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**Mathematics, Language, and Bilingual Latina/o Learners: A Review of the  
Empirical Research Literature**

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Abstract

This review considers empirical studies on student learning in mathematics for bilingual Latinas/os and examines how views of mathematics and language have constrained what we know about this population as mathematics learners. The purposes of the review are to describe views of mathematics and language evident in this research, critique these views in light of current research and theories, and propose recommendations for future research.

Mathematics, Language, and Bilingual Latina/o Learners: A Review of the Empirical  
Research Literature

Understanding the complex relationship between language and learning mathematics is crucial to designing mathematics instruction for bilingual Latina/o students. The design of mathematics instruction for this population should be informed by empirical research on how bilingual Latinas/os learn mathematics. What we currently know or don't know about the relationship between language and mathematics learning is based on how past research has conceived of mathematics and of language. This review considers empirical research on student learning in mathematics for bilingual Latinas/os and argues that narrow views of mathematics and vague notions of language in this research constrain what we currently know about this student population as mathematics learners.

The purposes of the review are to describe the views of mathematics and language evident in this research, critique these views in light of current research and theories, and recommend directions for future research. The review examines how twenty studies of Latina/o participants doing or learning mathematics conceived of mathematics, language, and the relationship between the two. The review shows that early research on Latinas/os and mathematics learning was constrained by narrow conceptions of mathematics and vague notions of language. These views of mathematics and language limit conclusions regarding the relationship between language and learning mathematics for this student population. The review also describes how future research can use broader notions of mathematics and language, in particular by integrating sociocultural<sup>1</sup> perspectives of mathematical activity, register, and bilingualism.

### Focus and Scope of the Review

There are multiple levels of phenomena to consider when examining bilingual Latina/o students learning mathematics. Cole and Griffin (1987) describe a set of concentric, overlapping, and embedded contexts that begin with learner and task in the center and expand to the teacher, lesson, school, and community. This review focuses on studies at the level of task and learner rather than levels such as teaching, lesson, school, and community. The rationale for this focus is that it allows a close analysis of the nature of the tasks used in the studies, develop an analysis of the mathematical content of the tasks and describe the views of mathematical activity.<sup>2</sup>

The search began with a bibliography created for using articles on the topic of diversity in mathematics education.<sup>3</sup> This review examined empirical studies reporting data on student reasoning and learning.<sup>4</sup> Twenty studies were selected from empirical studies with U.S. student populations,<sup>5</sup> which were published in peer reviewed journals and book chapters.<sup>6</sup> The studies involve empirical data both quantitative to qualitative ranging from results on assessments such as NAEP to interviews with individual children.

The majority of U.S. studies found in the search were with Latina/o participants; this review focuses on studies concerned with Latinas/os labeled as bilingual. The review focuses on Latinas/os, rather than other bilingual populations for reasons related to demographic trends and current issues for instructional practice in mathematics classrooms. Latinas/os are a large and growing sector of the U.S. population and an increasing number of school age children in the U.S. are from this population. The Latina/o population in the U.S. in 2006 was reported to total approximately 44 million or

14.8% of the total U.S. population (Pew Hispanic Center, 2008). This is an increase from 12.5% in 2000 (Therrien & Ramirez, 2000) and 14% in 2005 (Passel & Cohn, 2008). Projections for 2050 estimate that Latinas/os will then make up about 29% of the U.S. population. Latinas/os are a large portion of the foreign born population, 44.6 %, in 2006 was reported as Latina/o by region of birth (30.8 % from Mexico, 7.1% from Central America, and 6.7% from South America), compared to 23.6% from South and East Asia, the next largest foreign born population (Pew Hispanic Center, 2008). Latina/o children are a significant and growing student population in K-12 classrooms. In 2005-2006 Latinas/os accounted for approximately 19.8 % of all public school students, a 55% increase from 12.7% in 1993-1994 (Fry, 2007). Latina/o students in 200-2001 constituted the majority in virtually all the major urban school districts in the country (Young, 2002). It is reasonable to expect that as the Latino/a population grows and extends to other areas and regions in the U.S., many public school teachers in this country will be teaching Latina/o children. All of these are good reasons to begin by focusing on Latinas/os.

This set of studies presents a challenge in that the labels “Latina/o” and “bilingual” were used in problematic ways. While both of these categories are heterogeneous, studies treated these as if they were homogeneous. For example, participants in these studies included students some of whom were immigrants, others were born in the U.S. or from different countries of origin, and with different educational trajectories. The label “bilingual” was used in ambiguous ways and with multiple meanings. Students were labeled along a spectrum of language proficiencies ranging from monolingual to full bilingual, using different assumptions about what it means to be a bilingual person. Nevertheless, because these studies represent the empirical research

currently available that focuses on mathematics and language, it is important to review and critique these studies as a set.

It is important to note that the labels “bilingual,” “ESL,” and “English learner” have not been used in a consistent manner in the research literature in mathematics education. Early research in mathematics education may have labeled students bilingual who might now be labeled English learners. Sometimes these terms have been used to refer to different populations and other times to refer to the same group of children. Sometimes these labels have been used as if they are interchangeable although they are not. The review includes studies using all of these labels, not because they are equivalent but to ensure that the review included all studies on mathematics that considered language. In this article I will use only the label bilingual for the sake of simplicity and because the majority of the studies in the review used that label. It is also important to note that bilingual Latina/o students are only a subset of the Latina/o student population in the United States, since many Latina/o students are monolingual English speakers.

### *Overview of the Studies*

Table 1 lists 20 studies in alphabetical order, the mathematical topics, and the participants. Table 2 shows these same studies in chronological order lists to allow.

Overall, the 20 publications reviewed included 7 studies with elementary (K-7) students, 8 studies with secondary students, and 6 studies with adults or college students.<sup>7</sup> A majority of the publications (12) were concerned with arithmetic computation and/or solving traditional word problems (either arithmetic or algebra). Of the remainder, four studies used Piagetian tasks (Cuevas, 1983; De Avila, 1987; De Avila & Duncan, 1985; De Avila & Pulos, 1979); one study explored a topic related to geometry, spatial



visualization using geometric shapes (Dixon, 1995); one study examined logical reasoning (Mestre, 1986); and one study examined student arithmetic conceptions (Fuson, Smith, & Lo Cicero, 1997, examined student conceptions of two digit quantities).

Studies measured student performance in mathematics using many different instruments ranging from scores on full versions of exams such as the CTBS (Canadian Test of Basic Skills) or CAT (California Achievement Test), to selected problems taken from NAEP (National Assessment of Educational Progress) or CTBS exams, to problems designed by the researchers. As might be expected, studies with elementary-school age participants focused on arithmetic and studies with middle-school age children and adults focused on algebra.

Most studies used the term “language” to refer to proficiency in one or another national language. Six studies considered oral proficiency and four referred to reading comprehension. Four studies focused on the mathematics register, proposing that it is a source of difficulty for learning mathematics, especially for bilingual students. Studies assessed student language proficiency using many different instruments including the LAB (Language Assessment Battery), the LAS (Language Assessment Scales), and tasks designed by the researchers.

The review is organized into two main sections, views of mathematical activity and views of language. I begin by describing two views of mathematics evident in the studies, arithmetic computation and reasoning/problem solving. I then provide a summary of how narrow views of mathematical activity constrain what we currently know. I close the first section by considering how future research can broaden notions of mathematical activity and integrate sociocultural perspectives of mathematical activity. The second

section describes two views of language prominent in the studies, national language proficiency and the mathematics register. I then provide a summary of how vague notions of language constrain what we currently know and consider how sociocultural perspectives of register and bilingualism can inform future research. The article concludes with a summary of recommendations for future research.

### Views of Mathematics

This section describes how views of mathematical activity evolved using illustrative examples from two paradigmatic sets of studies, those examining arithmetic computation and word problems. Early research on the relationship between language and mathematics focused primarily on arithmetic computing and solving traditional word problems. These two mathematical topics seem to follow from the situations where language might be imagined to be relevant to learning mathematics for bilingual students. First, we can imagine bilingual students carrying out arithmetic computation in their first language and wonder how this may impact their mathematics learning. Another situation in which we might imagine that language might impact bilingual students is when solving word problems in English. These two scenarios, carrying out arithmetic computation and solving word problems, describe the majority of the studies.

One paradigmatic set of studies (several with adults and several with elementary students) focused on arithmetic calculation. Several studies explored adults' preferred language during computation and compared monolinguals and bilinguals in terms of response times. These studies were typically concerned with individual students calculating one, two, or three-step problems using the four arithmetic operations.

A second paradigmatic set focused on word problems. Several studies examined how students translated traditional word problems from English to mathematical symbols. A few studies mentioned reading comprehension while solving word problems as the central issue that second language learners are grappling with when learning mathematics. Another group of studies proposed the mathematics register as a barrier in constructing multiple meanings for words and expressions used in mathematics. (Studies focusing on the mathematics register will be discussed in greater detail in the section on views of language).

The views of mathematics reflected in these studies changed over time (see Table 2 for a list of studies in chronological order). Some early studies used narrow conceptions of mathematical activity and focused on quick performance on arithmetic computation. Table 3 provides the details for a review of studies conducted with monolingual learners (Aiken, 1971) to show that research on bilingual learners started with a similar view of mathematics as research on language and mathematics conducted with monolingual learners. A few early studies with young children used Piagetian tasks (see work by De Avila for examples). Later studies developed a broader view of mathematical activity, examining not only responses to arithmetic computation but also reasoning and problem solving (e.g., Mestre & Gerace, 1986), detailed protocols of students solving word problems (e.g., Spanos, Rhodes, Dale, & Crandall, 1988), the strategies children used to solve arithmetic word problems (Secada, 1991), and student conceptions of two digit quantities (Fuson et al., 1997). This progression is not surprising since the fields of cognitive psychology and mathematics education moved in a similar direction in framing mathematical activity. However, it is important to note that research with bilingual

learners followed a similar trajectory in terms of views of mathematics as did research with monolingual learners.

The next section describes a shift towards more complex views of mathematics by exemplifying two views of mathematical activity, as arithmetic calculation and as reasoning/problem solving. I use two studies (Marsh & Maki, 1976; McLain & Huang, 1982) to illustrate an early focus on arithmetic computation and three studies (Chamot, Dale, O'Malley, & Spanos, 1992; Mestre & Gerace, 1986; Secada, 1991) to illustrate how research in the 1980s and early 1990s began to use broader conceptualizations of mathematical activity.

#### *Mathematical Activity as Arithmetic Computation*

Two experimental studies with Spanish speaking adults support the claim that adult bilinguals report having a preferred language for carrying out arithmetic computation and that the preferred language is the language of instruction (Marsh & Maki, 1976; McLain & Huang, 1982). These two studies explored whether bilinguals have a preferred language for computing, response time, and error rates for bilinguals' performance on arithmetic operations.

One study concluded that arithmetic operations can be performed more rapidly in the preferred language, that switching from one language to another during an experimental session slowed reaction time, that the difference between performance in a preferred language and a non-preferred language was slight (0.2 seconds), and that the overall error rates for monolinguals and bilinguals did not differ significantly. A later experiment (McLain & Huang, 1982) compared response times in the preferred and non-preferred languages for Spanish bilinguals. They found the performance of monolinguals

and bilinguals using their preferred language did not differ significantly and that error rates were evenly distributed across conditions. This study showed that allowing participants to *choose* the language they use for computing decreased solution time and requiring bilingual participants to *change* from one language to another within an experimental session increased solution time. The researchers concluded that if bilinguals are required to use only one of their languages during an experimental session, the preferred language “advantage” could be eliminated.<sup>8</sup>

In summary, and as suggested by other reviews of this research, all we can safely say at this time is that “retrieval times for arithmetic facts *may* be slower for bilinguals than monolinguals” (Bialystok, 2001, p. 203). The two studies summarized here show that, for adults, the context of the task matters. Retrieval, response, or solution times may be minutely slower, but only when bilingual participants are not using their preferred language or are asked to switch from one language to another. Overall, the studies suggest that language switching during arithmetic computation does not affect the quality or accuracy of calculations. Because these two studies focused on computation, they say little regarding the role of carrying out arithmetic operations in two languages during word problem solution. However, evidence suggests that switching languages during arithmetic computation may not affect the quality of mathematical problem solving.<sup>9</sup>

In terms of the effect of bilingualism on mathematical performance, one researcher summarizes the current data as follows:

The most generous interpretation that is consistent with the data is that bilingualism has no effect on mathematical problem solving, providing that language proficiency is at least adequate for understanding the problem. Even

solutions in the weaker language are unhampered under certain conditions.

(Bialystok, 2001, p. 203)

*Mathematical Activity as Reasoning and Problem Solving*

In contrast to studies that used a narrow view of arithmetic, Secada (1991) provides an example of a study that used a complex view of mathematical activity, analyzing the *strategies* children used to solve arithmetic problems. The study considered not only performance on arithmetic procedures but also multiple strategies for solving arithmetic word problems. Thus, it was not the early focus on arithmetic exercises *in itself* that was problematic, but the narrow focus on procedures and quick performance, while excluding an analysis of multiple strategies and reasoning.

Mestre and Gerace (1986) provide another example of a study that considered mathematical activity as more than arithmetic calculation, examining complex and higher order thinking skills (as well as attitudes and beliefs about mathematics). This study used not only results from the CAT (California Achievement Test), exam but also other types of tasks. While many tasks in the CAT exam used in this study focus on arithmetic computation, the exam also includes tasks focused on concepts and applications. The study also included 18 Piagetian tasks focused on mathematical concepts and interviews where students translated English statements to algebra. The analysis of these interviews described how students explained the meanings of algebraic equations (such as  $6S = P$ ) and considered students' knowledge and comprehension of legal algebraic manipulations. Chamot et al. (1992) provide another example of a study that considered more complex aspects of mathematical problem solving, describing metacognitive strategies such as planning, monitoring, and evaluating.

*Summary*

The majority of the studies reviewed involved two paradigmatic scenarios, carrying out arithmetic computation and solving word problems. Therefore, conclusions should be limited to these two mathematical topics. It is not possible to generalize from studies on arithmetic computation and algebra word problems to other topics in mathematics such as geometry, measurement, probability, or proportional reasoning.

Most of the studies reviewed used narrow conceptions of arithmetic or algebra. Studies focusing on response time during arithmetic computation tell us little about strategies participants use to carry out these computations. Studies that focused on translating word problems to algebraic equations tell us little about participants' algebraic thinking. A few studies provided a broader view of mathematics, considering multiple strategies for arithmetic word problems, conceptions of two digit quantities, and logical reasoning.

Narrow conceptions of mathematics that focus on arithmetic computation or word problem translation while ignoring strategies, reasoning, and conceptual understanding constrain mathematical activity to lower order cognitive skills and limit our views of what constitutes mathematical proficiency. Since much of the research focused on lower order cognitive skills, and only a few studies went beyond computation and translating word problems, it is not possible to reach conclusions for this student population regarding higher order mathematical thinking or other aspects of mathematical proficiency such as conceptual understanding.

Studies that focused on the differences between bilinguals and monolinguals may have missed or de-emphasized the similarities in mathematical activity, for example, that

both groups have similar responses to syntactic aspects of algebra word problems. Many studies focused on the disadvantages bilingual learners may face, for example, in computation response time or with the mathematics register, and did not consider any possible advantages of bilingualism.

Future research for this student population should include broader notions of mathematical activity. This can be accomplished by expanding mathematical topics beyond arithmetic and algebra, considering conceptual understanding as well as procedural fluency, examining how learners use and connect multiple mathematical representations, and integrating sociocultural perspectives of mathematical activity.

#### *Sociocultural Perspectives of Mathematical Activity*

All the research studies reviewed treated mathematical activity as an individual cognitive phenomenon. In contrast, contemporary work in mathematics education (Cobb, Wood, & Yackel, 1993; Forman, 1996; Moschkovich, 2002 and 2007b; Nasir, 2002) provides a broader view of mathematical activity. Recent research in mathematics education provides a view of mathematical activity as developing socio-mathematical norms (Cobb, Wood, & Yackel, 1993), presenting mathematical arguments (Forman, 1996), participating in mathematical discussions (Lampert, 1990), and participating in mathematical discourse practices (Moschkovich, 2007b). Situated perspectives of cognition (Brown, Collins, & Duguid, 1989; Greeno, 1994; Lave & Wenger, 1991) present a view of learning mathematics as learning to mathematize situations, communicate about these situations, and use resources for mathematizing and communicating (Greeno, 1994). These perspectives assume that learning is inherently social and cultural “whether or not it occurs in an overtly social context” (Forman, 1996,



p. 117), that participants bring multiple views to a situation, that representations have multiple meanings for participants, and that these multiple meanings for representations and inscriptions are negotiated.

Although focusing on arithmetic computation and word problems may have been sufficient in the past, this emphasis does not include current views of mathematical activity or instructional practices in many classrooms. In some mathematics classrooms today, students are not grappling primarily with solving traditional word problems.<sup>10</sup> Students are expected to participate in classroom mathematical practices that go beyond carrying out computation exercises or solving word problems on a worksheet. In many classrooms teachers are incorporating many other forms of mathematical activity, such as working on projects and using multiple mathematical representations. Students are expected to participate in a variety of mathematical practices, such as explaining their solutions, describing conjectures, proving conclusions, and presenting arguments.

Research studies with other student populations conducted from sociocultural perspectives can provide examples for broadening notions of mathematical activity, even for arithmetic. Brenner's study of Hawaiian children's understanding of number concepts related to money (Brenner, 1998b), although conducted with a different population of students, illustrates how a sociocultural perspective involves looking at children's understandings in different contexts and settings, not only on responses to tests. Brenner's study found that young Hawaiian children not only knew more about money than might have been evident in the classroom, but they also understood money differently than expected. One interesting finding in this study was that young children paid little attention to pennies, because pennies have little value for making purchases.

Future studies examining arithmetic computation among bilingual Latina/o learners should include empirical data on what learners actually accomplish and understand rather than making generalizations about calculating skills only on the basis of experiments.

### Views of Language

The studies reviewed here considered multiple aspects of language including proficiency in each national language, proficiency in reading English, oral communication, and the role of the mathematics register.

#### *Language as Proficiency in English/Spanish*

Overall, the studies assessed proficiency in English or Spanish in many different ways and using many different instruments. Many of the studies with adolescents and adults used self-reports to describe language proficiency in each language and did not assess this proficiency directly. Of the seven studies that assessed learners' proficiency levels directly in *both* languages, five were with elementary age students (Cuevas, 1983; De Avila, 1987; De Avila & Duncan, 1985; De Avila & Pulos, 1979; Secada, 1991). One study with adolescents used the LAB (Language Assessment Battery) materials (Dixon, 1995) and another used the language portion of the CAT (in English) along with understanding of word problems in Spanish (Mestre & Gerace, 1986). Chamot et al. (1992), when describing the 32 students in the study, made no distinction among the students labeled Limited English Proficiency (LEP) in terms of their Spanish proficiency, English proficiency, or degree of bilingualism. This raises questions regarding how studies have defined the term "bilingual" and assessed proficiency in each language, particularly for adolescents.

Several studies used theories from second language acquisition to frame their results, in particular work by Cummins. Chamot et al. (1992) mentioned Cummins' work and the distinction between social and academic language (Cummins, 1981, 1984). Mestre and Gerace (1986) also mentioned Cummins' threshold hypothesis (Cummins, 1979) and the concepts of dominant bilingualism, additive bilingualism, and semilingualism.<sup>11</sup> Although subjects in this study were labeled as bilingual, the study did not report any criteria for using this label. For example, Experiment 5 seems to describe different kinds of bilingual participants. When presented with word problems in Spanish, two subjects read the Spanish problem and solved it; one subject who had no formal Spanish schooling was reported to "sound out the problem"; two subjects translated the problem into English and would not start the problem until they understood the English translation; and one subject who did not understand the Spanish version did not attempt a solution. Interviewers assisted any of the subjects when they did not understand a particular Spanish vocabulary word (see Mestre & Gerace, p. 154).

Secada (1991) is an example of one study that used a complex view of language proficiency and assessed language proficiency using multiple instruments: the Language Assessment Scales (LAS)<sup>12</sup> (De Avila & Duncan, 1981; Duncan & De Avila, 1986, 1987), oral story telling, and verbal counting up and down. The instruments assessed syntax, phonetics, lexicon, and pragmatics and included language tasks that are closely related to the specific mathematical thinking examined in the study. Results from this study include both similarities and differences between the two language groups. While on the one hand overall performance in English was higher than in Spanish, on the other hand "Hispanic first grade children seem remarkably similar to their English speaking

peers when it comes to solving addition and subtraction word problems” (Secada, 1991, p. 226).

Mestre and Gerace (1986) explored the impact of word problem syntax on performance. They found that variation in the syntax of simple word problems had an effect on the translation from word problems to equations. They contrasted responses to translating the sentence “a number added to 7 equals 8” (10/14 correct) with responses to translating the sentence “in seven years, John will be 18 years old” (4/14 correct). They concluded that sentences where the unknown was not readily discernible were more difficult than sentences where the unknown appeared clearly near the beginning of the problem. They also concluded that problems that could be worked out using left to right translation were easier than problems that did not follow a left to right syntax.

Lastly, two studies mentioned reading comprehension in English while solving word problems. Mestre and Gerace (1986) mentioned poor reading comprehension as a possible explanation for the low achievement of bilingual students in mathematics.

Chamot et al. (1992) reported assessing students on reading skills particular to mathematics, reading and understanding a word problem.

#### *Language as the Mathematics Register*

Several of the studies reviewed focused on the mathematics register and proposed that this register is a source of difficulty for native English speakers and an even greater source of difficulty for bilingual students (Cuevas, 1983; Mestre, 1986; Spanos et al., 1988; Spanos & Crandall, 1990). The evidence presented in these four studies to support this claim is not conclusive and difficult to summarize. The studies provide only a few

empirical examples of the actual difficulties that different words, phrases, or meanings presented for students in the studies.

Some of these studies actually seemed to show that bilingual learners face difficulties that are quite similar to those documented for native English speakers. For example, Mestre (1986) concluded, “The subtleties in language construction, jargon, and so on increase the likelihood of problems being misinterpreted by Hispanic bilinguals. These misinterpretations are not caused by an unfamiliarity with vocabulary per se” (p. 169). Mestre concluded that bilingual participants, like native English speakers, were translating word problems incorrectly from natural language to algebraic equations. One study (Mestre, Gerace, & Lochhead, 1982) reported a few mistakes that were specific to Spanish speakers, but overall difficulties for Spanish speakers seemed to parallel those that native English speakers face when translating from words to algebraic expressions.

Spanos et al. (1988) and Spanos and Crandall (1990) proposed a complex framework for analyzing what they called semantic, syntactic, and pragmatic aspects of mathematical language. They analyzed transcript excerpts for evidence of semantic, syntactic, and pragmatic difficulties with word problems. Many of the difficulties students experienced fell in the pragmatic category and were not difficulties with mathematical meanings of words or phrases. For example, they reported a student had difficulties understanding when a tax was applied to a bill rather than understanding the meaning of the word “tax.”

### *Summary*

Overall, the studies reviewed here used vague notions of language, multiple definitions of bilingualism, and a restricted view of the mathematics register. Although

reading comprehension was invoked in several studies, unclear definitions of reading comprehension prevent making any conclusions regarding the role of reading comprehension in solving word problems. Whether referring to national language or the mathematics register, the relationship between language and learning mathematics was viewed primarily in terms of language as an *obstacle* for doing or learning mathematics.

Multiple definitions of bilingualism are reflected in the many instruments used to assess language proficiency and the many ways to label participants. The variety of instruments and labels used in the studies makes comparisons across studies difficult. Participants in these studies were labeled along a spectrum of language proficiencies ranging from monolingual to full bilingual, using different assumptions about what the term bilingual means. Many studies did not report whether subjects spoke Spanish, did not assess students' proficiency in each language, or did not distinguish between proficiency in oral and written modes. Some studies did not assess English or Spanish proficiency *in general* but rather specifically for communicating in writing or orally about a particular mathematical topic. All of these issues raise questions regarding the definitions of bilingualism and language proficiency used in the studies.

The notion of the mathematics register was invoked in several studies. While there were a few descriptions of the hypothetical ways that the mathematics register was an obstacle, there were few empirical examples of these difficulties. It was also unclear what researchers actually meant by the mathematics register, in particular how this notion was theoretically framed.

*Situated and Sociocultural Perspectives of Language*

All of the research studies reviewed here treated language as an individual phenomenon. In contrast, contemporary sociocultural perspectives of language provide a view of language as a sociocultural activity. Instead of viewing language as separate from mathematical activity, research needs to consider how language is part and parcel of mathematical thinking and learning. Rather than viewing language only as an *obstacle* for learning mathematics, research should consider how language is one of multiple resources that learners use to understand mathematics and construct mathematical meaning.

In order to focus on the mathematical meanings learners construct, rather than the mistakes they make, researchers will need frameworks for recognizing the mathematical knowledge, ideas, and learning that learners are constructing in, through, and with language. Several sociocultural frameworks are available, for example functional systemic linguistics (O'Halloran, 1999; Schleppegrell, 2007), a communication framework for mathematics instruction (Brenner, 1994), and a situated and socio-cultural perspective on bilingual mathematics learners (Moschkovich, 2002 and 2007a). These can serve as frameworks for recognizing mathematical contributions by students and shift the focus from looking for deficits to identifying the mathematical discourse practices evident in student contributions (e.g., Moschkovich, 1999).

*Mathematics Register*

While invoking the mathematics register adds complexity to how language is conceptualized, this notion also presents several challenges. First, using the notion of

register requires that the concept be understood as it was proposed by Halliday (1978) rather than interpreted as lexicon, vocabulary, or a list of technical words and phrases.

A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the ‘mathematics register,’ in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

Some examples of registers are legal talk and baby talk. In mathematics, since there are multiple meanings for the same term, students who are learning mathematics have been described as learning to use these multiple meanings appropriately. Several examples of such multiple meanings have been described: the phrase “any number” means “all numbers” in a math context (Pimm, 1987). The mathematics register is sometimes described as a barrier for students learning mathematics. Multiple meanings of the same word are hypothesized as creating obstacles in mathematical conversations, because students often use the colloquial meanings of terms, while teachers (or other students) may use the mathematical meaning of terms. An example is the word “prime” which can have different meanings depending on whether it is used to refer to “prime number,” “prime time,” or “prime rib.”

The notion of register as proposed by Halliday (1978) depends on the situational use of much more than lexical items and includes also phonology, morphology, syntax, and semantics as well as non-linguistic behavior. The notion of register thus implies



considering the situational context of utterances. Although words and phrases do have multiple meanings, these words and phrases appear in talk as utterances that occur within social contexts. Much of the meaning of an utterance is derived from situational resources. For example, the phrase “give me a quarter,” uttered at a vending machine clearly has a different meaning than saying “give me a quarter” while looking at a pizza. When imagining that students face difficulties with multiple meanings in mathematical conversations, it is important to consider how resources from the situation, such as objects and gestures, point to one or another sense such as whether “quarter” means “a coin” or “a fourth.”

A second challenge in using the notion of register is that while it is easy to set up a dichotomy between the everyday and the mathematics registers, research should move away from construing everyday and school mathematical registers as a dichotomous distinction. During mathematical discussions students use multiple resources from their experiences across multiple settings, both in and out of school. Forman (1996) offers evidence of this in her description of how students interweave the everyday and academic registers in classroom discussions. Everyday practices and meanings should not be seen only as obstacles to participation in academic mathematical discussions. The origin of some mathematical meanings may be everyday experiences and some aspects of everyday experiences may actually provide resources in the mathematics classroom. For example, climbing hills is an experience that can be a resource for describing the steepness of lines (Moschkovich, 1996). Other everyday experiences with natural phenomena also may provide resources for communicating mathematically.

While differences between the everyday and mathematical registers may sometimes be obstacles for communicating in mathematically precise ways and everyday meanings can sometimes be ambiguous, everyday meanings and metaphors can also be resources for understanding mathematical concepts. Rather than emphasizing the limitations of the everyday register in comparison to the mathematics register, it is important to understand how the two registers serve different purposes and how everyday meanings can provide resources for mathematical communication and conceptual change.

### *Bilingualism*

Bilingualism is a notion that has different meanings depending on the theoretical perspective used to frame it. A researcher working from a psycholinguistic perspective might define a bilingual person as any individual who is in some way proficient in more than one language. This definition can include a native English speaker who has learned a second language in school with some level of proficiency but does not participate in a bilingual community. In contrast, a researcher working from a sociolinguistic perspective might define a bilingual person as someone who participates in multiple language communities and is “the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations” (Valdés-Fallis, 1978, p. 4). This definition defines bilingualism not only as individual but also as a social and cultural phenomenon that involves participation in language practices and communities.

Definitions of bilingualism range from native-like fluency in two languages, to the alternating use of two languages, to belonging to a bilingual community (Valdés-Fallis, 1978). Current scholars studying bilingualism see “native-like control of two or

more languages” as an unrealistic definition that does not reflect evidence that the majority of bilinguals are rarely equally fluent in both languages (Grosjean, 1999). Grosjean proposes that, instead of comparing each language used by a bilingual against a monolingual norm, we focus instead on the ways in which *individuals* who use more than one language operate along a continuum of *modes*. Thus, depending on whether they are speaking to a monolingual or another bilingual, bilinguals make use of one language, the other language, or the two together as they move along a continuum from monolingual to bilingual *modes*.

One common misunderstanding of bilingualism is the assumption that bilinguals are equally fluent in their two languages. If they are not, then they have been described as not true, real, or balanced bilinguals and sometimes labeled as “semilingual” or “limited bilingual.” The concept of semilingualism has been discussed by several educational researchers and strongly criticized by many (for a review, see Baetens Beardsmore, 1986 and MacSwan, 2000). This concept treats a bilingual person as the sum of two monolingual speakers and uses monolingual speakers as the norm. In contrast, researchers have proposed that a bilingual person should not be viewed as the combination of two monolingual persons nor should the bilingual speaker be judged only by monolingual norms. In particular, Cook (1992, 1999, and 2001) proposes we use the notion of ‘multi-competence’ to describe the knowledge of more than one language in the same mind and suggests, “L2 users be viewed as multicompetent language users rather than as deficient native speakers” (Cook, 1999, p. 185). This perspective is particularly important for classroom practice, since it would re-direct the focus of instruction from developing native like competence to setting goals that are appropriate for second

language learners, including situations and roles that are familiar to second language learners, and using teaching methods that acknowledge students' first language, and base teaching on descriptions of second language users (Cook, 1999).

#### Recommendations for Future Research

Research on language and learning mathematics for this student population should broaden views of mathematics and clarify notions of language. Sociocultural perspectives can frame studies on the relationship between language and mathematics by expanding views of mathematical activity, clarifying notions of language, and in general providing a more complex view of bilingual Latina/o mathematics learners. Future research should integrate contemporary views of mathematics and language, in particular sociocultural views of mathematical activity, bilingualism, and the mathematics register.

Future research for this student population should address broader aspects of mathematical activity. Research should expand mathematical topics beyond arithmetic and algebra, consider conceptual understanding as well as procedural fluency, examine how learners use and connect multiple mathematical representations, and consider multiple aspects of mathematical communication.

Research should stop framing the relationship between language and learning mathematics as primarily in terms of how language is an *obstacle* for learning mathematics. Instead, research needs to consider how language is one of the multiple resources that learners use to construct mathematical meaning. In order to focus on the mathematical meanings learners construct rather than the mistakes they make, researchers need to design studies that recognize the mathematical knowledge that learners are constructing in, through, and with language.

Research should explore all the resources that bilingual students use to construct meaning, such as gestures, objects, and inscriptions. For example, bilingual students' use of gestures conveying mathematical meaning has been documented in case studies (Moschkovich, 1999) and the use of gestures during mathematical discussions merits further study.

Future studies should clarify notions of language proficiency, distinguish between oral and written modes, and consider students' language proficiency for particular mathematical topics. There are serious challenges that such research will need to address, such as the complexity of defining a construct such as language proficiency, the lack of instruments that are sensitive to oral and written modes, and the scarcity of instruments that address features of the mathematics register for specific mathematical topics.

Research should describe proficiency in students' first language and experiences with mathematics instruction rather than assuming that all bilingual students share the same language proficiencies or educational experiences. Studies should document and report not only students' proficiency in each language but also their histories, practices, and experiences with each language across a range of settings and tasks. Studies should describe proficiencies in each language wherever possible in both oral and written modes. Researchers should consider assessing English or Spanish proficiency specifically for communicating about a particular mathematical topic. Students may have had different opportunities to talk about mathematics in each language, in informal or instructional settings, and about different topics. Future studies should also document and report not only students' proficiency in each language but also their experiences with each language and with mathematics at home and at school.

Definitions and assessments of language proficiency need to be considered for adolescents in more detail. Differences between early childhood and adolescence seem particularly important when considering second language acquisition. Students in elementary and secondary school have different experiences in terms of language development and instruction. Learning a second language is different in early childhood than in adolescence. Among other differences, adolescents have greater social development, larger short-term memory capacity, and already know and speak a first language (Cook, 2001). We cannot assume that the relationship between learning language and learning mathematics will be the same for elementary age children with little school experience in their first language and adolescents with school experiences in their first language. Young children and adolescents are likely to have had different experiences with language in and out of school and, in particular, different experiences with mathematics instruction in their first and second languages. For example, one student may be English dominant, have immigrated to the U.S. in early childhood, and have no experience in mathematics instruction in Spanish. Another student may have arrived in later childhood, had some instruction in Spanish but since arriving to the U.S. has spoken Spanish mostly at home. A third student may be a recent immigrant who arrived as an adolescent, has had extensive instruction in mathematics in Spanish, and may be in the beginning stages of acquiring English. Research with adolescents thus needs to develop assessments that distinguish among these different experiences and be informed by research on this particular sector of the Latina/o population (for example López & Stanton-Salazar, 2001; Portes & Rumbaut, 2001).

There are many questions to ask about bilingual mathematics learners and these

questions have complex answers: Do students themselves identify as bilingual or monolingual? Does the school identify students as monolingual or bilingual? If so, on what basis are students labeled bilingual or monolingual? Is a student a recent immigrant, first generation, second generation, or part of borderland communities? Do students come from rural, urban, migrant, or farming settings? What are students' previous schooling experiences? How many years have they been in school? How much mathematics instruction have students experienced in each language? What are students' informal mathematical experiences with activities such as selling and buying, games, or work related mathematics?

Studies should avoid deficit-oriented comparisons between monolingual and bilingual learners and consider any advantages that bilingualism might provide for learning mathematics. For example, since enhanced attention has been reported as one advantage for bilinguals, the role of attention in solving mathematical problems should be explored further. After reviewing research on the cognitive consequences of bilingualism, Bialystok (2001) concludes that bilinguals develop an “enhanced ability to selectively attend to information and inhibit misleading cues” (p. 245).<sup>13</sup> This conclusion is based, in part, on the advantage reported in one study that included a proportional reasoning task (Bialystok & Majumder, 1998) and another using a sorting and classification task (Bialystok, 1999). Although this advantage and these tasks seem to be closely related to mathematical problem solving, they have not been examined in detail in the context of Latina/o bilingual learners and mathematics.

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Footnotes

<sup>1</sup> I use the term “sociocultural” to refer to Vygotskian, neo-Vygotskian, and situated views of mathematical thinking and learning.

<sup>2</sup> The focus on tasks and learners means that several types of studies are not included in this review. Several authors (e.g., Gutierrez, 2002; Ortiz-Franco & Flores, 2001; Secada, 1992; Tate, 1997) have described the ways poverty, poor schools, lack of certified teachers, tracking, and other issues are significant for Latina/o students learning mathematics. While these issues are of the utmost importance, they are beyond the scope of this review, because they are not at the level of task and learner.

<sup>3</sup> An ERIC (Education Resources Information Center) search was conducted, using keywords like “mathematics education,” “bilingual,” “Latino,” and “English language learners.” When reading any publication, the bibliography was also used to generate further sources. From this extensive list, studies were selected that addressed language.

<sup>4</sup> Because of the focus on learners and tasks, studies that focused mainly on instruction were excluded from this review: studies examining classroom structures (e.g., Brenner, 1988a), teacher discourse (e.g., Khisty 1995, 1996; Khisty & Chval, 2002; Khisty, McLeod, & Bertilson, 1990), and participation in classroom discussions (e.g., Moschkovich, 1999).

<sup>5</sup> There are several studies from outside the U.S. that were not included in the review that are relevant to the topic of bilingualism and mathematics learning and

teaching, for example Adler (2001), Clarkson (1991), Ellerton and Clements (1991), Roberts (1998), Setati (1998), and Setati and Adler (2001).

<sup>6</sup> Although various bilingual education clearinghouses were contacted for articles or references, only articles published in journals or as chapters in books were included in this review. No project reports, conference presentations (e.g., Lane, Silver, & Wang, 1995), or conference proceedings (e.g., Fuson, Perry, & Ron, 1996) were included.

<sup>7</sup> The total is twenty-one rather than twenty because one publication included two studies, one study with ninth graders and another with college students.

<sup>8</sup> This study reports a puzzling and unexpected result. Bilinguals with English as their non-preferred language (Spanish-English) were faster at solving addition problems than bilinguals with English as their preferred language (English-Spanish).

<sup>9</sup> Qi (1998) provides a case study of one adult bilingual who switched to her first language (Chinese) for simple arithmetic computation while solving word problems and concluded that the switches were swift and highly automatic, that language switching facilitated rather than inhibited solving word problems in the second language, and that the demands of the task were as important in determining the choice of language as second language proficiency.

<sup>10</sup> This shift has occurred, in part, because traditional word problems are no longer seen as a paradigmatic case of mathematics learning.

<sup>11</sup> Currently, most scholars in linguistics (even early proponents of this notion) have discarded the concept on the grounds that the notion lacks empirical support and theoretical foundation. Perhaps the strongest argument against semilingualism is the empirical evidence that it is not possible to have limited or non-native ability in the

language of one's own home community. Linguists agree that "all normal children acquire the language of their speech community with some minor but ordinary degree of variation" and that "a native language is acquired effortlessly and without instruction by all normal children" (MacSwan, 2000, p. 25).

<sup>12</sup> Note that the tests used to assess language proficiency, the LAB and the LAS, are fraught with problems and recognized as such by those in the fields of bilingual and ESL education.

<sup>13</sup> The cognitive advantages of bilingualism seem to depend on some level of proficiency in both languages and "the extent to which an individual is fully bilingual is instrumental in mediating the effect on cognitive performance" (Bialystok, 2001, p. 205).

Table 1: Studies Reviewed in Alphabetical Order

Study	Mathematical topic	Subjects
1. Abedi & Lord (2001)	Word problems NAEP items 1992	1,174 8th graders 372 described as ELL 802 described as non-ELL and English proficient
2. Chamot et al (1992)	Word problems Meta-cognitive strategies: planning, monitoring, evaluating	32 students Described as: Elementary & secondary students in an urban school that was: 34% LM 16% LEP 52 national languages 69% of students in school are Spanish speaking
3. Cuevas (1983)	Number concepts MRT Metropolitan Readiness test Piagetian tasks (Conservation of length, volume, area, and mass, class inclusions, classification)	50 1 <sup>st</sup> graders First cohort of 25 followed for two school years Second cohort of 25 followed for one school year.
4. De Avila (1987 and 1988) These two publications report on the same study.	Conservation tasks CTBS (computation, concepts, and applications)	253 bilingual 2 <sup>nd</sup> , 3 <sup>rd</sup> , 4 <sup>th</sup> graders 300 monolingual 2 <sup>nd</sup> , 3 <sup>rd</sup> , and 4 <sup>th</sup> graders served as comparison group
5. De Avila & Duncan (1985)	Piagetian tasks	903 1 <sup>st</sup> to 3 <sup>rd</sup> and 5 <sup>th</sup> graders Described as: Urban Mexican-American Rural Mexican-American Puerto-Rican Cuban-American Chinese-American Franco-American (Cajun) Native-American (Navajo) Anglo-American Mexican
6. De Avila & Pulos (1979)	Piagetian conservation tasks	80 1 <sup>st</sup> graders
7. Dixon (1995)	Spatial visualization Paper folding Hole punching (3D) Card rotation (2 D)	9 classrooms of 8 <sup>th</sup> graders
8. Fuson, Smith, & Lo Cicero (1997)	Conceptions of 2 digit quantities	First graders 17 from a Spanish class 20 from an English class

<p>9. Marsh &amp; Maki (1976)</p>	<p>Arithmetic</p>	<p>40 adults undergraduate and graduate students                  20 bilinguals:                  10 S-E bilinguals (Spanish preferred, learned arithmetic in Spanish, English proficiency)                  10 E-S bilinguals (English preferred, learned arithmetic in English, Spanish proficiency)                  20 English monolinguals</p>
<p>10. McClain &amp; Huang (1982)</p>	<p>Arithmetic</p>	<p>40 bilingual adults                  Undergraduate and graduate students                  20 Spanish-English (born outside US)                  20 English-Spanish (most from Spanish speaking homes)                  20 monolinguals</p>
<p>11. Medrano (1986)</p>	<p>CTBS scores</p>	<p>278 7<sup>th</sup>-8<sup>th</sup> graders</p>
<p>12. Mestre (1986)</p>	<p>Test of General Ability-Computation (GTA)                  Formula Translation Examination                  Short Algebra Inventory                  Word Problem Inventory                   Test of Reading and Prueba de Lectura (GTA)                   SAT (ETS)                           Interviews for “Students and Professor” problem</p>	<p>Four studies, comparing Hispanic and Anglo science/engineering college students:                  #1: 133 total (60 H, 73A)                  #2: 95 (43H, 52A)                  #3: 113 (60H, 73A)                  #4: 134 (60H, 74A)                   Hispanics were described as:                  Half of Puerto Rican descent                  Other half South American, Central American, or Caribbean descent                  Many were dominant English speakers                    Interviews: 9 Hispanics, 11 Anglo students</p>

13. Mestre (1988)	Word problems Logical reasoning Negation	14 9 <sup>th</sup> graders: 6 Hispanics, En/Sp bilinguals enrolled in mainstream classes 3 Hispanics enrolled in advanced Algebra 5 En speaking monolinguals  37 College students: 20 monolinguals 17 Hispanics
14. Mestre & Gerace (1986)	Arithmetic (CAT arithmetic computation, concepts, and applications) Algebra (Translating English statements to Algebra, analyzing meanings of algebraic equations such as $6S=P$ , legal algebraic manipulations) Piagetian tasks	14 9 <sup>th</sup> graders
15. Mestre, Gerace, & Lochhead (1982)	Word problems “Students and Professors” problem	College students
16. Ortiz-Franco (1990)	Word problems (Pick the operation for a word problem) Embedded figures NLSMA (National Longitudinal Study of Mathematical Abilities) Math inventory (elementary math topics) Syllogistic reasoning	40 8 <sup>th</sup> graders 13-17 years old Hispanics 20 identified as Spanish speakers (assessed in Spanish) 20 identified as English speakers (assessed in English)
17. Ortiz-Franco & Flores (2001)	NAEP math proficiency scores	Comparison of data on Hispanic and White subjects from NCES
18. Secada (1991)	Arithmetic word problems (Strategies)	45 1 <sup>st</sup> graders Hispanic
19. Spanos & Crandall (1990)	Word problems	Secondary Students
20. Spanos et al (1988)	Algebra word problems	46 adults College students Described as some Hispanic, some LEP, some native English

Table 2:

Studies Reviewed in Chronological Order

Study	Mathematical topic	Subjects
1976 Marsh & Maki	Arithmetic	40 adults undergraduate and graduate students 20 bilinguals: 10 S-E bilinguals (Spanish preferred, learned arithmetic in Spanish, English proficiency) 10 E-S bilinguals (En preferred L, learned arithmetic in English, Spanish proficiency) 20 English monolinguals
1979 De Avila & Pulos	Piagetian conservation tasks	80 1 <sup>st</sup> graders
1982 McClain & Huang	Arithmetic	40 bilingual adults Undergraduate and graduate students 20 Spanish-English (born outside US) 20 English-Spanish (most from Spanish speaking homes) 20 monolinguals
1982 Mestre, Gerace, & Lochhead	Word problems “Students and Professors” problem	College students
1983 Cuevas	Number concepts MRT Metropolitan Readiness test Piagetian tasks (Conservation of length, volume, area, and mass, class inclusions, classification)	50 1 <sup>st</sup> graders First cohort of 25 followed for two school years Second cohort of 25 followed for one school year.
1985 De Avila & Duncan	Piagetian tasks	903 1 <sup>st</sup> to 3 <sup>rd</sup> and 5 <sup>th</sup> graders Described as: Urban Mexican-American Rural Mexican-American Puerto-Rican Cuban-American Chinese-American Franco-American (Cajun) Native-American (Navajo) Anglo-American Mexican
1986 Medrano	CTBS scores	278 7 <sup>th</sup> -8 <sup>th</sup> graders
1986 Mestre & Gerace	Arithmetic (CAT arithmetic)	14 9 <sup>th</sup> graders



	<p>computation, concepts, and applications)                      Algebra (Translating English statements to Algebra, analyzing meanings of algebraic equations such as <math>6S=P</math>, legal algebraic manipulations)                      Piagetian tasks</p>	
1986 Mestre	<p>Test of General Ability-Computation (GTA)                      Formula Translation Examination                      Short Algebra Inventory                      Word Problem Inventory</p> <p>Test of Reading and Prueba de Lectura (GTA)</p> <p>SAT (ETS)</p> <p>Interviews for “Students and Professor” problem</p>	<p>Four studies, comparing Hispanic and Anglo science/engineering college students:                      #1: 133 total (60 H, 73A)                      #2: 95 (43H, 52A)                      # 3: 113 (60H, 73A)                      #4: 134 (60H, 74A)</p> <p>Hispanics were described as:                      Half of Puerto Rican descent                      Other half South American, Central American, or Caribbean decent                      Many were dominant English speakers</p> <p>Interviews: 9 Hispanics, 11 Anglo students</p>
1987 & 1988 De Avila (These two publications report on the same study)	<p>Conservation tasks                      CTBS (computation, concepts, and applications)</p>	<p>253 bilingual 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> graders                      300 monolingual 2<sup>nd</sup>, 3<sup>rd</sup>, and 4<sup>th</sup> graders served as comparison group</p>
1988 Mestre	<p>Word problems                      Logical reasoning                      Negation</p>	<p>14 9<sup>th</sup> graders:                      6 Hispanics, En/Sp bilinguals enrolled in mainstream classes                      3 Hispanics enrolled in advanced Algebra                      5 En speaking monolinguals</p> <p>37 College students:                      20 monolinguals                      17 Hispanics</p>
1988 Spanos, Rhodes, Dale, & Crandall	<p>Algebra word problems</p>	<p>46 adults                      College students                      Described as some Hispanic, some LEP, some native English</p>

1990 Spanos & Crandall	Word problems	Secondary Students
1990 Ortiz-Franco	Word problems (Pick the operation for a word problem) Embedded figures NLSMA (National Longitudinal Study of Mathematical Abilities) Math inventory (elementary math topics) Syllogistic reasoning	40 8 <sup>th</sup> graders 13-17 years old Hispanics 20 identified as Spanish speakers (assessed in Spanish) 20 identified as English speakers (assessed in English)
1991 Secada	Arithmetic word problems (Strategies)	45 1 <sup>st</sup> graders Hispanic
1992 Chamot, Dale, O'Malley, & Spanos	Word problems Meta-cognitive strategies: planning, monitoring, evaluating	32 students described as elementary & secondary students in an urban school that was: 34% LM 16% LEP 52 national languages 69% Spanish speaking
1995 Dixon	Spatial visualization Paper folding Hole punching (3D) Card rotation (2 D)	9 classrooms of 8 <sup>th</sup> graders
1997 Fuson, Smith, & Lo Cicero	Conceptions of 2 digit quantities	First graders 17 from a Spanish class 20 from an English class
2001 Ortiz-Franco & Flores	NAEP math proficiency scores	Comparison of data on Hispanic and White subjects from NCES
2001 Abedi & Lord	Word problems NAEP items 1992	1,174 8 <sup>th</sup> graders 372 described as ELL 802 described as non-ELL and English proficient

Table 3:

A Review of Studies with Monolingual Participants

Study	Mathematical topic	Subjects
Aiken (1971)	Arithmetic Computation Word problems ITBS Stanford Achievement Test (arithmetic computation)	<i>Reviews 7 studies:</i> 1) 368 6 <sup>th</sup> graders 2) 119 6 <sup>th</sup> graders 3) 172 intermediate pupils 4) 18 3 <sup>rd</sup> graders 5) 269 6 <sup>th</sup> graders 6) 286 9 <sup>th</sup> graders 7) 210 11 <sup>th</sup> grade girls