Stokes-Mueller Formalism and the Poincaré Sphere for Wave Polarization State

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Abstract: Stokes parameters describe the polarization state of light. Mueller formalism can be used to manipulate these parameters as light passes through optical elements. The polarization state can then be visualized through the Poincaré sphere.

INTRODUCTION

In 1852, George Gabriel Stokes defined a set of parameters (which will be expressed in vector form) to describe, in a convenient fashion, the polarization state of light[1]. These parameters were named after Stokes later in 1942 by Francis Perrin after he discovered Stokes' work[2]. Quickly after that in 1943, physicist Hans Mueller developed a matrix method to manipulate the Stokes parameters as they pass through optical elements. This now allowed a mathematically convenient way of determining the polarization at the output Stokes vector of a system by knowing the initial Stokes vector and the elements in the system. Finally, the Poincaré Sphere, developed by mathematician Henri Poincaré in 1892, allows to visualize the polarization by extracting spherical coordinates from the Stokes parameters[3]. In the 21st century, Stokes-Mueller Formalism and the Poincaré sphere continue to be prominent methods to determine the polarization state of light. Furthermore, the Stokes-Mueller formalism continues to be improved, expanding its use cases in optics. As recently as 2019 a team from the University of Toronto developed Stokes-Mueller polarimetry for 3D non-linear cases[4].

METHODS

A. Stokes Parameters

The Stokes vector is a four-dimensional vector comprised of real, measured values defined as follows[3]:

$$\boldsymbol{S} = \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} \langle E_x^2 \rangle + \langle E_y^2 \rangle \\ \langle E_x^2 \rangle - \langle E_y^2 \rangle \\ 2\left\langle E_x E_y cos\left(\delta_y(t) - \delta_x(t)\right)\right\rangle \\ 2\left\langle E_x E_y sin\left(\delta_y(t) - \delta_x(t)\right)\right\rangle \end{pmatrix} = \begin{pmatrix} \langle I_{0^\circ} \rangle + \langle I_{90^\circ} \rangle \\ \langle I_{0^\circ} \rangle - \langle I_{90^\circ} \rangle \\ \langle I_{45^\circ} \rangle - \langle I_{135^\circ} \rangle \\ \langle I_{LC} \rangle - \langle I_{RC} \rangle \end{pmatrix} \quad (1)$$

Here, the brackets represent values that are time averaged values. S_0, S_1, S_2, and S_3 are the four Stokes parameters where S_0 is the light's total intensity, S_1 is the tendency for linear polarization in horizontal or vertical directions, S_2 is the tendency for linear polarization in either diagonal direction, finally S_4 is the tendency for circular polarization[3]. These parameters allow the derivation of necessary values to describe light's polarization.

B. Mueller Calculus

Mueller calculus allows the derivation of the output Stokes vector of a wave after passing through a system of optical devices. Each optical device has a 4 by 4 Mueller matrix (M_{device}) which contains all the information that describes the polarization characteristics of the device (birefringence, dichroism, and depolarization)[5]. For typical optical devices, Mueller matrices are already defined (see Ref. [6], Table 8.6). With an input Stokes vector ($\mathbf{S_{in}}$) and the aforementioned Mueller matrix we can find the output Stokes vector ($\mathbf{S_{out}}$) as follows:

$$\boldsymbol{S_{out}} = M_{device}\boldsymbol{S_{in}} \quad (2)$$

Conveniently, this can be applied to a multiple device system as follows:

$$\boldsymbol{S_{out}} = M_{device2}(M_{device1}\boldsymbol{S_{in}}) = M_{device2}\boldsymbol{S_{out1}} \quad (3)$$

C. Poincaré Sphere

With the Stoke parameters in-hand the focus can be turned onto the Poincaré sphere. The Poincaré sphere denotes a sphere of diameter S_0 with axes S_1, S_2, and S_3 as shown in Fig.1.b, which bounds the stokes vectors of any polarized light. A point on or within the Poincaré sphere denotes a polarization state with an ellipse as shown in Fig. 1.a bellow.

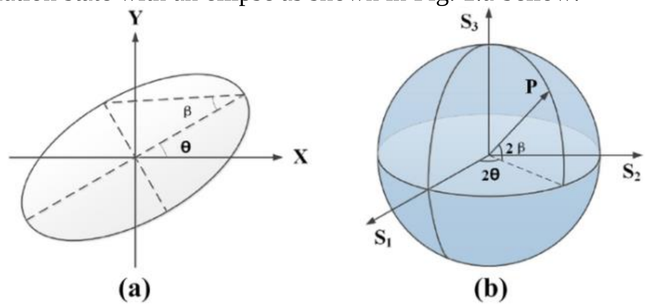

Fig. 1 (a) the polarization ellipse and (b) the Poincaré sphere (Ref. [7], Fig 2).

Where Azimuth angle (θ) and ellipticity angle (β) can be found from the Stokes vector as follows[3]:

$$\theta = \frac{1}{2}\tan^{-1}\frac{S_2}{S_1} \quad (4)$$

$$\beta = \frac{1}{2}\tan^{-1}\frac{S_3}{\sqrt{S_1^2+S_2^2}} \quad (5)$$

RESULTS AND INTERPRETATION

Through the use of Stokes-Mueller formalism, the output polarization state of a light wave through an optical system can be determined. Stokes vectors provide a mathematically convenient way of describing the polarization state of light while Mueller matrices provide the necessary information from optical devices. Consequently, Stokes-Mueller formalism uses matrix multiplication between Mueller matrices and an input Stokes vector. The output of the matrix multiplication is a Stokes vector that describes the polarization state of the light coming out of the optical system. The output Stokes vector can be mapped to a point in the Poincaré sphere which can then be used to determine the polarization ellipse of the light coming out of the system. This method provides a convenient way of calculating the effect of a system on the polarization state of light. Furthermore, optical devices can easily be swapped out or added by simply swapping out or adding the Mueller matrices of the device in the calculations, creating a sense of modularity in the calculations. When using a computer to perform Stokes-Mueller formalism this modularity proves to be very convenient.

CONCLUSIONS

In conclusion, by using Stokes vectors, it is possible to conveniently determine the polarization state of light. By further using Mueller matrices, the Stokes vectors can be modified to represent the output state of polarization of light going through an optical system. Finally, an understanding of the Poincaré sphere allows for the derivation of the polarization ellipse of a Stokes vector. Combining the three proves to be a useful and convenient method to visualize polarization state of light before and after it passes through a system. As time has passed, Stokes-Mueller formalism has been expanded to allow its use for an even wider variety of topics in the field of Optics not covered in this review, such as 3-dimensional polarimetry, which could be explored further.

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