1}{2}$ would result in a larger product, but it results in $\frac{1}{4}$ which has a smaller value than $\frac{1}{2}$. Integrated theory of numerical development posits that when students learn about fractions alongside whole numbers, they add to their understanding of real numbers (Siegler et al., 2011). Students expand their mathematical thinking by learning that there is an infinite amount of values between any two whole numbers allows students to expand their mathematical thinking. Integrated theory of numerical development asserts that in order to avoid these misconceptions, students need to be familiar with equal sharing to create fractional parts at an early age and to consistently integrate rational numbers throughout the curriculum.

Equal Sharing Context

Empson's research supports the use of equal sharing contexts (1999). Based on sociocognitive theory and the situative perspective, she argues that one way to help students make connections between their everyday lives and their fractions lessons at school is to use contexts and situations they have seen at home. These situations activate students' informal knowledge, or the knowledge that individuals bring on thinking and learning (Greeno &

Middle-School Math through Applications, 1997). One activity that students are often aware of is equal partitioning (Carpenter et al., 1993; Kouba, 1989). Students partition at home when they share an apple between four people or see someone cutting a cake into equal sized pieces. Using a context in which the students are already familiar allows them to use their knowledge of sharing to further understand the meaning of a fraction. Furthermore, the discussion surrounding an equal sharing story problem allows students to practice math discourse and reform thoughts as others share their problem-solving strategies. When teachers use story problems with the purpose of discussing equal sharing contexts, students are able to gain the understanding of the magnitude of a fraction or part, without losing sight of the whole entity.

Number Line to Teach Magnitude

Another way to combat the whole number bias is to use the number line when discussing fractions. By plotting ½ and ¼ on the number line, students will see that ½ is a greater than ¼ because it is closer to one whole. Research suggests that explicitly teaching fraction magnitude and plotting fractions on a number line in the third grade is correlated with student procedural knowledge of fractions (Bailey, 2012; Siegler et al., 2012; Fuchs et al., 2013). Fraction magnitude, or the value of the fraction seen as one number, allows students to compare values on a number line. Students are able to demonstrate the magnitude of a fraction by using the numerator and denominator of a fraction. For example, students can plot three-fourths on the number line by partitioning or dividing a line into four equal parts and counting over three parts. Once plotted, students can then compare three-fourths to whole numbers or other rational numbers.

Students who use the number line to model and prove the value of the fractions also have a concrete tool to demonstrate the value of improper fractions, another commonly misunderstood facet of fractions (Resnick et al., 2016). Teaching how to plot fractions onto the number line is a concrete strategy to then compare two values, or magnitudes. The strategy of using the number line to demonstrate fraction magnitude is a required skill for the Common Core State Standards for Mathematics.

Instead of utilizing the number line, Malone and Fuchs found that teachers most often address fraction magnitude through calculating cross products, a practice that does not incorporate conceptual knowledge (2017). As demonstrated by Malone and Fuchs, we see teachers continue to use strategies that impede student conceptual understanding of fractions during the Common Core era.

Gamoran (1994) and Stein et al. (1990) argue that there are three domains that affect what and how teachers determine instructional goals for their students: subject matter knowledge of teachers, pedagogical knowledge, and the teachers' knowledge of their students as learners. My study uncovered the decision-making process teachers use to determine what and how to introduce fractions.

Integrating Best Practices into the Common Core State Standards for Mathematics

The Common Core State Standards were the first nation-wide set of standards to be implemented in an effort to improve coherence among state standards, enhance rigor of content and understanding, and ensure that all students were prepared for college and career. With the input of all educational stakeholders, the National Governors Association and the Council Chief State School Officers developed the Common Core State Standards. Writers of the CCSS integrated the latest math research to ensure that the content allowed students to

develop a deep conceptual understanding of mathematics. Adopted by California in 2010, the CCSSM standards were created with the intent to improve student proficiency across the nation in math and ELA.

Besides narrowing the denominators, the Common Core State Standards for Mathematics integrated research supporting the importance of fraction magnitude taught on a number line. In addition, fourth grade students slowly add more denominators related to place value. Making an explicit connection between tenths and decimals or hundredths and per cents allows students to create a conceptual relationship between decimals and per cents.

Finally, the Common Core State Standards for Mathematics implemented integrated number theory to ensure students would see fractions more often throughout the academic year. Fraction concepts are part of numerous domains in each grade. For example, under the Measurement and Data Domain of the third-grade standards, students are required to plot values on a number line scaled by wholes, halves and fourths and measure to the nearest quarter inch. Also, in the Geometry Domain, third grade students must equally partition various shapes. Requiring the use of rational numbers in multiple domains will increase the opportunities students have with fractions thereby helping them realize fractions are not a separate subsection of mathematics, but vales that could be utilized in any of the domains.

The standards in fourth grade extend the concepts introduced in third grade by focusing on fraction equivalence, addition, subtraction and multiplication of fractions by a whole number. According to the CCSSM, fourth grade students should be exposed to visual models that represent equivalent fractions before being introduced to the general method for finding equivalent fractions. In addition, the CCSSM requires the practice of composing and decomposing fraction amounts to help students connect the properties of whole numbers and

fractions. By decomposing a number like three-fourths into $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ students not only see the addition that creates a larger sum, but also the repeated value being added together to make three groups of one-fourth. The CCSSM incorporated numerous studies that illuminated student misconceptions of fractions and encouraged more time developing conceptual understanding of fractions.

Creation of the California Assessment of Student Performance and Progress (CAASPP)

Soon after the CCSSM were rolled out, the California Department of Educated began testing students via the California Assessment of Student Performance and Progress (CAASPP). The CAASPP is an assessment created to analyze student achievement on the Common Core State Standards for Mathematics (CCSSM) and English Language Arts. Since 2015, students in the Golden State have taken the CAASSP, a computer-adaptive standardized test required for all 3rd through 8th and 11th graders. Proficiency, as defined by the creators of the CCSSM, is "the level at which a student is determined to be sufficiently educated at each grade level and upon graduation" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

Later that year, California state officials released the first year's results of the assessment 33.5% of students performed at or above grade level for math. Responding to this performance, in 2016, 70% of math teachers surveyed acknowledged that they needed to change their teaching practices in order to meet the demands of the Common Core State Standards (Swars & Chestnutt, 2016). On the CAASPP from 2017, students made a 4% gain in each subject. Although the standards have now integrated research-based practices with fractions, barriers with student conceptual understanding of fractions still exist.

Barriers to Effective Teaching of Fractions

New Policies Do Not Equate to a Change in Teaching Practices

Unfortunately, the creation of new mathematical policies fell short in the past as well. In 1990, David Cohen published a seminal work describing a case study of a teacher who claimed to be implementing the new California math framework. A teacher, Mrs. Oublier, claimed that she forced students to justify their answers and explain their solutions. However, upon observation, one noticed that Mrs. O. did not require students to extend their thinking and used many rote activities. Cohen's work demonstrated that a new policy was just one change necessary in education reform. Although policy changes create more challenging standards, teachers must understand and implement the rigor required to teach those standards. My study attempted to help identify how to help the elementary math teachers who are using the CCSSM so they could improve their students' conceptual understanding of fractions, thereby preventing a repeated case of Mrs. O.

Lack of Teacher Pedagogical and Content Knowledge

In a seminal study, Liping Ma's research (1999) suggested that elementary teachers from the United States require more content knowledge. In her study, Ma compared elementary teacher responses from the United States to that of Chinese teachers. Both were asked to answer elementary math questions, create story problems to match situations, and provide multiple representations of numbers. Her study revealed that although Chinese teachers receive less professional training, they have a stronger conceptual understanding of elementary mathematics. Students in China are taught to view each lesson as a vital part of a larger puzzle where all pieces develop a deep connection. They are taught to ask *how* and *why*. Ma argued that teachers draw upon their own elementary experience and as educators

continue to develop a conceptual understanding of mathematics through discourse with their colleagues (Ma, 1999). Due to the importance of childhood education experiences, my study draws upon teacher experiences in the classroom as a student as well as those as a professional educator.

Prior to Ma's work, Lee Shulman (1986) suggested that every teacher must not only know the subject matter content but know it so well that they could teach that content to another person. Coining the term *pedagogical content knowledge*, Shulman argued that the skills to help another individual learn something new is more than just the skills of teaching or the skills of math, but a mixture of both. Shulman stressed that we must bring back the importance of content knowledge as it is the foundation necessary to build pedagogical content knowledge. He explained that if one knows content well enough to teach it, they will use the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations and demonstrations (Shulman, 1986, p. 9). The teacher must also know curricular knowledge to help their students make connections, think critically, and explain their reasoning. Shulman stresses that teacher preparation programs often assume adults who have passed elementary mathematics do not need rigorous mathematics courses in how to teach those concepts. Although many years have passed since Shulman's work, one continues to notice few math curriculum and instruction courses in elementary teacher preparation programs. If elementary teachers do not have pedagogical content knowledge, their students will also lack opportunities to expand their mathematical thinking and the understanding of fractions will remain stagnant. My study will analyze how teachers use their pedagogical content knowledge to attend to their math curriculum.

Premature Use of Symbols and Representational Tools

According to National Research Council (2000), students who are introduced to symbolic representations without understanding the meaning of a fraction have difficulties using those symbols appropriately (p. 234). Symbols, part of the category of representational tools, are defined by Resnick, Bill, Lesgold, and Leer (1991) and Meira (1995) as any material or object used to enhance or aide in mathematical thinking. Because we use words like "one-fifth" to represent a magnitude, or value of a fraction, students must know the meaning of one-fifth before being expected to understand a written or numerical symbol. In particular, students must know that one-fifth means out of five equal sized parts, take one. If teachers do not take the time to ensure their students understand the meaning of a fraction before showing the symbolic representation, it could lead to difficulties as they progress.

Misaligned Curriculum

With the adoption of new state standards, new textbooks to teach the standards became available. In 2013, Choppin et al., found that 31% of educators do not find their textbooks are aligned to the Common Core Standards. In a study of 1,241 middle school math teachers across 42 states, only 61% of those teachers found the textbook materials supported student understanding (Davis et al., 2017). Corroborating these analyses, the Center for the Study of Curriculum identified that not one textbook was 100% aligned to the Common Core State Standards after analyzing 34 textbook series and 185 individual textbooks. Polikoff (2015) also analyzed seven textbooks and found that one-sixth to one-fourth of each were not aligned to the CCSS. Teachers do not have textbooks aligned to the Common Core State Standards.

Not only do teachers struggle to have general textbooks that address the standards, they also lack supplemental curriculum to reinforce skills and concepts. In Davis's study, 64% of teachers stated they use digital curriculum to create supplemental activities while 61% use the district-adopted materials (Davis et al., 2017). Unfortunately, piecing together curricula leads to repeated or missed curriculum. As MacCallum noted in 2012, disconnect within the curriculum can create confusion as students enter the next grade. When teachers cannot rely on school-adopted materials to teach the CCSSM, their students are at risk of gaining conceptual understanding.

Unfamiliarity with the Common Core State Standards for Mathematics

Another issue arises when teachers are so unfamiliar with the CCSSM that they are unable to judge the textbook's alignment to those standards. In 2016, a qualitative study of 73 urban elementary teachers, surveys and interviews revealed only 22% reported to be "very familiar" with the standards (Swars & Chestnutt, 2016). Hart Research and Associates found 50% were familiar in 2013 when interviewing 500 teachers from urban districts. Being uncertain of the meaning of standards impedes educators from effectively teaching the CCSS.

Lack of Discourse and Writing in the Math Classroom

Another obstacle teachers face is that the standards require written academic discourse throughout the curriculum and teachers are unfamiliar with writing in math. The importance of being able to communicate mathematical ideas has been encouraged by the National Council of Teachers of Mathematics since 1989. Empson's work also focused on the importance of social interaction while solving math problems. She posited that when others verbalize and interact with an idea, students learn to justify, argue and explain

differences in thought (1999). In addition, educators know that activities surrounding discussions about mathematical concepts have contributed to higher levels of achievement (Kosko, 2012; Mercer and Sams, 2006). Due to the CCSSM requirement of writing in math, various studies have now identified the current barriers when teachers try to incorporate language into their math classrooms.

A common misinterpretation of integrating written communication is simply assigning students to write out the steps needed to solve a problem. In contrast, Kosko & Gao (2017) point out that Common Core State Standards analyze the students' abilities to justify *why* a step was taken, not just write out *which* step was taken. Because mathematics has often been taught as an independent subject requiring the learner to use only numbers to explain, pre-service and veteran classroom teachers may struggle to create a collaborative language-based environment.

Besides helping students justify their thinking, teachers' questioning techniques further help students to verbalize their understandings. In an experimental study comparing two groups of educators, those who received additional professional development on engaging students in algebraic discourse and those who were traditionally trained, Jacobs et al. (2007) found that the teachers who participated in the extra training were better able to identify student understanding of concepts and elicit multiple strategies to solve critical thinking questions. These understandings then enabled the teachers to ask probing questions, such as, "What are some similarities between your solution and your partner's?" to help students reach a greater conceptual knowledge. Another study of middle school teachers revealed that educators do not connect communication with conceptual understanding (Davis

et al., 2014). Davis's study reveals that educators require professional development to strengthen their knowledge regarding the benefits and impact of academic discourse.

Teacher Preparation Programs Lack CCSSM Focus

Mirroring the concern of K-12 instructors, professors for teacher preparation programs proclaim to lack clarity of the CCSS as well. A study in 2015 revealed general education professors noted that they have not made many changes to their methods courses (Murphy & Marshall). This means even though the rigor of standards has greatly intensified, professors are not demonstrating how to help pre-service teachers reach these higher expectations. When Murphy and Marshall (2015) conducted focus groups with 22 pre-service teachers and asked if they were prepared to enter the classroom, many reported they felt like they needed to not only learn the CCSS themselves, but also anticipated the need to have to teach their prospective employers. Both general education and special education professors have not received adequate training to prepare pre-service teachers for the Common Core State Standards (Murphy & Marshall, 2015).

Conclusion

Although students have struggled to understand fractions for many years, California teachers are now following a set of math standards that integrated the latest researched-based teaching practices. By analyzing the experiences teachers have had since the introduction of the Common Core State Standards for Mathematics, my study illuminated the current third and fourth grade teacher understandings of fraction concepts and the ways in which introduce fraction concepts to their students.

CHAPTER THREE: METHODS

Introduction

In order to succeed in their future math courses, students need to have a strong conceptual understanding of fractions. Teacher decisions are vital to the success of their students as they provide opportunities to connect informal and formal knowledge thereby enhancing mathematical understanding. Previous studies suggest that what teachers know and believe about math is closely linked to their instructional decisions and actions (Bransford, 2000). I used a qualitative approach to learn more about how teachers build conceptual knowledge of fractions with third and fourth grade students and the pedagogical reasons behind those decisions. Participants engaged in a two-part interview where the first part focused on how they introduced fractions to their students, and the second part asked them to engage with a range of potential fraction activities as a way to both stimulate details of their reasoning and to provide them opportunities to think through approaches to teaching fractions they have not considered or considered and abandoned.

Research Questions

1. How do third and fourth grade teachers choose what and how to teach fractions in the context of the 2013 California Math Framework?
    a. Why do they make those pedagogical choices?
    b. What teaching and learning experiences have influenced third and fourth grade teachers' pedagogical choices when teaching fractions?
    c. What do third and fourth grade teachers perceive as challenges when teaching fractions?

Design Rationale

I used a qualitative design for this study. A qualitative approach allowed investigation into the details of participants' rationale behind their pedagogical decisions when teaching about fractions and get at the details of what they do and how they do it. By describing their decisions, teachers also made explicit their understanding of the math concepts and standards.

Participant teachers completed a short questionnaire and participated in a two-part interview. The questionnaire elicited information regarding the teachers' teaching experience, the curricula for math, school site information, and contact information. Part one of the interview was designed as a learning experience for both me, to gain insight into their current practices with fraction instruction. The first interview asked questions regarding the teachers' initial fraction activities, how they engaged students in learning fractions with those activities, how they decided to use those activities, the challenges they faced teaching fractions, and the resources they utilized.

Part two of the interview provided the opportunity to become familiar with and think through activities that the teachers might be able to use with their students to help build a strong conceptual understanding of fractions, but may not have considered or considered and abandoned. I described four fraction activities and then asked the participants what they did and did not appreciate, how they would adapt the activity, and if they would use the activity with their students. The four activities were: (1) strips of long paper that students would fold into halves, fourths, eighths, thirds, and sixths, (2) fair share story problem (3) choral counting by fractions, and (4) a picture book integrating fractions. These activities varied slightly for the third and fourth grade teachers. This part of the interview also allowed me to

understand the teachers' reasoning when deciding whether or not to implement a particular lesson or activity with their students and at the same time provided a learning opportunity for teachers as they considered what to do and why. Throughout this process, I looked for patterns among participants' responses in the ways they made sense of fraction activities and considered what to do and why to do it.

Data Collection Methods

Participants

The participants for my study were third and fourth grade teachers in the Los Angeles area. Participants needed to have taught fractions to third or fourth grade students previously in order to ensure the most complete answers to my interview questions, therefore, some first-year teachers, or teachers who previously taught other grades were omitted from this study. According to the 2013 California Math Framework, third grade is the first year that students are introduced to fraction concepts beyond equally partitioning halves, thirds, and fourths. During the third grade, students learn how to identify a fraction, analyze its value, and compare it to other fractions within the set of denominators of two, three, four, six, and eight. Third-grade teachers typically spend less time on fractions (and cover less fraction ideas) than teachers of older students and may have done little if any fraction work at the time of the interviews.

Fourth grade teachers have the responsibility of expanding the denominators that the students work with (fifths, tenths, twentieths, and hundredths). In addition, fourth grade standards introduce addition and subtraction of fractions with like denominators. Because third and fourth grades support learning the foundation to fraction concepts, these represent the best school years to do this study.

The participant selection process began by using the contact database created by UCLA's Center X which includes previous participants from their professional development initiatives, the Teacher Education Program (TEP) alumni, and the Principal Leadership Institute (PLI) alumni. This database provided access to teachers working in diverse school sites and varying teaching experiences. Including alumni of UCLA's TEP and PLI allowed the population of my study to access teachers who share the core principles of social justice and experience teaching in an urban school setting. Emailing school administrators who have graduated from UCLA's Principal Leadership Institute provided access to school sites that represent diverse populations of students with varying school policies, governing boards, and curricula. The Center X database provides access to a wide variety of potential participants which allows my data to shine light on a breadth of factors that teachers consider when making pedagogical choices surrounding fraction lessons.

After receiving a few participants, I then consulted the California School Dashboard website to locate LA County school websites, and from there, a staff directory of individual teacher email addresses. Colleagues also forwarded the recruitment email to third and fourth grade teachers. I then added a $15 Amazon gift card incentive for participants. After five months of recruitment, I connected with 20 teachers. Table 1 shows their responses to the short questionnaire which identified the number of years they have been teaching, their school-adopted curriculum, as well as how dependent they are on that curriculum. The school sites were kept confidential.

Table 3.1

Participant Pseudonyms, Teaching Experience, and School-Adopted Textbook Dependence

Teacher Pseudonym	Grade Taught	Years Teaching Experience	Years Experience Teaching 3rd or 4th Grade	Curriculum
Ms. Larsen	3rd	3 to 4	3 to 4	CGI, My Math
Ms. Lau	3rd	10+	9 to 10	CGI
Ms. Clark	3rd	1 to 2	1 to 2	CGI
Ms. Shore	3rd	10+	10+	My Math
Ms. Haskins	3rd	9 to 10	3 to 4	Engage New York, My Math
Ms. Lowe	3rd	10+	5 to 6	My Math
Ms. Tsui	4th	10+	10+	My Math
Ms. Jackson	3rd	10+	10+	Engage NY, My Math
Ms. Lee	4th	5 to 6	1 to 2	CGI, My Math
Ms. Lopez	4th	9 to 10	9 to 10	Engage NY, Bridges
Ms. Gonzalez	4th	10+	1 to 2	My Math
Ms. Ahmed	3rd	10+	3 to 4	Bridges
Mr. Allen	3rd	9 to 10	5 to 6	Bridges
Ms. Ortiz	4th	10+	10+	Bridges
Mr. Martin	4th	10+	3 to 4	Everyday Math
Mr. O'Brien	4th	10+	3 to 4	Pearson Realize
Ms. Baruch	3rd	5 to 6	3 to 4	Expressions
Ms. Levin	4th	1 to 2	1 to 2	Engage NY/Eureka
Mr. Jimenez	3rd	7 to 8	5 to 6	Engage NY/Eureka
Ms. Mancini	3rd	10+	5 to 6	Everyday Math

Management of My Role

Upon receiving interest from potential participants, I explained that I was a teacher myself, new to teaching third grade and had seen that fractions are a foundational concept introduced and expanded upon in third and fourth grades and therefore wanted to learn from each other how to best support students. I also informed the participants that I always had a strong interest in math and had witnessed the gaps in understanding fractions as a previous fifth and sixth grade teacher. I hoped to create buy-in and trust by explaining that this study was to better prepare upper elementary educators with a topic that many students find challenging. In the process, I helped them identify and discuss to what extent they understand the California Mathematics Framework and the fraction concepts within. My intention was to be open about my background and build a dialogue, so the participants felt free to be honest about their own lesson-planning process, priorities, challenges, and conceptions.

Procedures

Once the pool of participants was identified, I emailed each teacher to further explain the study, gain their consent, and set up dates and times for the interviews. The first interview was scheduled in December. The second interview was conducted within a few weeks of the first interview. Interviews continued through March. Interviews were recorded, transcribed, and coded for themes.

Measures

My study used a short questionnaire and two interviews.

Questionnaire

The questionnaire contained six questions. The questions asked how long they had been teaching, how long they had been teaching that grade level (third or fourth), their level of dependence on school-provided curriculum, their school-adopted math textbook, their school's name and their contact information. Teachers were asked to select a description of how often they utilize their school-adopted curriculum: (a) mostly school-adopted textbook, (b) use about 50% school-adopted textbook and 50% self-chosen or self-created, or (c) use mostly self-created. Using Qualtrics, I created a questionnaire that the prospective participant completed in about two minutes (see Appendix).

Interviews

Teachers participated in two 45-60-minute semi-structured interviews. Interviews allowed teachers to explain how and what they considered when teaching fractions. For most participants, both interviews were conducted over Zoom, an online video conference website, in order to ease the process of finding a time that fit the teachers' tight schedule and to enable video recording. Two teachers opted to meet in person for the interviews.

Interview Part One

The purpose of this interview was to discover the factors teachers consider when designing the introductory fraction lesson with their students. The decision to focus on the introductory lesson is that it provides insight into what they see as most critical to do to provide an entry point for students, the aspects of fractions that are supportive of future learning; it is one that teachers often remember and find easy to talk about, and this way all teachers will be talking concretely about a common aspect of teaching fractions.

Questions at the beginning of the interview were used to build a rapport with the participant. For example, I shared that this is only my third year of teaching third grade in

order to help them realize that I am not an expert in the expected skills and knowledge of my students, nor the pedagogy. I also asked them to share their favorite part about teaching third/fourth grade and in return shared that I love my students' motivation and love for learning. The teachers in my study needed to feel comfortable providing responses to the interview questions and not feel the need to give the "correct" answer. I needed to share information about myself to build trust and increase the credibility of the participant responses.

Afterward, teachers were asked a series of questions about their introductory fraction lesson and their decisions about that lesson. Participants were asked, "Could you describe your very first lesson with fractions?" and "If I observed this lesson, what would I see from beginning to end?" A series of prompts were used to ensure the teacher shared their decisions around the core constructs of (a) student- or teacher-led, (b) conceptual, procedural, or mixed approach to the mathematics, (c) use of familiar context for students, (d) differentiation, (e) approach to the mathematics, (f) symbolic vs. non-symbolic. These categories were used to analyze the data.

I also asked about the participants usage of the California Math Framework. For those who said they referred to the CA Math Framework, I asked if the teacher had used any of the fraction activities provided in the CA Math Framework. When teachers responded that they do not use the CA Math Framework, I moved on to the next question.

The third category of questions elicited challenges in teaching students about fractions. Specifically, I asked, "Which fractions concepts do you think are the most challenging for your students to grasp?" Asking the teacher to analyze the challenges they

faced required the teachers to describe student misconceptions as well as the teachers' own understanding of the concepts.

Interview Part Two

At the follow-up interview, participants were given a chance to analyze a set of pre-chosen activities to teach fractions and describe the reasons they would or would not implement any of four activities. Third and fourth grade teachers received a similar set of activities but with slightly different values to match their grade level content. The activities were chosen to show a broad range of approaches to teaching fractions, particularly ones that are consistent with the California State Mathematics Framework and the CCSSM.

The activities presented included a fair share story problem, choral counting (see Figure 3.1), fraction strip folding (See Figure 3.2), and a picture book (The Doorbell Rang) that introduced an increasing amount of people sharing the same amount of cookies. I first went over each of the four activities, one at a time, explaining the teacher and student actions. For each activity, I asked the participants to discuss whether or not they would consider implementing it, their rationale behind making these decisions, and the advantages and disadvantages they associate with each activity. After all activities had been explained, I asked which of the four activities they considered to be the most useful when introducing concepts and to justify their thinking. Their responses allowed me to understand teachers' decision-making processes when selecting instructional materials to teach fraction concepts.

0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$
6	$6\frac{1}{2}$	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$

Figure 3.1. An example of a choral count outline that teachers analyzed during Part 2 of the Interview.

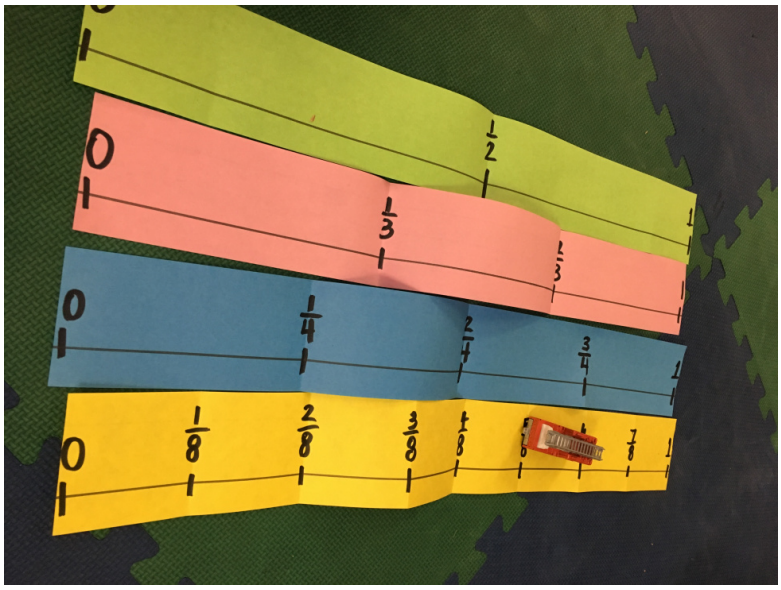

Figure 3.2. A photo of the fraction strips folding activity that teachers analyzed during Part 2 of the interview. (Smith, 2017).

Data Analysis Methods

Each interview was recorded using Zoom and then transcribed. First, I analyzed the questionnaire responses and created tables that described the participant demographics. Participants in this study ranged from second year teachers of third or fourth grade to those who had taught over ten years. Eight of the teachers taught fourth grade while the remaining

twelve taught third grade. Of the twenty teachers, four were males and 16 were females. They taught in various settings like large public school districts to small independent private schools. Their student demographics varied greatly ranging from 9-97% socioeconomically disadvantaged and 28-99% non-Caucasian.

Second, I analyzed the transcripts from both interviews for themes. Categories for both interviews were based on the factors that guided the teachers' pedagogical decisions. Coding was done on each interview separately and then compared to see if a change occurred from one interview to the next.

The first round of coding holistically categorized the teacher's current pedagogical decisions into the instructional approaches of conceptual, procedural, or a combination of both. The conceptual category includes teachers who are making decisions on the underlying understandings of fraction concepts. For example, teachers who began by asking how to equally share three cookies among two students are demonstrating a conceptual approach because they are allowing the students to see that we do not always get whole number quantities, and a fraction is how we represent those amounts. On the other hand, teachers who began lessons with a procedural approach focus more on the steps to solve a problem. For example, teachers might focus on shading in three-fourths of a circle by demonstrating that students first need to divide the circle into fourths and then shade in three of those parts. The steps would then be followed by the students. Finally, some teachers made pedagogical decisions that provided rationale that connects conceptual and procedural approaches and that goes in the "both" category. For example, a teacher who began with an equal sharing problem and then at the conclusion of the lesson reinforced that the numerator represents the number of parts needed or used and the denominator represents the number of parts that

make a whole. This first pass of coding helped me see how much of the decision-making process is related the extent to which they're driven more by one approach or the other.

The second round of coding looked deeper into the initial fraction lessons' description to account for the sub-categories of contextual vs. non-contextual, whole group teaching vs. differentiation, teacher- vs. student-led instruction, student thinking vs. mathematics, and symbolic vs. non-symbolic.

The contextual vs. non- contextual subcategories focused on the application of a problem. Teachers who purposefully chose to use a context in which the students already had informal knowledge were placed in the "contextual" category. On the contrary, teachers who presented a new math skill and then asked the students to apply that skill to a new context would be placed in the "non-contextual" category. When a teacher described a mixture of familiar and unfamiliar contexts, they were placed in the "both" category. The participants' description of how they chose or wrote the context of their math problems allowed me to analyze if they are helping students understand the math using a lens they can relate to outside the classroom.

The sub-categories of whole group instruction and differentiation allowed me to analyze the extent to which the teachers are addressing the needs of their students. Teachers using whole group instruction described teaching one lesson where all students are solving the same problem or doing the same activity. On the other hand, someone who differentiated decreased or increased the set of values in the moment, or had multiple activities going on at once to help each learner improve their understanding as individuals. Teachers who used half the class time to go over a whole group lesson and then half the class time to work with small groups or individuals, were placed in the "both" category. It was important to see how

teachers assess their students for understanding and make decisions in the moment to ensure that each student is increasing their conceptual understanding throughout the lesson.

The sub-categories of teacher- and student-led helped clarify when the teacher was using the students' questions, strategies, and solutions to guide the lesson. Teachers who made decisions to do a set of problems, lead the class discussions, show the strategies to solve a problem, or did not stray from their lesson plan may be assessed as a "teacher-led" lesson. Teachers who used the students' questions to guide the lesson's direction, used student work to help others refine their thinking, encouraged the students to work together, or posed questions to elicit student thinking were placed in the "student-led" category. Teachers who seemed to toggle between giving the students a chance to lead the direction of the lesson and taking the lead themselves were placed in the "both" category. When teachers enable students to participate in multiple ways, including discussion, asking questions to assess their comprehension, and posing possible solutions with their classmates, they develop a shared understanding which solidifies their conceptual understanding of fractions.

Another sub-category for the second pass of coding the data focused on student-thinking versus mathematics. For example, teachers who used their students' understanding to pace the lesson and plan activities that added onto or illuminated errors in their students' understanding were placed in the "student-thinking" category. On the other hand, teachers who paced their lessons or chose activities based on the school-adopted textbook, district requirements, or because that was the lesson/standard they needed to teach next were placed in the "mathematics" category. When a teacher used about half of the class time eliciting student thinking and then half the class time completing the lesson as written, they were placed in the "both" category.

Additionally, teachers were categorized based on a symbolic or non-symbolic approach. When teachers introduced fractions by writing them in numerical form on the board or writing equations including fractions as they began their instruction, they were adhering to a "symbolic" approach. In contrast, teachers who ensured that their students had a strong conceptual understanding before introducing the notations of fractions were utilizing a "non-symbolic" approach. Based on previous research, we know that introducing symbolic representations too early adds challenges to student understanding (Kilpatrick et al., 2001).

Finally, teachers were analyzed by similarities and differences within these categories. Three groups of teachers emerged: Group 1 contained the teachers who created their own curriculum and used student understanding to guide their instruction. Group 3 used the school adopted textbook and teacher-led lessons. Finally Group 2 were all the teachers who fell somewhere between groups 1 and 3, where their decisions and descriptions fell into both categories and they used a variety of instructional materials to teach their students.

Table 3.2. *Participant Groups*

Teacher Pseudonym	Curriculum Dependence	Student-Led vs. Teacher-Led	Group
Ms. Larsen	Creates/chooses	Student-led	Group 1
Ms. Lau	Creates/chooses	Student-led	Group 1
Ms. Clark	Creates/chooses	Student-led	Group 1
Ms. Shore	Creates/chooses	Teacher-led	Group 2
Ms. Haskins	Creates/chooses	mixed	Group 2
Ms. Lowe	Creates/chooses	mixed	Group 2
Ms. Tsui	Creates/chooses	Teacher-led	Group 2
Ms. Jackson	50/50	Teacher-led	Group 2
Ms. Lee	Creates/chooses	Teacher-led	Group 2
Ms. Lopez	50/50	Student-led	Group 2
Ms. Gonzalez	Creates/chooses	mixed	Group 2
Ms. Ahmed	Creates/chooses	Teacher-led	Group 2
Mr. Allen	50/50	Student-led	Group 2
Ms. Ortiz	50/50	Teacher-led	Group 2
Mr. Martin	Creates/chooses	Teacher-led	Group 2
Mr. O'Brien	textbook	mixed	Group 2
Ms. Baruch	Creates/chooses	mixed	Group 2
Ms. Levin	textbook	Teacher-led	Group 3
Mr. Jimenez	textbook	Teacher-led	Group 3
Ms. Mancini	textbook	Teacher-led	Group 3

Credibility/Trustworthiness

I addressed the credibility of my research in various ways. Prior to recruitment, I piloted the first interview protocol with three third grade teachers. This experienced allowed me to improve the wording of the questions and practice categorizing the data. After recording and transcribing their interview responses, I used direct quotes to capture factors that influence teacher understanding of fraction concepts or the standards. Another way I increased validity is by keeping my own field notes to help document interactions between myself and the participants including their facial expressions and body language.

Ethical Issues

I did not foresee any ethical issues with my study. I was not in contact with students and I used pseudonyms for all participants. All documents with the actual names were password protected and I will destroy the information three years after the study concludes. I detailed the purpose of the study for the participant and IRB to provide clarity and discussed any questions before beginning the interview process. I did not start the interview process until an informed consent form had been signed.

Conclusion

School administrators, teacher preparation programs, textbook companies, and professional development seminars could use the information gleaned from these interviews to front-load their training and seek ways to overcome fraction instruction challenges. They might also learn why teachers are resistant to implement certain activities or why students have trouble understanding particular fraction concepts. Finally, third and fourth grade teachers will be able to help each other regarding these challenges and provide advice to a larger population.

CHAPTER FOUR: FINDINGS

The findings in this chapter represent the broad differences in how the teachers discussed the decision-making process and challenges they face when implementing introduction fraction activities. In order to make sense of how teachers thought about the teaching of fractions, they were grouped based on how they described their introductory fraction lesson and their dependence on the school-adopted textbook. Three groups emerged: (1) teachers who supported *student-led thinking* to guide the instruction while designing their own lessons, (2) *teachers who led* their students through activities while using the school-adopted textbook, and (3) teachers who used a *mixture* of student-led and teacher activities while using the school-adopted textbook or activities they created or chose themselves. Group 1, student-led thinking group, had only three teachers with Ms. Larsen, Ms. Lau, and Ms. Clark. Group 3, teacher-led group, included Ms. Levin, Mr. Jimenez, and Ms. Mancini. The remaining 14 teachers were placed in group 2, mixed group, and represented decisions and curriculum that fell somewhere between those of groups 1 and 3. In comparing the responses of teachers in these three groups I found a number of important similarities as well as differences.

Similarities Across All Teachers

There were a number of similarities across the three groups of teachers. They included an openness to use the proposed fraction activities, taking into consideration their individual students and school dynamics, and demonstrating a concern for student success.

Openness to Using Proposed Activities

All teachers were open to using the suggested fraction activities with their students. Teachers expressed how they would use the proposed activities, reasons why they

appreciated the activities, and explained ways they could envision adapting the lessons to fit their students' needs. All teachers said they would use at least one activity, and all but four teachers said they would use every activity.

Fair Share Story Problem

All teachers said they would use was the fair share story problem. Teachers appreciated the way fair share activities drew on familiar contexts, provided experiences that would anchor their students' understanding of fraction sizes, and presented opportunities that could be easily differentiated. Ms. Levin, who fell into the teacher-led group, appreciated the way fair share story problems allowed students to see the connections between fractions and division. Meanwhile, Ms. Larsen, who was grouped with the student-led teachers, valued the way that fair share story problems brought in a familiar experience of sharing food. Mr. Jimenez made a similar comment as Ms. Larsen and acknowledged that this situation could come up at recess. Mr. Allen, Ms. Clark, Ms. Lau, Mr. Jimenez all stressed the importance of providing different number sets for the students to choose which one best fit their individual fraction understanding.

Implementation Considerations

Teachers mentioned that they liked that they could reuse the suggested activities with different numbers or easily extend the activities. While some teachers focused on inserting students' names and topics of interest into an activity to elicit engagement, others described ways to revisit or extend the activity. For example, Ms. Lowe, Ms. Tsui, Ms. Ahmed, Mr. Allen, and Ms. Ortiz all emphasized the importance of starting a choral count or fair share story problem by highlighting halves and then extending to fourths and eighths to help the students connect multiplication and division patterns. Ms. Shore suggested using choral

counts to not only help students develop and hear the language of fractions, but also to familiarize them with counting backwards by fractions, thereby addressing some subtraction conceptions. Ms. Lee, Ms. Larsen, Ms. Lowe, Ms. Lopez, and Mr. Jimenez all proposed the idea of having the kids write stories or story problems, which would allow for the freedom to create their own context while developing and understanding of the fraction concepts. Ms. Lopez discussed using Gator Pie not only to introduce fraction concepts to her fourth-grade students but also as a theme for their celebration of knowledge:

> I would also like to reread this book. We do a math celebration at the end of the unit and like everyone bring in pies or like you know, some food item and like actually you know, who wants this kind of pie, who wants this kind of pie? And then each kid has to divide their own pie like into equal slices. I think that would be really fun and real life. And yeah, I feel like pie is like something pie and pizza are like two things you actually do cut up evenly. So that makes sense. (Lopez)

Mr. Jimenez talked about using the fraction strip activity as more of an inquiry-based lesson where students would be asked to share what they noticed when looking at two different strips. Teachers appreciated the versatility of the proposed fraction activities.

Teachers went so far as to state exactly when they would use the activities. Ms. Shore said, "I'm already in my mind thinking 'Oh that's a good warm up for tomorrow,'" when referring to the fraction strips that you fold. Ms. Jackson also liked this activity and envisioned using it the following Monday.

Other teachers expressed urgency over buying the materials necessary to use that activity. Mr. O'Brien stated, "I wrote down the name of the title because I'm going to buy that book and use it in my classroom." Ms. Baruch also expressed excitement when analyzing the fraction strips, "I wish I could do this in my classroom with the students right now. I think this would actually be a really, really cool thing for them to have."

Decisions Were Based on School Culture and Dynamics

Another similarity existing between all teachers in the study was that they analyzed the activities in relation to their own school culture and dynamics. Teacher decisions were based on their school's common assessments, their colleagues' implementation of activities, and their student demographics.

Each teacher's available time and their school's use of common assessments influenced teacher decisions to implement activities. Ms. Gonzalez spoke to the pressure of state testing and the need to move on even when not all students have grasped the concept. Two teachers mentioned their school was part of a study which required students to take additional assessments and how those tests impacted their instruction. These extra assessments took instructional time and were given in isolation. Two different teachers mentioned the use of Interim Assessments from the CAASPP website and how they were used as common assessments.

Teachers valued coherence with their colleagues. Mr. Jimenez expressed the need to use the same activities as his fellow teachers and the importance of coherence with the school-adopted curriculum across grade levels. Ms. Lopez emphasized the unity that comes with doing the same thing as the other teachers in your grade level and the connotations of being the only teacher in a grade level doing something different. Mr. Martin praised his colleagues revealing they collaborated recently to develop a fraction unit. Ms. Clark discussed sharing all activities with her colleagues in a Google Drive so they can divide and conquer and improve lessons as a team.

Specifically Addressed Their Students' Needs

Teachers analyzed the activities by putting their students' individual needs at the forefront. Six teachers mentioned adapting the lessons to best suit English Language Learners, students with special needs, or those who entered with math skills below grade level. Teachers acknowledged the importance of unpacking, or helping students comprehend a word problem, before asking them to solve it. To help her students better understand the changing situations in The Doorbell Rang, Ms. Larsen suggested breaking up reading the book into multiple days. She explained:

> I'm worried, especially with my English language learners. The concept of the cookies staying the same and people coming, I feel like since they're not used to that kind of problem that they might struggle with that. And I can see if the other kids in my class kind of being like, "Wait, there's more kids?" And kind of grappling with that might be tough for them, like the changing numbers.

Ms. Larsen recognized the value of the book and how it would help her students better grasp fraction concepts, while also realizing she would need to help her students access the action of what was happening with the kids and cookies. She envisioned a set of lessons implementing The Doorbell Rang:

> First day we're working with exclusively wholes. Second day we're working with fractions. I don't know if it would be on the same day or the next day, I'd give them the opportunity or maybe we could talk through a few potential new numbers to kind of give them some ideas of what they could use and then give them some independent time to maybe write two to three and then get together with a partner and have them like switch papers or journals or whatever and solve through. And then we could talk about, we could have a whole class conversation about what was that like for you? What surprised you or like what did you notice when you were solving your partner's numbers? Maybe we could talk about like what numbers were easier for you, which numbers were more difficult for you to solve? Something like that.

Additionally, Ms. Tsui suggested ways to help her English Language Learners understand the various fraction activities and offered using a communication guide, a set of vocabulary, or sentence frames to assist her students in communicating their knowledge.

Other teachers analyzed and offered ways that they would meet the needs of their students to participate and demonstrate their knowledge. Teachers focused on ways to offer manipulatives like circle or rectangle paper cut outs that the students could use to represent cookies. Ms. Clark described how the hands-on materials allowed students to act out the situation and represent it in a drawing:

> So usually it's just a matter of taking a moment on the rug as a small group or just, you know, that student in myself and just unpack it further. So that's when I'll usually just bring in those circles and be like, okay, well let's just pretend this is happening right now. Let's pretend I'm your friend and where, you know, it's after school, we're sharing a snack, we have three cookies, what could we do with these? And so, with hesitant students, that's just like, okay, we put the paper, we put the word problem in aside, and then just talk about if this were in a real-life setting, how would we approach this? And then usually from there, then they kind of get an idea and then they go from like, okay, so and understood what we did. Now try to do that same thing on your paper. So, it's just usually it's just taking an extra second to unpack the problem further or to bring in those tools.

Second, teachers offered solutions to that perfectly folded strip of thirds and sixths to include small marks on which to fold. Third, teachers addressed the range of fraction conceptions in their students and the need to have multiple number sets.

Demonstrated Concern for Student Success

Finally, all teachers were concerned for their students' math success, although in different ways. Teachers expressed implementing activities that reached all learners and allowed for an entry point for their students. Teachers brought up the need for students to be able to access the lesson and lower the affective filter when addressing mathematics. These teachers wanted their students to feel emotionally safe and successful with the fraction activities and recognized the stigma some students carry with math, specifically fractions. Furthermore, 12 teachers in this study emphasized the importance of using activities that

demonstrated fractions in a familiar context. Most importantly, all of the teachers valued the importance of activities that allowed for all students to participate, whether it was through a strategy of their own, a choral count where they could hear and learn from others, or allowing various number sets to provide for a challenging math problem.

Teachers even held themselves accountable for their students learning stressing that they seek opportunities to improve their math instruction and demonstrated a history of actively seeking resources. Eight teachers had previously bought Extending Children's Mathematics: Fractions and Decimals: Innovations in Cognitively Guided Instruction by Susan B. Empson and Linda Levi as a key reference for introducing fractions to their students. Seven teachers curated a list of activities from various curricula they had used in the past or could find online.

Teachers in this study were aware of their students' challenges to understanding fractions concepts and sought resources for assistance. Twelve of the teachers identified fraction magnitude, or not understanding the value of a fraction or how the numerator and denominator create one value. Half of the teachers discussed looking online for activities that specialized in their students' misconceptions. Seven teachers worked with their math coaches, while others participated in professional development in order to improve their fraction instruction. Ms. Tsui praised her math coach for introducing her to activities that helped the students understand fractions and convincing her to buy another math book.

Five teachers were critical of their introductory fraction lesson they had described in the first part of the interview. Ms. Lee and Ms. Lopez both lamented that they wish they had done a midpoint recap or check for understanding so they gained a better analysis of their students' understanding. Ms. Gonzalez mentioned her personal goal was to see more of what

the students can do on their own and to decrease the amount of direction instruction. Ms. Ahmed was so open to changing her fraction instruction that she went online and started researching fraction instruction after the first part of our interview. Ms. Levin wished she had incorporated more activities surrounding the fraction language like morning meeting games, target English Language Development instruction as well as art projects.

All 20 teachers of this study were open to trying new fraction activities with their students, demonstrated a concern for student success, and analyzed the activity using the lens of their school culture and dynamics.

Differences in Teacher- and Student-Led Groups

The teacher-led group and student-led group represent the two ends of the continuum related to the teaching and learning of fractions. The teachers in the teacher-led group primarily used their school-adopted textbook to guide the instruction and described a teacher-led fraction introduction lesson. Ms. Levin, Mr. Jimenez, and Ms. Mancini made up the teacher-led group. In contrast, the teachers in the student-led group introduced fractions by discussing images and story problems that elicited student thinking to guide their instruction. Ms. Larsen, Ms. Lau, and Ms. Clark made up this latter group. Other differences include how they viewed the school-adopted textbooks, their lesson-planning process, and how they viewed student responsibilities of learning.

Describing the First Fraction Lesson

Recall and Repetition

One characteristic of the teacher-led group was that they described going through the tasks set forth by their school-adopted textbooks. Each teacher in this group referred to the set of activities as written by their textbook and then described how their students responded.

Ms. Mancini first helped her students recall what they had learned from second grade by drawing a shaded area model and asking prompting questions. She modeled how to write a fraction and identified the numerator and denominator. Afterward, she led a few examples of how to complete the problems from their student workbook and then allowed time to finish the workbook problems independently.

Both Ms. Levin and Mr. Jimenez focused on the structure of their lesson rather than the learning activity or how the students participated. They described how it started with a fluency activity, then shifted from concrete to abstract, and ended in a problem set and debrief. Upon further probing they described the learning activity with the lesson plan in hand. They ended the lesson because they went through the curriculum's activities, not because all students understood. When asked why she used this lesson to introduce fractions, Mr. Jimenez replied, "Cause it's in the book."

Connections with Math Concepts

On the contrary, the student-led group allowed for more connection between the math ideas and saw how students related that knowledge. Ms. Larsen explained how she unpacked a fair share problem with the students, so her students understood the context before attempting to solve the problem on their own. She emphasized that the context of the story problem needed to be familiar to her students so they could recall situations where they were sharing. This familiar situation allowed them to be successful when solving their first fraction problem of the year while also anchoring their understanding everyone must receive the same amount. She states:

> It's a context that kids have experienced with, like sharing cookies, or something like that…and then I feel like it's easy for them to understand the importance of equal

> halves. You don't want your sister to get a bigger piece than you do, so we've got to make sure we split it right down the middle so everyone's getting the same amount.

After about fifteen minutes of independent problem solving, she paired them with a partner, one she had chosen, so they could both practice verbalizing their strategies before the whole class share out. She then had a couple of students explain their strategies to their classmates. As they explained some students opted to write their math strategy on the poster themselves, while others verbally explained to her as she wrote out their solution on the poster. Ms. Larsen explained, "I am notating what they're telling me, so they're orally telling me…their thinking and their justification I'll write down." Student responses determined every action Ms. Larsen took.

Ms. Clark and Ms. Lau described very similar introductory fraction lessons. Ms. Clark began her lesson with a warmup where she projected an image of toast cut into equal pieces in multiple ways and asked students what they noticed while listening for the language they used. Ms. Clark in describing her lesson noted that she asked students questions to help them better understand the problem, share their strategies with a partner and intentionally chose students to share with the whole class. Ms. Lau did a nearly identical lesson as Ms. Clark; however, she emphasized the power of student work samples to discuss student conceptions. While walking around the room, Ms. Lau intentionally looked for students' work that demonstrated a strategy that could help the particular students or the class as a whole better understand fractional concepts.

> I might try and find different ways that they drew, like a couple different ways that drew. And if I'm feeling like there's some stronger fractional understanding in the class, I might also try and find somebody who solved it just numerically. (Lau)

All three of these student-led group teachers used student knowledge to guide their instruction. They made no assumptions of what the students already knew and used their students' work and vocabulary to plan future lessons. Unique to this group of teachers, they did not use the numerical or symbolic form of fractions in their introductory lessons and instead focused on the spoken language and word forms.

Textbook Perceptions

Trust in the School-Adopted Textbook

Another difference between these two groups was the level of faith teachers put in their school-adopted textbook. The teacher-led group referred to their textbook with high regard following all components of the lesson. Ms. Levin appreciated the ability of vertical articulation of the curriculum and expressed:

> I think that this curriculum has most of the elements that I think that the students need, and it's already compiled. It does save a little time and having to create my own curriculum… Eureka does a pretty good job connecting previous years with the present year, connecting fluency and conceptual understanding and concrete with the pictorial, like they do a really good job of it. (Levin)

Mr. Jimenez agreed with Ms. Levin, stating:

> I've taught it for five, six years, just seeing the growth every year, every school year I do feel like the group that I'm inheriting has a better understanding of math concepts. And I assume that's because kinder, first, second are using the same curriculum, so there's more consistency there. (Jimenez)

When asked why Ms. Mancini uses this as her first lesson, she replied, "It's the first lesson here and I think it works. I happen to be a big fan of this program." All three of these teachers are satisfied with the textbook's curriculum.

Textbooks Viewed as a Resource

Meanwhile, the student-led teachers discussed the collaboration and creation of lessons to best suit the needs of their students. All three of the teachers in this group do not use a textbook with their students. By doing so, they were able to spend more time on lessons that would directly address their students' understandings and draw upon their background knowledge and experiences.

Ms. Clark, Ms. Lau, and Ms. Larsen all discussed the textbook minimally. Ms. Clarke discussed how instead of using a textbook, her grade level team collaborates to create problems specifically designed to help their students.

> It's definitely a shared responsibility. Whenever we create a problem, then we share it amongst the four classes. And then just modifying them, changing the context slightly to fit the interests of my kids or changing the number set slightly based on what I was seeing in my classroom . . . So, it's very collaborative. I don't usually feel like I have to create completely on my own. And if I ever do, it's just because it's like something that I feel like my class really needs and then I still share it with the rest of my team. (Clarke)

Ms. Larsen answered that she used the textbook as a reference occasionally or to leave work for a sub. "I don't really ever open it. Sometimes if I'm having a hard time writing problems, I'll go in there just to get ideas for what else I can possibly write . . . or if there's a sub." Instead, she, like Ms. Clark, works very closely with her colleagues to backwards plan the year's units and lessons. Ms. Larsen described how she embeds a time within her lessons to make last minute changes based on student understanding:

> Generally, I plan out the word problems over the weekend so I can get them typed and printed and all of that. But I leave room for flexibility in my number sense routine. So, if I notice something that's happening while they're problem solving the day before, I'll find a way to incorporate it into the number sense routine for the next day. (Larsen)

Leaving room in her number sense routine, something she does daily with her students, allows her to have the time to directly address student conceptions whether they are

completely correct or not, or introduce concepts when the group is demonstrates sufficient understanding.

Ms. Lau told of a similar lesson-designing cycle working with her partner teacher and referencing the CCSSM. All three teachers in this group valued their own expertise and noticed that they could more effectively meet their students' needs by creating their own curriculum.

Teachers' Desired Student Takeaways

A third difference between these two groups of teachers was *what* they wanted their students to understand. The teacher-led group focused on the students' understanding the problems in the workbook. Meanwhile those in the student-led group focused on conceptual understanding.

During the first part of the interview, all participants were asked what they wanted the students to take away from their introductory lesson. Teachers in the student-led group responded with ways to help the students recall what they already knew about fractions and emphasize that they could be successful when solving fraction problems. Ms. Larsen expressed, "Just to expose them to fractions and want to make it familiar for them because I feel like the word 'fractions' freaks kids out and it can definitely be a daunting concept." Ms. Clark replied, "I want them to understand that they can, that whole objects can be broken up into smaller pieces to share equally." And Ms. Lau answered, "That everybody has some knowledge of fractions. Everybody has shared food." They all met the students where they were academically and used students' responses like an informal assessment.

Meanwhile, the teacher-led group focused on numerous ambitious math objectives. Ms. Levin listed:

> That a fraction is part of a whole, it's equal parts of a whole. I want them to know that fractions can be represented in multiple ways. I would want them to know that whenever they see the numerator as one and the denominator as a number, that that's a unit fraction and that …if the numerator is less than the denominator then it is proper, while if the numerator is greater than the denominator it is an improper fraction, which they will soon find can be renamed. (Levin)

Mr. Jimenez was not as specific as Ms. Levin and mentioned a broader goal of foundational understanding while referencing the worksheet he gave:

> I'm looking at their worksheet and one image has half of a cup fill the name that this is (one-half). It's not in numerical [form] yet. It's just unit form and it looks like they even have to be able to partition shapes being able to split. We use these bar models to diagram our math work and it looks like they have to be able to show what one half looks like using a bar model of one-third or one-fourth. (Jimenez)

Ms. Mancini also gave an overview of what she wanted her students to learn from this initial activity with fractions, stating, "Just a really solid, just bringing back their prior knowledge sort of revealing for themselves what they had learned before and now look to ways which we're going to apply this, that they've already learned to something new." The teacher-led group described general student objectives, while the student-led group described specific attainable goals.

Teacher Views of Student Responsibilities

Finally, student responsibilities looked different between these two groups of teachers. In the student-led group, the teachers talked about their students' responsibilities to take ownership over their learning. Ms. Larsen explains:

> We unpack the problems so that they can all understand context, and then I expect them to get started on solving the problem independently. They're expected to represent their thinking on their paper and write an explanation for how they solved a problem. (Larsen)

She continued to say that "At this point in the year I expect them to advocate for themselves if they know they need support getting started." Ms. Lau explained a very similar answer:

> They're expected to raise their hands, to be part of the conversation and not to dominate the conversation, need to solve the problem in at least two ways… if somebody asks them a question, they can't just ignore them…they need to be able to discuss or describe what they've worked on. (Lau)

The teacher-led group, on the other hand, discussed sitting with students until they were able to do it on their own. Ms. Mancini described:

> Once they got to something that was a little more difficult then I let them work with a partner. I wouldn't let them work with a partner until I was pretty sure that they had at least that beginning understanding of what a fraction was pretty solidly. (Mancini)

Ms. Levin and Mr. Jimenez defined student responsibilities as completing the workbook problems. While the teacher-led group described student responsibilities as finishing the practice problems the student-led group talked about the participatory actions that allowed the students to understand the math concepts.

Group 2 Similarities and Differences

Teachers who fell into this middle group had varied responses that did not fit completely in the teacher-led group nor the student-led group. These 14 teachers varied in responses about what guided their instruction, how they used their textbook, how they spoke of their learning objective for the introductory fraction lesson, and how they viewed student responsibilities. It is notable that there are 14 teachers in this group. It shows the range of different responses across teachers and how many of the teachers were managing in different ways teacher and student led, textbook driven and student thinking approaches.

What Guided Their Instruction

Teachers within the middle group expressed a spectrum of decisions ranging from teaching for student understanding to teaching to get through the curriculum. In terms of what guided their instruction, six teachers used student thinking to guide instruction, three allowed the stress of teaching the standards to impact their instruction, and five teachers fell somewhere in the middle. I discuss each group's statements and how these descriptions led me to determine placement within these categories.

Using Student Knowledge to Guide Instruction

Ms. Shore, Ms. Haskins, Ms. Lowe, Ms. Tsui, Ms. Lopez, and Mr. Allen all allowed student thinking to guide their instruction. They utilized their introductory lesson as a gauge into their students' understanding of fractions, much like the student-led group. Although these teachers differed in how they used the textbook, they valued student input when deciding when to move forward in the lesson, allowed for students to demonstrate their understanding in a variety of ways, and did their best to ignore the pressures of testing or finishing the curriculum. Simply put, they prioritized their students' understanding.

Ms. Haskins described how she had recently become more familiar with Cognitively Guided Instruction strategies and now allows her students to represent their thinking in a variety of ways. "With my CGI lens now I'm really trying to leave things more open-ended. If I can help students work within a concept without forcing their thinking in a certain strategy or a certain way," Ms. Haskins said. This came after she had just described her previous year's introductory lesson with fractions and how she had not allowed the students to show what they knew on their own. Ms. Tsui and Ms. Shore emphasized the importance

of making sure the students understand before moving on as well as knowing when to pull a small strategy group.

Balancing Student Understanding with Pressures of Assessment

The teachers who found themselves trying to balance using student knowledge to guide instruction while covering the whole math curriculum were Ms. Gonzalez, Mr. O'Brien and Ms. Baruch. Unlike Ms. Tsui's firm belief that we must ignore the pressures of state testing, Ms. Gonzalez admitted:

> I feel like there's a pace but it's self a pace that is fast, but it's self-imposed. Okay. Because I want to cover and when I say cover, I don't mean like deliver, move on, deliver, move on. I mean like I want to give them opportunities in each of the standards. In each of the topics we're learning each of the math domains, but I really to do all of them, need to work much faster because testing is, so early, like I don't have three trimesters to work. (Gonzalez)

Ms. Gonzalez's struggle to meet the demands of state testing while also ensuring her students' success was also addressed by other teachers in this group. Mr. O'Brien was stuck between his own values and the way that the school had set up students' classes. When discussing his fraction lesson, he explained how he used the introductory lesson as a formative assessment. He said:

> Well I guess the first way to address needs is to find out what they know first. And so that's, I structured it the way I have when they work with the whiteboards. That's a quick assessment. When they're working with their partners, I get to see how they play and talk about fractions with their partner. And that's another way for me to figure out what they know. (O'Brien)

Mr. O'Brien represents a teacher who is aware of the power of using student understanding to guide instruction and occasionally uses it but is not yet using it in all situations.

Standard-Guided Instruction

The last group of teachers, which included Ms. Jackson, Ms. Lee, Ms. Ahmed, Ms. Ortiz, and Mr. Martin acknowledged formative assessments as important to gauge student understanding, but ultimately focused on the standards to guide their instruction.

Ms. Ahmed shared conversations between her and her colleagues that focused on interpreting the standards:

> …We try to interpret it. Doesn't the standard ask for X, Y, and Z? And they'll say, no, it just asks for these things. So, then I'm thinking, well, I was just killing myself to get them to get to Z and they don't have to get to Z! And so, then I'm going to focus more on X and Y instead. Or I might say, 'Oh! Well, I didn't even realize they had to get to that!' So now I need to, you know, go back and, and work on that. (Ahmed)

Ms. Lee reflected on places where she wished she would have allowed for more student understanding in her initial fraction lesson and how she had succumbed to the pressure of getting through the curriculum:

> So, I wrote like, what are fractions or what are pieces that are the same size? And then I think they kind of struggled with that. I think I had two kids that maybe got what I meant by that initially. And so, I decided to prompt them. Well, what's the same as one-fourth? What's the same as one-half, and then they start to manipulate that foldable. Yeah. So that's all I wrote… I think they, I think I, I should've just kind of stopped that day with like, what do you notice? I think that I think the pressures that I was feeling, I tried to push them on, but I think it would have been good if, or better if I just stopped and focused on what do you notice about this? And just recording their answers and stopping there for that day. (Lee)

Mr. Martin described a direct instruction lesson to introduce fractions which came straight from his school's adopted textbook:

> I think the first day after really at about a 10-minute intro, define a couple of words, we really handed out, we did a worksheet about just naming fractions. So, having shaded pictures and I'm just naming the fraction. So basically, what part of the fraction is shaded? And really the first lesson was really to gauge who, what they kind of the basics of what they knew. Our school has used everyday math . . . First lesson we're really just focused on naming shaded parts and fraction . . . (Martin)

Textbook Perspectives

Teachers in this group viewed the textbook as a resource they would use sparingly with the exception of two teachers. Nine teachers of this group did not use the school-adopted textbook with their students, while three occasionally used it, and two primarily used it. For the majority (12) of teachers who used the textbook sparingly, they offered reasons they like and dislike their provided curriculum.

Textbook Lesson Plans and Problems are Confusing

Within this group, the teachers critiqued the worksheet nature of the school adopted textbook as well as the confusing nature of the tasks. Ms. Baruch mentioned occasionally using problems from her school's textbook but explained that she only used it if she and her colleagues felt that their students would understand the way it was presented. Ms. Tsui disliked her school's textbook because she preferred to use familiar contexts and projects with her students instead. She argued the contexts in the story problems were often unfamiliar to the students:

> So, most of the textbooks or the, My Math, that we use, is for children who generally have already a deep understanding. It's very worksheet related. The problems are not really in context. …I really need them to connect and have relationships with the learning. I don't want it just to be math time or writing time or speaking time. (Tsui)

Ms. Tsui complained that her textbook did not meet her students' needs.

Ms. Haskins and Ms. Ahmed offered a similar analysis of their school's adopted textbook but pointed to the confusing and busy nature of some of the learning activities. While Ms. Haskins focused on the actual wording or layout of the activities, Ms. Ahmed discussed how the lack of usage in previous grades made the learning activities too arduous to explain:

> I also don't want to waste time on something that I don't feel is necessary or that might confuse them. I think some of the resources that we have and the worksheets that we have just serve to confuse students. Or maybe there's too much on the page, so sometimes I'll omit things. (Haskins)

> And if the grades beneath me aren't using it, then the kids aren't familiar with it and I can't teach how to be familiar with the curriculum and the curriculum and there's just no way. So basically district, you know, conveyed that we need to follow the standards. So, I follow the standards. (Ahmed)

Ms. Gonzalez highlighted the confusing nature of the textbook to the point where at times she is unsure of the actual standard practiced. At first, she argued that the textbook was too prescriptive and limited the students' ability to think mathematically by requiring or only showing certain strategies. Later, she described how she found it easier to create her own problems rather than "mine" the ones that make sense and help students grapple with the math concepts.

> I find it like, it's just an exercise in frustration for me cause I'm like looking in the chapter and then I'm like, where is this in the standard? Because then I'm checking standard books. So, my happiest way to plan is to put my students in the forefront of my mind. Then I think of the standard that I want or that I'm covering. Then I think, okay, what mathematical practices do I want to see this week? (Gonzalez)

Ms. Lowe critiqued the heavy emphasis placed on worksheets. She defended her decision not to use the textbook by stating that students are not making sense of the textbook's learning activities:

> I mean algorithms are great and sometimes they're faster, but I want them to know what it is first. I think that's what elementary school is really for. So, they get the concepts and then they can learn all these different strategies for solving. But our math curriculum sucks so much… They don't, they can't determine reasonableness of an answer. Their estimation skills are completely non-existent. So, it's, I feel like it's just really old fashioned, really not current. (Lowe)

Textbooks Viewed as a Resource

Ms. Shore and Mr. Martin discussed how their previous experience with textbooks enabled them to feel more comfortable choosing and creating their own curriculum:

> Well for a long time I used [the school-adopted math curriculum] exclusively, well I wouldn't say exclusively, but mostly just what the school offered. Then I would have to supplement with things when they didn't get whatever was in the textbook. From that, I saw a lot of different lessons . . . Last year I decided that I was going to try not to use the school [adopted textbook] . . . I'm always online looking for some kind of fraction something or other depending on what my students understand and what they don't. So, it could be just something I found on Teachers Pay Teachers or some book I have or could be something out of the textbook. Old textbooks from years back, I like to keep the TEs [teacher-edition textbooks] just so I have activities and ideas. So, whatever I think will help them, I go looking for it, and that's what I'll use. (Shore)

Through Ms. Shore's explanation, she mentioned numerous resources she consults and her openness to try multiple activities to increase student understanding. Mr. Martin discussed using the textbook more of a guide or resource, while his units are still based off the school-adopted textbook.

Ease of Implementing the Textbook

Teachers in this group did not see their school-adopted textbook as an accumulation of the best teaching activities, but rather as a convenient resource they would pick and choose from. Three of the teachers in this group discussed the ease of using the school-adopted curriculum. One of those teachers, Ms. Lee, discussed only using the textbook for an occasional homework assignment, while Ms. Jackson and Mr. O'Brien liked how the curriculum provided differentiation and sufficient activities for the students, and use it often. Ms. Jackson honestly described her reason for using the textbook was due to the ease of implementation:

> Quite frankly, I don't have to create stuff and the homework's right there. So, if it meets my needs, I use it. We like Engage New York for the number line part of fractions because they do a better job. Engage does than My Math of teaching that

> number line part and there's more of it…So really it's just kind of how quick the kids are picking it up, how good the curriculum is, how lazy I am. Really because I don't always want to be copying, copying. (Jackson)

Mr. O'Brien, on the other hand, noted an alignment between his school-adopted textbook and CGI strategies, something he wanted to incorporate.

> So, we use Pearson Realize, and I think on the whole, I'm a fan of the way that the curriculum is structured. The way that Pearson would have you go through a lesson is that they give you, is they give the students what they call a solve and share. It is, it is a problem. And they just put it out there and they see what the kids can do. So are, are you familiar with CGI?. . . So it's like that in the sense that at the beginning of the lesson, they don't want the teacher to just start by, here's what you're going to learn today and now I'm going to show it's more of like, Hey, here's an interesting question, see if you can solve it. And then again, like CGI, they want you to ask the students to share their strategies with each other… (O'Brien)

Mr. Allen, Ms. Ortiz and Ms. Lopez used the problems from the textbook when the students had a strong conceptual understanding of the math lesson. Ms. Lopez specifically focused on the problems from the textbook that engaged her students in critical thinking. Meanwhile, both Mr. Allen and Ms. Ortiz explained that they used the school's textbooks in addition to fair share and multiple group story problems. While critiquing the school textbook, Ms. Ortiz shared how she determined to implement parts of the curriculum:

> Well, I use Bridges, but I don't like the way they laid out, like the framework of fractions. Like they get right into certain things and they jump around. So, I usually use Bridges. I like some of their games and I like some of their activities, but I don't use their like scope and sequence. And then I have written a ton of my own word problems just from our trainings, like from CGI problems. I also use like the CGI book with the practices of like the order of like how you should start. (Ortiz)

Learning Objectives

Learning objectives varied slightly in this group of teachers. Teachers either wanted their students to understand the part-whole relationship of fractions or the many ways fractions are used.

Exposure to Fractions

Seven of the teachers focused on exposing their students to a new number form called a fraction and the vocabulary used to discuss it. Ms. Lowe did this by introducing a game that students would play to try to cover up one whole. She reasoned that the vocabulary used to play the game and the task of creating one whole with different fraction amounts engaged the students in discussions using the language of fractions. Ms. Baruch summed up, "I think the greatest thing I want them to take away is that they're going to be learning about new numbers and they're fractions. I want them to know that a fraction is a number. I want them to, to not worry about, Oh, there's a top number and a bottom number." Ms. Shore explained, "For the first lesson probably my goal would just be to know how to say one-half, one-third, one-fourth, two-fourths, three-fourths, just how to say the fractions." Much like Ms. Shore's, Ms. Tsui used her first lesson simply to expose them to the language of fractions. Meanwhile, Ms. Gonzalez used the lesson to hear what the students said and the language they were using. All seven of these teachers acknowledged that fractions were a new concept and that their students would need time to become familiar with them.

Two teachers also had pure exposure goals in mind when implementing the first fraction lesson. Ms. Haskins was mindful of the upcoming Winter Break and wanted to have an activity the students would enjoy when introducing fractions. She decided to create pizzas cut into equal pieces and have the students talk about the fraction of their pizza with a

particular topping. She recognized that she didn't have enough time to fully delve into fractions and wanted a fun experience that used fractions for the students to recall when they returned in January.

Part-Whole Relationship

All but two of the teachers wanted their students to understand that fractions refer to equal-sized parts of a whole. Ms. Jackson recalled telling her students, "Fractions are fair, and you have to give equal amounts so you can't give one person a smaller piece and you a bigger piece, that fractions are fair." Ms. Lee also emphasized fair sharing as she hoped her students would gain a better understanding of the meaning of a fraction, not just that it's two numbers, but that those two numbers are something together.

Four teachers expressed they wanted their students to understand the part-whole relationship of fractions. Acknowledging one of the challenges with teaching fractions to her students, Ms. Ahmed explained her objective stating:

> That fractions are part of a whole, for example. That concept can be a little bit difficult for them to understand because they're seeing it in pieces rather than, I mean, they see the whole, but they're like, well, you only colored so many of them, so how is that the whole thing? That doesn't make sense. (Ahmed)

While Mr. Allen wanted his students to understand that fractions are less than one, Mr. Martin's learning objective was that students understand the part-whole connection. Finally, Mr. O'Brien emphasized, "I need them to understand what a fraction is, how to draw a fraction."

Fractions Are More than Area Models

Distinct from the rest of the group, Ms. Ortiz wanted her students to realize that fractions are more than just a part-whole relationship and can be used to represent survey results and parts of a set. She expressed, "I think also moving away from that idea like a fraction has to be a square or rectangle, a cookie, or a brownie." Ms. Lopez also used a different approach when introducing fractions. She introduced three story problems that students could solve using fractions or whole numbers. Problems ranged from sharing a bubble gum tape to using the clock's quarter hours. She stated, "I wanted them to understand how fractions are used in real life, how fractions can be used in different situations, and how we can persevere and use tools to help us with concepts we haven't even learned before."

Teacher Views of Student Responsibilities

Student responsibilities varied greatly within this group. Student responsibilities fell within two distinct categories: students who needed to copy a fraction model and students who activity solved problems and justified their answers. Ten teachers in this group described lessons where their students needed to follow a demonstration and create a similar model. The remaining four teachers in this group described lessons where their students worked collaboratively, shared strategies, and solved story problems.

Teachers Viewed Students as Self-Reliant Problem-Solvers

Ms. Lowe described how she honored all student attempts to solve as well as their observations. She elaborated:

> And then I always kind of give them the credit as like this mathematician came up with this theory and what do you think? And put up the ones that are wrong and the ones that are right and, and say, 'Okay, who agrees with that? Who doesn't agree? Why?' I try to never say that's right. Like I try to say, 'Okay, what do you guys think?' You know, just throw it back to them all the time. (Lowe)

Ms. Lopez praised her students' who needed more time to unpack the story problem, disclosing, "They would come up with actually the next level just cause it, they had to think like digest it for a long time." She continued, "They have to show their work in a clear way so that someone else can understand it. And then when they're chosen to speak, they have to be able to describe how they solve the problem." Mr. Allen emphasized the power of having the students analyze each other's work and giving feedback to each other to improve explaining:

> One thing I'll do is like I'll give examples of like, well, like of like a student work, but I won't put the name on so they can't take the name. And I'll say like, what, what could we, what could we how can we help this student? (Allen)

Using this teaching move allowed his students to hear the feedback from their peers while also informally assessing student conceptions. Ms. Lopez and Ms. Lowe, Ms. Gonzalez, and Mr. Allen described students rising to the challenge of taking ownership over their learning.

Teachers Viewed Student Responsibilities as Copying a Teacher's Model

Ten teachers in this group started their fraction lessons with a discussion and demonstration of a fraction model. These teachers introduced the number line, shaded and partitioned areas, or sets of a whole and then expected their students to be able to refer to fraction amounts using that same model or creating their own similar model. Ms. Baruch began her lesson by pointing out the fair share that needs to happen with fractions and then allowed students to choose a material like construction paper circles to create their own model of halves, thirds, or fourths. Ms. Haskins's first lesson was quite similar to Ms. Baruch's, but her students copied a model of a fraction with the choice of pizza toppings. Ms. Shore described the release of responsibility, stating:

> They would be making their own number line which would already have marks on it, and I would explain what each mark represented and have them fill in, like between zero and one, fill in where a half would go and then where thirds would go and where fourths would go and label them with the fractions. (Shore)

Other teachers in this group focused on students listening and copying what the teacher had written when discussing student participation. Ms. Lee described the process of using graph paper as a fraction model and having the students copy her cut marks and labels. Ms. Ahmed expressed the student responsibilities as, "Definitely to be attentive follow along. Not to go ahead. Be open to think, you know, think, pair, share, you're talking to your partner to be open to assisting if needed without answering before it's needed. Following directions." Ms. Jackson described student expectations similarly, "So they would need to be engaged and listening. They would need to be able to vocalize if they didn't understand." Mr. Martin's explanation echoed that of Ms. Ahmed:

> So, beginning a lesson they'll be taking notes and again, we'll really, we define the numerator and denominator and they'll draw a model of essentially a fraction, identify both parts and then even draw a picture to go with it. (Ahmed)

Teachers in this group varied in the amount of responsibility they gave their students.

Summary

Teachers in this study were separated into three groups based on descriptions of their first fraction lesson and their dependence on the school-adopted textbook. Although the fraction activities and teacher decisions varied greatly among this group, all teachers demonstrated an openness to implement new activities and a concern for student success and critiqued their own pedagogical decisions. Teachers were particularly inclined to use the fair share story problem and analyzed each activity with their individual students and school culture in mind. The teacher-led group primarily used recall and repetition during their

introductory fraction lesson, trusted the school-adopted curriculum to provide high quality lessons, and expected their students to copy teacher models in order to independently write fractions by the end of the lesson. The student-led group introduced fractions using fair share story problems, acknowledged and used student connections to guide their instruction, expected their students to actively participate, and did not implement a school-adopted curriculum. The middle group used a variety of resources to guide their instruction and struggled to stray from the part-whole narrative of fractions. While some teachers in this group viewed the textbook as a resource, others primarily used it for its on-hand resources and feasibility. Some teachers in this group grappled with providing time for the students to understand concepts while district assessment deadlines and a multitude of standards loomed. Finally, this mixed group of teachers varied in the way they viewed student learners. Perspectives ranged from students who bring background knowledge and can solve problems independently to those who need direct instruction and step-by-step assistance.

CHAPTER FIVE: DISCUSSION

Introduction

With 2019 NAEP results showing that the majority of our fourth-grade students are not proficient at math, changes must be made to our curriculum and instruction. This study suggests that teachers are interested in ways to improve their instruction but need help in finding and implementing effective learning activities for fractions. Additionally, the wide range of decisions and activities implemented signifies that students receive a range of classroom experiences which may or may not benefit conceptual understanding of fractions. To improve student math outcomes, teachers and educational leaders must provide professional development that utilizes conceptual understanding of fractions for teachers and students.

Summary of Findings

Interviews eliciting teacher decisions, challenges, and justifications around fraction instruction offered four findings. All of the teachers in this sample were open to using new fraction activities with their students, critically examined the proposed activities, and explained ways they would adapt the lessons to make sure their students' needs were met. At the heart of each decision, teachers considered their own school culture and dynamics. This study also provided insight on specific professional development topics that could assist students' fraction understanding.

Significance of Findings

These findings add to previous research while offering insight on how teachers can move forward in improving student understanding of fractions. First, this research highlights that teachers deeply care about their students' success and seek ways to improve their math

understanding. Teachers who had strong reasons for why they do not implement the school-adopted textbook relied on their PLC and work together to create learning activities that fit the needs of their students. The primary reason teachers created their own curriculum was to better meet the individual needs of their students and to use their student's thinking to guide the instruction. This study shows that teachers implement certain activities based on how they perceive them to be helpful for their students.

Finally, and most importantly, this research supports the use of analyzing fraction activities as an opening to professional development. In addition to the fair share story problem as promoted by Empson (1999), teachers in this study found value in discussing the pros and cons of choral counts, fraction folding strips, and children's books that integrate fraction stories. By allowing teachers to discuss these activities, conversations emerged about challenges they have seen and anticipate, evidence of a changing mind, and their decision-making process surrounding fraction activity implementation. Teachers critiqued their own decisions made in their first lessons, wishing they had stopped to check for student understanding before moving forward or had altered a lesson in the moment. Teachers appreciated topics surrounding story problems with a familiar context, using real-life situations in math problems, engaging, an activity that can be easily differentiated, and using activities that emphasized connections between fractions and other units of math such as division and measurement. Focusing on these topics for professional development could improve teacher understanding of the CCSSM, broaden their arsenal of fraction activities, and allow for teachers to begin analysis of student thinking of fractions.

Adding to the research of Ma (1990) and Shulman (1986), teachers disclosed that they do not know how to move forward in fraction instruction without stressing part-whole

relationship, the vocabulary of numerator/denominator, or quickly showing the number form of fractions. Shulman's pedagogical content knowledge applies to the teachers in this sample as they ask for better ways to understand and teach fractions to their students. One of the major differences between the student-led teachers and the mixed group of teachers was the way in which they allowed student thinking to guide the lesson. The mixed group of teachers reverted in some way back to the standards or textbook rather than hearing the student thoughts and pushing them forward. Teachers are seeking ways to use student thinking to improve their pedagogical content knowledge.

The results of this study demonstrate that teachers want to improve student math outcomes. Discussing the proposed activities of this study is one way to begin the conversation around improved fraction instruction for elementary students. Professional development could be specified to meet the needs of each type of teacher. Next I provide recommendations for all teachers described in this paper.

Teacher Recommendations

Teachers would benefit from using fraction activities similar to those used in this study as a catalyst to analyze and discuss their fraction instruction. These conversations need to be ongoing and allow time for reflection. Multiple conversations throughout fraction instruction allows time to verbalize the pros and cons of each activity, implement the activities with their students, and share observations with their colleagues. A second suggestion includes dedicating time to writing story problems and mirroring real-life tasks that integrate more than the part-whole interpretations of fractions, such as measurement, division, and ratio.

Because 12 teachers in this sample used the part-whole relationship in their initial lessons on fractions, we must introduce activities that use fractions in other ways just as both the research and standards suggest. By implementing fair share story problems, we are providing a real-life context for partitioning, a skill that is embedded in the CCSSM beginning in first grade. Empson's research of equal sharing contexts to introduce fractions is not only a widely shared practice, but one that teachers willingly accepted. These activities not only demonstrate the connection between fractions and division, but also allow teachers to gauge student understanding and form future lessons. Teachers could collaborate with caretakers to assign weekend or summer experiences to equally share an item with their family, take pictures of their plates, record parts of their discussions, and then discuss what they did when they return to the classroom. These conversations which naturally occur just need to be highlighted and used as teachable moments. Using a fair share context will also help address the challenge of understanding fraction magnitude as later lessons can focus on comparing amounts.

Teachers who use the textbook part of the time, or occasionally write their own curricula and use student thinking to guide their instruction would benefit from collaboration with colleagues so they have the support they need to develop activities together. As the teachers in this study pointed out, they value their colleagues' opinions and do not want to break the status quo alone. Teachers might decide to analyze the school-adopted curriculum to gauge alignment to the CCSSM and identify the lessons that need to be strengthened or that do not meet student needs. The time to work collaboratively on fraction instruction will allow teachers to explain their current practices, verbalize their questions and concerns about

writing their own curriculum and using student answers to elicit greater understanding, and give peace in knowing that they are providing coherence for their students.

Teachers who fall into a similar group as the student-led teachers from this study would benefit from in-the-moment training to help use their students' thinking to guide instruction and practice teacher moves or conducting lesson studies where teachers across the grade level observe each other teach, analyze student work and discuss noticings to improve their instruction.

These findings identify entry points to professional development opportunities and ways to improve fraction instruction in the elementary setting.

Research Limitations

Although this study explicated key areas for professional development, there are a few research limitations that future studies could address. First, future studies could focus on a more diverse group of teachers and a larger sample size. Adding more voices to this narrative working at different locations and school sites will allow for patterns to emerge and be analyzed. Second, future research could use a set of questions that focus on how teachers decide to move on from one skill or concept to another by eliciting student indicators of understanding. Third, interviews and observations could be done at the time of the fraction lessons to ensure teachers more fully remember their decision-making process and can recall student actions.

Implications of the Work for Research

This study illuminated a learning opportunity for teachers to discuss fraction activities as a way of learning more about fraction concepts and reflecting on their own pedagogical decisions. Future research could use different fraction activities that elicit teacher thoughts

on other interpretations of fractions such as measurement or ratios and proportions. In addition, research using student conceptions and misconceptions of fractions to guide instruction will benefit teachers. Likewise, researchers must work to discover the activities and teacher moves that best help students to conceptually understand fractions.

Additionally, different research methods could add meaning to the data supporting effective fraction instruction. Data collection and analysis of student work samples, recordings of classroom lessons, and recordings of PLC meetings that are undergoing lesson studies, all would provide more context to teacher decisions. Or, a participatory action research method could provide insight into the power of learning from your colleagues while teaching about fractions. Finally, a longitudinal study where we monitor students' understanding of fraction concepts from kinder to fifth grade would allow us to see change over time. These alternative research methods could create a more extensive narrative for student thinking and the teacher pedagogy that supports fraction understanding.

Implications of the Work for Educational Leaders

Fraction instruction must also become a priority for school administrators. First, school leaders need to allow more time for teachers to meet and discuss fraction lessons. Teachers benefit from talking with their colleagues, but their schedules are often full of other duties. Teachers expressed wanting to create their own learning activities but needing more time and resources. Administrators could supply more time for PLC collaboration, including time with a special education teachers, instructional assistants, and math coaches to better personalize the learning environment.

Second, school leaders need to become knowledgeable of the support their teachers need. They need to analyze their pedagogical decisions to see how and why they make decisions surrounding fraction instruction and then choose the appropriate support. The majority of teachers in this study fell into the middle group, with varied decisions surrounding fraction instruction and a focus on the part-whole narrative of fractions. This indicates a large portion of teachers who could benefit from ongoing professional development focused on best practices and the power of activities that aide student conceptual understanding of fractions. Improving student outcomes is the responsibility of the teacher and the school leaders.

APPENDIX A: EMAIL TO SOLICIT PARTICIPANTS

Good morning/afternoon/evening,

My name is Opal Chambers and I am a doctoral candidate for UCLA's Educational Leadership Program. I believe the best way to learn as a teacher is to share lessons and teaching strategies with each other. As a life-long learner, I am interested in discovering teachers' rationale behind their pedagogical decisions when teaching fractions to their students.

I am looking for **current third and fourth grade** teachers who teach math to their students and can donate about two hours of their time to help me grow as a learner. The initial interview will elicit information about how you currently teach fractions. The second interview will be scheduled later that same week and will allow us to discuss four activities to introduce fractions which may be helpful. Once the second interview concludes, I will email the activities we discussed.

If you are interested in participating, please complete the form attached which will be used to determine eligibility for my study. Responses to the questionnaire will be kept confidential whether or not you participate in the study. If you do not qualify for the study, the answers will be destroyed. If you qualify for the study and decide to participate and sign the research informed consent form, the answers to this questionnaire will be kept with the research record.

If you have questions about your rights as a research subject or if you wish to voice any problems or concerns you may have about the study to someone other than the researchers, please call the UCLA Office of the Human Research Protection Program at (310) 825-7122.

Feel free to forward this email to anyone in Los Angeles county who might be interested in participating in this study.

Thank you for your time,

Opal Chambers

Third Grade Teacher

APPENDIX B: PARTICIPANT QUESTIONNAIRE

Questionnaire Directions: Please answer all questions as accurately as possible. There are no right or wrong answers.

1. How long have you been a teacher?
    - This is my first-year teaching
    - 1-2 years
    - 3-4 years
    - 5-6 years
    - 7-8 years
    - 9-10 years
    - 10+ years
2. How long have you been teaching this grade level? (third or fourth grade)
    - This is my first year teaching this grade level
    - 1-2 years
    - 3-4 years
    - 5-6 years
    - 7-8 years
    - 9-10 years
    - 10+ years
3. What math textbook or curriculum does your school use?
4. How would you describe the curriculum you use to teach fractions?
    - Mostly school-adopted textbook

- About 50% from the school adopted textbook and 50% self-chosen or created
- I mostly create or choose my curriculum

5. Where do you teach? Please identify the name of your school._____
6. What email address would you like me to contact if you are selected to participate in this study?
7. What is your name?

APPENDIX C: INTERVIEW PROTOCOL PART ONE

Good morning/afternoon/evening _____.

Thank you so much for agreeing to be interviewed! I truly appreciate your time. As a teacher myself, I know how valuable your time is, so I thank you for taking the time to share your knowledge with me. In this interview, please keep in mind that there is no right or wrong answer, and I am asking these questions so we educators can continue the practice of being lifelong learners. The data that comes from this interview will help others better understand what other teachers do to make pedagogical decisions when teaching fractions.

This interview should last approximately 45 minutes. I will ask about 15 questions, so if you ever feel like you already answered one, just let me know and we will move on to the next question. I will be using an audio recorder so I can be more attentive to your responses. Your answers will be kept confidential, as I will be using a pseudonym for you as well as any other individuals that are named. If at any time you would like me to turn off the recorder, please let me know. Do you have any questions before we start? Great, let's begin.

<u>Building Rapport:</u>

1. I am only in my third year of teaching third grade, so I realize I still have a lot to learn about my students and which activities best support their learning. How long have you been teaching third/fourth grade?
2. What is your favorite part of teaching third/fourth grade?
    a. Share: My favorite part of teaching third grade is my students' love for learning. They find everything fascinating and have the motivation to learn about the world around them.

<u>Eliciting Fraction Activities and Rationale</u>

3. Since I taught sixth grade math for over five years, I often have to remind myself that my third-grade students are new to working with fractions. Because of this, I would love to use this as an opportunity to talk about how you introduce fractions. Could you describe your very first lesson with fractions?
    a. If I observed this lesson, what would I see from beginning to end?
        i. What would the students be doing?
        ii. What are some things that you might say?
        iii. What would you write on the white/chalk board?
        iv. Would everyone be working on the same activity?
    b. How do the students react to this lesson?
    c. What are the student responsibilities in this lesson?
    d. What do you want the kids to take away from this lesson?
    e. How do you address all student needs with this lesson? How do you differentiate with this lesson?
    f. How do you engage the students who are hesitant to participate in this lesson?
    g. How long is this lesson? (If under 20 minutes, what lesson happens after this one?)
    h. (If teacher uses a story problem) – Can you tell me what the story problem is and how the students go about solving it?
    i. If partitioning shapes – What vocabulary is used? Or clarify when the vocabulary is introduced.
4. Why do you use this as your first lesson?

a. How do you use student feedback to prepare the next lesson dealing with fraction concepts?

5. I was recently introduced to the California Math Framework. Are you familiar with the CA Math Framework?

Share the screen with the teacher to show them a grade level appropriate packet of the 2013 California Math Framework for reference. (Remember not to show the specific fraction activities.)

a. If so, have you done any of the fraction activities in it?

b. If not, move to next question.

Challenges:

6. What challenges, if any, do you face when teaching your students about fraction concepts?

Resources and Selection Process:

7. In the short questionnaire attached to the recruitment email, you mentioned that (you primarily use the school-adopted curriculum, you use a mixture of school-adopted curriculum and your own, or you primarily use your own curriculum instead of the school-adopted curriculum). Could you tell me about your decision?

a. How do you determine whether or not to use a lesson in your school-adopted curriculum?

b. (For participants who are mainly creating their own) How do you determine what to do?

i. What resources do you reference?

Closing: Thank you so much for your time and your openness to share your ideas and thinking. I'm already thinking about different activities I'd like to try with my students. If you think of anything else after we leave today, please feel free to email me. Before we leave, I just wanted to make sure that ____ was still a time that worked for you. (Make sure the upcoming date still works.)

APPENDIX D: INTERVIEW PROTOCOL PART TWO

Good morning/afternoon/evening_____.

It was great talking to you last _____(day) to discuss what you normally do to introduce fractions to your students. Thank you for agreeing to meet a second time with your already busy schedule. The data that comes from this interview will help me better understand what other teachers do to make pedagogical decisions when teaching fractions.

I want to be mindful of our time today, so although I will show you four activities, we will talk about each one for about eight minutes. I haven't used all of these activities myself, so this is an opportunity to gain your initial thoughts and discuss your reaction to them. Then at the end, I'll have you explain which one you'd be most open to trying with your students. I will be using an audio recorder so I can be more attentive to your responses instead of taking notes. Your answers will be kept confidential, as I will be using a pseudonym for you as well as any other individuals that are named. If at any time you would like me to turn off the recorder, please let me know. Do you have any questions before we start? Great, let's begin.

Share the screen with the participant to show them the PowerPoint of various activities exploring the value of fractions. Go through each activity and ask:

1. What are some things you appreciate about this activity?
2. What are some things you did not appreciate?
3. Would you consider using this activity with your students?
    a. Please explain why or why not.
4. If you were going to use this activity, how might you adapt it?

After going through each of the four activities individually:

5. Out of all the activities presented, which seem the most useful? Please explain why you think that.

APPENDIX E: DATA ANALYSIS CHART FOR INTERVIEW PART ONE

Directions: Use the transcript of Interview #1 to complete the table by placing an "x" in the appropriate category of each row. Provide evidence for each determination.

Student-led	mixed	Teacher-led x
Evidence: Teacher directions like: Show me one whole, show me one half. She emphasizes that a fraction is between 0 and 1.		
Conceptual	mixed	Procedural x
Evidence: The students follow her direction of where a half goes, she shows them where the half line goes when partitioning if struggling to do so on their own		
Familiar context	Somewhat familiar	Unfamiliar context x
Evidence: They read the story problem on their own, they read it as a class, as a class they discuss the vocabulary because sometimes they don't know words like "liter" and they need to solve something as ¼ of a liter		
High use of Differentiation	Some use of differentiation	No use of differentiation x
Evidence: everyone does the same lesson with same values and problems, the computer program Zearn will reteach it or pre-teach it at the speed they choose.		
Student-Thinking approach	mixed	Mathematics Requirements - Stress to teach standards approach x
Evidence: I do this because it is in the module, in our curriculum		
Non-symbolic	Some symbolic representations x	Stressed use of symbolic representations
Evidence: Written in words, not numbers		

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