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Journal

Journal of Cosmology and Astroparticle Physics, 2021(03)

ISSN

1475-7516

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Publication Date

2021-03-01

DOI

10.1088/1475-7516/2021/03/074

Peer reviewed

An Expansion of Well Tempered Gravity

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(Dated: December 9, 2020)

When faced with two nigh intractable problems in cosmology – how to remove the original cosmological constant problem and how to parametrize modified gravity to explain current cosmic acceleration – we can make progress by counterposing them. The well tempered solution to the cosmological constant through degenerate scalar field dynamics also relates disparate Horndeski gravity terms, making them contrapuntal. We derive the connection between the kinetic term K and braiding term G_3 for shift symmetric theories (including the running Planck mass G_4), extending previous work on monomial or binomial dependence to polynomials of arbitrary finite degree. We also exhibit an example for an infinite series expansion. This contrapuntal condition greatly reduces the number of parameters needed to test modified gravity against cosmological observations, for these “golden” theories of gravity.

I. INTRODUCTION

Current cosmological acceleration is an overwhelming characteristic of our universe, driving the expansion rate and shutting down the growth of large scale structure. Yet in seeking its origin, physics explanations almost invariably sweep under the rug the elephant in the room: the original cosmological constant problem that a much higher energy scale vacuum energy should have dominated the history of the universe, calling into question our understanding of how gravity reacts to vacuum energy [1–5]. Exploring a low energy cosmic acceleration by traversing a physics terrain where there is an elephant under the rug is an uncomfortable position.

Well tempering [6–9] aims to solve the original cosmological constant problem by employing a dynamical scalar field with certain degeneracy conditions in the equations of motion. To do so requires modifications of general relativity, which is also one of the favored approaches to explaining current cosmic acceleration. However, modified gravity is quite difficult to test against cosmological observations in a general manner, without a large number of model dependent assumptions and multiple free functions. By contraposing the two problems, well tempering shows a path toward resolving them both.

Modified gravity often works within Horndeski theory, the most general scalar-tensor theory giving second order equations of motion. While generally this involves four free functions, we take the simplest approach to setting the speed of gravitational waves to be equal to the speed of light so that $G_5 = 0$ and $G_4 = G_4(\phi)$. Then Horndeski gravity involves three free functions $K(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$ of two variables: the scalar field ϕ and its canonical kinetic form $X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2 = \dot{\phi}^2 / 2$ for a homogeneous scalar field in a Robertson-Walker spacetime.

To test against cosmological data, one could adopt the effective field theory approach [10–14] that reduces three functions of two variables to four functions of time, at the linear perturbation level. This has the drawback that one loses information from the nonlinear regime (including details of screening to satisfy solar system tests), where cosmological data can give great insight. If we wish to keep the full leverage, one generally has to assume a specific gravity theory, a specific functional dependence within that theory, and specific parameters within those functions. For example, one does not “test gravity” but says the data is or is not consistent with, say, “the class of $f(R)$ gravity with a specific function f and specific ranges of parameters appearing within that function f .” The conclusions tend to be highly model dependent. To put the general case of three functions of two variables in perspective, figuring out how to modify gravity yet obtain consistency with cosmological observations is like trying to figure out how to kick a football to score a goal when 1) the pull of gravity depends on the ball’s location and velocity, 2) the wind speed varies with the ball’s location and velocity, and 3) the ball’s mass depends on its location (e.g. $K(\phi, X)$, $G_3(\phi, X)$, $G_4(\phi)$ respectively).

In [8] we demonstrated that well tempering could not only address the original cosmological constant problem, but reduce the modified gravity parameter space from multiple functions to a small number (\sim four) of parameters. These give well defined, valuable theories that are predictive across the full range of cosmological observations, including on nonlinear scales, and could be regarded as the “golden” theories to use in cosmology, without climbing over the elephant under the rug.

The solutions in [8] were predominantly within shift symmetric theories, which have important protection against quantum corrections [15, 16], and gave particular solutions to the well tempered degeneracy equation by assuming various Ansätze for either $K(X)$ or $G_3(X)$, with the other then determined by the equations. The degeneracy equation

is a nonlinear differential equation and so other solutions can exist as well. Here we carry out a systematic analysis of all solutions having certain conditions discussed below.

In Section II we restate the key degeneracy equation giving well tempering and introduce a series expansion method. The remarkably simple solutions for a finite series are presented in Sec. III and we discuss some special cases as well, and how previous solutions are unified by our general expression. We present an example of an infinite series solution in Sec. IV. We summarize and conclude in Sec. V.

II. SERIES EXPANSION APPROACH

Well tempering works by reducing the equations of motion involving \ddot{a} and $\ddot{\phi}$, i.e. the expansion and scalar field evolutions, to have degenerate solutions for de Sitter spacetime, referred to as “on shell”. This empowers the scalar field dynamics to cancel the cosmological constant both on and off shell, i.e. for the full cosmological history. See [6] for details.

The equation guaranteeing the degeneracy is

$$\begin{aligned} & (M - 2g) \left\{ 3h\dot{\phi}K_X + 18h^2g - 6h^2M + \lambda^3 \right\} \\ & = (K_X + 2XK_{XX} + 6h\dot{\phi}g_X) \left[-h\dot{\phi}(M - 6g) + 2XK_X \right], \end{aligned} \quad (1)$$

where we have imposed shift symmetry so that $G_4 = (M_{\text{pl}}^2 + M\phi)/2$, and $G_3 = G_3(X)$, $K(\phi, X) = F(X) - \lambda^3\phi$. For notational convenience we write $g = XG_{3X}$, where a subscript X denotes a derivative with respect to X , and $h \equiv H_{\text{dS}}$ is the de Sitter value of the expansion rate.

This nonlinear differential equation relates the Horndeski terms K , G_3 , and G_4 . A variety of solutions were presented and analyzed in [8]. Here we look for more general solutions using a series expansion method like that introduced and used to good effect in [7]. Note that in their work they assumed $K = X - V(\phi)$, which turns the nonlinear equation into a simpler one (but their nonassumption of shift symmetry makes it a partial differential equation). Here we impose shift symmetry, justified for its protection against quantum loop corrections, but allow general $K(X)$ with the permitted $\lambda^3\phi$ tadpole term.

By resummation and judicious insight, a series expansion can provide the general functional dependence $K(X)$ or $G_3(X)$. This method can deliver an algorithm for generating new solutions, and some general principles.

Since the quantities entering the degeneracy equation are K_X and g , we will carry out series expansion of these; they can be rewritten in terms of K and $G_3(X)$ as desired. Thus,

$$g = \sum a_n X^{n/2} \quad (2)$$

$$K_X = \sum b_p X^{p/2}. \quad (3)$$

The sums go from n_{min} to n_{max} and p_{min} to p_{max} respectively, and we will see there are relations between these values. The degeneracy equation becomes

$$\begin{aligned} 0 &= 6h^2M \sum a_n \left(5 + n - \frac{\lambda^3}{3h^2M} \right) X^{n/2} - 36h^2 \left(\sum a_n X^{n/2} \right) \left(\sum a_n (1+n) X^{n/2} \right) \\ &+ Mh\sqrt{2X} \sum b_p (4+p) X^{p/2} - 2X \left(\sum b_p X^{p/2} \right) \left(\sum b_p (1+p) X^{p/2} \right) \\ &- 6h\sqrt{2X} \left[\left(\sum a_n X^{n/2} \right) \left(\sum b_p (1+p) X^{p/2} \right) + \left(\sum a_n (1+n) X^{n/2} \right) \left(\sum b_p X^{p/2} \right) \right] \\ &- 6h^2M^2 + M\lambda^3. \end{aligned} \quad (4)$$

III. FINITE SERIES SOLUTION

One solves Eq. (4) by equating terms order by order in X to obtain relations between a_n and b_p , and then summing the series to derive functional relations between K and G_3 (or g), with or without M from the G_4 term. We begin by considering a finite series.

A. General Results

The results are

$$K_X = -3h\sqrt{2}X^{-1/2}g + h\sqrt{2}X^{-1/2}\left(M - \frac{\lambda^3}{6h^2}\right) \quad [\text{Branch A}] \quad (5)$$

$$K_X = -3h\sqrt{2}X^{-1/2}(g - a_0) - \frac{3h\sqrt{2}}{2}X^{-1/2}\int dX \frac{g - a_0}{X} + b_{-1}X^{-1/2} \quad [\text{Branch B}]. \quad (6)$$

Appendix A shows the steps in the derivation but one can verify the solutions by direct substitution into Eq. (1).

The coefficient b_{-1} entering as $K_X^{[-1]} = b_{-1}X^{-1/2}$ plays a special role (since $K_X^{[-1]} + 2XK_{XX}^{[-1]} = 0$ in the degeneracy equation) and helps distinguish the two branches. For Branch A,

$$b_{-1} = h\sqrt{2}M - \frac{\lambda^3}{3h\sqrt{2}} - 3h\sqrt{2}a_0 \quad [\text{Branch A}], \quad (7)$$

i.e. b_{-1} is specifically connected to a_0 , the constant term in g or the contribution $G_3^{[0]} = a_0 \ln X$ (recall from [8] that constant g is a hallmark of Brans-Dicke type scalar-tensor theories as well as $f(R)$ gravity and No Slip Gravity). The quantity b_{-1} is also connected to the G_4 mass M and the tadpole scale λ for Branch A, so all the functions are woven together.

Branch B has different contrapuntal conditions between the functions, with

$$b_{-1} = \{-h\sqrt{2}M, \text{arbitrary}\}, \quad a_0 = \frac{M}{2}, \quad \lambda^3 = 3h^2M \quad [\text{Branch B}]. \quad (8)$$

Here the connections are more concrete between G_3 , G_4 , and K , i.e. a_0 , M , and λ , but b_{-1} also has the possibility of being arbitrary, as we discuss in the next section.

Equations (5) and (6) are general solutions. They cover many of the particular solutions given in [8] (hereafter called HV). For example, HV Eq. 3.36 is a Branch A solution; Eqs. 3.34 and 3.38 are Branch B solutions. We have essentially succeeded in deriving a unified solution for all the individual solutions obtained in HV.

Some particular solutions need or merit special treatment and we deal with these in the next subsections.

B. Special Case: Truncation Below b_{-1}

Under the standard solutions of the previous section, for some a_n , even $a_{n_{\min}}$, there must be a b_{n-1} , as shown in Eq. (A3). However, we see that the $a_{n_{\min}}^2$ contribution (the second term in Eq. 4) vanishes when $n = -1$ and so this term is an exception: if we set $n_{\min} = -1$ we are free to set $b_{-2} = 0$ (and all $b_{p < -2}$ are zero likewise). This may be considered attractive in that it means that K does not have any negative powers of X , so when the field rolls slowly there is no blow up of the kinetic term. Thus we consider this special case $n_{\min} = -1$, i.e. $a_{n < -1} = 0$.

With $b_{-2} = 0$, for the next higher order equation, which involves $X^{-1/2}$ terms, b_{-1} actually cancels out of the degeneracy equation. We continue going through the intermediate powers and find that for Branch A, b_{-1} takes the form in Eq. (7), *unless* $a_{n > 0} = 0$ and $a_0 = M/2$, in which case b_{-1} remains arbitrary. The value of a_{-1} does not affect b_{-1} under these conditions. For Branch B, Eq. (7) reduces to $b_{-1} = -h\sqrt{2}M$; however, if the above conditions hold and we also require $a_{-1} = 0$, then b_{-1} is arbitrary. This explains the “ b_{-1} arbitrary” possibility mentioned in the previous subsection.

Cases with arbitrary b_{-1} can be seen in HV Eqs. 3.31, 3.35, and 3.39, all Branch B solutions, and Eq. 3.37, a Branch A solution but where the arbitrary b_{-1} arises from an arbitrary term in a_0 , i.e. it still follows Eq. (7).

C. Special Case: $n_{\min} = 0$, $n_{\max} = 1$

One other special case of note is truncation where $n_{\min} = 0$ and in addition $n_{\max} = 1$, i.e. $g = sX^{1/2} + a_0$. This gives only three equations, so in addition to determining b_{-1} and b_0 they must fix a_0 . The result shows that despite the tight restrictions this case nevertheless follows the general Eqs. (5) and (6). That is, Branch A gives

$$g = sX^{1/2} + \frac{M}{3} - \frac{b_{-1}}{3h\sqrt{2}} - \frac{\lambda^3}{18h^2} \quad (9)$$

$$K_X = -3h\sqrt{2}s + b_{-1}X^{-1/2}, \quad (10)$$

with b_{-1} arbitrary and $\lambda^3 \neq 3h^2M$, following from Eq. (5), and Branch B, i.e. Eq. (6), yields

$$g = sX^{1/2} + \frac{M}{3} + \frac{\lambda^3}{18h^2} \quad (11)$$

$$K_X = -6h\sqrt{2}s - \frac{\lambda^3\sqrt{2}}{3h}X^{-1/2}. \quad (12)$$

We can show this as follows. The X^1 equation gives $b_0 = \{-3, -6\}h\sqrt{2}a_1$, as expected from Eq. (A2); for the choice $b_0 = -3h\sqrt{2}a_1$, i.e. following Branch A, the $X^{1/2}$ equation gives Eq. (7) for b_{-1} , but for the choice $b_0 = -6h\sqrt{2}a_1$ then b_{-1} cancels out and instead the equation imposes

$$a_0 = \frac{M}{3} + \frac{\lambda^3}{18h^2}, \quad (13)$$

so this is a hidden version of Branch B. The hidden aspect arises because of the lack of extra equations that would impose the usual additional consistency conditions such that $\lambda^3 = 3h^2M$ and hence $a_0 = M/2$. Finally, the X^0 equation determines b_{-1} following Eq. (7), which would only lead to Eq. (8) upon setting the additional consistency conditions $a_0 = M/2$, $\lambda^3 = 3h^2M$, which do not apply here. Thus we have $g = sX^{1/2} + a_0$ leading to HV Eq. 3.37 for Branch A and HV Eq. 3.33 for the ‘‘hidden’’ Branch B.

D. Special Case: $\ln X$ Terms in K_X

A finite expansion in powers will not always work. Before we move on to an infinite series in Sec. IV, let us consider the special case where K_X contains terms involving $\ln X$. This is particularly of note since [8] did find solutions involving $\ln X$. Equations (21) and (22) below provide the final solutions. To derive them we begin by expanding g and studying the form of the resulting degeneracy equation, writing

$$g = \sum a_n X^{n/2} \quad (14)$$

$$K_X = X^{-1/2} B(X), \quad (15)$$

where K still has a tadpole term $-\lambda^3\phi$.

The degeneracy equation becomes

$$0 = XB_X \left(4B + 12h\sqrt{2} \sum a_n X^{n/2} - 2h\sqrt{2}M \right) + h\sqrt{2}B \left[6 \sum a_n (1+n) X^{n/2} - 3M \right] \\ - 6h^2M \sum a_n \left(5+n - \frac{\lambda^3}{3h^2M} \right) + 36h^2 \left(\sum a_n X^{n/2} \right) \left(\sum a_n (1+n) X^{n/2} \right) + 6h^2M^2 - M\lambda^3. \quad (16)$$

A term involving $B \sim X^m \ln X$ has nothing to cancel against if $m \neq 0$, so we allow only a term like $B \sim \ln X$, besides standard powers. Writing

$$B = b_r \ln X + \sum b_p X^{p/2}, \quad (17)$$

the degeneracy equation for the terms involving $\ln X$ becomes

$$0 = 4b_r \ln X \left(b_r + \frac{1}{2} \sum b_p p X^{p/2} \right) + h\sqrt{2}b_r \ln X \left[6 \sum a_n (1+n) X^{n/2} - 3M \right]. \quad (18)$$

If we set $b_r = 0$, there is no $\ln X$ term and we return to the power series of the previous sections. The solution for general $p \neq 0, -1$ is

$$a_p = b_p \frac{p}{3h\sqrt{2}(1+p)}. \quad (19)$$

However, looking at the terms in the degeneracy equation involving only powers and not $\ln X$, i.e. X^p , we find the only consistent solution is $b_p = 0$ for all $p \neq 0$. For $p = 0$, we find b_0 is arbitrary and

$$a_0 = \frac{M}{2} - \frac{b_r\sqrt{2}}{3h}. \quad (20)$$

For the X^0 order (without $\ln X$), the solution requires either $a_0 = M/2$ (implying $b_r = 0$ and hence reducing to the pure power expansion without $\ln X$ as in the previous sections) or $\lambda^3 = 3h^2M$.

Thus the final solution is $B = b_0 + b_r \ln X$, or

$$K_X = b_0 X^{-1/2} + b_r X^{-1/2} \ln X, \quad (21)$$

with a tadpole term $-3h^2M\phi$ in K , and

$$g = \frac{M}{2} - \frac{b_r \sqrt{2}}{3h} + a_{-1} X^{-1/2}, \quad (22)$$

where a_{-1} is arbitrary. This is the unique solution where K_X involves a $\ln X$ term, and is equivalent to HV Eq. 3.20. When $g = 0$, so $b_r = 3hM/(2\sqrt{2})$, we reproduce HV Eq. 3.11, and when $g = rM$ (as for $f(R)$ and No Slip Gravity), we obtain HV Eq. 3.19.

IV. INFINITE SERIES SOLUTION

When the series expansion is infinite then one must use the Cauchy product to determine terms at a certain order. Since there is no finite $n_{\max} = N$, we are no longer constrained by the lack of a counter term to $a_N^2 X^N$. This breaks the relation between a_n and b_{n-1} . There is very little one can say in general for such a situation. However, if one restricts the series in some way then some progress can be made. For example, consider the case where $b_{n \neq -1} = 0$. Then the only series product comes from the second term in Eq. (4), which we evaluate using the Cauchy product, except for separating out where one term has $a_{-1} X^{-1/2}$. If we choose a_n as a semi-infinite series, with $a_{n < -1} = 0$, then we can solve the equation.

Starting with the $X^{-1/2}$ power equation and working up, we can obtain all a_n and hence g . A particularly compact form obtains for $b_{-1} = -\lambda^3 \sqrt{2}/(3h)$, in that then the (semi)infinite series sums to the solution

$$g = \frac{M}{2} + \frac{c}{2} X^{-1/2} \left[1 \pm \sqrt{1 + \frac{2(3h^2M - \lambda^3)}{9h^2c} X^{1/2}} \right]. \quad (23)$$

Note that the lower root gives $a_{-1} = 0$, and hence g involves only nonnegative powers of X . (For the upper root $c = a_{-1}$.) When $\lambda^3 = 0$, the entire kinetic term $K = 2b_{-1} X^{1/2} - \lambda^3 \phi = 0$ and we have HV Eq. 3.45. When $\lambda^3 > 3h^2M$ we have a ‘‘speed limit’’ on X , i.e. the scalar field motion $\dot{\phi}$, to keep the function real (see for example [17–19]).

V. CONCLUSIONS

The reduction of the description of modified gravity from three functions of two variables to a single function of one variable, or a handful of constant parameters, could open up powerful leverage on scanning theory space to compare to observational data. Remarkably, we have shown this can be done with the bonus of solving the original cosmological constant problem – through well tempering – and protecting from at least some quantum corrections – through shift symmetry. By contraposing two highly challenging problems we solve both.

Equations (5) and (6) give general solutions relating the Lagrangian terms K , G_3 , and G_4 for a wide range of gravity theories, derived using a power series expansion under well tempering. We show that they unify the disparate solutions found piecemeal in [8], while going well beyond them, extending monomial or binomial cases to arbitrary finite polynomials. Logarithmic terms are included as well. Branch A solutions are what [8] referred to as the (\checkmark) class that gives a scalar field equation that becomes trivial on shell, while Branch B solutions are fully well tempering.

Allowing the series expansion to be infinite gives formal solutions but ones difficult to sum to a compact functional form. We exhibit one example where this can be done, generalizing a case from [8]. For this result, the full gravity theory can be described by four constant parameters: M , λ , h , and c , which can readily be sampled for likelihood estimation compared to data.

Such theories that possess highly desirable characteristics for fundamental physics – solving rather than neglecting the cosmological constant problem, and protecting against quantum corrections – soundness, and robust ability for full comparison with observations (including nonlinear scales, in principle) could be regarded as the favored ‘‘golden’’ gravity theories to work with. While exciting work remains to investigate further their detailed properties, they represent a significant step away from arbitrary functions toward true benchmarks.

ACKNOWLEDGMENTS

SAA is supported by an appointment to the JRG Program at the APCTP through the Science and Technology Promotion Fund and Lottery Fund of the Korean Government, and was also supported by the Korean Local Governments in Gyeongsangbuk-do Province and Pohang City. EL is supported in part by the Energetic Cosmos Laboratory and by the U.S. Department of Energy, Office of Science, Office of High Energy Physics under contract no. DE-AC02-05CH11231.

Appendix A: Derivation for Finite Series

To begin the analysis of the finite series case order by order, let us look at the maximum powers for each series. For $N \equiv n_{\max}$ we have one term going as X^N . To match this we must have $P \equiv p_{\max} = N - 1$, unless $N = 0$. That is, the K_X series must cut off at one less power than the g series. We find

$$b_{N-1} = -3h\sqrt{2}a_N \left(1 + \frac{1 \mp 1}{2N}\right) = \left\{-3h\sqrt{2}a_N, -3h\sqrt{2}a_N \frac{N+1}{N}\right\}. \quad (\text{A1})$$

Thus there are two branches of solutions, what we call Branch A and Branch B. We can then proceed to the next lowest power, $X^{N-1/2}$ and determine b_{N-2} from a_{N-1} . By continuing this process for X^n , for $N/2 < n \leq N$, we find

$$b_p = \left\{-3h\sqrt{2}a_{p+1}, -3h\sqrt{2}a_{p+1} \frac{p+2}{p+1}\right\} \quad \text{for} \quad 0 \leq p \leq N-1. \quad (\text{A2})$$

There are N equations, for $X^N, X^{N-1/2}, \dots, X^{(N+1)/2}$, and these define the N coefficients $b_{N-1}, b_{N-2}, \dots, b_0$. Now let us jump to the most negative power of X , i.e. $L \equiv n_{\min} < 0$ and evaluate the b_p with $p < 0$ working upward. One obtains basically the same expression as Eq. (A2), and again $p_{\min} = n_{\min} - 1$:

$$b_p = \left\{-3h\sqrt{2}a_{p+1}, -3h\sqrt{2}a_{p+1} \frac{p+2}{p+1}\right\} \quad \text{for} \quad L-1 \leq p < -1. \quad (\text{A3})$$

Again there are two branches, and we must choose the same branch for the negative powers as the positive powers. These determine the L coefficients $b_{L-1}, b_L, \dots, b_{-2}$.

There are several equations for the powers X^n with $-L/2 \leq n \leq N/2$ and only a single parameter b_{-1} left to determine. All these equations must give a consistent solution for b_{-1} . Note that more and more terms from Eq. (4) enter into these equations, but a consistent solution occurs for each branch. For branch A, we need

$$b_{-1} = h\sqrt{2}M - \frac{\lambda^3}{3h\sqrt{2}} - 3h\sqrt{2}a_0 \quad [\text{Branch A}], \quad (\text{A4})$$

and for branch B, the requirements are

$$b_{-1} = \{-h\sqrt{2}M, \text{arbitrary}\}, \quad a_0 = \frac{M}{2}, \quad \lambda^3 = 3h^2M \quad [\text{Branch B}]. \quad (\text{A5})$$

To summarize, all b_p are determined by Eq. (A2) (or equivalently Eq. A3), except for b_{-1} – special since for this order $K_X + 2XK_{XX} = 0$ – which is given by Eq. (A4) or (A5) for the respective branch.

We can now try to sum up the series to obtain a functional relation. For Branch A,

$$\begin{aligned} K_X &= \sum b_p X^{p/2} = -3h\sqrt{2} \sum_{p \neq -1} a_{p+1} X^{p/2} + b_{-1} X^{-1/2} = -3h\sqrt{2} X^{-1/2} \sum_{p \neq -1} a_{p+1} X^{(p+1)/2} + b_{-1} X^{-1/2} \\ &= -3h\sqrt{2} X^{-1/2} \sum_{n \neq 0} a_n X^{n/2} + b_{-1} X^{-1/2} \\ &= -3h\sqrt{2} X^{-1/2} g + h\sqrt{2} X^{-1/2} \left(M - \frac{\lambda^3}{6h^2}\right) \quad [\text{Branch A}]. \end{aligned} \quad (\text{A6})$$

For Branch B,

$$\begin{aligned}
K_X &= -3h\sqrt{2} \sum_{p \neq -1} \frac{p+2}{p+1} a_{p+1} X^{p/2} + b_{-1} X^{-1/2} = -3h\sqrt{2} X^{-1/2} \sum_{p \neq -1} \frac{p+2}{p+1} a_{p+1} X^{(p+1)/2} + b_{-1} X^{-1/2} \\
&= -3h\sqrt{2} X^{-1/2} \sum_{n \neq 0} \left(1 + \frac{1}{n}\right) a_n X^{n/2} + b_{-1} X^{-1/2} \\
&= -3h\sqrt{2} X^{-1/2} (g - a_0) - \frac{3h\sqrt{2}}{2} X^{-1/2} \int dX \frac{g - a_0}{X} + b_{-1} X^{-1/2} \quad [\text{Branch B}]. \tag{A7}
\end{aligned}$$

Recall that for Branch B, $\lambda^3 = 3h^2 M$ and the constant part of g is simply $a_0 = M/2$. Thus we have derived our general solutions Eqs. (5) and (6).

Substituting these back into the degeneracy Eq. (1), we find that Branch A gives a zero for the first factor on the right hand side, $K_X + 2XK_{XX} + 6h\dot{\phi}g_X$. This indicates the coefficient of $\ddot{\phi}$ in the scalar field equation vanishes on shell. This is what we called (\checkmark) models in [8]. Branch B does not zero out coefficients of $\ddot{\phi}$ and gets a full \checkmark .

Note that if desired we can set $M = 0$, removing the coupling to the Ricci scalar and making the G_4 term standard. For Branch B this will also make $\lambda^3 = 0$ and $a_0 = 0$.

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