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ABSTRACT

The nonlinear resonant response of a uniform plasma to an external plane-wave field is formulated in terms of the mismatch Δ_{NL} between the driving frequency and the time-dependent, complex, nonlinear normal mode frequency at the driving wavenumber. Our formalism is applied to computer simulations of this process, yielding a deduced nonlinear frequency shift. We analyze the time dependence of the nonlinear phenomena at frequency Δ_{NL} and at the bounce frequency of the resonant particles. The interdependence of the nonlinear features is described by means of energy and momentum relations.

While much study has been devoted to the topic of nonlinear plasma waves, propagating freely,[1] relatively little attention has been paid to the problem of driven nonlinear plasma waves.[2] Such a process arises in two situations of current interest: (a) The nonlinear interaction of two high-frequency electromagnetic waves $(\omega_1,\vec{k}_1)$, $(\omega_2,\vec{k}_2)$ occurs through an effective beat potential[3] at $(\omega_0 = \omega_1 - \omega_2$, $\vec{k}_0 = \vec{k}_1 - \vec{k}_2)$, which can excite a nonlinear Langmuir wave if the beat potential is strong and nearly resonant, i.e., if $\epsilon(\omega_0,\vec{k}_0) \ll 1$. (b) A single nonlinear Langmuir wave $(\omega_1,\vec{k}_1)$ may be amplitude-modulated at $(\omega_0,\vec{k}_0)$, and thereby drive a nonlinear ion wave[2a] if $\omega_0 \approx k_0 c_s$.

In the present paper, we study the nonlinear excitation in detail, taking the driving potential as given (i.e., we ignore the reaction back on it) and treating the excitation of a one-dimensional Langmuir wave for definiteness, with the nonlinearity due to electron trapping. We relate the plasma response to the frequency mismatch between the driving frequency ω_0 and the time-dependent complex eigenfrequency $\omega_{NL}(t)$ of a normal mode (i.e., a freely propagating wave), whose wave number $\vec{k}_0$ is that of the driver, and whose amplitude is that of the driven response.

The nonlinear normal mode frequency has a complicated time dependence, influenced by the interaction of resonant particles with the wave[4] and by the time dependence of the response amplitude. We shall demonstrate in the following that the nonlinear response is determined by the frequency mismatch $\Delta_{NL}(t) \equiv \omega_0 - \omega_{NL}(t)$, and therefore a fully self-consistent calculation of the nonlinear response and normal mode frequency is demanded. In our simulations, oscillations in ω_{NL} are observed at both the bounce or trapping frequency of a deeply trapped particle and at Re Δ_{NL}. Expressions will be derived for this process, which formally relate particle and wave momentum to nonlinear dissipation, and particle and wave energy to a nonlinear frequency shift.

To illustrate this physical process, and to apply our analysis, we have performed a set of computer simulations for a one-dimensional electron plasma, whose unperturbed distribution is Maxwellian [with thermal speed $v_e \equiv (T/m)^{\frac{1}{2}}$]. We have chosen the phase velocity of the driver $\omega_0/k_0 = 3v_e$. The corresponding linear normal-mode frequency is $\Omega_L = 1.17\omega_p$, and the linear Landau damping rate is $-\gamma_L = 0.03\omega_p$. The driving frequency was chosen to be $1.00\omega_p$, so that the linear frequency mismatch is $\Delta_L \equiv \omega_0 - \Omega_L \approx -0.2\omega_p$ [Fig. 1(a)]. The ratio of the response amplitude (related to the total electric field: self-consistent Coulomb field plus driver) to the driver amplitude is, on the basis of steady-state theory [see Eq. (2)], given by $\omega_p/2\Delta_{NL}$; so the linear theory predicts ~ -2.5 for this ratio.

In Fig. 2, we show results of a simulation where the driver potential amplitude was chosen as $2\,\tilde{\Phi}_0(t) = 0.05\,(m/e)(\omega_0/k_0)^2$, or equivalently $E_0^2/8\pi n_e T = 0.01$, where $\Phi_0(x,t) \equiv 2\,\tilde{\Phi}_0(t)\cos(k_0 x - \omega_0 t)$. The response $\tilde{\Phi}(t)$, $\Phi(x,t) \equiv 2\,\tilde{\Phi}(t)\cos(k_0 x - \omega_0 t)$, is seen to be about $(+5)\tilde{\Phi}_0$, rather than $-2.5\tilde{\Phi}_0$. The "resonance region" in velocity-space is given approximately by $v_T \equiv |e\,2\,\tilde{\Phi}/m|^{\frac{1}{2}} \sim (5 \times 0.05)^{\frac{1}{2}}\,\omega_0/k_0 \sim 0.5\omega_0/k_0 \sim 1.5v_e$, so that the resonance region is extensive, viz. $(3 \pm 1.5)v_e$ or $(1 \pm 0.5)\omega_0/k_0$ [see Fig. 1(b)]. Trapping has been verified by tracking individual particle orbits. In Fig. 3 are shown typical orbits in phase space, velocity vs. position, as observed in the frame traveling at the driven-wave phase velocity

ω_0/k_0. In Fig. 3(a) a particle becomes trapped and then is detrapped after only one bounce period. The orbit pictured in Fig. 3(b) is for a fairly deeply trapped particle from whose observed bounce period is determined a bounce or trapping frequency ω_T^0, which is slightly less than the standard bounce frequency $\omega_T^s \equiv k_0 v_T$.

To appreciate the relation between how deeply a particle is trapped and its observed bounce frequency, we consider the case of a particle in a time-independent field with energy given in the wave frame by

$$H = mv^2/2 - e\Phi \cos k_0 x .$$

The particle is trapped if $H < e\Phi$, and the phase space orbit can be described in terms of the usual Jacobian elliptic functions. For a trapped particle the orbital period τ is given by

$$\tau = 2\pi/\omega_T = 4K(\kappa)/\omega_T^s ,$$

where $0 \leq \kappa \equiv |(H + e\Phi)/2e\Phi|^{\frac{1}{2}} \leq 1$ and $K(\kappa)$ is the complete elliptic integral of the first kind. For a relatively large range of particle energies $0 \leq \kappa \leq 1/2$, and hence also of finite phase space excursions $\oint v dx$ in the wave frame [Fig. 3(b)], the corresponding bounce frequencies fall in a fairly narrow range $\omega_T^s \geq \omega_T \gtrsim 0.8\omega_T^s$.[5] Particles for which $1/2 < \kappa \leq 1$ are defined to be "weakly trapped" and have lower bounce frequencies satisfying $0.8\omega_T^s \gtrsim \omega_T \geq 0$.

The distribution of particle velocities and positions in phase space is shown in Fig. 2(b) and the velocity distribution function averaged over position in Fig. 2(c). We see that the trapping effect is quite large, so a large negative nonlinear frequency shift may be expected.[4] As shown below, this is deduced to be

$\sim -0.25\omega_0$, so that the nonlinear eigenfrequency is shifted from above ω_0 to below ω_0, and nearer to it [Fig. 1(a)]. Hence the response is larger than and opposite in sign to the linear prediction.

To proceed to an analytic formulation, we express the driving potential as $\Phi_0(x,t) \equiv \Phi_0(t) \exp(i\, k_0\, x) + \text{c.c.}$, and consider the self-consistent plasma potential $\Phi_{sc}(t)$ at the same wave number k_0 (suppressing harmonics of k_0; see Fig. 1a). The latter is determined by the Poisson equation $\Phi_{sc}(t) = (4\pi/k_0^2)\, \rho(t)$, while the charge density $\rho(t)$ is determined by the relevant kinetic equation. We postulate a (weakly) nonlinear susceptibility kernel $\bar{\chi}_{NL}(\tau)$ relating the charge density at t to the <u>total</u> potential $\Phi \equiv \Phi_0 + \Phi_{sc}$ at time $t - \tau$: $(4\pi/k_0^2)\, \rho(t) = -\int_0^\infty d\tau\, \bar{\chi}_{NL}(\tau)\, \Phi(t-\tau)$. (The nonlinearity is <u>implicit</u> in $\bar{\chi}$.) We now factor out the dominant (ω_0) time-dependence for each function of time, e.g., $\rho(t) \equiv \tilde{\rho}(t) \exp(-i\, \omega_0\, t) + \text{c.c.}$, and obtain

$$
\begin{aligned}
(4\pi/k_0^2)\, \tilde{\rho}(t) &= -\int d\tau\, \bar{\chi}_{NL}(\tau) \exp(i\omega_0\tau)\, \tilde{\Phi}(t-\tau) \\
&= -\int d\tau\, \bar{\chi}_{NL}(\tau) \exp(i\omega_0\tau - \tau d/dt)\, \tilde{\Phi}(t) \;.
\end{aligned}
$$

Substituting this into the Poisson equation $\tilde{\Phi} - \tilde{\Phi}_0 = (4\pi/k_0^2)\tilde{\rho}$, and using the usual definitions $\epsilon(\omega) \equiv 1 + \chi(\omega)$ and $\chi(\omega) \equiv \int_0^\infty d\tau\, \bar{\chi}(\tau) \exp(i\omega\tau)$, we obtain the formal equation for the total response amplitude $\tilde{\Phi}(t)$:

$$
\epsilon_{NL}(k_0, \omega_0 + i\, d/dt)\, \tilde{\Phi}(t) = \tilde{\Phi}_0(t) \;. \qquad (1)
$$

An analysis generalized to species-dependent driving potentials Φ_0 is presented in Ref. 6.

To introduce the nonlinear normal mode frequency ω_{NL} (at wavenumber k_0), we let the driver Φ_0 vanish, whence Eq. (1)

reduces to $\epsilon_{NL}(k_0,\omega) = 0$. We define ω_{NL} as the complex root of the nonlinear dielectric function $\epsilon_{NL}(\omega)$. Defining the nonlinear increments $\delta\epsilon \equiv \epsilon_{NL} - \epsilon_L$ and $\delta\omega \equiv \omega_{NL} - \omega_L$, where ω_L is the linear normal mode frequency [i.e., the complex root of the linear dielectric function $\epsilon_L(\omega)$], we have the relation $\delta\omega = -\delta\epsilon/\bar{\epsilon}_L(\omega_L)$ to lowest order, where $\bar{\epsilon} \equiv \partial\epsilon/\partial\omega$.

Returning to Eq. (1), we expand $\epsilon_{NL}(\omega_0 + i\, d/dt)$ in a Taylor series about ω_{NL} (where ϵ_{NL} vanishes), and obtain to first order

$$(\Delta_{NL} + i\, d/dt)\, \tilde{\Phi}(t) = \tilde{\Phi}_0(t)/\bar{\epsilon}_{NL} \tag{2}$$

where $\bar{\epsilon}_{NL} \equiv \partial\epsilon_{NL}/\partial\omega$ evaluated at ω_{NL} and $\Delta_{NL} \equiv \omega_0 - \omega_{NL}$. We note that ω_{NL} is implicitly a function of $\tilde{\Phi}$, so that this equation of evolution for $\tilde{\Phi}$ is nonlinear. Writing Eq. (2) in the form $\tilde{\Phi}(t) = (\Delta_{NL} + i\, d/dt)^{-1}\, \tilde{\Phi}(t)/\bar{\epsilon}_{NL}$, we see that, in the limit of slow variation, the amplitude of the response varies reciprocally as the complex nonlinear frequency mismatch Δ_{NL}. Thus for small mismatch, the response is large, thereby modifying ω_{NL} and the mismatch, and either enhancing or reducing the response (depending on the relative signs of the mismatch and of $\delta\omega$).

We can integrate Eq. (2) to obtain a formal expression for the potential:

$$\tilde{\Phi}(t) = -i \int_0^t dt'\, \bar{\epsilon}_{NL}^{-1}\, \tilde{\Phi}_0(t')\, \exp\left[i \int_{t'}^{t} dt''\, \Delta_{NL}(t'')\right] .$$

For $\tilde{\Phi}_0$ a constant, $\bar{\epsilon}_{NL} \approx \bar{\epsilon}_L$, and to lowest order $\Delta_{NL}(t) \approx \Delta_{NL}^{(0)}$ a constant, this expression becomes $\tilde{\Phi} \approx \tilde{\Phi}_0[1 - \exp(i\Delta_{NL}^{(0)}t)]/\bar{\epsilon}_L\Delta_{NL}^{(0)}$. This omits the time-dependent part of $\Delta_{NL}(t)$; therefore it somewhat distorts and exaggerates the modulation in the potential response produced by the frequency mismatch. In fact, $[1 - \exp(i\Delta_{NL}^{(0)} t)]$ periodically vanishes. However, $\tilde{\Phi}$ is never observed to vanish in the simulations, although it does exhibit considerable modulation (Fig. 4). In any case, the resonant particles $(v \approx \omega_0/k_0)$ see a time-dependent electric field with large variation at frequency $|\mathrm{Re}\, \Delta_{NL}|$. In the limit $|\Delta_{NL}| \ll \omega_T^s$, the modulation in $\tilde{\Phi}$ due to Δ_{NL} causes recurrent trapping and detrapping close to the separatix, but has only an adiabatic effect on the deeply trapped particles (Fig. 3). Consequently, there may then be additional modulation of the potential response due to the deeply-trapped particle oscillations.[4]

Since ω_{NL} depends on the history of $\tilde{\Phi}$, not just on its instantaneous value, and since no theory yet exists for this relationship (except qualitatively,[4a] or asymptotically in time[4]), we invert Eq. (2) to <u>deduce</u> the evolution of $\omega_{NL}(t)$ in terms of the complex response ratio $R(t) \equiv \tilde{\Phi}(t)/\tilde{\Phi}_0(t)$. Specifically, we have $\omega_{NL}(t) = \omega_0 - [R(t)\bar{\epsilon}_{NL}]^{-1} + i\, \mathring{R}/R$, when $\tilde{\Phi}_0$ is a step function in time. For $\bar{\epsilon}_{NL}$ we take $\bar{\epsilon}_{NL} = \bar{\epsilon}(\omega_{NL}) \approx \bar{\epsilon}_L[1 + \partial(\delta\epsilon/\bar{\epsilon}_L)/\partial\omega] \equiv (1 + \beta)\bar{\epsilon}_L$ and assume $|\beta| \ll 1$.

To illustrate the application of these ideas, we return to the simulation discussed above, for which Fig. 2 displayed its state at $\omega_0 t = 300$. In Fig. 4(a), we show the time-dependence of the relative amplitude of the response $R(t) \equiv r(t) \exp[i\,\Theta(t)]$. Equation (2) is then used to deduce the time-dependent complex frequency of the corresponding normal mode. The curve in Fig. 4(b) shows the total nonlinear damping rate $-\gamma_{NL} \equiv -\mathrm{Im}\,\omega_{NL}(t)$, while Fig. 4(c) shows the nonlinear frequency shift $\delta\Omega \equiv \mathrm{Re}\ \omega_{NL}(t) - \Omega_L$.

We observe that r, Θ, $\delta\Omega$ and γ_{NL} all oscillate at the same frequency, which corresponds to the average deduced mismatch $\langle \mathrm{Re}\, \Delta_{NL} \rangle = \langle \omega_0 - \mathrm{Re}\, \omega_L - \delta\Omega \rangle \sim 0.1\, \omega_0$. In the traces for the response r and γ_{NL} shown in Fig. 4, there are finer scale oscillations, which persist and whose frequency corresponds to the trapping frequency of deeply trapped particles $\omega_T^0 \lesssim \omega_T^s \sim 0.5\omega_0$. The frequency mismatch is considerably smaller, $|\mathrm{Re}\, \Delta_{NL}| \lesssim \omega_T^s/5$, and is directly related to the dominant modulation of the wave amplitude and phase as described by Eq. (2). Figure 1(a) shows the relative ordering of the various frequencies. There is little or no fine structure with frequency ω_T^s in the traces for $\delta\Omega$ and Θ in Fig. 4. This is in contrast to the case of freely propagating, weakly nonlinear waves, for which theoretical studies[4a] have shown that γ_{NL} oscillates at ω_T^s and $\delta\Omega$ at $2\omega_T^s$. As deduced in our simulations, $\delta\Omega$ and Θ evidence steepening, which indicates some harmonic content at frequencies $n|\mathrm{Re}\, \Delta_{NL}|$, $n \geq 2$, and further illustrates strongly nonlinear behavior.

Asymptotically in time the nonlinear damping vanishes, and the frequency shift approaches a constant value $\delta\Omega^{(0)}$. The latter may be compared with theories[4] of the frequency shift due to trapping in freely propagating waves of nearly constant amplitude, which require that $\omega_0/k_0 \gtrsim 4v_e$, $v_T = |2e\Phi/m|^{\frac{1}{2}} << k_0 v_e^2/\omega_0$, and hence $|\delta\Omega| << \omega_T^s$. These various assumptions are violated by our choice of parameters (which were dictated by economic considerations) and by the variation in Φ produced by Δ_{NL}. However, because $|\Delta_{NL}| << \omega_T^s$ the orbits of the deeply trapped particles and the associated nonlinear effects on the dielectric response ought to be similar to those for a freely propagating, nearly constant amplitude wave. Furthermore, the analytic theories indicate that it is the deeply trapped resonant particles which give the largest contribution to the nonlinear dissipation and frequency shift.[4] Therefore, we expect some qualitative agreement between the deduced frequency shift in our simulations and the analytic theories.

These theories predict for the ratio of asymptotic frequency shift to the standard bounce frequency: $\delta\Omega^{(0)}/\omega_T^s \sim -(1/3)u^5 \exp(-u^2/2)$, where $u \equiv \omega_0/k_0v_e$ is the ratio of phase velocity to thermal speed. For our parameters ($u = 3$), this predicts $\delta\Omega^{(0)}/\omega_T^s \sim -0.8$. Figure 4(c) indicates that $\delta\Omega^{(0)} \sim -0.25\omega_0$ and from the observed amplitude of the response [Fig. 4(a)] we calculate that $\omega_T^s \sim 0.5\omega_0$, or $\delta\Omega^{(0)}/\omega_T^s \sim -0.5$. The analytic theories also predict that, for fixed u, the asymptotic frequency shift varies as the trapping width (i.e., as $|\tilde{\Phi}|^{\frac{1}{2}}$). In our set of simulations we have varied the amplitude of the driver, so that the trapping width varied from $v_T = 0.7v_e$ to $1.9v_e$. The ratio of the deduced asymptotic frequency shift to the trapping width was found to be constant within 25%. Considering how deeply the trapping region digs into the thermal distribution [see Fig. 2] and the substantial amplitude and phase modulation of $\tilde{\Phi}$, the qualitative agreement with analytic theory of both the magnitude of the deduced $\delta\Omega^{(0)}$ and its dependence on $|\Phi|^{\frac{1}{2}}$ are quite encouraging.

In the Morales-O'Neil study[4a] considerable progress was made in theoretically understanding the effects of the deeply trapped particles by skillful use of energy and momentum relations. To further illustrate the important new qualitative features that arise when a wave is resonantly excited, we derive similar relations, assuming the following ordering: $\omega_0, \Omega_L \sim \mathcal{O}(\omega_p)$; $\delta\Omega$, $(\omega_0 - \Omega_L)$, $\beta\omega_0$, $(\partial/\partial t) \sim \mathcal{O}(\eta\omega_p)$; and γ_L, $\delta\gamma \sim \mathcal{O}(\eta^2\omega_p)$, where $\eta \ll 1$.

The momentum density P of the electrons evolves, under the influence of the driver, as $dP/dt = \langle \rho(x,t)(-\partial_x)\Phi_0(x,t) \rangle$, the brackets representing a spatial average. With manipulations similar to those leading to Eq. (1), we can express this, to $\mathcal{O}(\eta^2)$, as

$$(d/dt)(P - k_0 J) = -2\gamma_{NL} k_0 J , \tag{3}$$

where $J \equiv (\mathrm{Re}\ \bar{\epsilon}_L)\ k_0^2\ |\tilde{\Phi}|^2/4\pi$ is the wave action density. We interpret the expression on the left, total momentum less wave momentum, as the momentum of the resonant electrons. Its increase must be balanced by the decrease $(-\gamma_{NL})$ of the wave momentum.

A similar expression can be derived for the evolution of the particle kinetic energy density K. It is most convenient to choose the driver frame $(\omega_0 \to 0,\ \Omega_L \equiv \omega_0 - \Delta_L \to -\Delta_L)$, whence we obtain, after some algebra, to $\mathcal{O}(\eta^3)$,

$$(d/dt)\{K - (-\Delta_L J - k_0^2|\tilde{\Phi}|^2/4\pi)\} =$$

$$-[2d\delta\Omega/dt + \delta\Omega(d/dt)]J + \mathrm{Re}\ \bar{\epsilon}_L(k_0^2/4\pi)\quad [i\ \tilde{\Phi}^*(d^2/dt^2)\tilde{\Phi} + \mathrm{c.c.}] \tag{4}$$

The quantity in the parentheses on the left side represents the wave energy in the driver frame as modified by the linear frequency mismatch $\Delta_L = \omega_0 - \Omega_L$, less the field energy, and so is the sloshing energy. The expression in curly brackets is thus interpreted as the energy of resonant electrons. Its variation is related to the frequency shift $\delta\Omega$, on the right side of Eq. (4), as explained by Morales and O'Neil for the case of a freely propagating wave.[4a]

Equations (3) and (4) describe relations between the time dependence of the wave-induced resonant-orbit perturbations as represented by the left sides and the time dependence of the wave amplitude and phase, the dissipation, and the frequency shift on the right. Due to the modulation induced by the finite mismatch frequency $\Delta_{NL}(t)$, these relations are significantly more complicated than the corresponding ones discussed by Morales and O'Neil. Because $\omega_T^s \gg |\Delta_{NL}|$ in our simulations, there emerge relatively fast, small-scale oscillations at the trapping frequency and slower, large-scale ones at the mismatch frequency in the momentum and kinetic energy densities of the resonant particles, the wave amplitude, dissipation and frequency shift, as qualitatively suggested by Eqs. (3) and (4).

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FIGURE CAPTIONS

Fig. 1(a) The relative frequencies are shown for a typical simulation (see Fig. 2) in which the driving frequency ω_0 is chosen equal to ω_p. the linear normal mode frequency $\Omega_L = 1.17\omega_p$, and the linear Landau damping $-\gamma_L = -0.03\omega_p$. The asymptotic nonlinear normal mode-frequency is deduced to be $\langle \mathrm{Re}\ \omega_{NL} \rangle \sim 0.9\omega_p$ and hence the mismatch $\langle \mathrm{Re}\ \Delta_{NL} \rangle \equiv \omega_0 - \langle \mathrm{Re}\ \omega_{NL} \rangle \sim 0.1\omega_p$. The standard bounce frequency for a deeply trapped particle, $\omega_T^s \equiv k_0(2|e\tilde{\Phi}|/m)^{\frac{1}{2}} \sim 0.5\omega_p$, slightly exceeds the largest observed bounce frequency ω_T^0.

(b) The phase velocity of the driving potential ω_0/k_0 is chosen to equal $3.0\ v_e \equiv 3.0\ (T/m)^{\frac{1}{2}}$. The velocity $v_T \equiv (2|e\tilde{\Phi}|/m)^{\frac{1}{2}} \sim 0.5\ \omega_0/k_0$, which is a measure of the effective trapping strength of the wave, accounts for a resonance region extending from $(1 - 0.5)\omega_0/k_0$ to $(1 + 0.5)\omega_0/k_0$.

Fig. 2 Simulation of the resonant response of a Maxwellian (thermal speed v_e) electron plasma (periodic with uniform positive background, 2500 particles, $\omega_p\Delta t = 0.2$, and $k_0\Delta x = 0.1$) to a plane wave driving field, with parameters described in Fig. 1. For a typical simulation we exhibit the following at $\omega_p t = 300$:

(a) The driving field E_0 and the total field E, as functions of x, in natural units. Note that E is larger than E_0 by ~ 5 times, is nearly in phase with it, and has some harmonic structure.

(b) The electron distribution in phase space, which is strongly perturbed and has a large hole centered at the bottom of the potential well.

(c) The velocity distribution, in arbitrary units, which has an extended tail, in accord with (b).

Fig. 3 Phase space trajectories from a typical simulation, with parameters identical to those of Fig. 2, except for having a larger driving field by a factor of 2. Relative velocity, in arbitrary units, and position, in natural units, are measured as functions of time, from $\omega_p t/2\pi = 18.7$ to 28.0 in the driven-wave frame.

(a) A resonant particle becomes trapped, bounces once, and then is detrapped.

(b) A well trapped particle is observed executing many bounces. The velocity scale has been expanded compared to that in (a).

Fig. 4 For the same simulation as in Fig. 2, we show, as functions of time,

(a) the magnitude r and phase θ of the relative response function $r \exp i\,\theta \equiv \tilde{\Phi}/\tilde{\Phi}_0$, with modulation period $\tau_M = 2\pi|\mathrm{Re}\,\Delta_{NL}|^{-1}$ and bounce or trapping period $\tau_T = 2\pi(\omega_T^s)^{-1}$ indicated for reference;

(b) the deduced nonlinear damping rate $-\gamma_{NL}(t)$, and for reference the linear damping $-\gamma_L = -0.03\omega_p$; and

(c) the nonlinear frequency shift $\delta\Omega(t)$ and mismatch

$\mathrm{Re}\,\Delta_{NL}(t) = \omega_0 - \mathrm{Re}\,\omega_{NL}(t) = \omega_0 - \Omega_L - \delta\Omega(t)$.

The linear frequency mismatch $\Delta_L = \omega_0 - \Omega_L = -0.17\omega_p$ is smaller in magnitude than Δ_{NL} and opposite in sign.

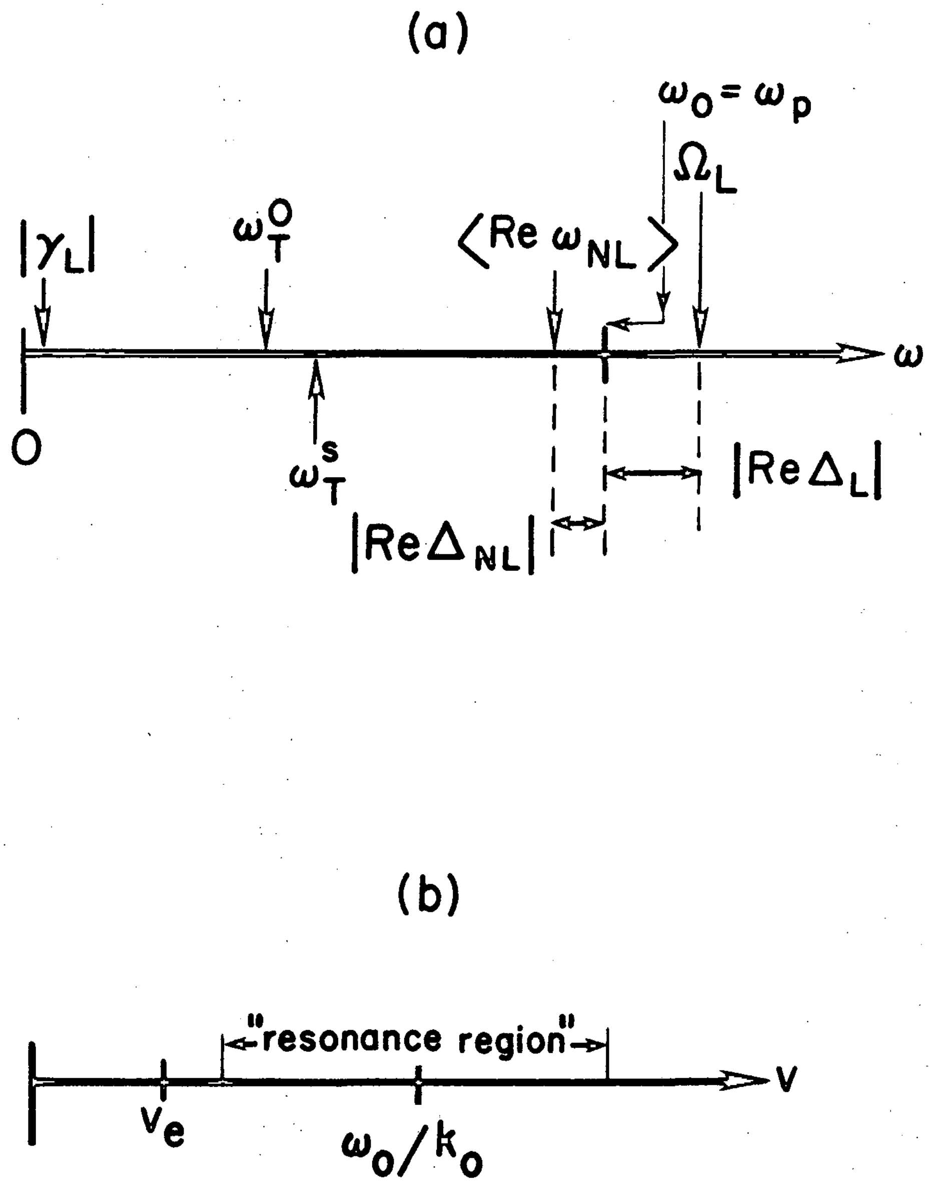

Fig. 1

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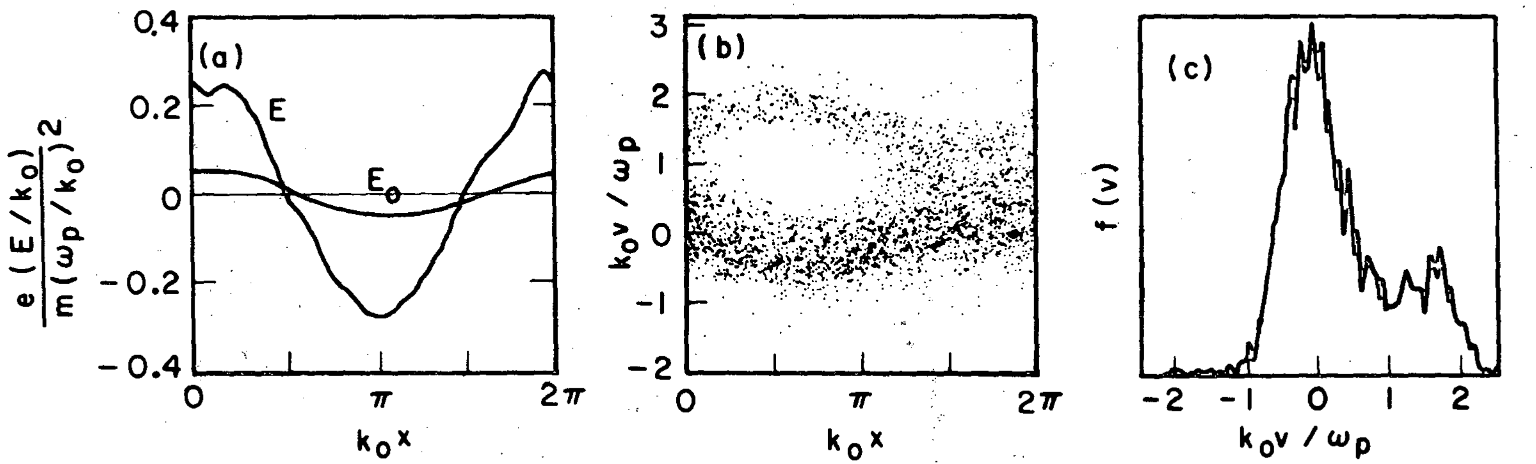

Fig. 2

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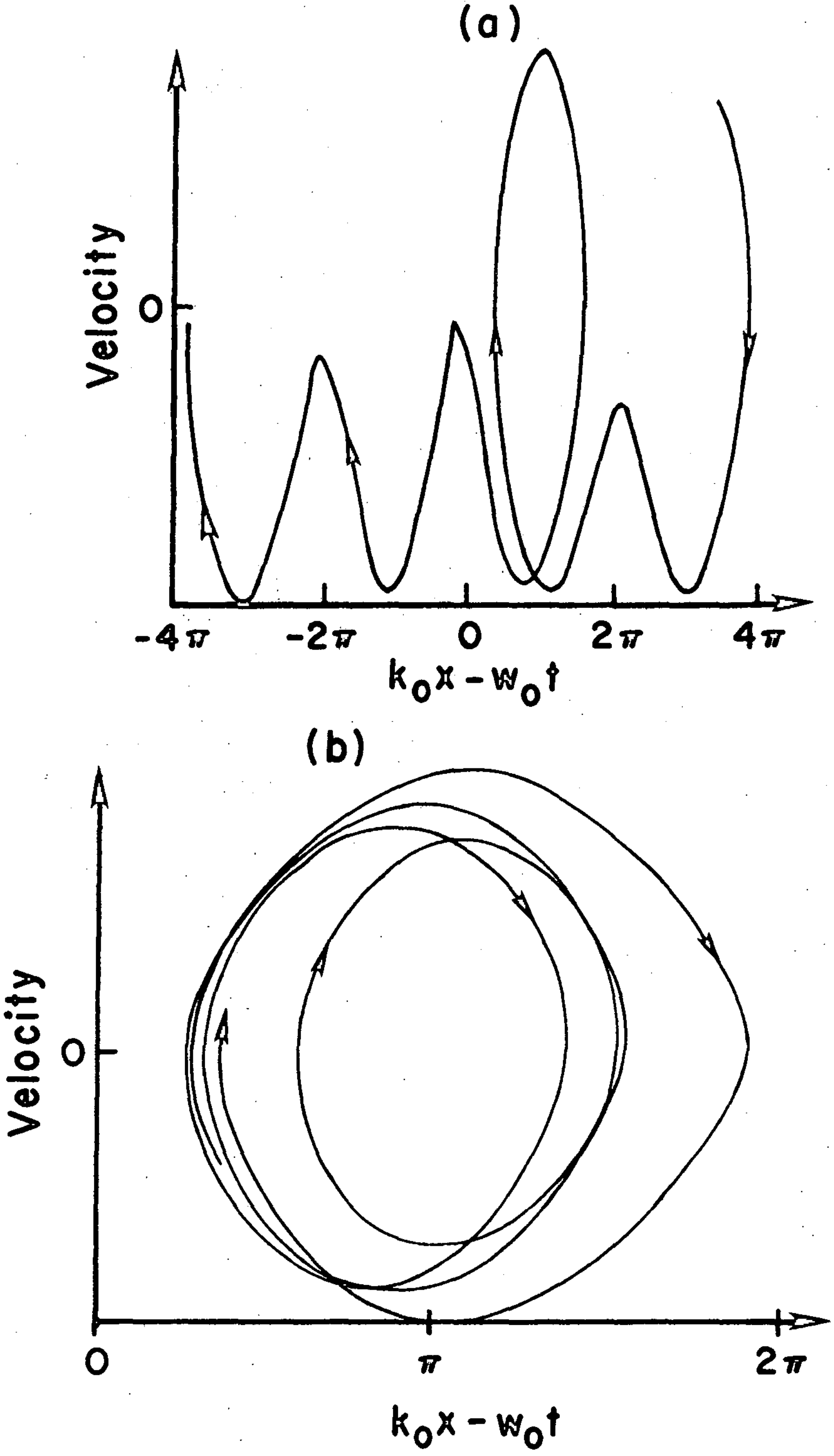

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Fig. 3

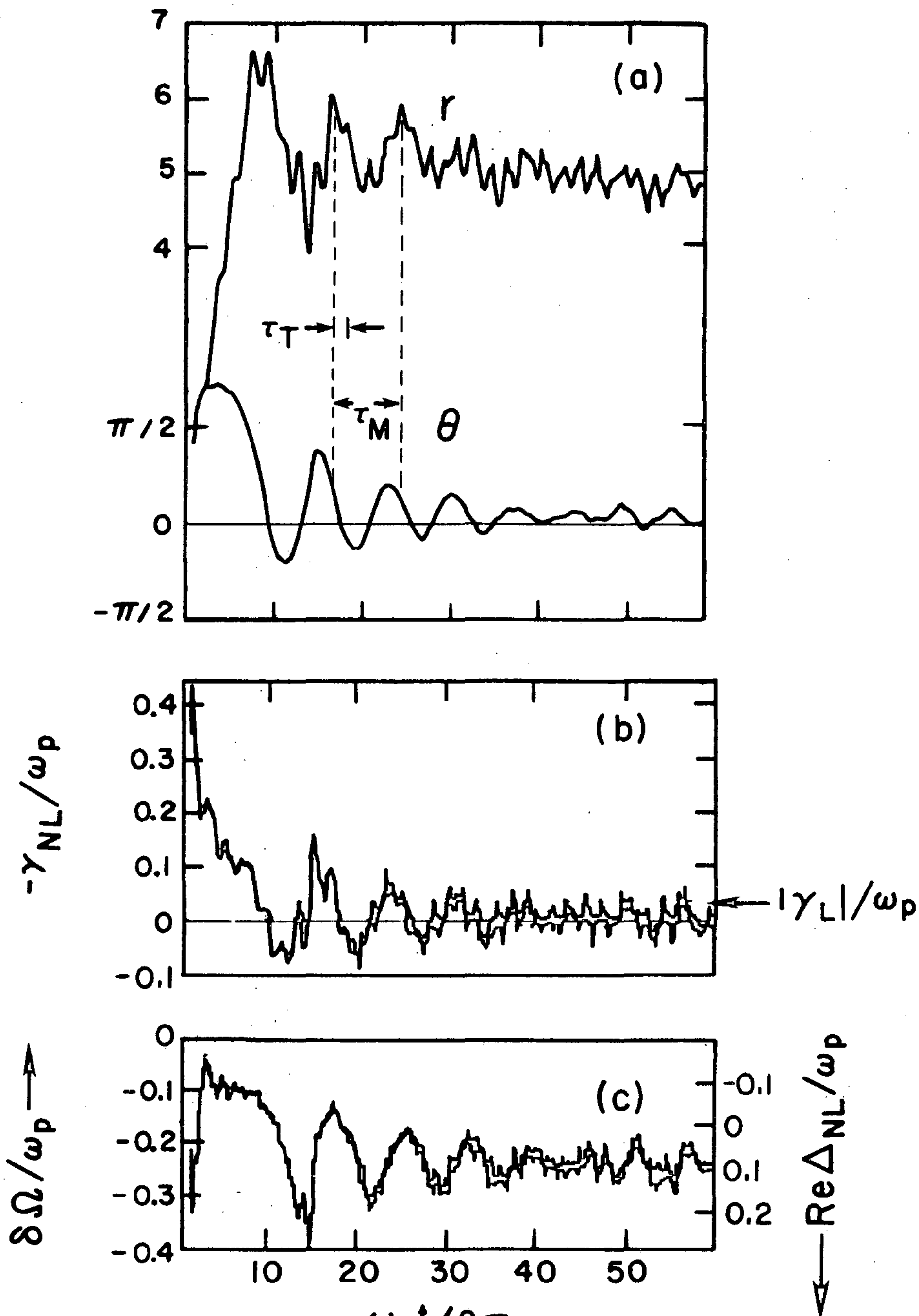

Fig. 4

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