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On the Matrix Description of Calabi-Yau Compactifications

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We point out that the matrix description of M-theory compactified on Calabi-Yau threefolds is in many respects simpler than the matrix description of a T^6 compactification. This is largely because of the differences between D6 branes wrapped on Calabi-Yau threefolds and D6 branes wrapped on six-tori. In particular, if we define the matrix theory following the prescription of Sen and Seiberg, we find that the remaining degrees of freedom are decoupled from gravity.

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1. Introduction

The matrix model proposal for M theory [1] aims to describe a space-time theory with gravity via a much simpler theory which is manifestly better-defined. With no dimensions compactified, the theory is described by the large-N quantum mechanics of D0-branes in the limit $g_s \to 0$ (vanishing string coupling). However, it has become clear that compactification requires the inclusion of additional degrees of freedom. Thus, to learn what degrees of freedom are needed in the full formulation of the theory, we require an understanding of the fully compactified theory (or theories, depending on the level of connectedness of M theory backgrounds). The degrees of freedom that are needed in various other simple compactifications have been determined. On tori of dimension $p \leq 4$ the matrix description can be written as a quantum field theory (for $p \leq 3$ it is $p+1$-dimensional Yang-Mills theory with 16 supercharges [1,2]). On T^5 the theory can be defined as the "little string theory" on a wrapped fivebrane [3,4].

The M-theory compactification on T^6, however, seems overly complicated [5,6]. One finds that the finite-N DLCQ description of M theory on T^6 is given by a theory of ALE sixbranes (that is, of the $6+1$-dimensional core of an A_{N-1} ALE singularity in eleven dimensions) in M-theory [7,5,6], which fails to decouple from bulk gravity [5,6]. We are then left with the unappetizing prospect of defining the 6-dimensional compactification of M-theory via M-theory (*cf* also [8]).

One of the promising features of the matrix model is the independence of its motivating arguments from supersymmetry. Both the infinite momentum frame and DLCQ arguments for decoupling of $p_{11} \leq 0$ modes do not refer to supersymmetry. In particular, the prescription of [9,6] applies to any compactification. However, one might expect that breaking supersymmetry complicates matrix theory compactifications even further.

Nonetheless, we show in this note that for compactifications on six-manifolds, the situation in fact simplifies upon reducing the amount of supersymmetry. In particular, the difficulties with T^6 compactifications do not persist for generic six-manifold compactifications with eight space-time supercharges (for which the D0-brane theory has four supercharges). Using well-known facts about Calabi-Yau compactifications, we find that the DLCQ compactification of M-theory on a generic Calabi-Yau threefold following [9,5,6] is described by a theory decoupled from gravity. When we need a concrete example we will always use the quintic threefold in $\mathbb{P}^4$, but we expect our results to generalize in a straightforward way to many other Calabi-Yau spaces.[1] Our discussion relates issues of singularity resolution, which have played a large role in the string duality story, to matrix theory.

2. T^6 vs CY_3

2.1. Review of the T^6 case

Let us review first the situation on T^6 [5,6]. One considers N D0-branes in type IIA theory on a six-torus whose cycles are each of fixed size in eleven-dimensional Planck units, in the limit of vanishing string coupling ($g_s \to 0$). In this limit, we wish to focus on states with energy of the order $g_s^{2/3}/\ell_P$, where ℓ_P is the eleven-dimensional Planck scale; these correspond to M-theory states with finite light-cone energy. We are therefore in the DKPS regime [10], and we will call this energy $E_{\rm DKPS}$. In the four noncompact dimensions, the D0-branes are also separated by distances fixed in Planck units. The light open string modes stretching between the D0-branes have energy $E_{\rm DKPS}$, and are therefore relevant in this regime. Closed strings decouple because their interactions with the D0-brane quantum mechanics are velocity-suppressed. Oscillator modes of open strings also decouple: their masses go like $1/l_s = E_{\rm DKPS}/g_s^{1/3}$.

In analyzing the compactification, one needs to consider the full spectrum of states, including wrapped D-branes. The hierarchy of masses is as follows:

$$\mathrm{D6-branes:}\ \ M_6 \sim g_s^{2/3} E_{\rm DKPS} V_6 \tag{2.1}$$

$$\mathrm{stretched\ open\ strings:}\ \ M_1 \sim E_{\rm DKPS}\delta x \tag{2.2}$$

$$\mathrm{D4-branes:}\ \ M_4 \sim E_{\rm DKPS} V_4 \tag{2.3}$$

$$\mathrm{string\ oscillator\ modes:}\ \ M_{osc} \sim \frac{E_{\rm DKPS}}{g_s^{1/3}} \tag{2.4}$$

$$\mathrm{D2-branes:}\ \ M_2 \sim \frac{E_{\rm DKPS}}{g_s^{2/3}} V_2 \tag{2.5}$$

Here V_2,V_4, and V_6 are the volumes of the two-, four-, and six-cycles respectively, in eleven-dimensional Planck units (and presumably a few orders of magnitude at most in

[1] In the special case of toroidal orbifolds and their deformations, the analysis will be somewhat different.

these units). δx is the separation between D0-branes, also in eleven-dimensional Planck units. As we can see, in the "matrix theory limit", one scales out the wrapped D2-branes as well as the oscillator modes; both have masses which are parametrically larger than the lowest open string modes as $g_s \to 0$.

As explained by Sen and Seiberg, the light D6-branes signal a serious problem in the matrix formulation of this compactification. They have several important properties, in addition to their vanishing mass:

1) The BPS multiplet corresponding to the wrapped D6-brane has states with spin 2 in the transverse dimensions, which therefore become gravitons propagating in the bulk as they go to zero mass. One can see this by relating them to D0-branes via T-duality.

2) There is a bound state of k D6-branes for any k. Again, T-duality relates this the the bound states of D0-branes. This identifies the light D6-branes as Kaluza-Klein modes of a new large dimension.

Finally, the substringy six-torus with N D0-branes has a T-dual description in terms of a large six-torus, $\widetilde{T}^6$ with N wrapped D6-branes, which we will denote as $\widetilde{D6}$-branes to distinguish them from the wrapped D6-brane states on the original torus. The original D6-branes map to $\widetilde{D0}$-branes. This gives a third important fact about the matrix formulation of the toroidal compactification:

3) It can be described as a 6+1-dimensional theory, at least before considering the effects of the bulk.

This theory does not decouple from the bulk graviton $\widetilde{D0} = D6$ states. In fact the $\widetilde{D6}$-branes, which start out as Kaluza-Klein monopoles in eleven dimensions, decompactify in the DKPS limit into an ALE space which has no reason to decouple from the $\widetilde{D0}$-brane gravitons [6].

Because the scaling (2.1) should hold for any six-manifold (if we understand how to define V_p and δx), it is tempting to conclude that any such background will have the same problem. However, as we will now explain, the matrix compactification on a generic Calabi-Yau threefold has much simpler properties, and in particular there is no issue of coupling to Kaluza-Klein gravitons.

2.2. The DKPS limit of the Calabi-Yau compactification

We must first determine what the prescription of [6] entails for the Calabi-Yau case. We will for concreteness take the Calabi-Yau to be the quintic hypersurface in $\mathbb{P}^4$, which has a one-dimensional Kähler moduli space. In the DKPS limit, we are instructed to study type IIA string theory on a sub-stringy quintic. The point in the moduli space of the quintic where the overall volume shrinks to zero is the (mirror) conifold point in Kähler moduli space [11] (for a nice discussion see e.g. [12] and references therein). Often we will slightly abuse terminology and refer to the finite distance singularity in the Kähler moduli space of the quintic as the conifold point. We will take the theory of N D0 branes near the conifold as the definition of matrix theory on the quintic. This choice will be further motivated in the following discussion. In §2.3 we will summarize and enumerate how points 1),2), and 3) above differ in the Calabi-Yau case.

In the case of T^6, before modding by the $SO(6,6;\mathbb{Z})$ modular group one has tori of arbitrarily small radii, which we can measure via the masses of string winding modes, and there is an infinite distance singularity at very small radius. This is identified by the modular group with a very large T-dual $\widetilde{T^6}$. The analogous discussion for the quintic is more complicated. If one were to work with the *classical* Kähler cone, one would find a very small quintic which is not related by any elements of the stringy duality group to a very large quintic. However, since the Kähler moduli space receives stringy corrections and the proper notion of distance is subtle and probe-dependent [13,12], we should also try to verify our choice for the DKPS limit in other ways.

If we begin by describing the theory as M-theory on the Calabi-Yau threefold times S^1, we would find that classically, the masses of the wrapped branes and of the string states scale as in Eqs. (2.1)- (2.5). Thus we would find that the D6-brane is becoming light and the other branes have finite or large mass relative to $E_{\rm DKPS}$. However, the masses will receive corrections from membrane instantons (in particular, factorization of the Kähler and complex structure moduli spaces tells us that they will be membranes wrapping the S^1 and two-cycles of the Calabi-Yau). This will generically shift the masses of the wrapped D2- and D4-branes, and in fact for the quintic there is no point in the moduli space for which these masses vanish [12]. At the conifold point the mass of the wrapped D6-brane vanishes even including such corrections.

Thus, the hierarchy of branes suggests that the conifold limit is the correct DKPS/matrix theory limit. Still, one could ask why this is true intrinsically. This is the only point at which any size is truly going to zero, rather than being of order ℓ_s.[2] But this direct discussion is subtle due to the worldsheet instantons.

We can in fact avoid complications coming from worldsheet instantons, by going to the mirror quintic to discuss the limit. On the mirror, we work with the complex structure moduli space, which receives no worldsheet instanton corrections. There, before dividing by the modular group of the complex structure moduli space, there is only one infinite distance "large complex structure" limit. The conifold locus is not identified with any other infinite distance singularity, as it is at finite distance [14]. The conifold locus is the only special point mirror to a "small" Calabi-Yau, and so we expect it to be singled out as the DKPS/matrix theory limit. This point of view again supports our conclusion that the DKPS limit on the quintic involves a quintic near the conifold point, and in addition it cannot be "T-dualized" to a large quintic.

Of course, one *can* use mirror symmetry to relate the theory of N D0 branes on the original quintic to N $\widetilde{D3}$ branes wrapping the fibers of a $\widetilde{T^3}$ fibration on the mirror quintic [15]. This gives a 3+1 dimensional theory, instead of the 6+1 dimensional theory which arises in T^6 compactification.

In the limit we have defined, we have only the D6-branes becoming light, and this gives rise to precisely *one* massless hypermultiplet, as in [16]. There are no multi-D6 brane bound states, as we can see by mirror symmetry; the mirror of the wrapped D6-brane is the D3-brane wrapped around the vanishing three-cycle [11,12]. Consistency requires that there only be one such state [16] and this was indeed found to be the case [17]. Also, one finds *no* extra massless particles from wrapped D2 or D4 branes in this limit. Although the overall volume has shrunk to zero, the volumes of 2 and 4 cycles in this limit are actually of order l_s^2 and l_s^4, respectively [12]. This surprising assertion is a property of the quantum corrected volume in compactified string theory, and is a direct consequence of mirror symmetry.

We should also ask about the masses of the light open string modes. If we start at large radius with N D0 branes separated by sub-stringy distances, and then shrink the Calabi-Yau, we might expect the lightest stretched strings between the D0 branes to remain much lighter than the oscillator modes. This is because we do not expect that

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[2] In particular, if one chose any other point in the Kähler moduli space, the situation would only *improve* – there are no extra light degrees of freedom. In this sense, we are making the most dangerous choice and arguing that the matrix theory is still simple.

the effective distance between the zero branes would *increase* as we shrink the Calabi-Yau. (For example, the sizes of two-cycles are bounded below for the quintic, but they do decrease monotonically as we move from the large-radius limit to the conifold point). Still, at the (mirror of the) conifold point, worldsheet instanton effects are strong and to understand what degrees of freedom are present, one will have to understand the effects of string instantons. It is perhaps easier to think about this problem in the 3+1 dimensional theory on the mirror.

On the mirror, the D0 branes map to N $\widetilde{D3}$ branes wrapping the fiber $\widetilde{T^3}$, while the base S^3 is becoming very small – the moduli are the locations of the $\widetilde{T^3}$ on the base and the Wilson lines of the D-brane theory around the cycles of $\widetilde{T^3}$ [15]. We can be at a generic point in the Kähler moduli space of the mirror, so in particular (for "large enough" complex structures on the quintic) we can take worldsheet instanton effects to be suppressed on the mirror. Then, distance is measured using the classical metric. In this set-up, the stretched strings between three-branes which are very close together on the base S^3 are much lighter than the string oscillator modes, because their masses are well approximated by the naive classical formula. These lightest strings give the W bosons of the 3+1 dimensional quantum field theory, and although their mass might be renormalized by a significant numerical factor it will not be parametrically enhanced to M_s.

2.3. Problems (1)-(3) revisited

We have argued in the previous subsection that the conifold regime is the DKPS regime we should use to define the DLCQ quantization of matrix theory on the Calabi-Yau. Given this, there are very precise differences between the T^6 case and the Calabi-Yau case. In particular, the properties 1)-3) of T^6 compactifications which complicate their matrix theory description have very different analogs in the Calabi-Yau case.

1) On the Calabi-Yau, the D6-brane has the quantum numbers of a *hypermultiplet*, instead of a gravity multiplet, in the transverse spacetime. Indeed, the light D6-brane is the monopole state that resolves the conifold singularity in the IIA string theory compactification on the CY [16]. This also implies:

2) Here the wrapped D6-branes do not form multi-sixbrane bound states, unlike in the torus case.

3) Here there is no T-duality symmetry (more precisely, no element of the discrete symmetry group of the CY compactification) which maps D0-branes on the CY in the conifold regime to $\widetilde{D6}$-branes on a large CY. The conifold singularity is at finite distance from the

interior points on the moduli space [14], whereas the large radius singularity is at infinite distance. Therefore there is no reason to believe the physics is that of a 6+1-dimensional theory.[3] One *can* mirror symmetrize so that the D0-branes become $\widetilde{D}3$-branes wrapping a $\widetilde{T}^3$ on the mirror Calabi-Yau [15]. This leaves us with a 3+1-dimensional theory, which can be well defined without gravity and which by the arguments we have given does not seem to couple to gravity.

We see from 1) and 2) that the matrix description of Calabi-Yau compactifications does not involve gravity, and the limit of [5,6] does not entail the growth of an extra dimension. We will present tentative arguments that the light D6-brane may also decouple from the D0-branes in the next section. Note that even if this is not the case, the matrix formulation is a considerable simplification over the original description of the M-theory compactification, as the relevant degrees of freedom do not seem to include gravity.

3. Coupling of the D0 branes to the Wrapped D6 Hypermultiplets

Although the matrix description does involve a considerable simplification, we still should wonder to what extent the heavy D0 branes will couple to the light wrapped D6 branes. This issue is still not clear to us. However, we will present two indications that the D6-branes might in fact decouple from the D0-brane theory in this case.

One indication is that the open string theory that lives on the D0-branes (perhaps mirror symmetrized to $\widetilde{D}3$-branes) may be consistent by itself. If this theory is non-singular, then we would not expect the theory to contain non-perturbative light states. Before considering this open string theory in the DKPS limit, let us recall first how things worked for the ordinary IIA string theory compactified on the Calabi-Yau. This closed string theory becomes ill-defined at the conifold singularity [14]. One can see this using mirror symmetry as in [14]. Another very useful way to get a handle on such singularities is to study the compactification using the gauged linear sigma model technique introduced in [18]. There, the signal of the singularity is the emergence of a noncompact branch (a throat) in the target space of the two-dimensional sigma model (indeed, as argued in [18], the appearance of a noncompact direction is the only possible source of a singularity in such models). As in [18] we will refer to the complex coordinate on this branch as

[3] For Calabi-Yau compactifications realized as orbifolds of T^6, one may perform T-duality on all six directions of the orbifold, so as noted above the story will differ from the cases we are discussing here.

σ. The field σ is a neutral scalar in the $U(1)$ gauge multiplet of two-dimensional $(2,2)$ supersymmetry. The existence of the σ branch does not by itself imply that the closed-string physics is singular, but this turns out to be the case. In particular, one finds that σ is the vertex operator for the closed string state corresponding to the Kähler modulus, whose correlation functions diverge at the singularity [19]. In general, there are multiple fields $\sigma_a,\ a = 1,\ldots,h^{1,1}$ corresponding to the elements of $H^{1,1}$. The wavefunctions for other states (for example complex structure moduli) are not supported down the various σ branches.

Now let us consider the situation for the open strings. This can be approached by studying the linear sigma model of [18] on worldsheets with boundaries [20]. In principle we need to undertsand the details of this model to analyze the singularity structure, but we can already make a strong qualitative argument. One finds that at the singularity the open string theory also has an extra branch, where the open strings have Neumann boundary conditions [20]. Here it is one real dimensional; we will call it σ_R, as it arises from the real part of σ. But in this case the open string states do not correspond to elements of $H^{1,1}$, and therefore have no natural relation to the σ_a fields. In particular, at large radius the vertex operators for the open strings are linear in the charged fermions ψ (or their bosonic partners $\partial\phi$ depending on the picture). Since they are charged these vertex operators are never just proportional to the neutral field σ_a, even at small radius. The corresponding wavefunctions of open string states are suppressed down the σ_R branches since the charged fields become extremely massive for large σ; thus, we do not expect these vertex operators to have singular correlation functions at the conifold singularity. This suggests that the open-string theory living on the D0-branes is consistent by itself, without extra light degrees of freedom such as the D6-branes which propagate in bulk.

Another way to approach this issue is to study the D0- and D6-brane states in the low-energy effective action of the spacetime theory. D0- and D6-branes carry dual electric-magnetic charges in the low-energy N=2 supersymmetric Yang-Mills theory. From this point of view, an analogous question has been studied in quantum field theory. We can think of the heavy D0 branes as being very massive magnetic monopoles, and the light D6 branes as being light electrically charged particles. The question is then, to what extent do the heavy and slowly moving monopoles create electron-positron pairs? Although the analogy is not precise (the D0 branes have spin 2, and move around on a Calabi-Yau target space!), we can get some intuition by considering this case. The suppression of the Schwinger pair production effect for small fields (and therefore small monopole velocities) suggests that the bulk D6-branes are not pair-produced by the D0-branes in our setup.

4. Conclusions

We conclude that in the DKPS limit of Calabi-Yau compactifications, which we are instructed to take in order to find a matrix formulation [5,6], there is no sign of the infinite tower of light gravitons which complicate the matrix description of T^6 compactification. Instead, there is a single massless hypermultiplet [16]. So although the matrix description is by no means trivial, it is a considerable simplification over the spacetime theory which contains gravity.

At this point we should begin to ask what sort of theory this limit describes. Now that we know that states which would make the theory intractable do not haunt us, we need to know what states to include. One might ask, for example, whether the DLCQ of this theory has a finite or infinite number of degrees of freedom – in other words, is it a higher-dimensional field theory or non-gravitational string theory, and if so in how many dimensions does it live?

In the case of the torus, we know that we have 6+1-dimensional theory because there is an infinite tower of winding modes corresponding to each dimension of the torus, with masses which are multiples of $E_{\rm DKPS}$. These can then be described as momentum modes in some dual theory.

For curved backgrounds we do not have topologically stable winding sectors, but we may still have stationary string wavefunctions corresponding to strings with non-trivial spatial extent and enough of these could give us a field theory's worth of degrees of freedom. As an example, there seems to be a field-theoretic description of compactification on a general (i.e. non-orbifold) K3 surface which one may derive via a Fourier-Mukai transformation corresponding to T-duality in all four directions of the K3 [21]. The field theory should have low-energy modes corresponding to excitations of the gauge theory, and they should be dual to states of stretched open strings (since T-duality generally exchanges kinetic and tensile energy of the string).

It is not completely clear what happens in our case, as the string theory lives in the regime where quantum geometry is very important. There is a significant difference between our case and the torus and K3 cases. For us the DKPS limit (the conifold point on the moduli space) is at finite distance. On the torus and on K3, the relevant singularities are at infinite distance. This is related to the tower of winding strings coming down to zero mass there.

Ideally, one would be able to look at the annulus diagram for the open string states and count the states which have energies of order $E_{\rm DKPS}$ or less; we could do this by measuring the conformal dimensions of the Virasoro primaries as a function of the Kähler moduli, as we approach the conifold. Of course, in the quintic the only regimes where we know how to do this calculation are at the large radius limit and at the Landau-Ginzburg orbifold point. Indeed, some preliminary work has been done on describing D0-branes in the latter case [22]. Since we are interested in the conifold regime, these calculations do not directly apply to our setup.

Another issue which deserves study is the re-emergence of classical geometry. Our goal is the description of M-theory, rather than string theory, compactified on a Calabi-Yau. At finite N we describe the theory via type IIA with vanishing string coupling, and it appears that membrane instantons wrapped around S^1 times a two-cycle are important. In the large N limit such effects should go away, and we will need to understand the mechanism by which such effects decouple.

Finally, we should note that we have not addressed the problems pointed out in [23,24] with compactifications that break supersymmetry. It seems that one sensible interpretation, following [25], is that there is no reason to expect a simple correspondence between supergravity and matrix theory computations for finite N.

It is tempting to speculate that the simplification with respect to the T^6 case means that the matrix model in some sense prefers backgrounds with reduced supersymmetry. It will be interesting to consider further supersymmetry breaking down to $4d$ N=1 supersymmetry. There instanton effects will be of interest, but one can again begin in the DKPS regime in type I' theory on a Calabi-Yau. In particular, worldsheet instantons of the type I/type I' theory do not contribute to the superpotential and therefore do not lift radial moduli. Of course there are also qualitatively different problems in light-cone gauge compactifications down to four and fewer dimensions. Perhaps the story there will also depend in a crucial way on the number of unbroken supercharges in a given compactification.

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