A Familiarity-dependent Retrieval Threshold in ACT-R

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Abstract

In their current functional form, ACT-R's retrieval equations do not account for the left side of the RT-distance relation, that is, that as memory activation decreases, so does response time for retrieval failures. To accommodate this effect, I propose that the memory system uses the familiarity of the encoded object to gauge how much effort it should devote to retrieval. I quantify the degree of familiarity through the match score, which is the output of a global matching process. Familiarity, in turn, directly determines what the retrieval threshold should be. Adding a familiarity process orthogonal to recollection is in line with neuroimaging results, which uncover parallel familiarity and retrieval processes. The developments in this paper extend ACT-R's memory theory into a dual process theory.

Keywords: ACT-R, declarative memory, familiarity, retrieval threshold

Introduction

Perhaps uniquely among current theories of memory, ACT-R's memory theory (Anderson & Schooler, 1991; Schooler & Anderson, 1997) assumes a recall process, but no familiarity process. Despite being a single-process theory, it has successfully accounted for both responses and response times (RTs) of not only various recall processes (e.g., Anderson, Fincham, & Douglas, 1999; Anderson & Rader, 1999), but also of various recognition processes (e.g., Anderson, Bothell, Lebiere, & Matessa, 1998; Schneider & Anderson, 2012). Yet, at least one aspect of recall that ACT-R does not currently account for is the shape of the RT curve of "No" responses. Here I put forth a proposal of extending this memory theory such that it can also accommodate the RT distribution of recall failures. This proposal is consistent with recent neural evidence of separate familiarity and recall processes (e.g., Borst and Anderson, 2015). Specifically, I suggest that (1) familiarity in ACT-R is modeled with a global-matching process and (2) the retrieval threshold is strategically varied as a function of familiarity.

The Memory Theory behind ACT-R

ACT-R makes the distinction between representations, which inhabit the symbolic level, and the equations governing them, which lie at the subsymbolic level. At the symbolic level, ACT-R represents items in declarative memory as chunks, which are a collection of one or more slot-value pairs. Facts, such as "Otters hold hands" and "Cherry coke tastes like cyanide", and experiences such as "I rappelled off a bridge on Sunday" are all stored as chunks in declarative memory.

At the subsymbolic level, several equations determine whether chunks are likely to be retrieved or not and how long that will take. These equations take into account the prior history of encounter of the episodes or facts encoded in chunks as well as their relevance to the current context, and bind those together into a single quantity – a chunk's activation. Each chunk *i* has an activation, A_i, associated with it that quantifies its strength. Activation is a dynamic quantity that models the logarithm of the odds (i.e., the *log-odds*) that a chunk is needed at this point in time in this context to achieve the goal the agent strives for. Activation is composed of *base-level activation*, B_i, the *spreading activation*, SA_i, and noise, ε:

$$A_i = B_i + SA_i + \varepsilon \qquad (1)$$

The base-level activation is a function of the chunk's history:

$$B_i = \ln \sum_{k=1}^{n_i} t_k^{-d}, \qquad (2)$$

where the *decay parameter*, d[1], specifies the rate of forgetting over time, which is modeled with a power function. The power function was chosen, because the likelihood of encountering items in the real world also decays as a power function as time passes and the memory system is hypothesized to have adapted to this regularity (Anderson & Schooler, 1991). The parameter n is the number of encounters with the information that chunk *i* represents, and t_k is the time since the k^{th} encounter.

Spreading activation SA_i assesses a chunk's relevance to the current context, where the current context consists of all chunks currently in the focus of attention (i.e., all chunks currently in the *buffers* of the various *modules* that ACT-R consists of). SA_i assumes that chunks in declarative memory related to or previously encountered with chunks in buffers are more likely to be needed than those that are not. The amount of spreading activation to chunk *i* in declarative memory is a function of the associations between that chunk and the currently attended to chunks *j*:

$$SA_i = \sum_j W_j S_{ji}, \qquad (3)$$

where the associative strength, S_{ji}, between chunks *i* and *j* is weighted by the source activation, W_j, of chunk *j* in a buffer. The associative strengths, S_{ji}, between chunks is approximated by

$$S_{ji} = S - \ln(fan_j), \qquad (4)$$

where S denotes the maximum associative strength and fan_j is the number of chunks associated with a chunk *j*. The more

[1] Typically set to $d = 0.5$.

chunks are associated with a chunk in memory, the lower the associative strength between it and each of its associates becomes. Equation (4) is approximation of the Bayesian memory analysis that ACT-R is based on (Anderson & Milson, 1989) which assumes that each association of chunk *j* is equally likely to be needed. This approximation usually accounts sufficiently well for experimental regularities, but in some cases the full Bayesian equation needs to be summoned (see Anderson & Reder, 1999).

Equations 2-4 determine the activation components summed in Equation 1, which then determines probability of retrieval and retrieval failure as well as retrieval time. Specifically, whenever A_i is above the *retrieval threshold* τ, the chunk can be retrieved, while if it is below that threshold, the chunk is not sufficiently active to be retrieved, resulting in a probability of retrieval p_i as a function of threshold:

$$p_i = \frac{1}{1+e^{-\frac{\mu_{A_i}-\tau}{s}}}, \tag{5}$$

where $\mu_{A_i} = B_i + SA_i$ is the mean of the activation distribution. Retrieval time is also exponentially related to activation:

$$t_{retrieval} = Fe^{-A_i}. \tag{6}$$

The *latency factor F* scales the resulting quantity into units of seconds. If the activation is below the retrieval threshold, the resulting retrieval failure time is constant:

$$t_{retrieval\ failure} = Fe^{-\tau}. \tag{7}$$

ACT-R and Familiarity

Recognition tasks in ACT-R have typically been modeled with the same retrieval process that recollection is modelled with, but with different parameters (see Anderson, Bothell, Lebiere, & Matessa, 1998), whereby no fluency or familiarity processes are mentioned. Yet, familiarity processes are explicitly mentioned in at least one (decision) model constructed in ACT-R, the *fluency heuristic* (Schooler & Hertwig, 2005).

The Fluency Heuristic

The fluency heuristic is a memory-based decision strategy that infers which of two alternatives scores higher on a criterion by choosing the alternative that is more fluent (i.e., more familiar). The fluency heuristic does not require a separate familiarity processes running in parallel to retrieval to model fluency. Instead, in its original definition, the fluency heuristic operationalizes fluency as the time it takes an object to be retrieved (Schooler & Hertwig, 2005). Later, this heuristic was redefined to rely on the newly developed ACT-R module for prospective time interval estimation (Taatgen, van Rijn, & Anderson, 2007), thus comparing the subjectively perceived retrieval times of the two alternatives and choosing the one that is subjectively faster to retrieve (see Dimov, Marewski, & Schooler, 2017; Fechner et al., 2016; Marewski & Schooler, 2011). Still, even in its updated version, the fluency heuristic does not assume a separate familiarity process, but continues to rely on a recall process paired with a process for estimating the time recall takes.

Neural Evidence of Familiarity

While, for the most part, the memory and decision tasks modeled with ACT-R did not necessitate two separate mnemonic processes, recently several neuroimaging studies examining the time course of associative recognition have provided evidence in favor of two processes operating in parallel: a familiarity process and a recollection process. Specifically, due to fMRI not providing the temporal resolution necessary to observe sub-second retrieval processes, both EEG (Borst and Anderson, 2015) and MEG (Borst, Ghuman and Anderson, 2016) were used to record brain signatures during this retrieval task. The brain signatures during associative recognition indicate the existence of a familiarity process commencing in parallel with a recollection process and finishing typically before, but not substantially before the recollection process. How can we model this familiarity process with ACT-R?

A Global-Matching Process in ACT-R to Model Fluency

My first proposal is that familiarity in ACT-R is related to *blending* (Lebiere, 1998). Blending is a process in ACT-R's declarative memory that produces a weighted average of a quantity over all chunks in memory that hold a value of that quantity, whereby the contribution of each chunk is weighted by its activation. The output of blending is a chunk holding the weighted average value. This mechanism has been used to model mistakes that children make when engaging in arithmetic (Lebiere, 1999), choices in dynamic decision making tasks (e.g., Gonzalez & Dutt, 2011; Gonzalez, Lerch, & Lebiere, 2003) and belief updating in repeated games (Spiliopoulos, 2013) among others.

At the subsymbolic level, the blended chunk is described with a *match score M*, which is the analogue of activation for the blended chunk. Just as the blended value, the match score is a function of the activations of the set of all chunks included in the blending process (called the *match set MS*):

$$M = \ln \textstyle\sum_{i \in MS} e^{A_i}. \tag{8}$$

At first sight unintuitive, Equation 8 becomes clearer once we consider that activation is on a logarithmic scale (see Equation 2) and that all observables (Equations 5-7) are related to the exponent of activation. Summing the exponents of all relevant chunks' activation and then taking the logarithm renders the resulting match score equivalent to the activation resulting from the cumulative experience of all blended chunks. For example, if we consider only base level activation, the resulting match score would be:

$$\begin{aligned} M &= \ln \textstyle\sum_{i \in MS} e^{A_i} = \\ &= \ln \textstyle\sum_{i \in MS} e^{\ln \sum_{k=1}^{n_i} t_k^{-d}} = \\ &= \ln \textstyle\sum_{i \in MS} \sum_{k=1}^{n_i} t_k^{-d}, \end{aligned} \tag{9}$$

which is the activation a chunk would have had it had the prior history of all blended chunks.

While activation is interpreted as the log-odds of a chunk being needed, the match score is the log-odds of *any* chunk in declarative memory being needed. The specific relationship that I propose is that the familiarity of an input is quantified by the match score produced by the blending process, that is, familiarity serves as a coarse gauge if any of the input is relevant to the task at hand.

The RT-distance Relation and ACT-R

In a recognition task, responses are classified as Hits and False Alarms (whenever the response is "yes") and Misses and Correct Rejections (whenever the response is "no"). Whether responses are correct or not, there is a well-established relation between the time that those responses take (RT) and how frequently the item was presented in the experiment or encountered in life: the RT-distance relation (Koppell, 1977). This relation states that response time is fast whenever items were presented very frequently or very rarely, resulting in a memory trace with a very high or very low strength. However, whenever the memory strength is in the middle ground, close to the retrieval threshold, responses take more time. In other word, RT decreases as the memory strength of an item lies further away (either to the left or to the right) from the retrieval threshold (see Figure 1 for an idealization). Consequently, both Hits and False Alarms become faster the higher memory strength is of retrieved items. Moreover, both Misses and Correct Rejections speed up the lower memory strength of the items that fail to be retrieved.

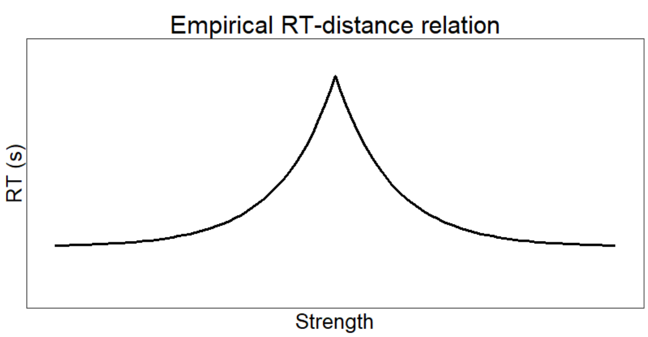

Figure 1: RT-distance relation in recall. A memory item with a medium strength provides ambiguous information about whether it has been encountered in the past or not and, consequently, requires a longer time to be retrieved. Memory items with either very high or very low strength are both responded to quickly.

In its current form, ACT-R's memory theory accounts for half of the RT-distance relation: that related to successful retrievals (Hits and False Alarms). Yet, following from Equation 7, when an item of memory fails to be retrieved (Misses and Correct Rejections), ACT-R predicts a constant RT (see Figure 2), which contradicts the empirically found RT-distance relation.

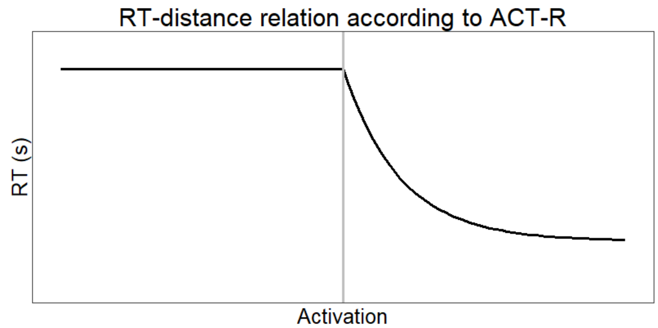

Figure 2: Relation between chunk activation and response time according to ACT-R. The grey line indicates the location of the threshold. Above the threshold, successful retrievals get progressively faster as one moves away from the threshold. Below the threshold, retrieval failures take a constant time irrespective of their distance from the threshold.

Fluency Determines Threshold

My second proposal aims to modify ACT-R's memory to account for the RT-distance relation using the fluency process earlier introduced. Specifically, I propose that the retrieval threshold τ is not constant, but that it is a function (i.e., the negative) of an item's familiarity (which I proposed to model with the match score):

$$\tau = -M. \tag{10}$$

Since M is the log odds that any chunk in memory is needed, -M is the log odds of no chunk being needed:

$$-M = -\log\left(\frac{need}{\neg need}\right) = \log\left(\frac{\neg need}{need}\right) \tag{11}$$

In plain language, my proposal can be interpreted as the memory system dynamically adjusting the amount of effort it is willing to invest into a retrieval (as described by the retrieval threshold) as a function of how likely it is that no chunk in memory is ever needed, which is estimated via the fluency signal. If the global fluency signal is weak, that is, if the odds that any chunk in memory is needed at this moment is low, then the system will invest less resources into a retrieval attempt and abort it earlier. On the other hand, if the fluency signal is strong, the memory system will be ready to invest a lot of time into retrieval as it is more certain that it will retrieve a relevant chunk, even though in practice it will invest very little time as a successful retrieval will soon arrive.

Resulting RT-distance relation

When chunks are very distinct or, in the extreme case, when all chunks spread 0 activation to each other, the predominating factor in the match score is the activation of the chunk being probed as only this chunk will be included in the match set MS. In this case, the resulting RT-distance relation is almost entirely symmetric (see Figure 3)[2].

[2] The code used to generate Figures 2, 3 and 4 can be found in the Appendix.

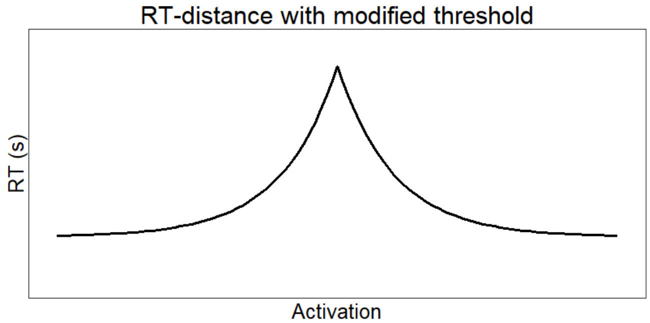

Figure 3: Relation between chunk activation and response time according to our modification of the retrieval threshold in ACT-R. I assume that the only chunk in the match set is the chunk being probed.

On the other hand, when other chunks are similar to the probed chunk and, thus, included in the match set, they increase the likelihood that any chunk in declarative memory will be needed. Consequently, the activation of the chunk representing the item being probed crosses the retrieval threshold at a lower value and, moreover, RT on retrieval failures decreases less and less steeply (see Figure 4)[3].

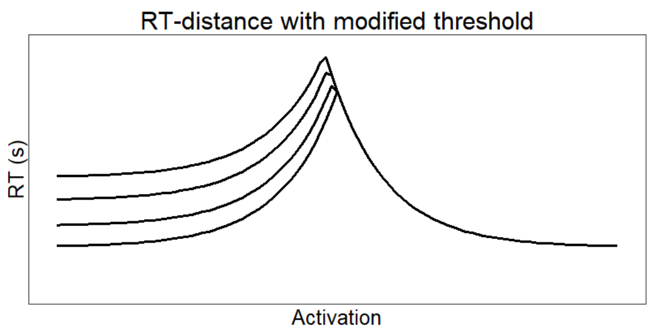

Figure 4: Relation between chunk activation and response time according to our modification of the retrieval threshold in ACT-R. The different curves correspond to various contribution to the match score of chunks that represent other items than the item being probed. As the similarity to the probed chunk increased, RT of retrieval failures increases.

Discussion and Conclusion

I proposed that familiarity in ACT-R is modeled with the match score from blending. This extends ACT-R's memory theory to a dual-process theory of memory. Moreover, I hypothesize that the memory system relies on the familiarity signal to assess the amount of effort it should invest in retrieval before aborting it. This allows ACT-R to account for the RT-distance relation. Finally, the prediction that retrieval failures will take longer when the probed chunk is confusable with other chunks in memory also follows from this new formulation. The modification interprets the blending module as a global-matching component of ACT-R's memory and puts it in the tradition of many a memory models in psychology, which include TODAM (Murdock, 1982), MINERVA 2 (Hintzman, 1984) and SAM (Gillund & Shiffrin, 1984; Raaijmakers & Shiffrin, 1980; for an overview of global-matching models, see Humphreys, Pike, Bain, & Tehan, 1989). I will proceed by briefly comparing the proposed extension of ACT-R to two related theories of memory and discuss the potential issues with the current proposal.

Comparison to Source of Activation Confusion

A theory that shares its lineage with ACT-R's is *Source of Activation Confusion* (*SAC*, Diana, Reder, Arndt, & Park, 2006). This theory has been used to model a wide variety of memory phenomena in various tasks, among which cued recall (Reder, Park, & Kieffaber, 2007), perceptual match effects (Diana, Peterson, & Reder, 2004) and feeling of knowing (Schunn, Reder, Nhouyvanisvong, Richards, & Stroffolino, 1997).

SAC is not based on the rational analysis of memory, yet many of the processes that it assumes are the same as those of ACT-R. First, it assumes that events and objects are encoded as chunks. Second, those chunks' activations are also separated into a base-level and spreading-activation components. Third, base-level activation decays with time as a power law, while spreading activation is a function of co-occurrence frequencies. Yet, there are at least two points of departure between ACT-R and SAC. First, SAC assumes that spreading activation slowly decays with time once the chunk that spreads activation is removed from the focus of attention, while in ACT-R this happens instantaneously. Second, in SAC a working memory of a limited capacity is populated with all chunks above a certain level of activation.[4]

Unlike ACT-R, SAC is a dual-process theory: it includes both a familiarity and a recall process. The familiarity (or feeling-of-knowing) process stems from retrieval of the concept node, which is the internal representation of the probed item. Activation then spreads to associated nodes, which leads to cued recall. Thus, unlike the current proposal of extending ACT-R's memory theory, in SAC familiarity does not result from a global-matching process, but from a retrieval of a single chunk.

Note, however, that adding a global-matching process that determines the retrieval threshold can also benefit SAC. First, just like ACT-R, SAC does not model the RT-distance relation related to retrieval failures, because it assumes a constant threshold. By adding a threshold that is inversely related to the global-matching signal, SAC should also be able to accommodate this relation. Second, SAC assumes that the familiarity (those related with retrieval of the concept node) and recollection processes (those related to retrieval of episode nodes) rely on different thresholds, whereby the

[3] I have relied on a single parameter to quantify the total amount of spreading activation that comes from chunks not corresponding to the presented item. See the Appendix for model code.

[4] Note that ACT-R's notion of working memory is more complicated in that it includes the buffers of all of its modules and potentially the content of the imaginal module, which stores task-relevant information.

concept threshold is typically higher than the episode threshold. I posit that Equation 10 would accommodate that a higher threshold for the concept than for the episode node. Specifically, during retrieval, activation spreads from the concept and the context into the episode node. If the activation of the episode is high enough, it will be retrieved. In this case the high activation of the episode node would also imply a high overall match score and, consequently, a low retrieval threshold. On the other hand, if the episode node cannot be retrieved and, instead, the concept node is relied upon, this would imply that the episode node has a lower activation. Consequently, the overall match score will be lower, implying a higher retrieval threshold.

Comparison to Retrieving Effectively from Memory

Retrieving effectively from memory (*REM*; Shiffrin & Steyvers, 1997) is a memory model that stems from the tradition of SAM. REM is a global matching model – it assumes that the recall probe is matched to all memory traces in parallel. Similar to ACT-R's memory theory, which is based on rational analysis, REM assumes that the memory system is optimally weighing signal and noise, and computing the likelihood that the probe has been encountered before in order to respond whether a presented item is recognized or not.

REM has been extended to also model various cued recall (e.g., Diller, Nobel, & Shiffrin, 2001) and free recall (e.g., Lehman & Malmberg, 2013) phenomena. To this end, REM was complemented with a trace recovery process, which is executed if the global matching process indicates a likely past experience with the probe. The current extension of ACT-R to include familiarity as a global matching process is similar to REM in that (1) it is a dual process model, (2) familiarity is a global matching process, (3) recollection is the recovery of a single memory trace, (4) whether effort is invested in recollection is strategically determined by the familiarity signal.

In addition to these similarities, there are several core differences between the two models. First, ACT-R assumes that base-level activation decays with time, while in REM and its extensions memory decay is generally absent. Second, ACT-R assumes that memory traces monotonically increase in activation with the number of encounters of the objects or events that they represent, while in REM a new trace can be created to store the encoded event/object or an already existing trace can be updated to store a more complete representation of the object. After a certain number of presentations, the object is perfectly encoded and no further updates of the memory trace(s) takes place. Which of those approaches provides a better description of memory phenomena is subject to further investigations.

Limitations of the Current Proposal

ACT-R's theory of memory assumes that our memory system makes a guess about which items of memory are most likely to be needed, what the cost and benefits of retrieval will be, and optimally combines those. The current analysis does not take into consideration costs and benefits. Yet, this might be problematic as the blending process is much more computationally intensive than the retrieval process itself: the activations of all chunks are computed and inserted into Equation 8 and, moreover, the blended value needs to be computed. Perhaps one way to alleviate these considerations would be to separate the computation of the match score from the computation of the blended value. This way the familiarity process would only require the computation of the match score, which sums chunks activation – values that need to be computed for retrieval in any case. Moreover, this would make the proposed familiarity process as complex as that of any other global matching theory.

To conclude, the current analysis is limited to only memory processes. Yet, neural data indicate that recollection, in addition to having a different neural signature than familiarity, also includes an additional decision phase (Borst, Ghuman, & Anderson, 2016). My analysis does not speak to the nature of these decision processes. Future work should focus on better understanding them.

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Appendix

Here I include the R code used to generate Figures 2, 3, and 4. The two parameters that I specify are (1) the latency factor F and (2) the perceptual-motor time t_{pm}. The precise values of these parameters (0.35 s and 0.8 s) were chosen to be realistic. Yet, their values do not change the functional form, which is what we are ultimately interested in.

To generate Figure 2, I used the standard ACT-R equation (Equations 6 and 7), which assumes a constant RT below the threshold τ (here τ is set to 0) and an activation-dependent RT above threshold:

```
F = 0.35;
t_pm = 0.8;
RTACTR <- function(A){
  tau <- 0
  if (A < tau){
    return(F + t_pm)
  } else {
    return(F*exp(-A)+t_pm)
  }
}
```

To generate the data for Figures 3 and 4, I the modified equation that I propose:

```
RTACTR_new <- function(A,A_rest){
  M <- log(exp(A)+exp(A_rest))
  tau <- -M
  if (A < tau){
    return(F*exp(-tau)+t_pm)
  } else {
    return(F*exp(-A)+t_pm)
  }
}
```

where A_{rest} is the contribution to the match score of all non-target items. To generate Figure 3, I assumed that A_{rest}=0, while I used values of -10, -2, -1.2, and 0.8 to generate the plot in Figure 4.