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Preferred Mental Models in Spatial Reasoning

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Preferred Mental Models in Spatial Reasoning

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Abstract

The assessment of whether a statement is consistent with what has gone before is ubiquitous in discourse comprehension. One theory of the process is that individuals search for a mental model of a situation in which all the statements in the discourse are true. In the case of spatial descriptions, individuals should prefer to construct models that retain the information in the description. Hence, they should use strategies that retain information in an efficient way. If the descriptions are consistent with multiple models then they are likely to run into difficulties. We report two experiments in which the participants judged the consistency of spatial descriptions. The participants made more errors when later assertions in the description conflicted with the preferred models of earlier assertions. The results shed light on the sequential integration of relational assertions, and they show that participants exploit implicit constraints for strategical chunking.

Reasoning with Relations

When we think about which alternative to choose, we often integrate relational information. For example, before buying a camera you might read several published tests that each ranks cameras. One test might tell you that camera A is better than camera B, another that camera B is better than camera C. You can easily infer that camera A is also better than camera C. The predominant strategy to draw such transitive inferences is to integrate relations in a mental representation – an idea that goes back to William James and was revived by De Soto, London, and Handel (1965) and Huttenlocher (1968). Relational assertions are interpreted by constructing a spatial representation (a *mental model*) that serves as a structural analog of the situation under description and that supports inferences (Johnson-Laird, 1983). Neuroimaging studies (e.g., Knauff, Fangmeier, Ruff, & Johnson-Laird, 2003) have confirmed the involvement of spatial representations in relational reasoning. Verbal processes in reasoning with relational assertions (cf. Clark, 1969) may be a precursor to the construction of models. But, the use of mental logic (Rips, 1994) has received less empirical support.

The difficulty of relational reasoning problems varies with the number of possible interpretations. The camera tests in the example yield a determinate ordering of the cameras: A B C. However, a third test might tell you that camera D is better than camera B and consequently, camera A or camera D might be the best. There are two possible rank orders, ADBC and DABC, but the tests are not inconsistent. Hence, the information is indeterminate.

With an indeterminate set of assertions, multiple possibilities have to be considered to validate a general conclusion. Multiple model problems are reliably harder than single model problems in relational reasoning. The general fact that individuals have difficulty in keeping track of multiple possibilities holds across a range of deductive reasoning tasks (Goodwin & Johnson-Laird, 2005).

The multiple models of an indeterminate set of assertions might be narrowed down by further information. Imagine that in order to arrive at a decision between camera A and camera D, you head for a shop. You ask a sales assistant and learn that they do not sell camera A, but that camera C is better than camera D in any case. This is interesting, because this assertion is inconsistent with what you have read about the tests. According to the model theory, individuals notice such inconsistencies because they sequentially integrate assertions in a mental model, searching for a possibility in which all the asserted relations are true (Johnson-Laird, Legrenzi, Girotto, & Legrenzi, 2000).

Multiple possibilities are on the same footing logically, but they might not be equally obvious. In the introductory example, the information in the first two tests that yielded the ordering: A B C, can be summarized as “Camera B is between camera A and camera C,” where it is clear from the context that camera A is the best. However, in isolation such an assertion is indeterminate. For example, if a grocer gives his apprentice three crates and tells him to put the apples between the oranges and the grapes, he probably does not care whether the oranges end up to the left or to the right of the apples. If it matters, then a further assertion or the context itself may eliminate one possibility. Otherwise, "between" refers to a one-dimensional layout, and there are two possibilities:

oranges apples grapes
or
grapes apples oranges

Are these two possibilities equally likely and equally salient? In reasoning from a verbal description, individuals are likely to preserve the order in which the terms are written (Jahn, Johnson-Laird, & Knauff, 2005). Hence, given the assertion, "The apples are between the oranges and the grapes," they should construct first the order:

oranges apples grapes

We refer to this phenomenon as the *order of mention preference*. It probably reflects a culturally determined habit to scan from left to right (e.g., Spalek & Hammad, 2005). If individuals enter the objects in spatial models from left to right, the result is an order of mention preference (cf. Huttenlocher, 1968).

Such preferred interpretations of indeterminate descriptions can affect the difficulty of reasoning (Knauff, Rauh, & Schlieder, 1995; Rauh et al., in press). In a judgment of consistency, for example, if the preferred interpretation of the first assertion is consistent with subsequent assertions, they can be integrated into the model smoothly. But, if a later assertion is inconsistent with the preferred model, then the alternative model of the initial indeterminate assertion has to be checked to determine whether the later assertion is consistent with it. The model theory accordingly predicts that the evaluation of the consistency of a set of assertions should be more difficult if a mental model of earlier assertions is inconsistent with a later assertion (Johnson-Laird et al., 2000).

Experiment 1 – The Difficulty of Reordering

In Experiment 1, we compared problems that could be solved by a smooth extension of the preferred model with problems in which the preferred model ran into an inconsistent assertion. The participants had to write down a possible arrangement consistent with three assertions (see Table 1). The first assertion referred to a "between" relation among three entities in all the problems. Hence, the order of mention preference should bias participants towards one model. In the *neutral* problems, the second and the third assertion could be integrated in the preferred models of the first assertion. In the *reordering* problems, the third assertion was inconsistent with the preferred model, and the solution called for an alternative model of the first assertion (see Table 1). If participants started reordering problems with the preferred model, the third assertion forced them to reorder the tokens. The theory therefore predicts that the reordering problems will be more difficult than the neutral problems.

Method

Participants. Twenty-four students of the University of Tübingen served as paid participants. The mean age of the 19 women and 5 men was 23.1 (*SD* = 4.1).

Materials, Procedure, and Design. The reasoning problems consisted of three assertions that referred to horizontal one-dimensional layouts of four objects. Examples of experimental problems are shown in Table 1. In all problems, the first or second assertion referred to a "between" relation among three entities. The terms A, B, C, and D were replaced by the names of common objects in the experiment.

Table 1: Examples of reasoning problems in Experiment 1

Assertions		Possible Layouts			
		Neutral			
C between B and D					
A left of B	4	**ABCD**	DCAB	DACB	ADCB
C next to D	3	**ABCD**	DCAB		ADCB
		Reordering			
C between B and D					
A left of B	4	**ABCD**	DCAB	DACB	ADCB
C next to A	2		DCAB	DACB	
		Neutral			
C between B and D					
D next to A	4	**BCDA**	**BCAD**	DACB	ADCB
B next to C	4	**BCDA**	**BCAD**	DACB	ADCB
		Reordering			
C between B and D					
D next to A	4	**BCDA**	**BCAD**	DACB	ADCB
A left of B	2			DACB	ADCB

The initial two assertions in the experimental problems had four possible layouts (see Table 1). In problems with *neutral* third assertions there was at least one layout possible that was consistent with all three assertions and that preserved the order of mention. In contrast, in problems with *reordering* third premises, the third assertion ruled out all layouts that preserved order of mention.

In addition to 30 experimental problems, 12 problems had inconsistent sets of assertions. 6 further consistent problems were included in the study but are not discussed in this paper. The problems and instructions were in German. We constructed 24 different sets of objects. The four objects in a set were from a single domain and of comparable size, e.g., hammer, saw, drill, and pliers. We assigned these contents four times at random to the forms of problems in Table 1, and tested equal numbers of participants with each assignment.

The problems were displayed in black on white on a LCD screen. The presentation was self-paced. Each trial began with the initial two assertions. When participants pressed the space-key, the third assertion replaced the initial assertions. The third assertion was presented together with the prompt "Is there a layout for which all are true?". After the participants had responded "yes" or "no" with one of the response keys, they used the initial letters of the four objects to write down a layout on the answer sheet. The next trial started as soon as participants hit the space-key.

Every participant evaluated each of the 48 problems once. The trial sequence was pseudo-random to ensure an even distribution of inconsistent problems. Each session started with two practice trials.

Results and Discussion

The drawings for problems with neutral third assertions indicated whether participants were biased towards solutions that preserved the order of mention in the first assertion. The drawings were correct if they matched one of the layouts that were possible after the third assertion. The percentage of correct drawings for the neutral problems was 84.5% (see Figure 1). Among these correct drawings, the percentage of drawings that preserved the order of mention in the between-assertion was 94.4%. Thus, the presumed order of mention preference was confirmed. All 24 participants showed this bias ($.5^{24}$ Binomial test).

The mean reading time for the initial two assertions was 33.3 s for correctly solved problems (*SE* 2.4). These long reading times suggest that participants made an effort to memorize the information in the initial assertions in order to be able to consider alternative models.

Figure 1 shows the percentages of correct drawings and the response times for correctly solved problems. As predicted, participants solved neutral problems more often than reordering problems. Neutral problems were solved more often by 21 participants, only 3 participants solved reordering problems more often, Binomial test, $p < .0001$.

Reordering problems took longer to solve than neutral problems, $t(22) = 5.82$, $p < .001$ (there were 22 degrees of freedom, because 1 out of 24 participants did not solve any reordering problem).

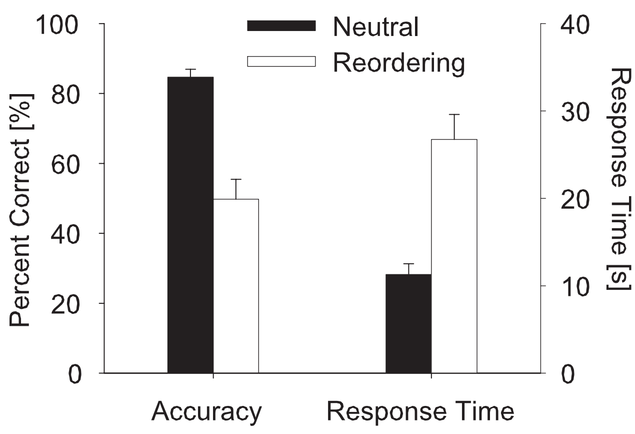

Figure 1: Accuracy and mean response times in Experiment 1 (error bars indicate the standard error).

Despite the lengthy encoding of assertions, which is shown in the reading times, the participants often failed to construct the alternative solution that was necessary for reordering problems. The successful attempts took two to three times as long as for neutral problems.

As Table 1 shows, two models remained possible after the third assertion in reordering problems. The remaining possibilities in reordering problems were less than in neutral problems, in which three or four models remained possible. However, among the possible models in neutral problems, the models that did not conform to the order of mention were hardly ever suggested by participants. Hence, the slightly lower number of possibilities in reordering problems in itself does not explain their difficulty. Reordering problems were difficult because participants had to change their interpretations of the initial assertions.

Experiment 2 – Adjacency and Linked Items

Indeterminate spatial relations such as "between" and "next to" are not the only way to introduce multiple possible orderings. For example, the assertions "The grapes are to the right of the oranges. The apples are to the left of the grapes." are consistent with two orderings:

oranges apples grapes
or
apples oranges grapes

Effects of indeterminacy in relational reasoning have been studied mainly with such assertions, which each determines the order of two terms. With a first subset of reasoning problems in Experiment 2, we tested whether the two possibilities in the example are equally likely to be envisaged by reasoners. Individuals may have a tendency to interpret "to the right of" as if it meant "to the right of and adjacent to" and likewise for "to the left of". Hence, the second assertion in the example creates an *adjacency conflict*. Adjacency can be fulfilled for only one assertion in an ordering.

In the first ordering of the example above, adjacency is fulfilled for the second assertion "The apples are to the left of the grapes". The apples that are introduced in the critical second assertion have been inserted inside. They break up the adjacency of oranges and grapes that have been introduced in the first assertion "The grapes are to the right of the oranges". In contrast, in the second ordering, the apples have been put outside and oranges and grapes remained adjacent. We designed problems to explore preferred solutions of such adjacency conflicts (see Table 2). The critical assertions in this set of problems introduced objects either leftward or rightward. We also manipulated whether the critical assertion was the second one (early) or the third one (late). Late adjacency conflicts might be resolved less often with inside insertions, because participants may be reluctant to break up an established ordering of three objects after two initial assertions.

Experiment 2 also examined a strategy that individuals might adopt to lower the difficulty of reordering problems. This strategy is possible if two adjacent objects in an ordering can be treated as though they were a single unit (cf. Halford, Wilson, & Phillips, 1998). Thus, orderings of four objects would be reduced to orderings of three objects. To examine this hypothesis, we used a second set of problems that consisted of reordering problems similar to the ones in Experiment 1. For *linking* problems, participants might notice that they could link two objects that formed the inner pair, and treat them as constant in all possible models ("A" and "C" in the examples in Table 2). If so, they had to consider only three entities when they had to reorder a preferred model because of an inconsistent assertion. Such linking problems should be easier than problems that contained no such linked items.

Table 2: Examples of reasoning problems in Experiment 2

Assertions		Possible Layouts
		Adjacency Conflict
		* indicates the preferred solution (see Table 3)
		Early conflict, leftward insertion of B
C right of A		
B left of C	2	*A**B**C **B**AC
D right of C	2	*ABCD BACD
		Late conflict, rightward insertion of D
B left of C		
A left of B	1	ABC
D right of A	3	A**D**BC *ABC**D** AB**D**C
		Reordering
		Linking possible (A and C)
C between B and D		
A next to C	4	**BACD BCAD** DACB DCAB
A right of D	2	DACB DCAB
		No linking
C between B and D		
A next to D	4	**BCAD BCDA** ADCB DACB
A left of B	2	ADCB DACB
		Linking possible (A and C)
A between B and D		
C between B and D	4	**BACD BCAD** DACB DCAB
D left of B	2	DACB DCAB
		No linking
A between B and D		
A between B and C	4	**BACD BADC** DCAB CDAB
D left of B	2	DCAB CDAB

Method

Participants. Twenty-three students of the University of Tübingen who had not taken part in the first experiment served as paid participants. The mean age of the 20 women and 3 men was 24.3 (SD = 5.1). One additional participant performed far below the average accuracy (only 6% correct overall) and was not included in the analyses.

Materials, Procedure, and Design. As in Experiment 1, the problems consisted of three assertions that described horizontal layouts of four objects. Examples of problems are listed in Table 2. A first subset of problems aimed at model preferences for sets of assertions with *adjacency conflicts*. The conflict either arose *early* in integrating the second assertion or *late* in integrating the third assertion. In half of the problems, the critical object could be inserted at the leftmost position (*leftward* problems), in the remaining problems, the critical object could be inserted at the rightmost position (*rightward* problems).

With a second set of 5 problems, *reordering* and *linking* were studied. In all reordering problems, the preferred layouts that fulfilled the order of mention principle were ruled out by the third premise as in the reordering problems in Experiment 1. In two problems, the inner two objects ("A" and "C") could be identified after the second assertion, which yielded linking of these two objects possible (*linking* problems). In contrast, the remaining reordering problems were *no linking* problems. In addition to the experimental problems, 5 inconsistent problems were constructed. The same 24 sets of objects as in Experiment 1 were used to replace A, B, C, and D.

The procedure in Experiment 2 was similar to the procedure in Experiment 1. On each trial, the participants indicated with "yes" or "no" whether there was a possible layout, and finally produced a drawing of this layout on an answer sheet. Each participant evaluated 36 problems in total (16 problems with an adjacency conflict, 15 reordering problems, and 5 inconsistent problems). Six pseudo-random trial sequences and four random assignments of object sets to problems were used.

Results and Discussion

The participants' drawings showed their preferences. The adjacency conflicts in the first subset of problems were early or late, and the insertion of the critical object was leftward or rightward. Both variables affected the percentage of insertions inside an established model. Inside insertions sacrifice adjacency in the existing model in favor of adjacency of the critical assertion that introduces the conflict. Table 3 shows the percentages of inside insertions. They were more frequent early in a sequence of assertions when the established model consisted of two instead of three tokens. This bias held for leftward and rightward insertions. In addition, leftward insertions were more often inside insertions, rightward insertions were more often outside insertions.

Table 3: Percentages of inside insertions of the critical object to resolve adjacency conflicts in Experiment 2. With inside insertions, adjacency is preserved for the assertion that introduces the critical object.

Insertion of critical object	Inside insertions [%]	*N*	$\chi^2(1)$	*p*
Early leftward	69	71	10.27	.001
Early rightward	51	72	0.06	.807
Late leftward	58	78	1.85	.174
Late rightward	35	71	6.21	.013

What the data imply is first that three token models of two initial assertions are less often broken up by inside assertions than two token models of a single assertion. Hence, late insertions tend to be outside. Second, rightward insertions tend to be outside. Individuals construct models from left to right and so it is easier to add an entity on the right-hand end of a model than to add it on the left-hand end, which entails a mental shift of the model rightwards. The left to right construction of linear models is also shown by the order of mention preference and has already been noted in early experiments on linear reasoning (e.g., Huttenlocher, 1968).

Turning to reordering problems, we tested whether the opportunity to link items improved performance. As Figure 2 shows, mean accuracy of drawings was higher, if linking was possible, but the difference was not reliable. However, the reading times for the initial assertions in correctly solved problems were lower for linking problems than for control problems in which linking was not possible, $t(16) = 3.79$, $p = .002$ (there were 16 degrees of freedom, because 6 out of 23 participants did not solve any linking and/or "no linking" problem). In mean response times, the advantage with linking problems was also noticeable, but it was less pronounced, $t(16) = 1.55$, $p = .14$.

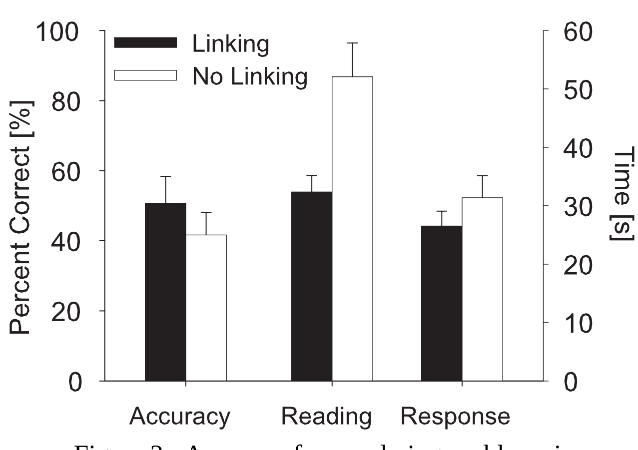

Figure 2: Accuracy for reordering problems in Experiment 2 with the respective mean reading times for the initial two assertions, and mean response times (error bars indicate the standard error).

Thus, as the reading times showed, the predicted advantage for linking items mainly affected the encoding of the initial assertions. But, trends in line with the expected advantage for linked items were observed in all performance measures.

General Discussion

The main aim of the experiments was to study the difficulty of changing an initial model in reasoning to consistency with spatial relations. According to the model theory, individuals focus on one possibility and run into difficulties if they have to switch to another possibility. We used a preference in interpreting "between"-assertions to bias participants towards one possibility. In Experiment 1, we corroborated the order of mention preference and showed that participants had difficulties in constructing the alternative possibility if the third assertion called for it. Experiment 2 showed that when a linking strategy was feasible, the initial assertions of reordering problems were easier to encode as reflected in reading times. Performance slightly improved with linked items, but it remained hard to change the initial model.

There are several ways in which individuals can try to keep track of alternative spatial layouts. They can try to hold all the possibilities in mind at the same time. This task is highly demanding. A strategy that places less of a load on working memory is to annotate just those items whose relative positions are indeterminate (Vandierendonck, Dierckx, & De Vooght, 2004). Or, as Rauh, Hagen, Knauff, Kuß, Schlieder, & Strube (in press) suggest, reasoners start with the preferred mental model and then vary this model to find alternative models that are also consistent with the assertions. Still another strategy is to build a single model and to try to remember the premises more or less verbatim if there are other possibilities. Our participants probably varied in the strategies that they used. Yet, our evidence suggests that they relied chiefly on constructing just a single model. The third author has implemented this strategy for temporal reasoning (see, e.g., Schaeken, Johnson-Laird, & d'Ydewalle, 1996). The program refers back to the premises when it encounters an inconsistency with its current model.

Participants' solutions for a subset of problems in Experiment 2 (problems with adjacency conflicts) shed light on the process of integrating spatial relations in mental models. The construction of models proceeds from left to right. This asymmetry is consistent with a culturally based left-to-right bias in visual attention. One of its consequences, which Experiment 2 demonstrated, is that it is easier to add an item to the right-hand end of a model than to the left-hand end of a model. This finding is in accordance with a recent computational theory of spatial reasoning (Ragni, Knauff, & Nebel, 2005).

Adjacency conflicts arise from a tendency to interpret "to the right of" as if it meant "to the right of and adjacent to" and likewise for "to the left of". Where does this principle of adjacency come from? A spatial description of the interior of a room usually proceeds in the sequence in which objects are encountered if the room is visually scanned ("gaze tours", e.g., Linde & Labov, 1975). It follows that assertions relating objects are likely to refer to objects that are adjacent in the room. If the assertions are taken as instructions for how to place objects rather than as descriptions of their location, it is justified to interpret them as implying adjacency. It is reasonable to assume that the speaker specifies the intended placement precisely and chooses the closest reference object. In discourse, spatial descriptions and instructions referring to a linear ordering of objects usually proceed in one direction and adjacency conflicts are avoided.

The integration of spatial relations is also affected by more subtle linguistic principles that we tried to counterbalance in our materials (Hörnig, Oberauer, & Weidenfeld, in press). This control was imperfect, but the effects of adjacency and left-to-right processing were so strong that such linguistic factors do not provide an alternative explanation for our results.

The problems that we have investigated are artificial. They have to be in order to control many variables that might affect performance. Nevertheless, individuals do reason about spatial relations in daily life. There are obvious cases in which individuals also use spatial representations to represent other sorts of relation, such as temporal order (cf. Boroditsky, 2000). The model theory postulates that spatial representations may even underlie the human ability to reason in general (Johnson-Laird, 1983).

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