Title

Cost Benefit Analysis of Rehabilitation Services Programs: A First Model and its Sensitivity Analysis

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COST BENEFIT ANALYSIS OF REHABILITATION SERVICES PROGRAMS: A FIRST MODEL AND ITS SENSITIVITY ANALYSIS

by

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with

Konrad Stahl and Charles Cole

November 1971

Working Paper No. 163/RS001

Frederick C. Collignon, Project Director - Michael B. Teitz, Principal Investigator

Project for Cost Benefit Analysis and Evaluation of Rehabilitation Services

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Errata Slip

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Page 10, line 11, the symbol i stands for the discount rate in the definitional formula of the diagonal matrix D.

Page 10, the symbol Σ stands for the summation of the terms of the vector following the sum.

Page 23, first table, underlined values are extreme values in the first column and in the whole table.

FOREWORD

Cost benefit analysis is a major area of activity in our policy research on rehabilitation services. We define cost benefit analysis broadly, embracing assessment of the relative effectiveness and value of alternative strategies, programs, and policies in the rehabilitation field.

Because of the frequent misuse of traditional cost benefit analysis among vocational rehabilitation agencies and researchers, we want to set forth a model cost benefit analysis which would be a guide to future agency studies and which would open new avenues for research. Thus, we are anxious to consider the value of homemaking services, child care, and unpaid work as well as paid employment. We seek to project lifetime earnings in a more sophisticated manner than previous studies which simply extrapolate earnings at closure into the future, adjusting for general productivity increases and perhaps for subsequent dropping out of the labor force after closure of rehabilitants. We also want to incorporate assorted follow-up survey findings which have appeared in the last five years; these surveys have looked both at the experience of rehabilitants and other handicapped who might comprise a valid control group. We will be looking at payback period models, which emphasize the perspective of the taxpayer and the redistributional aspects of rehabilitation programs, as well as at the more traditional GNP-models of cost benefit analysis. We will seek to make explicit the value assumptions inherent in cost benefit analysis and to identify the proper and improper uses of the evaluation technique. Our cost benefit estimates will be disaggregated for various client groups, but will not be aimed at reallocating resources among client groups. We will also explore alternative analytical approaches to measuring and understanding client impact and the efficiency and effectiveness of service delivery.

This paper summarizes the work done by Christian Averous, Konrad Stahl and Charles Cole, three Ph.D. candidates in the Department of City and Regional Planning, during the first phase of our research in the summer of 1971. Anticipating that the Rehabilitation Services Administration (R.S.A.) and various state agencies would have some need for cost benefit estimates prior to the time we could complete our planned research, we decided to put on a computer a cost benefit model summarizing the best work which has been done to date. In our judgment, this work is that of Dr. Ronald Conley. While we have made it possible to modify Conley's conceptual and empirical assumptions in our model, the model is designed so that which can be gathered from standard R-300 program data on clients, to generate a cost benefit estimate which is as sophisticated an estimate as has hitherto been possible in the routine program analyses of rehabilitation agencies. The model is also designed to make possible changes in key parameters if new survey data becomes available or if agencies wish to specify their own assumptions and prices for valuing client impacts, e.g., the price at which homemaking services are to be valued. In the first programmatic use of the model, we assisted R.S.A. in generating cost benefit estimates for rehabilitation services to public assistance recipients.

A second reason for developing the model, quite apart from wishing to respond to short-run needs of operating agencies, was our desire to conduct sensitivity analysis of the various assumptions and components of a cost benefit model in rehabilitation services. We were anxious to see which assumptions and which components had the greatest impact upon the overall cost benefit estimates for rehabilitation programs. Too often in the past, the significance of cost benefit research has been lost amidst the nit-picking of researchers and critics over assumptions which are only of marginal importance for the overall cost benefit estimate. Mr. Averous' paper summarizes the findings of this sensitivity analysis. The findings have been used within our research project to set priorities for further research effort in improving the data which are the basis of assumptions.

Finally, the model as presented here, provides the framework for our subsequent research. We are currently seeking to improve various components of the model, especially the treatment of homemaking and nonpaid work, the projection of future earnings, the handling of repeater costs, changes in the labor force participation of other family members, and costs borne by agencies other than vocational rehabilitation agencies. Working closely with the Rehabilitation Services Administration, we will use the improved model to generate new cost benefit estimates. These estimates will be disaggregated for various client groups and will make explicit social costs and benefits which do not currently enter the program accounts of most rehabilitation agencies.

Comments, criticisms, and suggestions are welcomed on this working paper.

Frederick C. Collignon
Project Director
Project for Cost Benefit Analysis and
Evaluation of Rehabilitation Services

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INTRODUCTION

The following study is part of the research effort aiming at the production of a cost benefit analysis of the rehabilitation service programs of the Rehabilitation Service Administration.

The goals of this study are:

(1) To provide a sensitivity analysis of the cost benefit studies already developed by Ronald W. Conley, in his book The Economics of Vocational Rehabilitation[1] and in his subsequent article "A Benefit Cost Analysis of the Vocational Rehabilitation Program."[2] This sensitivity analysis answers the following questions: how reliable is the cost benefit estimate of a given rehabilitation program? and what are the critical assumptions which most contribute to this imperfection? One can note that those questions are not to find out how the internal structure of the program affects the cost benefit estimate. In other words, this sensitivity analysis aims to evaluate the quality of the cost benefit tool, but does not aim to use the cost benefit tool for proposals of modification of the rehabilitation program.

(2) To obtain a cost benefit estimate of the rehabilitation service program, which would include more effects of the program than previously done. Therefore this cost benefit estimate is an improvement compared to previously available estimates. The new program effects which are included in this paper follow closely the directions pioneered

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[1] Baltimore: Johns Hopkins Press, 1965.

[2] Journal of Human Resources, Vol. IV, No. 2, pp. 226-252, 1969.

by Ronald W. Conley in his more recent cost benefit research on mental retardation.[3]

(3) To provide an operational computerized model which could produce in a short time cost benefit estimates for various populations of rehabilitants. The sensitivity analysis, the cost benefit estimate and the operational model are preliminary, since they provide both experience and insights for the construction of a future cost benefit analysis based on more refined assumptions and disaggregations. In the short term it was also necessary to have cost benefit estimates and an operational model.

The present study 1) develops the conceptual framework, 2) details the quantitative model used, 3) analyzes the sensitivity of the results to assumptions, 4) concludes with a specific cost benefit estimate and draws conclusions for the future cost benefit analysis. The conceptual framework, basically the one of Conley, considers concept and components of benefits, concept and components of costs, and the choice of a c/b index. The formal quantitative model allows for the manipulation of several sets of hypothesis and variables, and embodies as a special case the quantitative model of Conley.

[3] "The Economics of Mental Retardation," Baltimore, Johns Hopkins, forthcoming.

CHAPTER I. CONCEPTUAL FRAMEWORK FOR THE COST BENEFIT MODEL

In the first chapter of this paper the framework of the study is presented. It is basically the framework of economic evaluation of the rehabilitation program, developed by R. Conley.

I.1. General Comments

The framework used by Conley is a cost benefit analysis framework, thus with a unique objective function. Several variables are discussed but dropped in the quantitative analysis. A few features characterize the approach:

(1) Benefits and costs are considered for the society as a whole; the study does not disaggregate to take explicit account of various groups, (rehabilitants or not, geographical impact, etc.).

(2) The approach is mainly a G.N.P. approach which expresses an emphasis on economic rather than other social consequences of the program, and an emphasis on productive rather than distributive consequences.

(3) The choice is made for a before-after rehabilitation approach to analyze the benefits and costs as more tractable than approaches using a control group or differential results of different programs.

I.2. Benefits

As usual in an evaluation scheme, the choice of objective functions is crucial. The uniqueness of the objective function used in c/b analysis, which leads either to a partial framework or to abusive aggregation of various impacts based on ultra rough valuation of the relative worth, has been felt as a major limitation of c/b analysis. This study is indeed a partial study: the impact analyzed is the impact on the national G.N.P. The idea of maximization of G.N.P. refers to a specific economic objective. Other social or humanitarian considerations as for instance the integration of disabled people in the society, the improvement of their health, eventually the lowering of the mortality rate due to better medical care, the well being of the disabled and their families, etc. are not present. Other economic objectives could have been considered as well, as for instance distributive effects.

The benefit concept used here is the "gross national gain to society" in other words the gross contribution to the G.N.P. of the rehabilitation program, which ideally would compare the G.N.P. with the program and what the G.N.P. would be if the rehabilitation services had not been rendered. This comprises four main terms:

DB	=	DOR	+	DOF	+	DOO	-	DAC
incremental social benefit		incremental output due to the program produced by rehabilitants		incremental output due to the program produced by the family of rehabilitants		incremental output due to the program produced by other persons		variation of the associated costs of the program

Let us disaggregate some of them:

$$DOR \approx DER + DNPOR$$

DOR the incremental output of the rehabilitants refers to the discounted stream of incremental output produced in the rehabilitants life after rehabilitation. Such a productivity can be remunerative or not. Paid output will be roughly approximated by earnings, which does not go without basic underlying assumptions on the concept of earnings, the concept of value in economic theory. Unpaid output of rehabilitants comprises the output of homemakers, family workers, or agricultural workers.

$$DAC = DMC + DNC + DCC$$

DAC on the other hand includes variations in medical care, nursing care and custodial care. Finally let us examine DOO, variation of output of other persons not rehabilitants, and not in their families. First the existence of the rehabilitation program affects "others" since it is not obvious that the positions that rehabilitants get after rehabilitation are specific to them and would not have been performed by "others." More specifically, if a job is performed by a rehabilitant, either 1) it would not have been performed by anybody else (if full employment exists for instance), 2) or it would have been performed by "other" persons (if unemployment exists for instance). Whether a c/b study should evaluate the full benefits of a rehabilitation program in a world of full employment relates to the controversial concept of economic branches where one agency acts in its branch as if the optimum was achieved in all other branches. If one wants to evaluate the benefits of a rehabilitation program in the real world, with its unemployment, then some positive incremental output of rehabilitants is offset by some negative incremental output of "other persons." Second DOO would also include people directly affected by the program, but who are not rehabilitants. For instance the training program has effects both inside and outside the R.S.A. since trained people do not work only in vocational rehabilitation after training.

A summary formula of incremental benefits is thus:

$$DB = (DER + DNPOR) + DOF + DOO - DAC$$

We will later focus our attention on DER, DNPOR and DOO.

I.3. Costs

Following Conley, the incremental social costs DC of the program are different from the nominal yearly program costs R, due to several additive terms:

$$DC = R - DRC - DRM + DRR + DRP + RR + RT + OF$$

R	program costs including expenditures under section 2 and 3 of the V.R. Act
DRC	increase in current year's expenditures not attributable to current year's closures
DRM	maintenance exp. paid out of the program funds, unconnected to the program effort
DRR	special service (Repeaters) costs
DRP	program costs not born by the agency
RR	costs of research in rehabilitation
RT	gov't. expenditures for the training of professional in rehabilitation
OF	output forgone during the rehabilitation process

There is no need at this point to discuss in more detail the cost side, since the conceptual framework is exactly similar to the one discussed by R. Conley in "The Economics of Vocational Rehabilitation." The magnitude of these terms and eventually their more disaggregated expression will be presented in the following pages.

CHAPTER II. THE QUANTITATIVE COST BENEFIT MODEL

The conceptual framework presented in Chapter I is there used for the quantitative cost benefit analysis of the rehabilitation program, which is an expansion of the c/b analysis of R. Conley, because 1) it introduces some variables not explicitly considered in "The Economics of Vocational Rehabilitation," 2) it formalizes the expression of some terms (like the variation in the earnings of rehabilitants due to rehabilitation) with more explicitness and more disaggregation. This later allows the separation of the effects of variables (like productivity and labor force participation), 3) it quantifies with various degrees of precision some of the variables only qualitatively discussed by Conley.

This chapter will focus mainly on the detailed expression of

(1) The variation of earnings of the rehabilitated population: DER.

(2) The variation of nonpaid output of the rehabilitated population: DNPOR.

(3) The forgone earnings and nonpaid output of the rehabilitated population during the rehabilitation period: OF.

II.1. Variation of earnings of the rehabilitated population

This will be done in two steps. First, estimation of the number of man-years of life after acceptance, of surviving rehabilitants below age 65, by 5 year time periods, then estimation of the variation in earnings of the rehabilitated population, both total and by 5 year time periods after acceptance.

II.1.1. Estimation of the number of man years of life, after acceptance of surviving rehabilitants, below age 65, by 5 year time periods for a given program

Let us define:

P_t column vector (10x1) of the number of rehabilitated surviving at the beginning of period t, by 5 year age groups from 15 to 65.

P population matrix of rehabilitated (10x10). Its rows are the transposed of the P_t vectors, and thùs give the structure of the rehabilitated population by 5 year age groups from 15 to 65, at the beginning of the corresponding period (period t for row t).

M mortality diagonal matrix (10x10), with the mortality rate of the age group i as element M_{ii} of the diagonal

A and B are (10x1) vectors as follows

$$A = \begin{bmatrix} 5 \\ 5 \\ \vdots \\ 5 \\ 2.5 \end{bmatrix} \qquad B = \begin{bmatrix} 2.5 \\ 2.5 \\ \vdots \\ 2.5 \\ 1.25 \end{bmatrix}$$

J is a (10x10) matrix of decalage: with zero everywhere except in the sub-diagonal full of one

$$J = \begin{bmatrix} 0 & & & & \bigcirc \\ 1 & 0 & & & \\ & 1 & \ddots & & \\ & & \ddots & 0 & \\ \bigcirc & & & 1 & 0 \end{bmatrix}$$

Then the evolution of the population through time is:

$$\boxed{P_t = J\,(I - M)\,P_{t-1}}$$

Since (I - M) is the survival matrix, knowing P_1 the population of rehabilitants entering the program, one obtains by iteration $P_2 \ldots$ to P_{10}. Two remarks however should be made:

1) P_1 will be approximated by the population structure of rehabilitated clients at the end of the first year of period 1.

2) the formula above assumes steady mortality rates through time. This assumption is quite safe for the whole U.S. population, it should be checked for rehabilitated populations. However the bias,

if any substantial one is present, tends to understate the benefits, since most likely mortality rate would tend to decrease through time.

Now the number of man-years of life, by time periods of 5 years, of surviving rehabilitants below 65, after acceptance is the (10x1) column vector: $P (I - M) A + PMB = P [A + M (B - A)]$

Indeed some people are surviving from the beginning of a given period to the beginning of the next period and thus live five years in this given period, while some others are dying in this given period and assuming a constant death rate over the five years, thus live an average 2.5 years in this period. The first term $P(I - M)A$ corresponds to the people surviving from a period to the next one, the second term PMB corresponds to the people dying during a period. For the age group 61-65, the situation is somewhat different due to the fact that we are concerned with only those people of age below 65. Assuming a constant distribution of the people in this age group, those surviving will live an average of 2.5 years in the period, with an age below 65, those dying in the period will live an average 1.25 years in the period with an age below 65.

The total number of man-years of life can, if necessary, be obtained by summing the elements of $P [A + M (B - A)]$, and used to get to the percentages of man-years of life by 5 year periods. This however will not be the way taken; subsequently, the vector being used rather than the sum to take advantage of the disaggregated information it carries.

II.1.2. Estimation of the variation in discounted earnings of the rehabilitated population of a given program

It is here useful to distinguish between the 1965 Conley's computational scheme and the computational scheme that we used, and to show that the former is a specific case of the latter:

a) Conley's Formulation

Call D, the diagonal discounting matrix, of which the terms are those of column 4 (or 5) of Conley's table 4.7. (p. 78)

He gets to the "estimated percentage of the total future increase in output" by multiplying · the vector of "estimated man-years of life as percent of total man-years of life of surviving rehabilitants below age 65" (table 4.7, column 3, p. 78), that is

$$\frac{P\ (A + M\ [B - A])}{\Sigma P\ (A + M\ [B - A])}$$

· by the discounting matrix D of general element

$$D_{tt} = \frac{1}{5} \sum_{k=5t-5}^{k=5t-1} \frac{1}{(1+i)^k}$$

· and summing over all terms gets: $\Sigma D \cdot \frac{P\ (A + M\ [B - A])}{\Sigma P\ (A + M\ [B - A])}$

Now in order to compute this "total future increase in output," he multiplies:

· the "first year's rise in earnings" that is $(e_{ra} - e_{rb}) \cdot \Sigma P_1$ where e_{ra} and e_{rb} are the after and before mean annual earnings of the total rehabilitated population, and where ΣP, is the total rehabilitated population.

· by the "estimated average number of working years of remuneratively employed rehabilitants, which is 17" that is

$\frac{\Sigma P\ (A + M\ [B - A])}{\Sigma P_1} - 0.9$ this is the consequence of the assumptions that rehabilitants still working 5 years after rehabilitation, work until 65 or death, and that "losses of earnings due to declines in employment are roughly offset by increases in real earnings of those still employed." The adjustment of -0.9 years corresponds to the period of rehabilitation during which clients are not productive.

Summarizing, we get to:

$$DER = \underbrace{(e_{ra}-e_{rb})\ \Sigma P_1}_{\text{first year's rise in earnings}} \underbrace{\left[\frac{\Sigma P\ (A+M\ [B-A])}{\Sigma P_1} -0.9\right]}_{\text{average number of working years of remuneratively employed rehabilitants}} \underbrace{\Sigma D \cdot \frac{P\ (A+M\ [B-A])}{\Sigma P\ (A+M\ [B-A])}}_{\text{estimated percentage of total future increase in output}}$$

This can be simplified since ΣP_1 and $\Sigma P\ (A + M\ [B - A])$ cancel in the main term, and rewritten:

$$DER = (e_{ra}-e_{rb})\ \Sigma D \cdot P\ (A + M\ [B - A]) - \text{adjustment for the 0.9 years without earnings.}$$

b) <u>Expanded Formulation</u>

In order to test the sensitivity of the results to the variations of some parameters through time, which seems a priori possibly substantial, it is useful to carry out a disaggregation of some parameter through time. Also, it is valuable to separate various phenomena in various terms, then it will be easier to isolate the influence of each of them. This is why the term $(e_{ra}-e_{rb})$ has to be analyzed in greater detail. It embodies assumptions on:

-- the level of earnings at acceptance and closure

-- the variations in productivity through time (for with and without rehabilitation)

-- the variations in the level of remunerated employment through time (for with and without rehabilitation)

-- the variations in the level of total productive activities through time (for with and without rehabilitation).

These assumptions have to be made explicit and isolated in the formulation, to evaluate their impacts.

To do this, 1) the term $(e_{ra}-e_{rb})$ is replaced by a more complex term, 2) and any summation over time is postponed.

We have the following formulation

$$\boxed{DERV = (W1A.EA - W1B.EB)\ D\ \ P\ [A + M\ (B - A)] \quad (1)}$$

each of the terms being either vector or matrix of the shown dimension.

-- D.P [A + M (B - A)] is exactly similar to what we had before: as number of man-years of life, after acceptance, of surviving rehabilitants, below age 65, by 5 year time periods, discounted.

-- EA is a diagonal matrix of average yearly earnings of remuneratively employed rehabilitants after rehabilitation, by 5 year periods: its terms have been taken as:

$$EA_{ii} = \frac{1}{5} \sum_{k=5i-4}^{5i} ERA\ (1 + \alpha)^{k-1} \qquad i = 2, 3, \ldots .\ 10.$$

where : - ERA is the level of mean yearly earnings of remuneratively employed rehabilitants at closure.

- α is the annual rate of change of yearly earnings through time reflecting variations in productivity.

$$EA_{11} = \frac{1}{5} \left[\sum_{k=1}^{5} ERA\ (1 + \alpha)^{k-1} - 0.85\ ERA \right]$$ which takes care of the adjustment for the unproductive time of rehabilitants during rehabilitation itself.

-- EB is a similar diagonal matrix of average yearly earnings of remuneratively employed rehabilitants before rehabilitation by 5 year periods.

$$EB_{11} = \frac{1}{5} \left[\sum_{k=1}^{5} ERB\ (1 + \beta)^{k-1} - 0.85\ ERB \right]$$

$$EB_{ii} = \frac{1}{5} \sum_{k=5i-4}^{5i} ERB\ (1 + \beta)^{k-1} \qquad i = 2, 3, \ldots 10$$

where: ERB is the level of mean yearly earnings of remuneratively employed rehabilitants at acceptance.

$\underline{\beta}$ is the annual rate of change of yearly earnings through time reflecting variations in productivity.

-- WIA is a diagonal matrix of the proportions of rehabilitants remuneratively employed after rehabilitation, by 5 year periods. Various inputs can be used here. We used the simple assumption of a constant rate of decrease through time ρ

$$W1A_{ii} = \frac{1}{5} \; W1A_1 \sum_{k=5i-5}^{5i-1} \frac{1}{(1+\rho)^k} \quad \text{where}$$

$W1A_1$ is the proportion of rehabilitants remuneratively employed at closure.

-- W1B is a similar diagonal matrix of the proportions of rehabilitants that would have been remuneratively employed if no rehabilitation program had existed.

$$W1B_{ii} = \frac{1}{5} \; W1B_1 \sum_{k=5i-5}^{5i-1} \frac{1}{(1+\rho)^k} \quad \text{where}$$

$W1B_1$ is the proportion of rehabilitants remuneratively employed at acceptance.

Considering the fact that $W1B_1 = \varepsilon 1 \; W1A_1$ (which defines ε1), we can rewrite:

$$W1A.EA - W1B.EB = W1A.E1$$

with:

$$E1_{11} = \frac{1}{5}\left[\sum_{k=1}^{5} \left(ERA\ (1+\alpha)^{k-1} - \varepsilon 1.ERB\ (1+\beta)^{k-1}\right) - 0.85\ (ERA - \varepsilon 1.ERB)\right]$$

$$E1_{ii} = \frac{1}{5}\left[\sum_{k=5i-4}^{5i} \left(ERA\ (1+\alpha)^{k-1} - \varepsilon 1.ERB\ (1+\beta)^{k-1}\right)\right] \quad i=2,\ldots 10$$

Then:

$$\boxed{DERV = W1A.E1.D.P.\ [A + M.\ (B-A)] \qquad (1')}$$

-- DERV is the vector of variation in the discounted earning stream of rehabilitants due to the program, by 5 year periods of time. The sum of its terms is thus DER.

The formula (1') is a subcase of formula (1), and has been used because it fits better the available data.

II.2. Variation of nonpaid output of the rehabilitated population

To provide an estimate of the variation in nonpaid output of rehabilitants, the following expression has been used:

$$DNPORV = NPP.\ W2A.E2.D.P\ [A + M\ (B - A)] \quad (2')$$

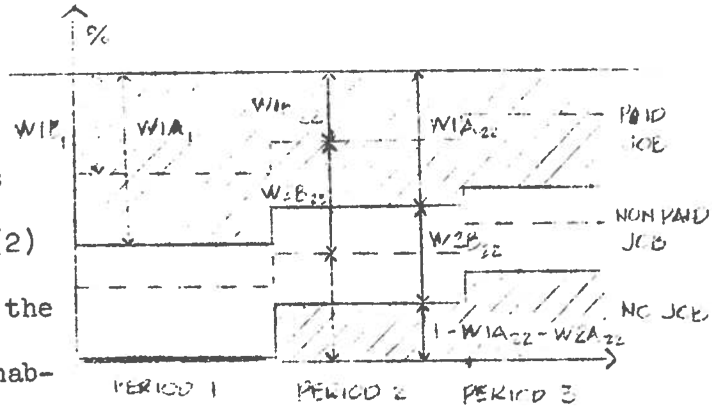
as contraction of

$$DNPORV = NPP\ (W2A.EA - W2B.EB)\ D.P.\ [A + M\ (B - A)] \quad (2)$$

They are similar to (1') and (1) respectively except that:

1) NPP is the ratio of productivities of a nonpaid job to a paid job as measured by earnings.
2) W2A and W2B replace W1A and W1B with the same meaning for rehabilitants nonremuneratively employed.
3) E2 replaces E1 with the same meaning, ERB being adjusted by $\varepsilon 2$ rather than $\varepsilon 1$.

Let us develop on the employment structure of the rehabilitant population through time, that is on W1A, W1B, W2A, W2B, which are diagonal matrices.



This can be schematized on the nearby diagram, or expressed as follows: the formula (1) and (2) allow to consider as variables the employment structure of the rehabilitant population over time, both as it is or as it would have been if no rehabilitation program had existed. Considering the preliminary character of this study and the lack of information on this employment structure, the formula (1') and (2') have been used on the basis of the assumptions of:

Some variations through time of the employment structure both with or without rehabilitation, and of constant decreasing rate of productive population and of remuneratively employed population:

$$\begin{bmatrix} W1A_{11} & & & \bigcirc \\ & W1A_{22} & & \\ & & \ddots & \\ \bigcirc & & & W1A_{10,10} \end{bmatrix} + \begin{bmatrix} W2A_{11} & & & \bigcirc \\ & W2A_{22} & & \\ & & \ddots & \\ \bigcirc & & & W2A_{10,10} \end{bmatrix} = \begin{bmatrix} \ddots & & \bigcirc \\ & \frac{1}{5} \sum_{k=5i-5}^{5i-1} \frac{1}{(1+\sigma)^k} & \cdots i \\ \bigcirc & & \ddots \end{bmatrix}$$

Since $\quad W1A_{ii} = \frac{1}{5} \; W1A_1 \sum_{k=5i-5}^{5i-1} \frac{1}{(1+\rho)^k} \qquad W1A_1 + W2A_1 = 1$

The following input will be necessary:

$W1A_1$ proportion of rehabilitants remuneratively employed at closure

$W1B_1$ " " " " " at acceptance

σ rate of increase of the nonproductive population of rehabilitants

ρ rate of decrease of the remuneratively employed population of rehabilitants

Note that one gets then $\varepsilon 1 = \frac{W1B_1}{W1A_1}$ and $\varepsilon 2 = \frac{W2B_1}{W2A_1}$ or equivalently we get $0.1 \frac{1-W1B_1}{1-W1A_1}$ assuming that among the population not remuneratively employed, 10% is productive, 90% unemployed.

Finally DNPOR is the sum of the terms of the vector DNPORV.

To summarize, the expanded formulation allows for the analysis of the influence of:

-- the ratio of the productivity of a nonpaid job to a paid job.

-- the level of earnings both at closure and acceptance for the remuneratively employed population.

-- the variations of those earnings through time due to productivity changes.

-- the variations of the employment structure through time "with" rehabilitation and "without" rehabilitation.

II.3. Forgone earnings and nonpaid output of the rehabilitants during the rehabilitation period

The output forgone during rehabilitation OF, is approximated as follows:

$$OF = + 0.85 \cdot (\underbrace{W1B_1}_{\text{earnings}} + \underbrace{NPP\ W2B_1}_{\text{nonpaid output}})\ ERB \cdot \sum_{i=1}^{10} P_{1i} \qquad (3)$$

or

$$OF = + 0.85 \cdot (\varepsilon 1 .\ W1A_1 + NPP\ \varepsilon 2\ W2A_1)\ ERB \sum_{i=1}^{10} P_{1i} \qquad (3')$$

If rehabilitation had not taken place, the output would have been the one of the total population of rehabilitants at the beginning of the period, among them some proportion $W1B_1$ was remuneratively employed and earned ERB, some proportion $W2B_1$ was productive but not paid and produced NPP. ERB, the others were unproductive. The output is forgone in approximately 0.85 years according to Conley. This explains the above formulae (3) and (3').

CHAPTER III. SENSITIVITY ANALYSIS

III.1. General Remarks

The analysis of the variables has been developed in 3 steps:

1) identifying a variable in the conceptual framework
2) evaluating roughly its quantitative contribution to judge of its significance
3) evaluating in detail with more accurate analysis of its components, use of available data, accurate computation.

This has been used for instance in realizing the order of magnitude of the cost components: since R and OF are the main elements they deserve an accurate treatment, the other corrections are of low magnitude and can be treated as % of the program costs R. This avoids waste of computational time and efforts and guarantees some homogeneity in the analysis.

This procedure is closely related to the distinction between say simple and complex components of the benefit sum and the cost sum:

-- simple terms of the additive expression can be manipulated by hand and more easily treated with a range of percentages of the main terms.

Also the additive combination of several simple terms is straightforward.

-- complex terms of the additive expression can only be manipulated by use of the computer, and must be expressed in absolute dollars.

The range of variation of the sensitized variables can be attributed to several causes:

1) imperfect knowledge due to simplifying assumptions used in the model

2) imperfect knowledge due to imperfect data base, or mere absence of data on some facts of the past.

3) imperfect knowledge on the future, due to uncertainty.

The last point is important both for the analysis of the impact of a given program, and the inference from past programs analysis of the value of future programs. In this scope, the sensitivity analysis should be performed for rehabilitation programs of several different years. The sensitivity analysis has been performed for the years 1960, 63, 66, 69, 70 and the range of variations of some sensitized variables includes the historical range of variation. However in the absence of extensive follow up studies, uncertainty on the effects of the rehabilitation programs through time remains one basic difficulty.

III.2. Sensitized variables -- Input data

The presentation of the sensitized variables and input data is done as follows: First the terms of the cost side, second those of the benefit side, third data varying with the program year. In each case a summary table is presented and comments are added.

Table 1 COST SIDE

Variable	Sensitized	Values	Data Source
R	known	see table 3	see table 3
DRC	yes	.005R, .02R, .05R basic	Conley's estimate (article 69)
DRM	yes	.07R, .10R, .13R basic	Conley's estimate (article 69)
DRR	no	0	
DRP	no	.04R	Conley's estimate (article 69)
RR	known	see table 3	see table 3
RT	known	see table 3	see table 3
OF	yes	complex variable sensitized by NPP, ERB, $\varepsilon 2$ in formula (3')	

-- Further disaggregation of DRC has been considered but is not useful because of its low magnitude

-- Both DRC and DRM are used for negative correction

-- The effect of repeater costs does not deserve treatment, if consistent assumption on the variation of earnings through time is made (decrease in productivity for instance)

-- for RR and RT the expenses of a given year have been fully attributed to this year. However two questions can be raised:

1) Shall we somehow spread the research expenditures and training expenditures over several years, since their benefits spread over several years?

2) Shall we allocate them fully to the program, since they have quite substantial by products, as for training expenditures? Are these expenditures indivisibly attached to the program or divisible.

Table 2 BENEFIT SIDE

Variable	Sensitized?	Values	Data Source
DAC	no	0	Conley 65 assumption
DOO	yes	0, .1.DER, .2DER basic	
DOF	no	0	
DNPOR	yes	formula (2')	see details below
DER	yes	formula (1')	see details below
i	yes	0, 4%, 8%, 12% basic	
M_{kk}	yes	railroad disabled, U.S. pop., average basic	see Conley 65 p. 77
ERB	yes	see table 3	see table 3
NPP	yes	.25, .50, .75, 1.00 basic 1.25	
α	yes	--.015, 0, 0, .025, basic .025	
β	together	--.015, -.015, 0, 0 .025	
σ=p	yes	0, 0.005 basic	

-- DAC is composed of elements some positive, some negative, so partially offsetting each other. It should be estimated in more detail.

Table 3 DATED INPUT

	1960	1963	1966	1969	1970
ERB	$ 967	$ 1,435	2,171	2,991	3,099
	450	900	1,700	2,500	2,500
	1,380	1,800	2,600	3,500	3,500
ERA	$ 2,443	2,532	2,801	3,666	3,823
R thousand $	80,500	117,050	216,800	458,000	561,300
RR	5,513	10,338	} 55,000	} 63,402	} 58,275
RT	6,117	12,108			
W1A1	0.83	0.83	0.83	0.83	0.83
ε1	0.95	0.33	0.22	0.26	0.27
ε2	0.12	0.42	0.46	0.46	0.46
P_1	17.09			23.3	24.2
	11.88			14.1	14.2
	9.51			8.7	8.8
	10.34	same	same	10.2	9.2
	11.01			8.9	8.4
	11.09			8.9	8.4
	10.35			8.9	8.6
	7.94			6.9	6.6
	6.07			6.2	6.1
	2.99			3.1	3.0

-- Variations in DOO correspond to full employment, and assumptions of 10% and 20% of jobs occupied by rehabilitants which would be otherwise occupied by nondisabled people.

-- DOF is not treated in the model, and should be roughly estimated.

-- NPP has been varied on wide range since the analysis and the data base is very rough.

-- α and β vary together, the productivity variations being assumed better after than before, or similar.

<u>Comments on table 3</u>

1960 and 1963 values are those used by Conley in his 1965 study.

1969 and 1970 values are from an information memorandum (RSA-IM-72-4) of July 19, 1971.

1966 values are those used in the article of Conley 69, except RR and RT. 2 is relative to the structure of employment of the population of rehabilitants: typically there are three groups of rehabilitants, those who were remuneratively employed, those who were productive nonremuneratively, those who were unemployed. $\varepsilon 2$ is the proportion of those in group 2 to the total of group 2 and 3. On the basis of 1969 and 1970 data we assumed 10% in group 2, 90% in group 3.

III.3. Results of the sensitivity analysis

The formulation of the model allowed to test the sensitivity of the results to several variables. In the following, we review each of them, by presenting tables of results for the benefit cost ratio ($BCR = \frac{DB}{DC}$), that is the ratio of incremental social benefits to incremental social costs, and for the net present value (NPV=DB-DC) and comment.

III.3.1. Tables of results

a) Variation of α and β

BCR&NPV α	β	1960	1963	1966	1969	1970	
-.015	-.015	7.11	8.78	7.23	7.01	6.78	cost/benefit ratios
0	-.015	8.51	10.25	8.45	8.29	8.04	
0	0	8.07	9.96	8.20	7.98	7.73	
.025	0	11.20	13.25	10.93	10.88	10.57	
.025	.025	10.21	12.61	10.37	10.18	9.86	
		all figures are ratio					and
-.015	-.015	891,465	1,394,460	2,078,842	4,054,826	4,616,907	net present values
0	-.015	1,095,026	1,657,685	2,486,742	4,917,001	5,615,109	
0	0	1,030,878	1,605,966	2,403,121	4,708,029	5,368,134	
.025	0	1,487,154	2,195,976	3,317,416	6,663,835	7,635,941	
.025	.025	1,343,369	2,080,049	3,129,983	6,189,790	7,074,839	
		all figures in thousand dollars					

Changes in productivity, also assumed happening with very small rates are substantial. Note that all these results are obtained varying (α,β) but keeping all the other parameters constant, at their "basic" level.

b) Variations of the discount rate

The changes in the discount rate affect heavily the level of the c/b ratio for instance, as shown by the table:

BCR&NPV discount rate	1960	1963	1966	1969	1970
0	19.60	24.19	19.91	20.15	19.61
4%	11.66	14.39	11.84	11.68	11.33
8%	8.07	9.96	8.20	7.98	7.73
12%	6.15	7.59	6.24	6.04	5.84
0%	2,711,803	4,156,117	6,312,994	12,917,404	14,856,990
4%	1,554,022	2,399,633	3,619,966	7,205,514	8,247,251
8%	1,030,878	1,605,966	2,403,121	4,708,029	5,368,134
12%	750,480	1,180,570	1,750,908	3,398,234	3,862,304

c) Variations of mortality rates

BCR mortality rates	1960	1963	1966	1969	1970
high (railroad)	7.16	8.84	7.27	7.06	6.84
intermediate	8.07	9.96	8.20	7.98	7.73
low (US)	9.12	11.26	9.27	9.05	8.77

Although not drastic, the changes are substantial.

d) Variations of the ratio of productivity in nonpaid jobs to paid jobs

BCR / NPP	60	63	66	69	70
.25	6.56	9.00	7.65	7.45	7.19
.50	7.06	9.33	7.84	7.63	7.38
.75	7.55	9.65	8.02	7.81	7.55
1.00	8.07	9.96	8.20	7.98	7.73
1.25	8.57	10.26	8.37	8.14	7.89

Despite the wide range of variation of NPP, the changes keep roughly at respective +.50, +.30, +.20, +.18, +.18 by .25 increase of NPP for each year.

e) Variations of the mean earning of rehabilitants at acceptance

BCR / ERB varying:					1960	1963	1966	1969	1970
60	63	66	69	70					
$967	$1435	2171	2991	3099	8.07	9.96	8.20	7.98	7.73
450	900	1700	2500	2500	12.75	12.01	9.09	8.77	8.65
1350	1800	2600	3500	3500	5.70	8.77	7.46	7.22	7.16

Drastic changes are experienced due to variations of ERB.

f) Other changes

BCR		60	63	66	69	70
DOO:	0	8.73	10.87	8.97	8.73	8.45
	0.1 DER	8.07	9.96	8.20	7.98	7.73
	0.2 DER	7.40	9.05	7.42	7.23	7.00
DRC:	0.005 R	8.00	9.86	8.12	7.90	7.65
	0.02 R	8.07	9.96	8.20	7.98	7.73
	0.05 R	8.21	10.16	8.36	8.14	7.89
DRM:	.07 R	8.07	9.96	8.20	7.98	7.73
	.10 R	8.21	10.16	8.36	8.14	7.89
	.13 R	8.35	10.37	8.53	8.32	8.07

As it was a priori forecasted, only minor changes are experienced in costs due to corrections for maintenance costs and closure effects. The influence of the full employment assumption is significant.

g) Additional experiments were performed to analyze the sensitivity to the variables (σ,ρ) and $\varepsilon 2$. As shown by the following tables the couple (σ,ρ) of rates of decrease of total labor force, and of labor force with earnings, is significant considering the small variation of (σ,ρ). Similarly the influence of the employment structure before is important.

BCR		60	63	66	69	70
$\sigma=\rho$	0	8.07	9.96	8.20	7.98	7.73
	0.005	7.63	9.55	7.89	7.67	7.42
$\varepsilon 2$	case 1*	7.63	9.55	7.89	7.67	7.42
	case 2*	7.35	6.40	4.33	3.95	3.85

* see text

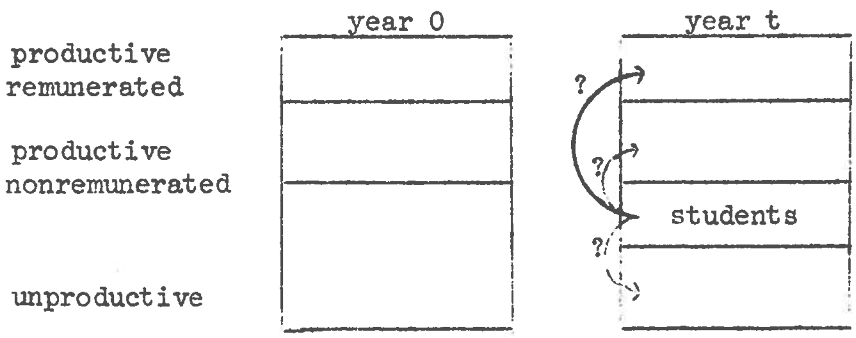
III.3.2. Comments on the results

The above tables make clear a certain number of points:

-- On the benefit side, variations in productivity through time, variations in the discount rate, the mortality rates, the mean earnings at acceptance, the assumption on the employment level in the economy, the employment structure of the population before rehabilitation and its variation through time are all inducing important changes. The influence of the ratio of productivity in nonpaid jobs to paid jobs is not essential apparently.

-- On the cost side, variations for costs of closure effect, maintenance expenses are of small effect. A similar conclusion can be drawn for costs not born by the agency, research and training costs since a variation of 10% for costs which amount to 4 to 10% of the program costs implies just a total variation of 0.4 to 1% of the program costs. The conclusion for the effect of variation in earnings forgone is not so clear, and should be further investigated.

-- The effect of variations of $\varepsilon 2$ deserves a special comment: $\varepsilon 2$ embodies an assumption about the proportion of the rehabilitants productive, nonremuneratively and of the rehabilitants unproductive to the population of rehabilitants without earnings, and this before acceptance. The case 1 refers to 10% productive 90% unproductive rehabilitants among those without earnings. The case 2 refers to 50%, 80%. If the first figures seem more realistic in view of the 1970 data on rehabilitants before acceptance, it should not be overlooked that they are used to assess what would have been the output of the group, if no program had existed. In particular they are the basis for a computation all through time. Eventually the before rehabilitation employment structure of rehabilitants would have changed. One group is specially concerned by those changes, it is the one of students substantial in 1970 (16% of the total of rehabilitants) who will after completion of school go into any of the three employment groups, productive-remunerated, productive-nonremunerated, unproductive. Such movements will affect substantially the amount of output paid or nonpaid produced. A more accurate treatment of this question involves the analysis of a control group of nonrehabilitated, and the use of formula (1), (2) and (3) rather than (1'), (2') and (3'), which allows for detailed inputs on the labor force.



REPARTITION OF THE REHABILITANT POPULATION AMONG EMPLOYMENT GROUPS IN THE HYPOTHETICAL CASE WITHOUT REHABILITATION

To summarize, better knowledge of the structure of the employment of the rehabilitated population before rehabilitation, and of its variations through time "without rehabilitation" seems important.

-- The detailed output of the computer program provides the values of BC1 benefit cost ratio which groups on the cost side only the program costs, and on the benefit side everything else including costs counted as negative. The variations in the ratio are substantial, and stress the sensitivity of a cost benefit ratio to the distinction between benefits and costs.

For basic values of parameters	1960	1963	1966	1969	1970
BC1	13.80	14.72	12.08	11.28	10.56
BCR	8.07	9.96	8.20	7.98	7.73
NPV (millions of $)	1,030	1,606	2,403	4,708	5,368

The net present value (NPV), is not subject to this criticism. The results exhibit a strong increase through time due to the expansion of the program.

-- If we consider the index used by Conley, that is $\frac{DER}{R}$ we obtained the following results slightly higher than the one reported by Conley, with similar assumptions, and discounting at 8%.

	1960	1963	1966
DER/R our results	10.6	12.3	10.5
DER/R Conley's results	10.4	11.7	10.2

This is explained by the small difference in the discounting procedure, which goes as follows: Conley discounts back to a year zero, the year before the program: the costs are assumed to occur in this year zero (undiscounted), the benefits of the program year (year 1) are discounted by a factor $\frac{1}{1+i}$. In our model, we discount back to the first year, and consider that costs and benefits of the program year occur during the first year.

CHAPTER IV. CONCLUSIONS

IV.1. A Single c/b estimate

The following single c/b estimate is based on the assumptions:

costs: DRC = .02R, DRM = .07R, DRR = 0, DRP = .04R

R, RR, RT are given, OF is computed from (3').

Benefits:
- $\alpha = \beta = \sigma = \rho = 0$
- NPP = .75
- mortality rates: average between disabled railroad workers and U.S. population.

Results are shown for 0, 4 and 8% discount rates.

		1960	1963	1966	1969	1970
Incremental Social Costs (in million $)		145	179	334	673	797
Variation in discounted earning streams: DER (in million $)	0%	2,356	3,970	6,277	12,793	14,707
	4%	1,401	2,361	3,734	7,417	8,497
	8%	970	1,635	2,585	5,066	5,793
Variation in discounted nonpaid output stream DNPOR (in million $)	0	552	571	747	1,555	1,810
	4	329	340	445	903	1,050
	8	211	220	286	575	785
Incremental Social Benefits (in million $)	0	2,908	4,541	7,024	14,348	16,517
	4	1,730	2,701	4,179	8,320	9,547
	8	1,181	1,855	2,871	5,651	6,578
Cost Benefit Ratios	0	20.0	25.3	21.0	21.3	20.7
	4	11.9	15.1	12.5	12.4	12.0
	8	8.1	10.4	8.6	8.4	8.3

IV.2. Implications for a new c/b analysis

In light of this study, a good number of points can be made for the forthcoming design of the new c/b study. Let us go through them.

1) Need for a better knowledge of the structure of employment of rehabilitants both at acceptance and through time if no program existed (as outlined before) and probably also of the structure of employment after rehabilitation through time.

2) Need for a better knowledge of the mean earnings of rehabilitants with earnings both at acceptance and through time, with or without program. This corresponds to the influence of ERB and of assumptions on productivity through time. In both points 1) and 2), there are problems of level at one moment in time which can be observed and thus involve mainly data questions, and problems of future variation through time which can not be known surely and thus involve mainly uncertainty questions. The way to handle this information in the c/b study is simple.

3) Need for a better knowledge of the mortality rates.

4) Need for a display of results for various discount rates, since the value of the discount rates influences substantially and since there is no clear agreement among what such value should be.

5) The evaluation of nonpaid output, although rather rough, has shown that it seems that the question is less to estimate the productive output of an individual in terms of $ per year, but to analyze the structure of the nonpaid population and its size.

6) With respect to costs, no extended efforts and time should be devoted to any other term than the output forgone during the program.

7) Disaggregation of the cost benefit analysis by groups of disabled would be most helpful for increasing the accuracy of the results, and

allowing the analysis of the distributive effect of the program. This disaggregation could be by sex (useful for the analysis of homemakers output), by age (useful for the analysis of the earnings through time), by type of disabled (useful for the analysis of productivity), etc.

8) An even rough evaluation should be conducted on the impact of the program of people directly related to it, but <u>nonrehabilitants</u>. In the area of training the changes in earnings could be considered as those are of rehabilitants. This question has been untouched by this study so far.

9) Finally a few more points are related to the choice of one index for emphasis among c/b ratio, net present value, payback period (work actually in progress), to the full employment assumption to investigate more thoroughly at the theoretical level, to the gross production approach, to the evaluation of associated costs.

GLOSSARY

Note:	D refers to the variation of a quantity with/without the program.
DB	gross social benefits of the rehabilitation program of a specific year
DOR	incremental output of rehabilitants
DOF	" " of the family of the rehabilitants
DOO	" " of others
DAC	incremental associated costs
DER	variation in the discounted earning stream of rehabilitants due to the program
DNPOR	variation in the discounted nonpaid output stream of rehabilitants due to the program.
DERV	same as DER, but vector by 5 year period
DNPORV	same as DNPOR, but " " " " "
DC	incremental social costs of the rehabilitation program of a specific year
R	program costs including expenditures under section 2 and 3 of the VR Act
DRC	increase in current year's expenditures not attributable to current year's closures
DRM	maintenance expenditures, paid out of the program funds, unconnected to the program effort
DRR	special service (repeaters) costs
DRP	program costs not borne by the agency
RR	costs of research in rehabilitation
RT	government expenditures for the training of professionals in rehabilitation

OF	output forgone during the rehabilitation process
P_t	vector (10x1) of the number of rehabilitants surviving at the beginning of period t, by 5 year age groups from 15 to 65
P	population matrix (10x10) (see page 8 for details)
M	mortality diagonal matrix (10x10) (see page 8)
J	matrix of zero but the subdiagonal of 1
I	unit matrix
A	vector as page 8
B	vector as page 8
D	discounting diagonal matrix as page 10
e_{ra}, e_{rb}	after and before mean annual earnings of the total rehabilitated population
ERA, ERB	after and before mean annual earnings of the rehabilitants with earnings
EA, EB	diagonal matrix of average yearly earnings of remuneratively employed rehabilitants after (before) rehabilitation, by 5 year periods
α, β	annual rate of change of yearly earnings through time (before or after)
W1A, W1B	diagonal matrix of the proportion of rehabilitants remuneratively employed after (before) rehabilitation, by 5 year periods
$W1A_1$, $W1B_1$	proportion of rehabilitants remuneratively employed at closure (acceptance)
W2A, W2B	similar as W1A, W1B for nonremunerated but productive rehabilitants
$\varepsilon 1$	$W1B_1/W1A_1$
$\varepsilon 2$	$0.1 \frac{1-W1B_1}{1-W1A_1}$
NPP	ratio of productivity of a nonpaid job to a paid job
σ	rate of increase of the non productive population of rehabilitants
ρ	rate of decrease of the remuneratively employed population of rehabilitants