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STUDY OF  $w \rightarrow n^+ n^-$  IN  $K^- p \rightarrow w$  FROM 1.2 TO 2.7 GeV/c

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STUDY OF  $\omega \rightarrow \pi^+ \pi^-$  IN  $K^- p \rightarrow \Lambda \omega$  FROM 1.2 TO 2.7 GeV/c\*

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January 8, 1969

Abstract

A general phenomenological method for studying a two-pion mass spectrum is applied to data containing more than 8000  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events. The lower-momentum half of the sample, which shows a significant  $\omega \rightarrow \pi^+ \pi^-$  signal, was published previously; but is here re-analyzed with quite different, but still significant, results for the branching ratio. The new data at higher momenta show no significant  $\omega \rightarrow \pi^+ \pi^-$  signal. The results from the various momenta are shown to be consistent.

## I. Introduction

The decay of  $\omega$  into  $\pi^+\pi^-$  has been of continuing theoretical and experimental interest<sup>1</sup> because of its possible revelations concerning electromagnetic mixing between the  $\rho$  and the  $\omega$ . However, although it is generally agreed that  $\omega \rightarrow \pi^+\pi^-$  has been seen,<sup>2</sup> no quantitatively precise results have been obtained because of the complication of interference between the production of the two-pion state via  $\omega$  and via other channels.

The experimental results have been not only imprecise, but even somewhat mysterious; though significant results have been reported by several individual experiments,<sup>2</sup> when compilations<sup>3,4</sup> are made, no significant  $\omega \rightarrow \pi^+\pi^-$  signal is seen, even at a much smaller level.

The experiment reported here contains what is probably the largest individual sample of  $\omega \rightarrow \pi^+\pi^-\pi^0$  events in existence, namely a total of about 8000 events. Data corresponding to 5900 events are used in the two-pion-decay analysis; this can be compared with the compilation by Lütjens and Steinberger,<sup>3</sup> which had about 3500 events from six different reactions.

The events discussed in this paper are from the reaction  $K^-p \rightarrow \Lambda\omega$  as seen in the 72-inch hydrogen bubble chamber. About half the events, in the momentum region 1.2 to 1.7 GeV/c, have been previously published.<sup>5</sup> They show a significant  $\omega \rightarrow \pi^+\pi^-$  signal, which was reported to imply a branching ratio  $R = \Gamma(\omega \rightarrow \pi^+\pi^-)/\Gamma(\omega \rightarrow \pi^+\pi^-\pi^0)$  between 1 and 10% (90% confidence level). The other half of the events,

in the momentum region 1.7 to 2.7 GeV/c, are analyzed here. The present analysis shows that:

1. The previous quantitative analysis for the lower-momentum half of the data was incorrect as well as limited in generality. It is not possible to set an upper limit on R at all, and the lower limit should be lowered to 0.2%.
2. The new data show no significant  $\omega \rightarrow \pi^+ \pi^-$  signal.
3. When data are separated into four momentum regions (1.5, 1.7, 2.1, and 2.6 GeV/c) the only significant  $\omega \rightarrow \pi^+ \pi^-$  signal is seen in the 1.5-GeV/c sample.
4. Despite the above, the four momentum regions are consistent.
5. The results are consistent with those of the compilations by Lütjens and Steinberger<sup>3</sup> and Roos,<sup>4</sup> although it must be emphasized that a significant effect is seen in this experiment.

## II. The Data

Between 1961 and 1965 more than 1.5 million pictures of  $K^-$  incident on hydrogen in the 72-inch bubble chamber were gathered. The  $K^-$  momenta were spread from 1.2 to 2.7 GeV/c. Many results have come from this film and it is still proving fruitful today. The analysis of the vee-two-prong topology has been described in detail elsewhere;<sup>5</sup> here only the measurements pertinent to a study of  $\omega \rightarrow \pi^+ \pi^-$  are discussed.

The two reactions of interest are  $K^- p \rightarrow \Lambda \pi^+ \pi^- \pi^0$ , where the dominant  $\omega$  decay mode into  $\pi^+ \pi^- \pi^0$  is seen, and  $K^- p \rightarrow \Lambda \pi^+ \pi^-$ , where the two-pion mass spectrum is studied. In the latter reaction there is

strong production of  $\Sigma(1385)\pi$ ; in order to raise the signal-to-noise ratio in the two-pion spectrum, the  $\Sigma(1385)$  events are eliminated by requiring both  $\Lambda\pi$  masses to be greater than 1430 MeV. If the incident beam momentum and the two-pion mass are fixed, then the  $\Lambda\pi$  mass cut-offs correspond to restrictions on the angle between one of the pions and the  $\Lambda$  in the two-pion rest frame. In order to find out how many  $\omega \rightarrow \pi^+\pi^-\pi^0$  events correspond to a given two-pion mass spectrum, it is necessary to place the same restrictions on the angle between the normal to the  $\omega$  decay plane and the  $\Lambda$  in the  $\omega$  rest frame. This has reduced the effective  $\omega \rightarrow \pi^+\pi^-\pi^0$  events by about 30%, but has reduced background considerably. The cutoff is much less damaging to the high-momentum samples than those at low momentum because the  $\Sigma(1385)$  covers a significantly smaller portion of the Dalitz plot at high momentum.

Table I lists the total number of  $\omega \rightarrow \pi^+\pi^-\pi^0$  events in the samples; the number of  $\omega \rightarrow \pi^+\pi^-\pi^0$  after restrictions on the decay angle have been applied; and the number of  $\Lambda\pi^+\pi^-$  events after elimination of  $\Sigma(1385)$ . As mentioned in the introduction, some of the data (the "old" sample) have been previously published and are here reanalyzed. Those data covered 1.2 to 1.7 GeV/c beam momenta, while the "new" data cover 1.7 to 2.7 GeV/c. In the next section an explanation is given for the division of the data by incident beam momentum into four samples--1.5, 1.7, 2.1, and 2.6 GeV/c--where the 1.5-GeV/c sample contains data labeled 1.4 and 1.5 GeV/c in previous publications, and events at 1.6 GeV/c have been omitted.

Figures 1 through 7 show the histograms of the two-pion mass-squared for the various samples of the data. In all the figures clear evidence for the  $\rho$  meson is seen. In Figs. 1, 2, and 4 a definite spike at the  $\omega$  mass is seen (remember that Fig. 4 is a subsample of Fig. 2, which is a subsample of Fig. 1).

Before the analysis is discussed the following should be stated: No dependence on the polarization of the  $\Lambda$  or on the production angle of the  $\Lambda$  in the center of mass that would distinguish them from the  $\rho$ -meson events has been discovered for the events in the spike.

### III. The Analysis

In the past many methods have been used to analyze two-pion mass spectra. Originally fits were made with  $\rho$ ,  $\omega$ , and background terms adding incoherently. However, it has been pointed out by Harte and Sachs<sup>6</sup> that the  $\rho$  and  $\omega$  have a "natural" coherence due to their electromagnetic mixing, and that this coherence would be washed out only by a fortuitous cancellation. Some more recent analyses considered the possibility that the  $\rho$  and  $\omega$  were completely coherent, with background added incoherently. And several different expressions for the amplitudes themselves have been used.

An attempt is made here to be as general as possible. The only qualification that should be stated immediately is that no concerted effort is made to understand the  $\rho^0$  meson, beyond finding a formula which fits the shape reasonably well. The sole purpose of the analysis is to discover and parameterize any anomaly in the  $\omega$ -mass region. Because of



the narrowness of the  $\omega$ , the task is made much simpler by this point of view.

A general amplitude for two-pion production may be written

$$A_{2\pi} = B + \psi_1 \frac{|m_\rho^2 - \rho_0|}{k^2 - \rho} \left( \frac{\Gamma_\rho}{\Gamma_{\rho_0}} \right)^{1/2} + \psi_2 \frac{|m_\omega^2 - \omega_0|}{k^2 - \omega} \left( \frac{\Gamma_\omega}{\Gamma_{\omega_0}} \right)^{1/2},$$

where  $k^2$  is the two-pion mass squared,  $m_\rho$  and  $m_\omega$  are the masses of the  $\rho$  and  $\omega$ ,  $B$  is the background amplitude, and  $\psi_1$  and  $\psi_2$  are complex numbers (in general functions of  $k^2$ ). Also

$$\Gamma_\omega = \Gamma_{\omega_0} \left( \frac{k^2 - 4m_\pi^2}{m_\omega^2 - 4m_\pi^2} \right)^{3/2}, \quad \Gamma_\rho = \Gamma_{\rho_0} \left( \frac{k^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2},$$

$$\rho = (m_\rho - i\Gamma_\rho/2)^2, \quad \omega = (m_\omega - i\Gamma_\omega/2)^2;$$

$\rho_0$  is  $\rho$  evaluated when  $k^2 = m_\rho^2$  and  $\omega_0$  is  $\omega$  evaluated when  $k^2 = m_\omega^2$ .

The square of the amplitude is now

$$|A_{2\pi}|^2 = \alpha_1 + \alpha_2 \frac{|m_\rho^2 - \rho_0|^2}{|k^2 - \rho|^2} \left( \frac{\Gamma_\rho}{\Gamma_{\rho_0}} \right) + \alpha_3 \frac{|m_\omega^2 - \omega_0|^2}{|k^2 - \omega|^2} \left( \frac{\Gamma_\omega}{\Gamma_{\omega_0}} \right)$$

$$+ \text{Re} \left\{ C_1 \frac{|m_\rho^2 - \rho_0|}{k^2 - \rho} \left( \frac{\Gamma_\rho}{\Gamma_{\rho_0}} \right)^{1/2} + C_2 \frac{|m_\omega^2 - \omega_0|}{k^2 - \omega} \left( \frac{\Gamma_\omega}{\Gamma_{\omega_0}} \right)^{1/2} \right.$$

$$\left. + C_3 \frac{|m_\rho^2 - \rho_0| |m_\omega^2 - \omega_0|}{(k^2 - \rho)(k^2 - \omega)} \left( \frac{\Gamma_\rho \Gamma_\omega}{\Gamma_{\rho_0} \Gamma_{\omega_0}} \right)^{1/2} \right\}.$$

Unfortunately, two very important simplifications can now be made. The word "unfortunately" is used because the simplifications are the result only of the lack of precision in presently possible experiments. First, present experiments lack statistics, and second, they lack perfect mass resolution. Both these effects mean that the exact shape of the experimental mass spectrum is not known well, which allows the following simplifications:

1. The terms multiplied by  $C_2$  and  $C_3$  are indistinguishable. They are the interference terms between the  $\omega$  and either the  $\rho$  or background, and because of the small width of the  $\omega$  the shape of these terms is overwhelmed by the  $\omega$  Breit-Wigner amplitude. ("Shape" refers to the distribution in  $k^2$ .) Therefore the  $C_3$  term can be dropped, and its effects are incorporated in the  $C_2$  term.
2. The parameters  $C_1$  and  $C_2$  are complex, and they multiply Breit-Wigner amplitudes. Thus

$$\text{Re} \{C (BW)\} = (\text{Re } C) (\text{Re } BW) - (\text{Im } C) (\text{Im } BW).$$

Because of the experimental limitations already mentioned, it is a fact that the imaginary part of a Breit-Wigner is indistinguishable from the Breit-Wigner-squared plus a small background. But the Breit-Wigner-squared is already included in the amplitude squared (the  $\alpha_2$  and  $\alpha_3$  terms).

The square of the amplitude can now be succinctly presented:

$$|A_{2\pi}|^2 = \alpha_1 + \alpha_2 |BW|_{\rho}^2 + \alpha_3 |BW|_{\omega}^2 + \alpha_4 \text{Re}(BW)_{\rho} + \alpha_5 \text{Re}(BW)_{\omega},$$

where

$$|BW|_{\rho}^2 = \frac{|m_{\rho}^2 - \rho_0|^2}{|k^2 - \rho|^2} \left( \frac{\Gamma_{\rho}}{\Gamma_{\rho_0}} \right),$$

$$|BW|_{\omega}^2 = \frac{|m_{\omega}^2 - \omega_0|^2}{|k^2 - \omega|^2} \left( \frac{\Gamma_{\omega}}{\Gamma_{\omega_0}} \right),$$

$$\text{Re}(BW)_{\rho} = \frac{|m_{\rho}^2 - \rho_0|}{|k^2 - \rho|^2} \left( \frac{\Gamma_{\rho_0}}{\Gamma_{\rho}} \right)^{1/2} \left\{ k^2 - m_{\rho}^2 + \frac{1}{4} \Gamma_{\rho}^2 \right\},$$

$$\text{Re}(BW)_{\omega} = \frac{|m_{\omega}^2 - \omega_0|}{|k^2 - \omega|^2} \left( \frac{\Gamma_{\omega_0}}{\Gamma_{\omega}} \right)^{1/2} \left\{ k^2 - m_{\omega}^2 + \frac{1}{4} \Gamma_{\omega}^2 \right\}.$$

Figure 8 shows these four universal functions of  $k^2$ , with the masses and widths of the  $\rho$  and  $\omega$  set to 765 MeV, 783.4 MeV, 120 MeV, and 12.2 MeV.

Thus far a pure state has been assumed; that is, all variables other than  $k^2$  have been fixed (for example, momentum transfers, polarizations, etc.). The mixed-state case is treated by taking the expectation value of  $|A_{2\pi}|^2$  over all variables other than  $k^2$ .

Since variables other than  $k^2$  appear only in the  $\alpha$  parameters, the form of the expression for  $|A_{2\pi}|^2$  remains the same when expectation values are taken; the only change is in the relative size of the  $\alpha$  parameters. In general the  $\alpha$  parameters can also be functions of  $k^2$ , but the assumption is made that they are slowly varying near  $k^2 = m_{\omega}^2$ . Hence the  $\alpha$  parameters are assumed to be constants, and the form of  $|A_{2\pi}|^2$  as a function of  $k^2$  remains as valid for a mixed state as it was

for a pure state. However, for a pure state the  $\alpha$  parameters have a definite algebraic relationship; for a mixed state only inequalities can be given.

The actual two-pion spectrum is obtained by multiplying by phase space. Because of the cuts on the  $\Lambda_\pi$  mass, phase space is a linear function of  $k^2$ ; however, for simplicity in parameterizing the background, the final two-pion spectrum is obtained by the equation

$$dN/dk^2 = |A_{2\pi}|^2 \left\{ 1 + \alpha_6 (k^2 - m_\omega^2) + \alpha_7 (k^2 - m_\omega^2)^2 \right\}.$$

Now the final assumption that all the  $\alpha$ 's are constant makes the above a simple expression which reproduces the salient features of any two-pion spectrum and has the unique feature that it is capable of representing any degree of coherence of the  $\omega$  with the other amplitudes.

Finally the experimental mass resolution ( $\approx 10$  MeV FWHM) is folded in.

Several important characteristics of the final result should be emphasized:

1. The parameters  $\alpha_2$  and  $\alpha_3$  may be negative. One might think that a Breit-Wigner squared must make a positive contribution, but one must remember that these terms contain contributions from the imaginary parts of the Breit-Wigners which can give negative contributions. Therefore dips could be seen in the two-pion spectra instead of peaks. If no peak or dip is seen it could be that a negative contribution has canceled a positive contribution, as Lütjens and Steinberger point out; therefore without assumptions no conclusion can be drawn from the absence of a peak.

This is a consequence of the experimental inability to distinguish the imaginary part of the  $\omega$  Breit-Wigner from its square. Of course if one assumes complete incoherence or coherence of the  $\omega$  with other amplitudes, then an upper limit can be set on the  $\omega$  production.

2. Observation of either an  $\alpha_3$  or  $\alpha_5$  term allows one to set a lower limit on the  $\omega \rightarrow \pi^+ \pi^-$  branching ratio.

3. The  $\alpha_1$  through  $\alpha_5$  have dimensions  $M^{-2}$ , so that, for example,  $\alpha_3$  represents the actual height (in events/ $0.01 \text{ GeV}^2$ ) of the  $\alpha_3$  term's contribution to the spectrum at the  $\omega$  mass-squared.

4. The hypothesis of no  $\omega$  production can be easily treated by setting  $\alpha_3 = \alpha_5 = 0$ .

5. If it were possible to determine a pure state for the production of  $\rho$  and  $\omega$ , without background, so that complete coherence could be assured, then one could solve for the amplitude of pure  $\omega$  production.

Let  $\alpha_2' = \alpha_2 |\text{BW}|_{\rho}^2$  evaluated at  $k^2 = m_{\omega}^2$ . Then

$$|\psi_{\omega}|^2 = (\alpha_3 + 2\alpha_2') \pm \left[ (\alpha_3 + 2\alpha_2')^2 - (\alpha_3^2 + \alpha_5^2) \right]^{1/2}.$$

Then the branching ratio  $\omega \rightarrow \pi^+ \pi^- / \omega \rightarrow \pi^+ \pi^- \pi^0$  is  $|\psi_{\omega}|^2 (\pi m_{\omega} \Gamma_{\omega_0}) / N_{\omega}$ , where  $N_{\omega}$  is the number of  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events corresponding to the fitted sample.

Unfortunately, even if present-day experiments had enough data to restrict  $s, t$ , the decay angle of the two-pion system, and all decay angles of other final-state particles (in this case the  $\Lambda$ ), still a pure state would not necessarily be achieved because of background in the two-pion system.

On the other hand one might attempt to create a completely incoherent case, where all interference effects have washed out. There are two objections to this; first, it is quite difficult to be assured of having an incoherent sample (as mentioned previously Harte and Sachs<sup>6</sup> maintain that even if the production processes of  $\rho$  and  $\omega$  were incoherent, which would not be easy to prove, the final two-pion spectrum would in general exhibit interference from the very nature of  $\rho$ - $\omega$  mixing); second, the effect may be so small as to be undetectable, whereas in an interference term, small amplitudes can have large effects.

Therefore it seems worthwhile to try to restrict as many kinematical variables as possible, in order to see what effect they have. For this reason the data have been split into four parts, each part having a particular value of  $s$ . The variable  $s$  was chosen because the data are very close to the threshold of the reaction, and are therefore perhaps more susceptible to  $s$ -channel rapidly varying effects than anything else.

6. The branching ratio calculated in point (5) can be used, without assumption of a pure state, to find a lower limit on the  $(\omega \rightarrow 2\pi)/(\omega \rightarrow 3\pi)$  ratio.

#### IV. The Results

In order to determine whether a significant anomaly exists in the data at the  $\omega$  mass, the following is done: The terms representing the  $\omega$  are set to zero, and the other five  $\alpha$ 's, along with the  $\rho$  mass and width, are allowed to vary in a fit to the data, which yields a minimized  $\chi_p^2$ . Then another similar fit is made, but with the  $\omega$  parameters free to

vary, which yields a minimized  $\chi_{\omega}^2$ . Since the  $\omega$  mass and width are fixed at their accepted values (783.4 MeV and 12.2 MeV) this second fit has only two more parameters than the first. The significance of an  $\omega$  signal is measured directly by the difference between the two  $\chi^2$ ; that is  $\chi^2 = \chi_{\rho}^2 - \chi_{\omega}^2$ , which is a  $\chi^2$  for two degrees of freedom.

A confidence level can be calculated from this  $\Delta\chi^2$  for two degrees of freedom. The confidence level thus determined is the confidence level for the theory that no  $\omega$  signal exists in the data. This should not be confused with the confidence level for the "p alone" fit, which has 63 degrees of freedom. Even if the "p alone" fit failed by a tremendous  $\chi^2$ , it would not prove the existence of the  $\omega \rightarrow 2\pi$  decay, since the reason for failure may be unassociated with the  $\omega$ . On the other hand just looking at the goodness of the "p alone" fit is not a sensitive test of an  $\omega$  signal, because the fit could be relatively quite good in  $\chi^2$ , but fail miserably in the bins near the  $\omega$ . Therefore the most sensitive test of the  $\omega$  is the  $\Delta\chi^2$  test. The number of standard deviations from zero,  $N_{\sigma}$ , for an  $\omega$  signal can also be computed from  $\Delta\chi^2$  (remember, two degrees of freedom!).

Table II lists the various subsamples of analyzed data, with the fitted values of  $\alpha_1$  through  $\alpha_7$ , the fitted  $\rho$  mass and width, the  $\chi^2$ , the confidence level, and the number of standard deviations from zero for the  $\omega$  signal. The fits without  $\omega$ , along with the separate contributions from the  $\rho$  and from background, are shown in Figs. 1 through 7. The fits with the  $\omega$  parameters included are shown in Figs. 9 through 15, with the  $\omega$  contribution also shown. It is clear that the only undeniably significant  $\omega$  effect appears in the 1.5-GeV/c data (and of course

exhibits itself in the "old" and "total" samples). The  $\chi_\omega^2$  contours for the 1.5-GeV/c data, plotted in  $(\alpha_3, \alpha_5)$  space, are shown in Fig. 16.

From Fig. 16 a lower limit for the  $\omega \rightarrow \pi^+ \pi^-$  branching ratio can be found with the help of the equation derived in Section V. First, since the  $\omega$  can interfere with background as well as the  $\rho$ , the quantity  $\alpha_2'$  must be replaced by  $\alpha_2' + \alpha_1$ . Secondly, if  $|\psi_\omega|^2$  is imaginary from the equation (which is physically impossible), then a pure state is not allowed by the data, and a minimum  $|\psi_\omega|^2$  is found by a search of all mixed state possibilities.

Thus,

$$\Gamma(\omega \rightarrow \pi^+ \pi^-) / \Gamma(\omega \rightarrow \pi^+ \pi^- \pi^0) > 0.2\%$$

at a 90% confidence level, which is different from, and lower than, the value given in Ref. 5. The difference arises solely from the analysis. In Ref. 5,  $m_{\rho_0}$  and  $\Gamma_{\rho_0}$  were fixed; in this analysis they were allowed to vary, thus the limit is weakened here. Also interference with background was completely neglected in Ref. 5. Both effects were important.

The other three samples of data show no significant  $\omega$  signal. An investigation into the dependence on momentum transfer, and also on  $\Lambda$  polarization, by splitting the data into smaller subsamples also yielded no significant  $\omega$  signals. Since we have shown that any given sample can set only a lower limit on  $\omega \rightarrow \pi^+ \pi^-$ , never an upper limit, naturally there is no contradiction between samples. In fact it is not surprising to see the  $\omega$  signal appear at only one energy, since the effect is probably due not to a simple  $\omega$  signal, but to interference between



a very small  $\omega$  amplitude, and the  $\rho$  + background amplitude.

One might ask why the 2.2-std. dev. effect in the new data is ignored, while the 2.7-std. dev. effect in the old data is considered significant. First the 1.5-GeV/c subsample of the old data has a 3.4-std. dev. effect that cannot be ignored. Second, the effect in the new data is in fact associated with the two bins in the middle of the  $\rho$  which are so low, and though the  $\omega$  fit lowers the curve somewhat in this area, it really seems that these two bins have little to do with an  $\omega$  anomaly.

These results should be compared with those of a large compilation of pion-induced reactions: In 1967 Roos<sup>7</sup> published a compilation which claimed a three-standard deviation effect in  $\pi^- p \rightarrow \pi^+ \pi^- n$ , but no branching ratio could be set because of the unknown  $\omega \rightarrow \pi^+ \pi^- \pi^0$  rate. However in a later paper<sup>4</sup> the claim was withdrawn due to changes in some of the experimental data.

Lütjens and Steinberger,<sup>3</sup> in an earlier compilation, set an upper limit of 0.8% on  $\omega \rightarrow \pi^+ \pi^-$ . Even though their limit is consistent with the result of this paper, it should be mentioned that they assumed no interference. (They had to assume something about interference; otherwise, as the present analysis shows, and as they specifically pointed out, they could not set any upper limit.)

## V. Conclusions

Harte and Sachs<sup>6</sup> have shown, within a simple and believable interpretation of the behavior of quantum-mechanical states under mixing, that, in a reaction where  $\rho$  and  $\omega$  are produced, the  $\pi^+ \pi^-$  system will almost always produce nonnegligible interference effects, no matter

how many reactions are added together. Thus assumptions of no interference may not be valid. Consideration of this problem has led to the development of a general method for analyzing the  $\omega$  contribution to a two-pion spectrum, without any assumptions about coherence.

It has been shown that if no assumptions about coherence are made it is impossible for present experiments to set an upper limit on  $\omega \rightarrow \pi^+ \pi^- / \omega \rightarrow \pi^+ \pi^- \pi^0$ .

The method has been used to analyze a sample of the reaction  $K^- p \rightarrow \Lambda \pi^+ \pi^-$ , where the sample corresponds to 5900  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events. An  $\omega \rightarrow \pi^+ \pi^-$  signal is seen, and the final result at a 90% confidence level is

$$\frac{\Gamma(\omega \rightarrow \pi^+ \pi^-)}{\Gamma(\omega \rightarrow \pi^+ \pi^- \pi^0)} > 0.2\%.$$

This result is based in part on a reanalysis of previously published data,<sup>5</sup> and supersedes all previous upper and lower limits stated in previous publications; differences are solely a result of the more general analysis employed here.

## VI. Acknowledgments

The author thanks John Harte for several stimulating discussions on the theory of  $\rho$ - $\omega$  mixing, and Harte and R. G. Sachs for providing a copy of an unpublished paper. Also the encouragement and support of Professor Luis Alvarez, Professor M. Lynn Stevenson, and Dr. Frank Solmitz are greatly appreciated.

Footnotes and References

\*Work done under auspices of the U. S. Atomic Energy Commission.

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Table I. Number of events in the experiment. The column labeled " $\omega \rightarrow 3\pi$ " lists the total number of  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events in that subsample after background subtraction. The column labeled " $\omega \rightarrow 3\pi$  with restriction" lists the number of  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events remaining after a restriction is made on the  $\omega$  decay angle (the normal to the  $\omega$  decay plane with respect to the  $\omega$  line of flight) that corresponds to the elimination of  $\Sigma(1385)$  events in the  $\Lambda \pi^+ \pi^-$  samples. The third data column lists the number of events of  $K^- p \rightarrow \Lambda \pi^+ \pi^-$  with  $m^2(\pi^+ \pi^-) < 1.2 \text{ GeV}^2$  and  $m^2(\Lambda \pi) > 2.05 \text{ GeV}^2$  in the subsample. The samples at individual energies, taken together, do not represent the total sample because events at 1.6 GeV/c were eliminated. The "new data" events have been weighted for  $\Lambda$  escape from the chamber, which accounts for the larger number than listed in Ref. 8. The number of unweighted new  $\omega \rightarrow 3\pi$  events is 4020.

Sample	$\omega \rightarrow 3\pi$	$\omega \rightarrow 3\pi$ with restriction	$\Lambda \pi^+ \pi^-$ without $\Sigma(1385)$
Total	9132	5920	10479
Old data	3706	2050	2997
New data	5426	3870	7482
1.5 GeV/c	2980	1650	2218
1.7 GeV/c	1919	1160	1857
2.1 GeV/c	1581	1080	2426
2.6 GeV/c	2283	1840	3697

Table II. Fitted parameters and  $\chi^2$  in the  $\omega \rightarrow \pi^+\pi^-$  analysis. The parameters are defined in Section III; their physical identification is indicated above each one. "p Int" means the term representing  $\rho$  interference with background. " $\omega$  Int" represents  $\omega$  interference with background or the  $\rho$ . The units of  $\alpha_1$  through  $\alpha_5$  are events per  $0.01 \text{ GeV}^2$ . The units of  $\alpha_6$  and  $\alpha_7$  are  $\text{GeV}^{-2}$  and  $\text{GeV}^{-4}$  respectively. The units of  $m_\rho$  and  $\Gamma_{\rho_0}$  are MeV. Each sample has two rows; the first represents the fit without the  $\omega$ , the second with the  $\omega$ . The number of degrees of freedom are 63 and 61 respectively. The column labeled  $N_\sigma$  lists the number of standard deviations from zero for the  $\omega$  signal, as derived from the differences in  $\chi^2$  between the two rows of each sample. The column labeled CL lists the confidence level for the theory that no  $\omega$  signal exists in the data.

Sample	Bkgd	$\rho$	$\omega$	$\rho$ Int	$\omega$ Int	Bkgd	Bkgd	$m_\rho$	$\Gamma_{\rho_0}$	$\chi^2$	$\Delta\chi^2$	CL (%)	$N_\sigma$
	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$						
Total	77	108	--	-8	--	-1.2	-1.1	771	134	77.5			
											21.0	0.005	4.2
	76	98	62	-10	-36	-1.1	-0.9	770	142	56.5			
Old	20	40	--	-2	--	-2.8	-0.1	782	120	88.5			
											10.0	0.7	2.7
	22	33	26	1	-7	-2.7	-0.6	775	124	78.5			
New	52	82	--	-14	--	-0.4	-0.1	767	124	62.1			
											7.3	2.5	2.2
	55	69	38	-7	-27	-0.4	-0.7	763	141	54.8			
1.5 GeV/c	16	31	--	0	--	-3.2	-2.0	785	116	75.0			
											14.7	0.07	3.4
	19	22	20	1	-18	-2.9	-3.4	787	122	60.3			
1.7 GeV/c	16	18	--	-3	--	-1.1	-1.2	771	134	69.7			
											2.4	30.	1.0
	16	17	11	-2	3	-1.1	-1.0	760	135	67.3			
2.1 GeV/c	19	20	--	-4	--	-0.2	-0.8	780	125	57.9			
											0.7	70.	0.4
	19	18	8	-3	2	-0.3	-1.0	772	129	57.2			
2.6 GeV/c	23	47	--	-6	--	-0.3	-0.3	763	124	59.9			
											5.2	7.5	1.7
	24	43	19	-6	-13	-0.2	0.4	759	126	54.7			

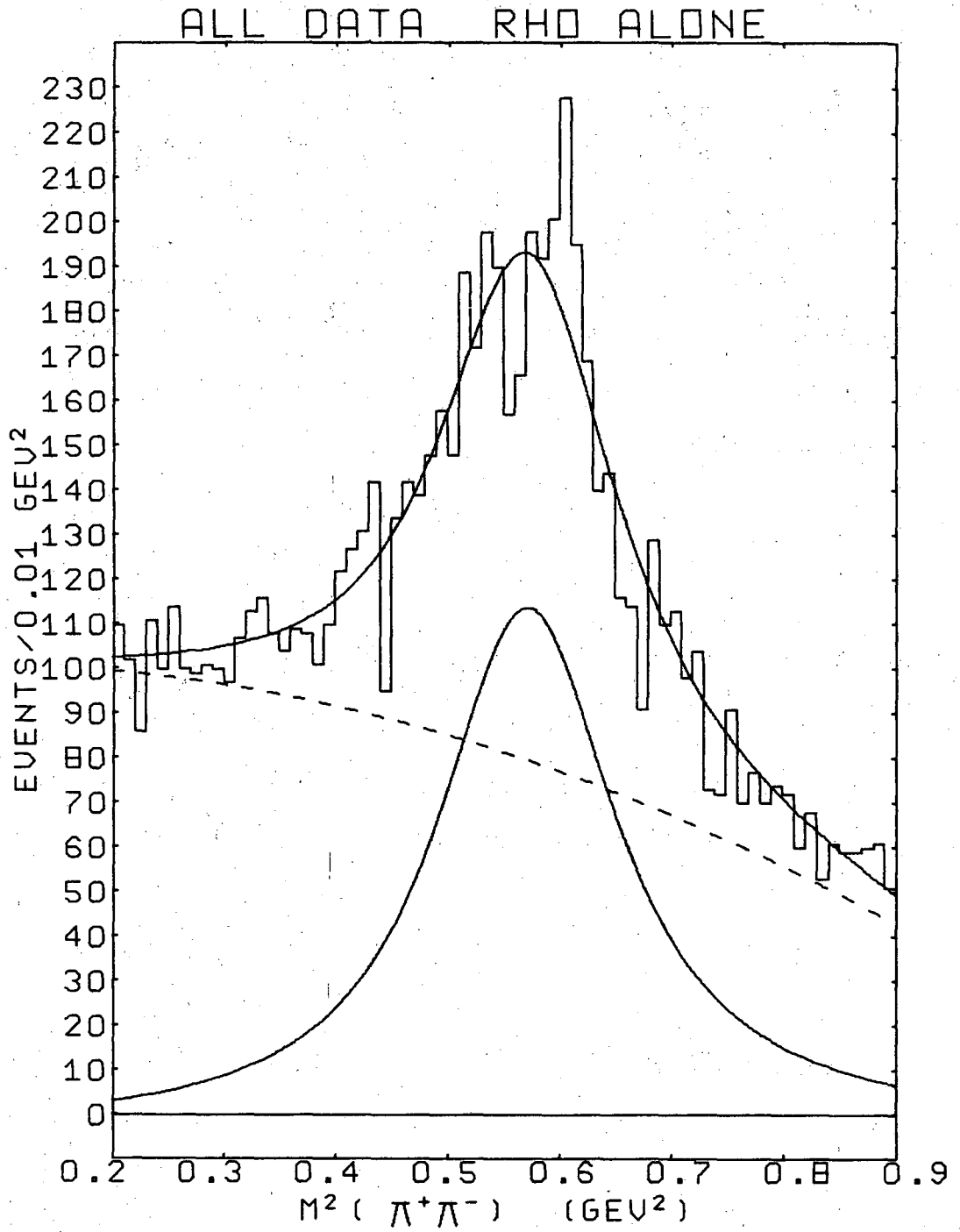
Figure Captions

- Fig. 1. Two-pion mass-squared spectrum for the total sample of  $K^- p \rightarrow \Lambda \pi^+ \pi^-$  events from 1.4 to 2.7 GeV/c with the  $\Sigma(1385)$  removed. The sample corresponds to 5920  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events. The curves represent a fit including a  $\rho$  meson and background, as described in the text. The top solid curve is the complete fit; the bottom solid curve is the contribution from the  $\rho$  plus the interference term between the  $\rho$  and background; the dashed curve is the background contribution.
- Fig. 2. Two-pion mass-squared spectrum for the part of the  $K^- p \rightarrow \Lambda \pi^+ \pi^-$  data that has been previously published. The beam momenta ranged from 1.4 to 1.7 GeV/c, the  $\Sigma(1385)$  events have been removed, and the number of corresponding  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events is 2050. The curves represent the  $\rho$  plus background fit as described in Fig. 1.
- Fig. 3. Two-pion mass-squared spectrum for the new  $K^- p \rightarrow \Lambda \pi^+ \pi^-$  data, with beam momenta from 1.7 to 2.7 GeV/c. With  $\Sigma(1385)$  removed, the sample corresponds to 3870  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events. The curves represent the  $\rho$  plus background fit as described in the text.
- Fig. 4. Two-pion mass-squared spectrum for the 1.5-GeV/c data (sub-sample of Fig. 2). The number of corresponding  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events is 1650. The curves represent the  $\rho$  plus background fit as described in Fig. 1.
- Fig. 5. Two-pion mass-squared spectrum for the 1.7-GeV/c data (sub-sample of both Fig. 1 and 2). The number of corresponding  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events is 1160. The curves represent the  $\rho$  plus background fit as described in Fig. 1.

- Fig. 6. Two-pion mass-squared spectrum for the 2.1 GeV/c data (sub-sample of Fig. 3). The number of corresponding  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events is 1080. The curves represent the  $\rho$  plus background fit as described in Fig. 1.
- Fig. 7. Two-pion mass-squared spectrum for the 2.6-GeV/c data (sub-sample of Fig. 3). The number of corresponding  $\omega \rightarrow \pi^+ \pi^- \pi^0$  events is 1840. The curves represent the  $\rho$  plus background fit as described in Fig. 1.
- Fig. 8. The four universal functions which can be used to represent a two-pion mass spectrum near the  $\omega$  mass. Each curve is the corresponding function described in the text, multiplied by 100 events/ $0.01 \text{ GeV}^2$ .  $|\text{BW}|_{\rho}^2$  and  $|\text{BW}|_{\omega}^2$  are the Breit-Wigner-squared of the  $\rho$  and  $\omega$  respectively; while  $\text{Re}(\text{BW})_{\rho}$  and  $\text{Re}(\text{BW})_{\omega}$  are the real parts of the Breit-Wigner formulas for the  $\rho$  and  $\omega$  respectively. The  $\text{Re}(\text{BW})_{\rho}$  term represents the interference of the  $\rho$  with background, and the  $\text{Re}(\text{BW})_{\omega}$  represents the interference of the  $\omega$  with either the background or the  $\rho$  amplitude.
- Fig. 9. Two-pion mass-squared spectrum for the total sample as in Fig. 1. The curves represent the fit including the  $\omega$  meson as described in the text. The top solid curve is the fit; the dashed curves are, first, the background, and second, the  $\rho$  (including the  $\rho$  interference term) contribution; the bottom solid curve is the  $\omega$  (including the  $\omega$  interference term) contribution.
- Fig. 10. Two-pion mass-squared spectrum for the old data as in Fig. 2. The curves represent the fit including the  $\omega$  meson, as described in Fig. 9.

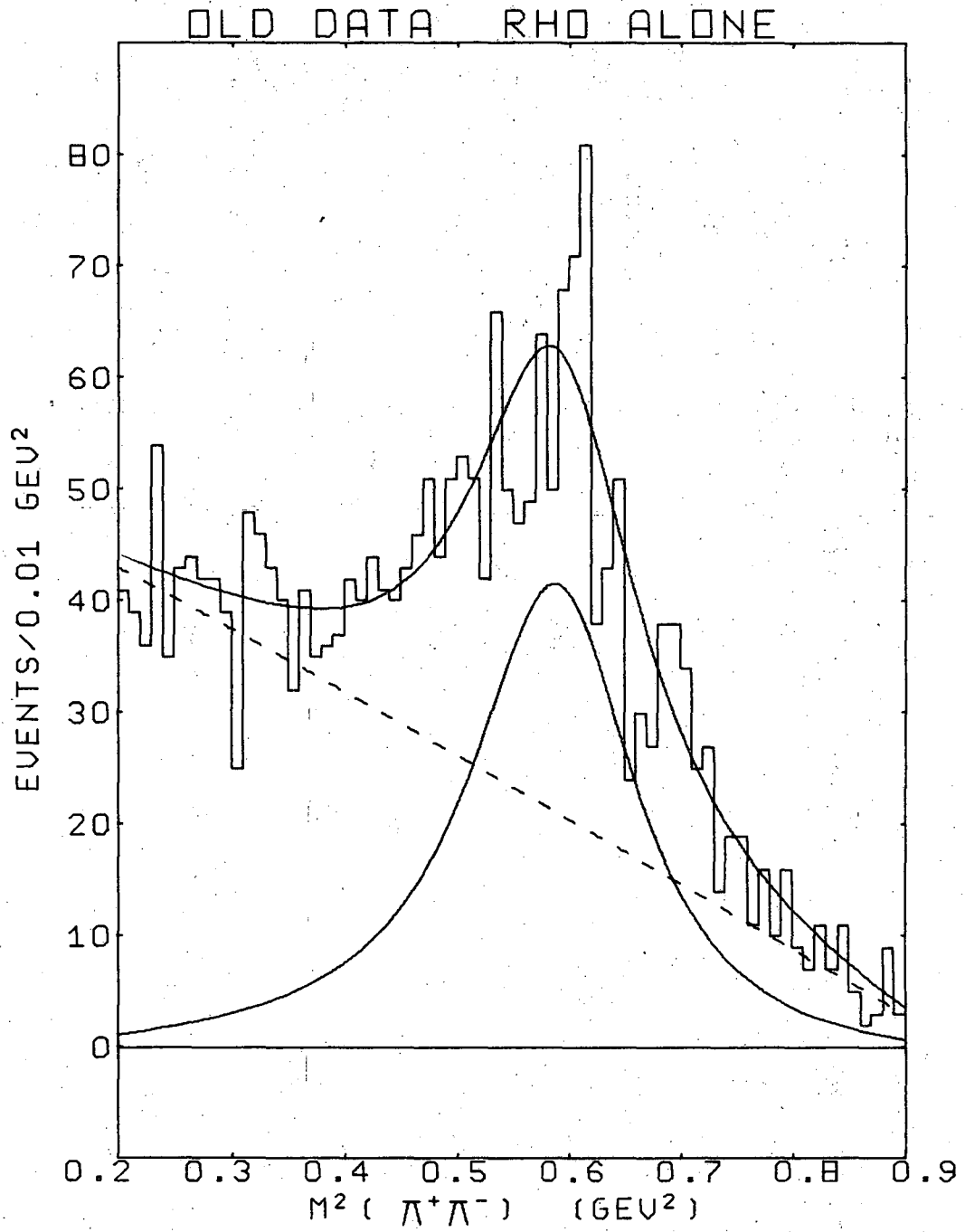
- Fig. 11. Two-pion mass-squared spectrum for the new data as in Fig. 3. The curves represent the fit including the  $\omega$  meson, as described in Fig. 9.
- Fig. 12. Two-pion mass-squared spectrum for the 1.5-GeV/c data as in Fig. 4. The curves represent the fit including the  $\omega$  meson, as described in Fig. 9.
- Fig. 13. Two-pion mass-squared spectrum for the 1.7-GeV/c data as in Fig. 5. The curves represent the fit including the  $\omega$  meson as described in Fig. 9.
- Fig. 14. Two-pion mass-squared spectrum for the 2.1-GeV/c data as in Fig. 6. The curves represent the fit including the  $\omega$  meson, as described in Fig. 9.
- Fig. 15. Two-pion mass squared spectrum for the 2.6-GeV/c data as in Fig. 7. The curves represent the fit including the  $\omega$  meson, as described in Fig. 9.
- Fig. 16. Contours of  $\chi^2$  for the 1.5-GeV/c data. The variables  $\alpha_3$  and  $\alpha_5$  represent the  $|BW|_{\omega}^2$  and  $\text{Re}(BW)_{\omega}$  terms; they are not strongly correlated with the other variables in the fit. The contours are labeled by the differences of  $\chi^2$  from the best fit value, which is 60.3 for 61 degrees of freedom.





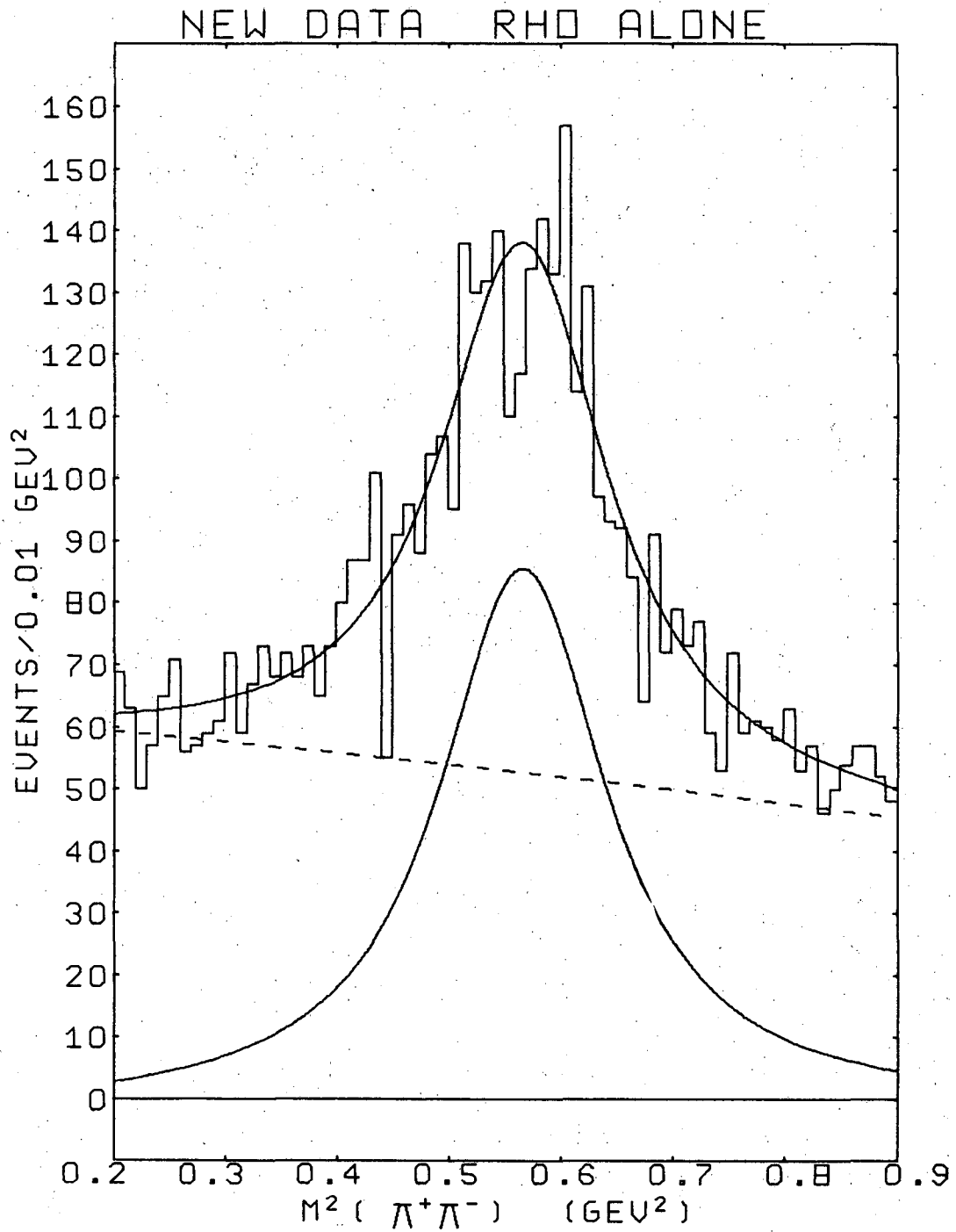
XBL 691-54

Fig. 1



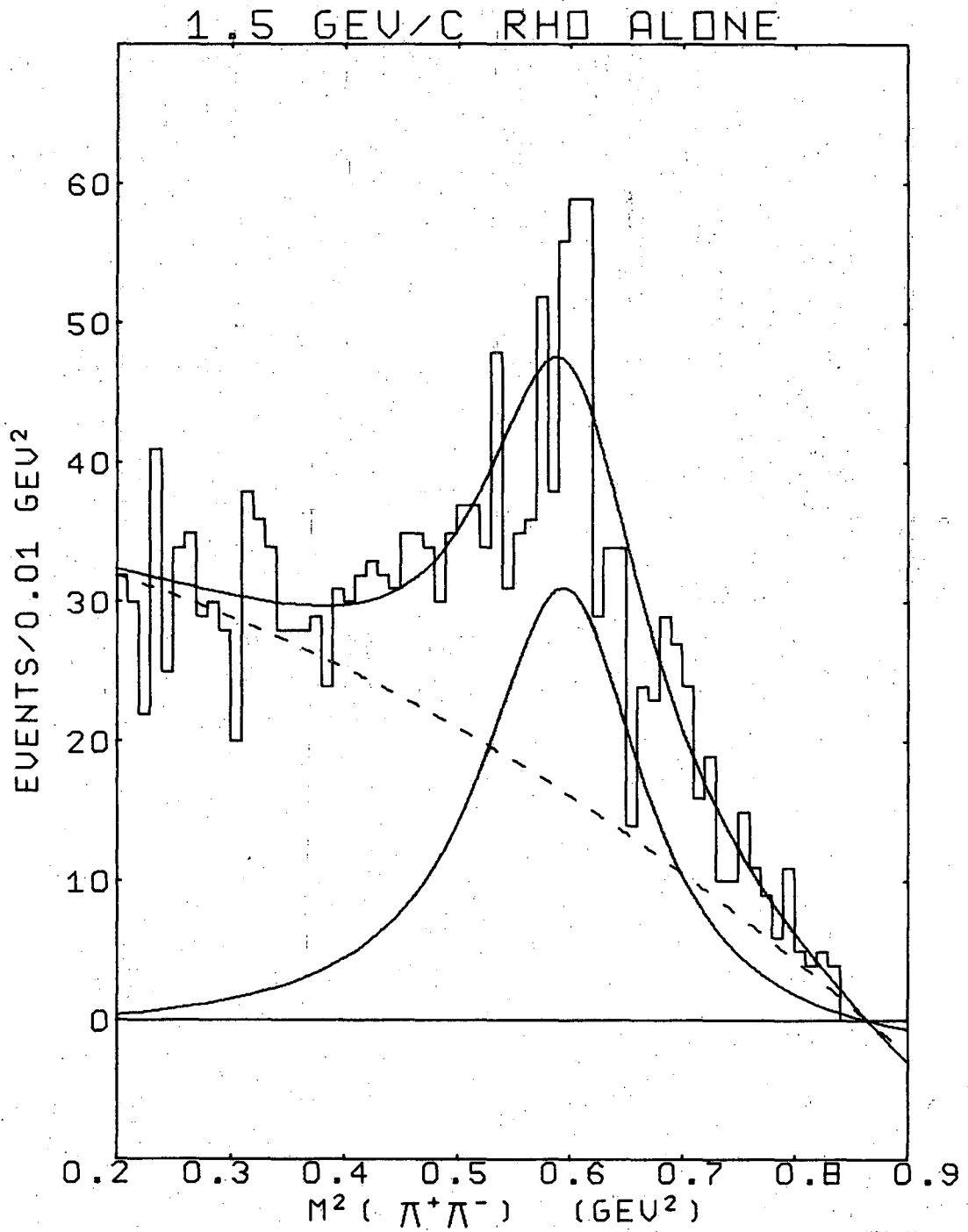
XBL 691-55

Fig. 2



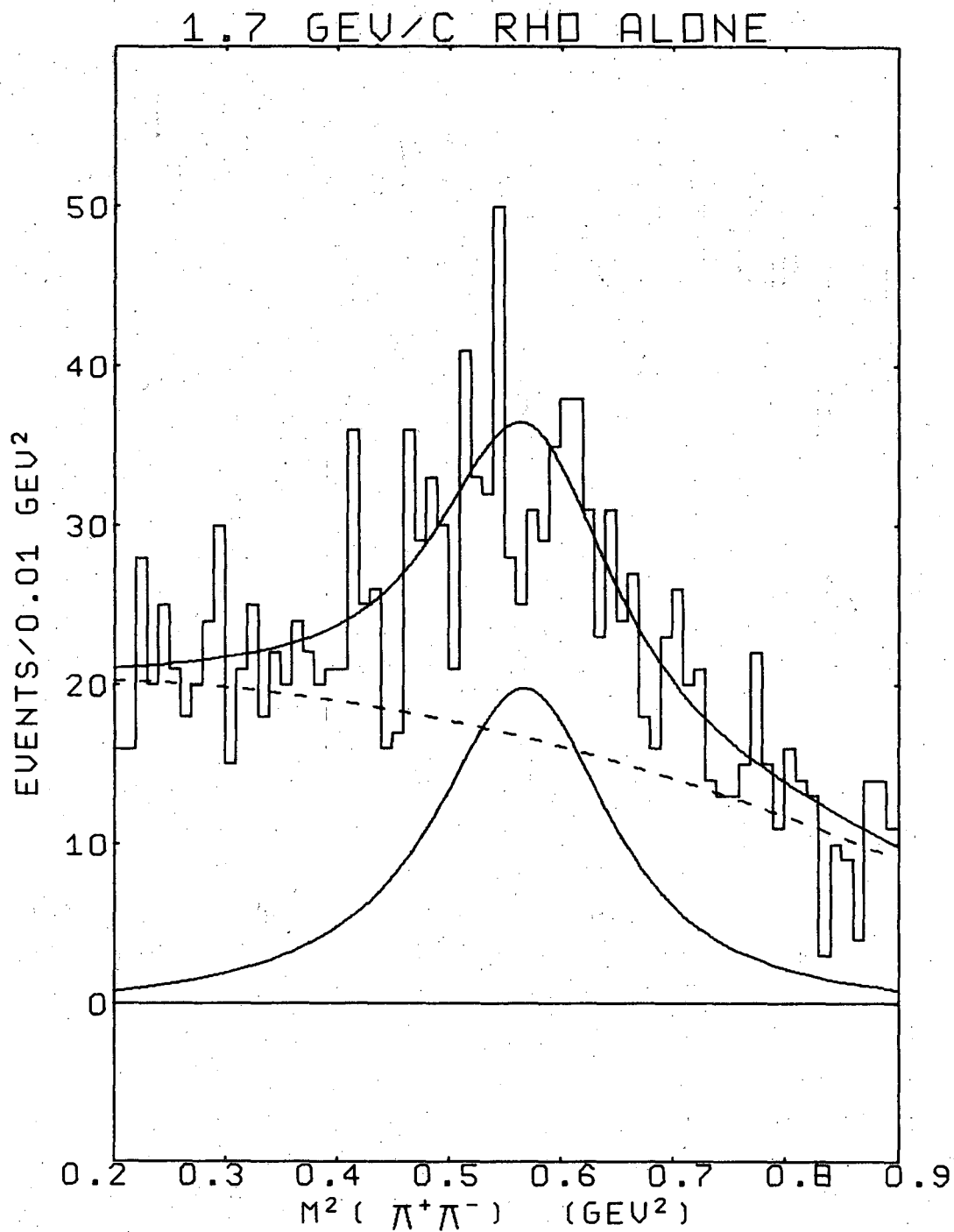
XBL 691-56

Fig. 3



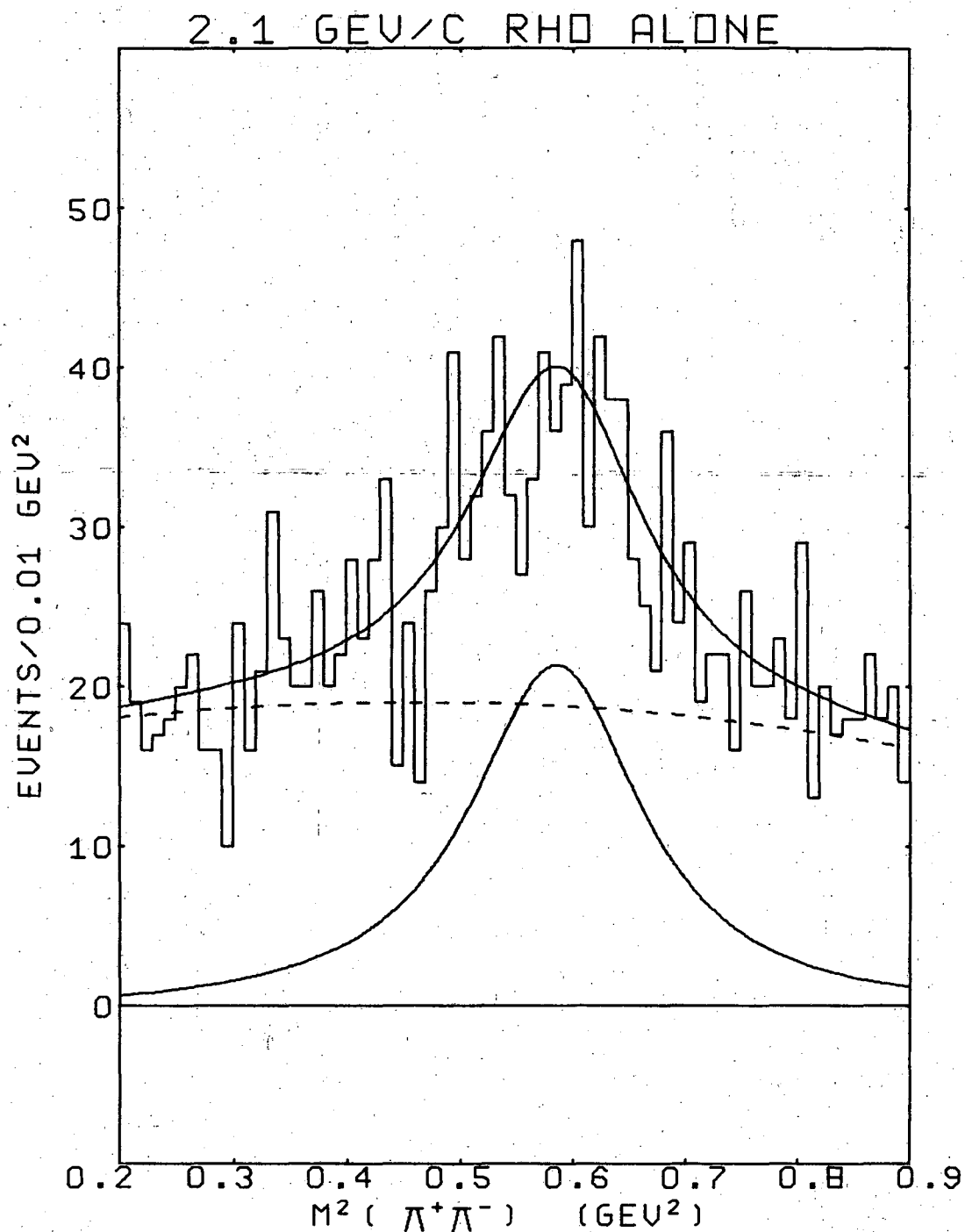
XBL 691-57

Fig. 4



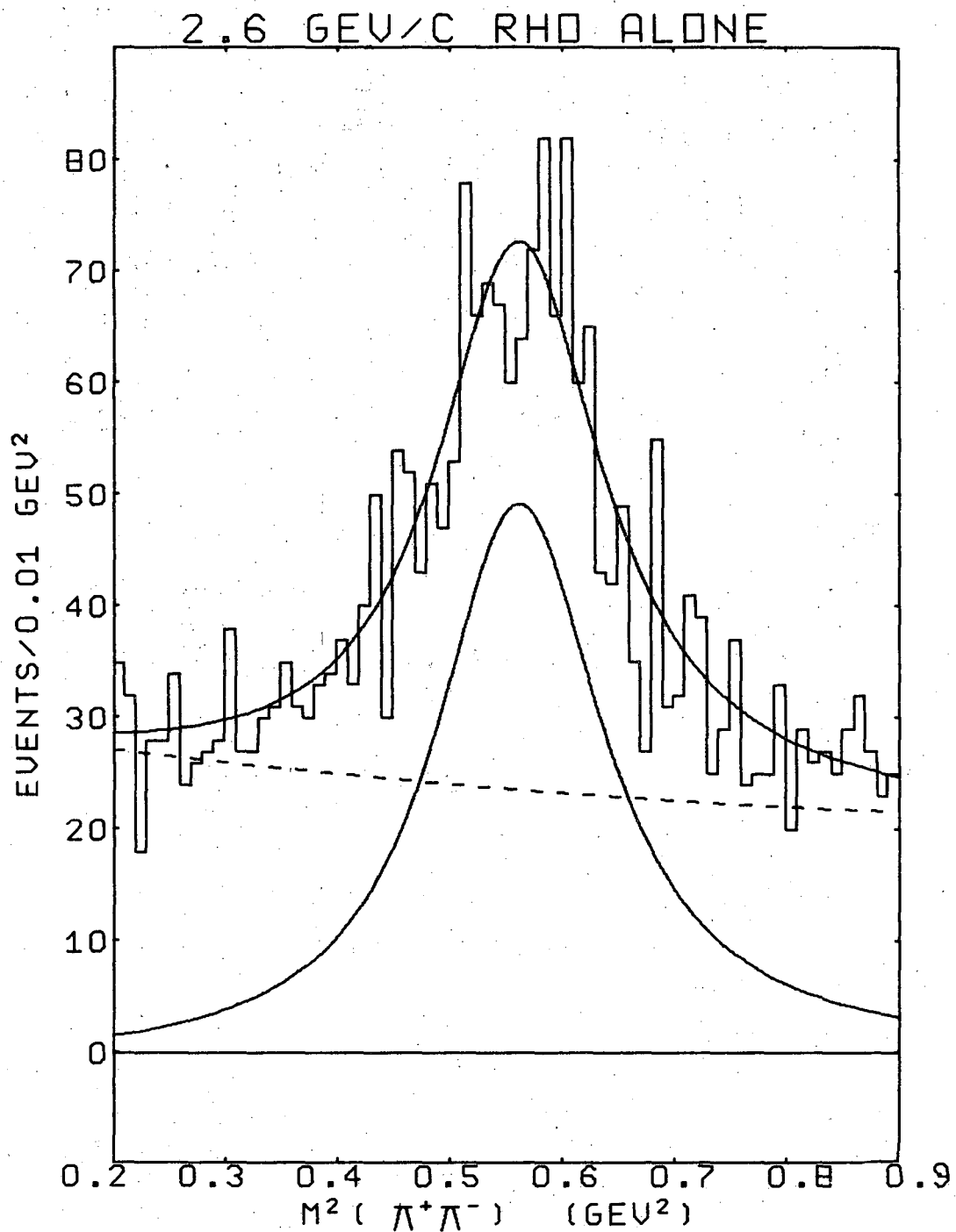
XBL 691-58

Fig. 5



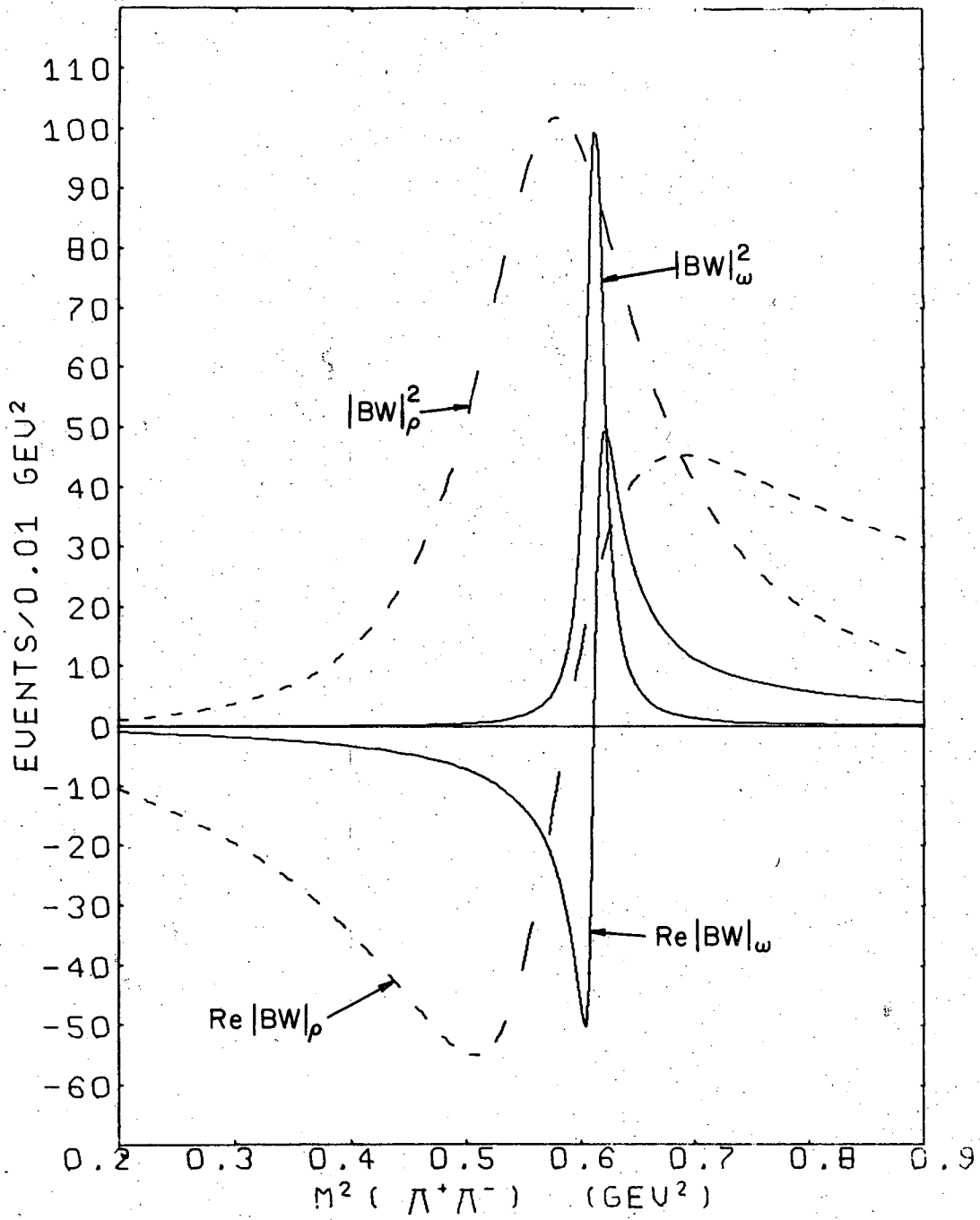
XBL 691-59

Fig. 6



XBL 691-60

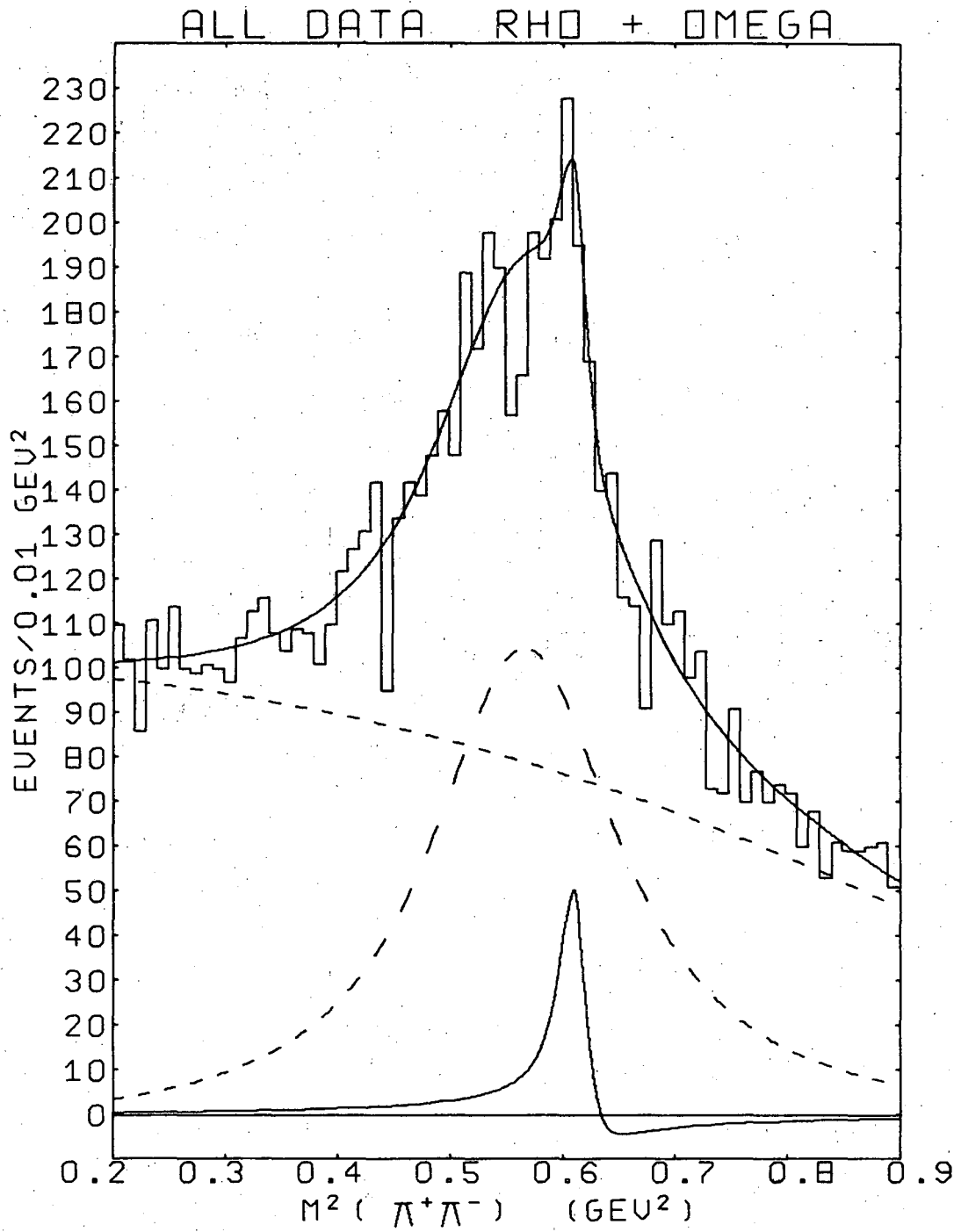
Fig. 7



XBL 691-61

Fig. 8

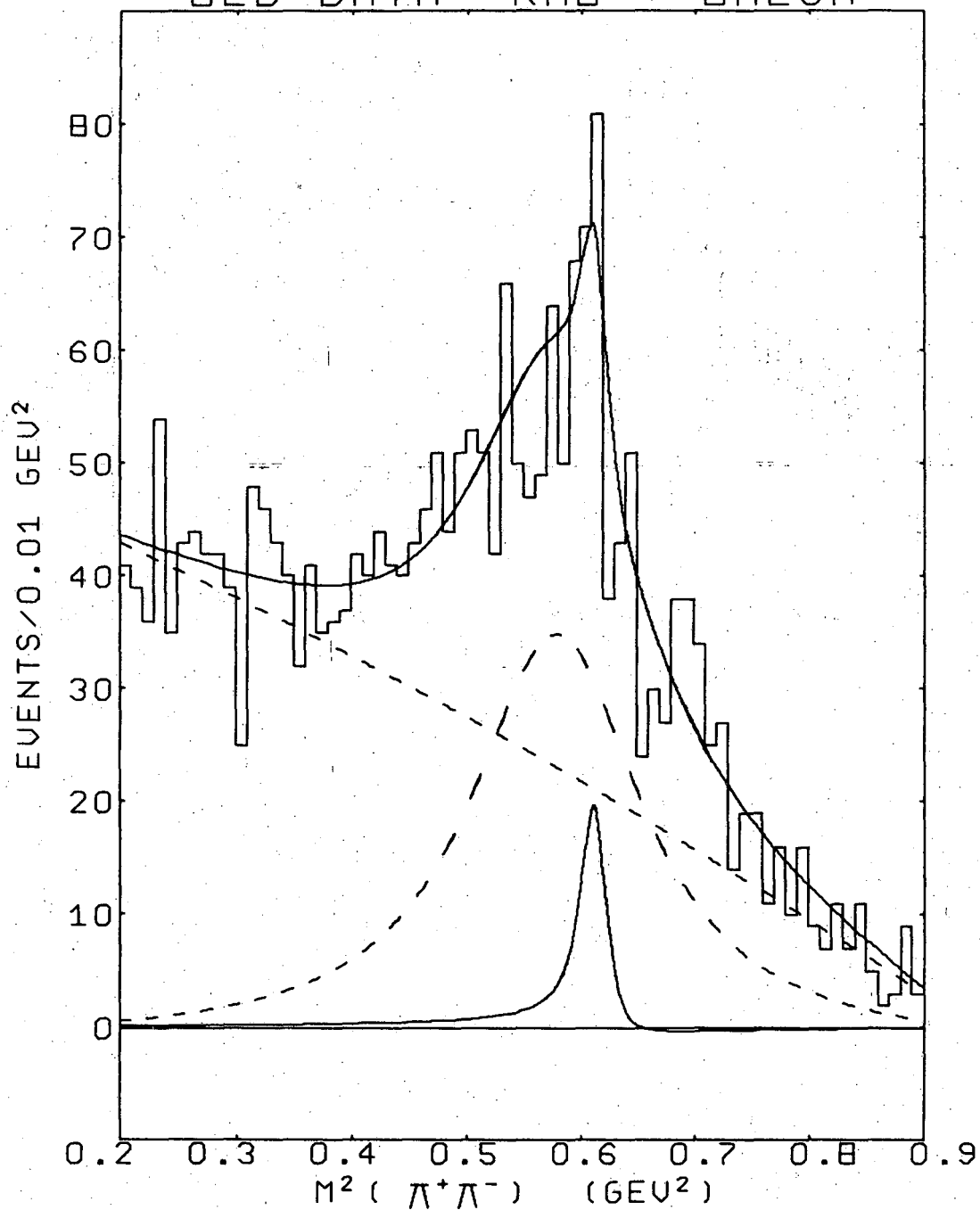




XBL 691-62

Fig. 9

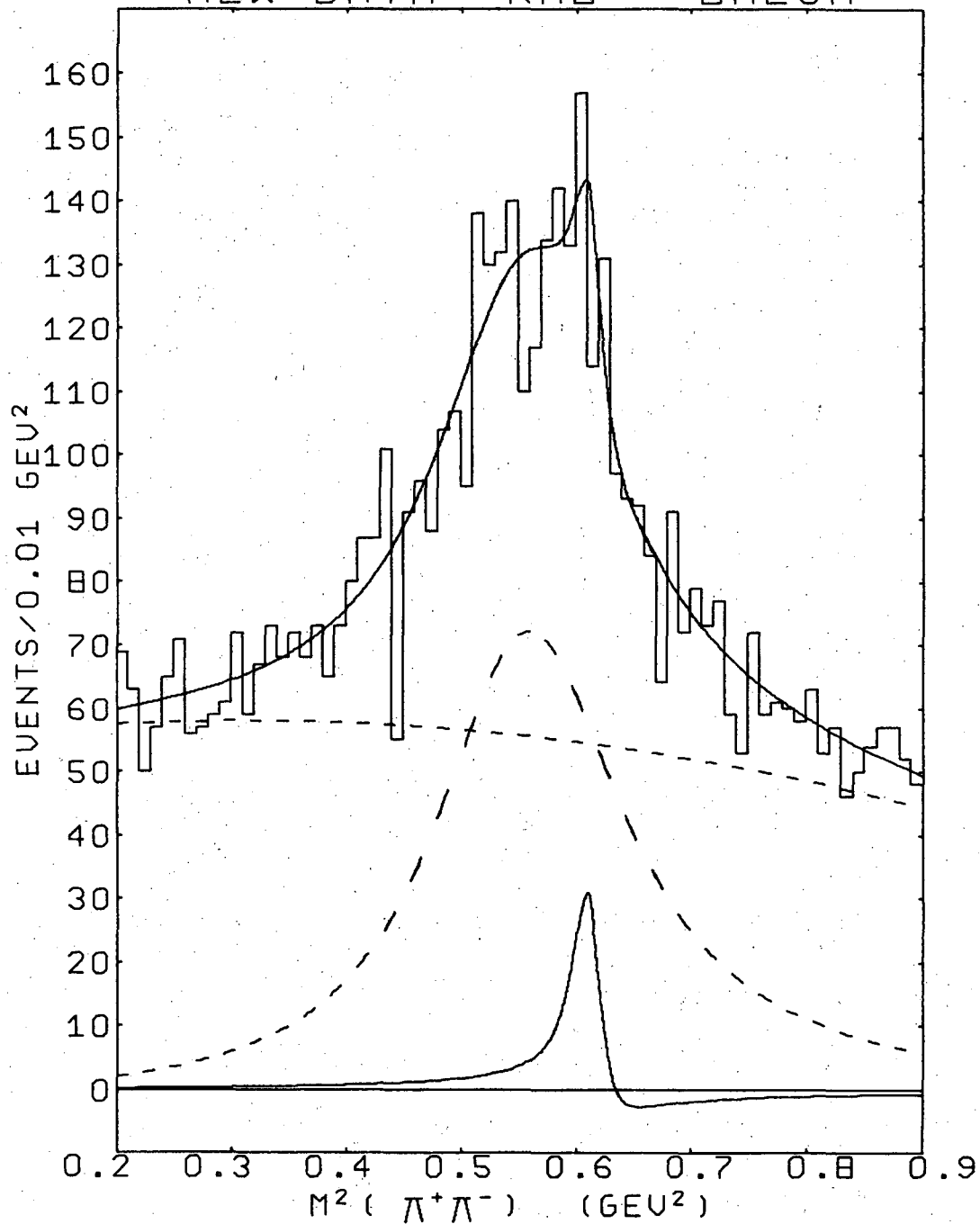
OLD DATA RHO + OMEGA



XBL 691-63

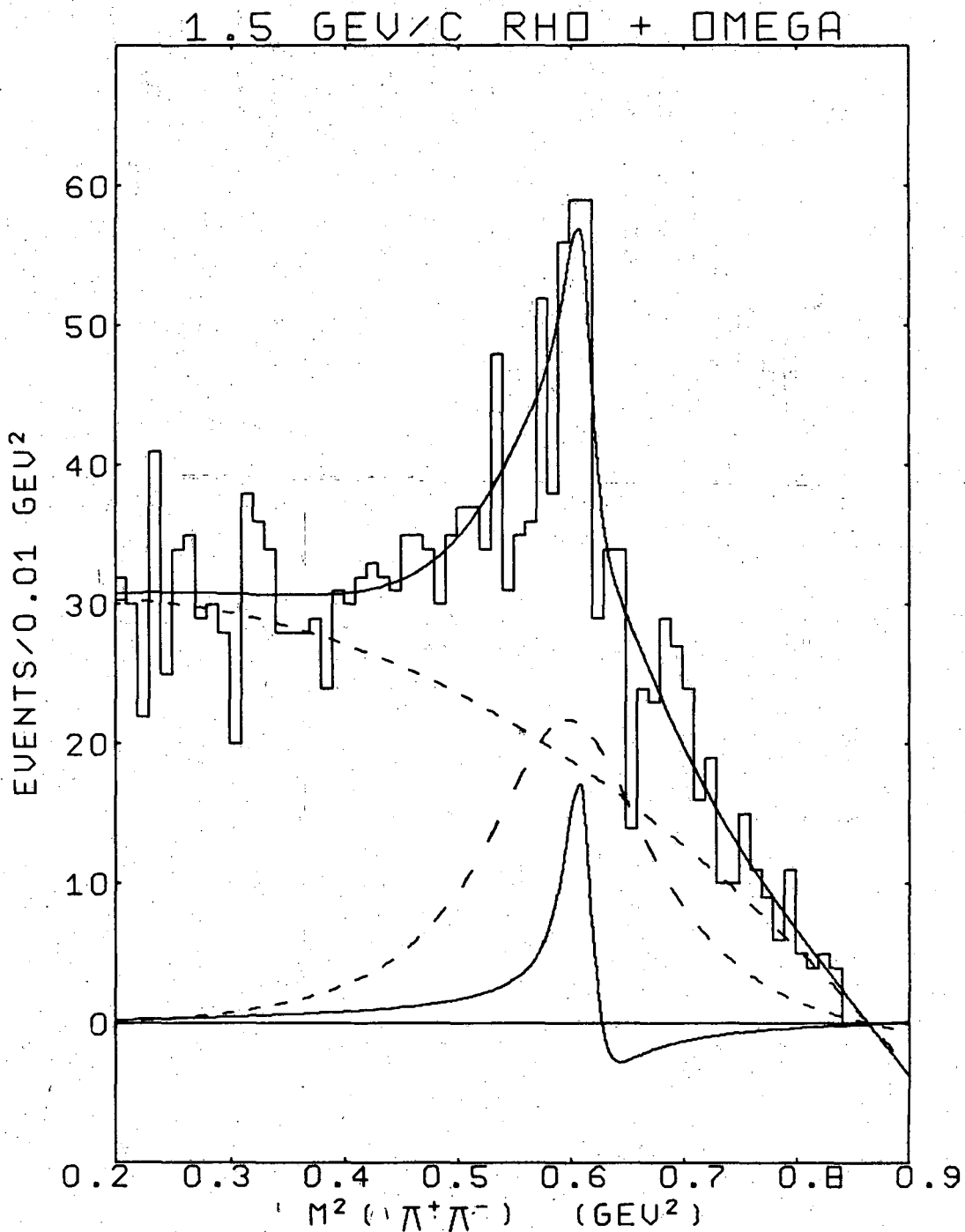
Fig. 10.

NEW DATA RHO + OMEGA



XBL 691-64

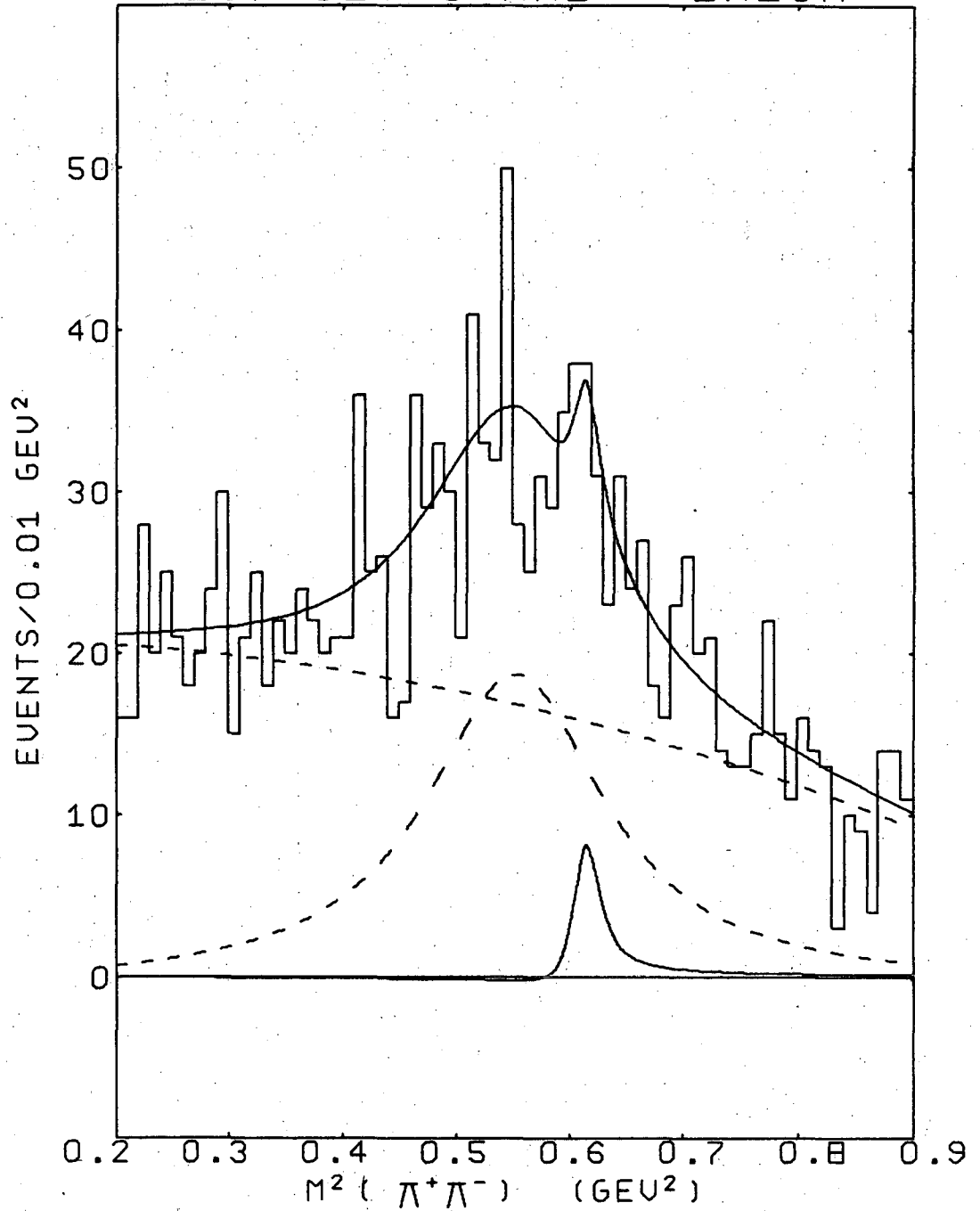
Fig. 11



XBL 691-65

Fig. 12

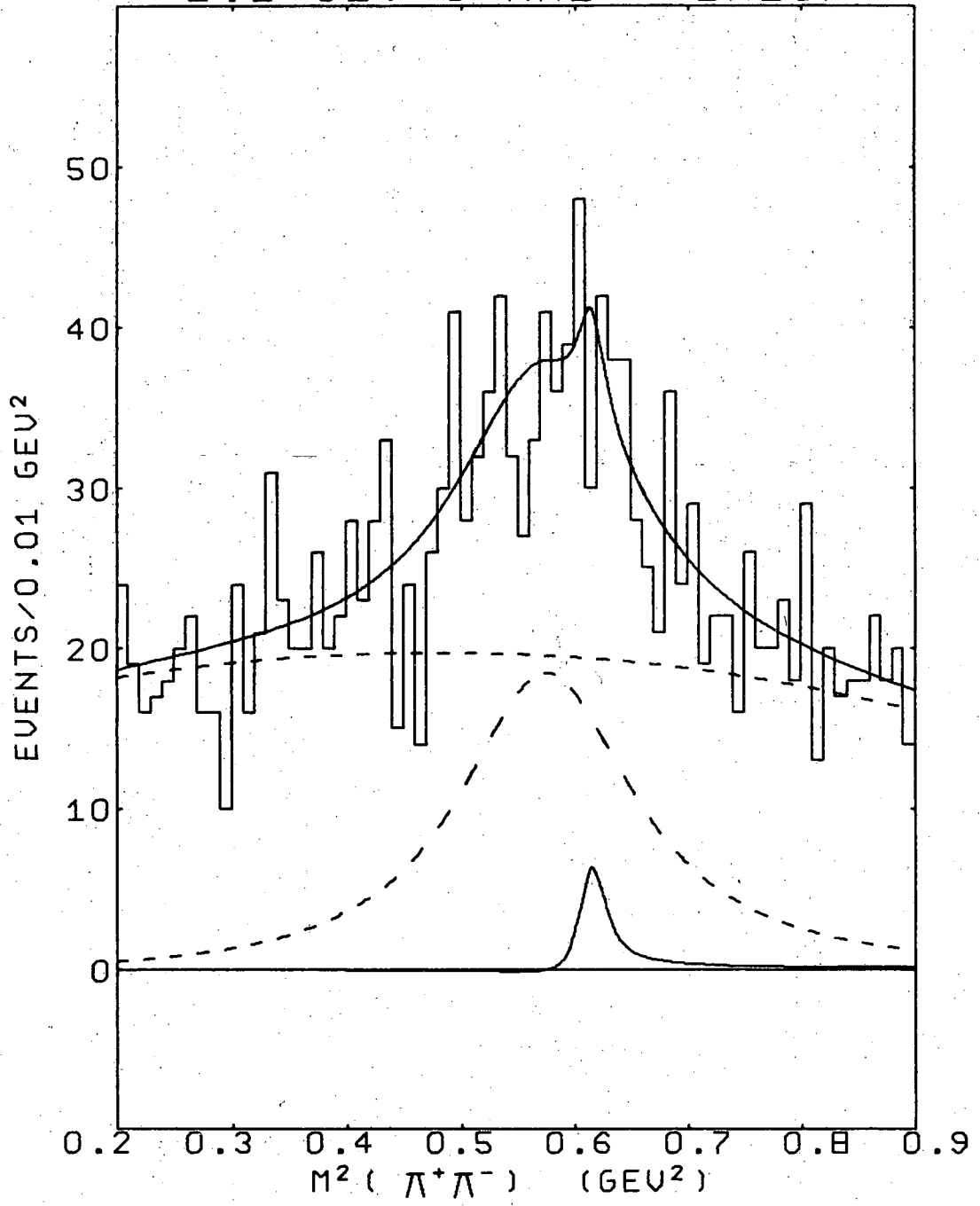
1.7 GEV/C RHO + OMEGA



XBL 691-66

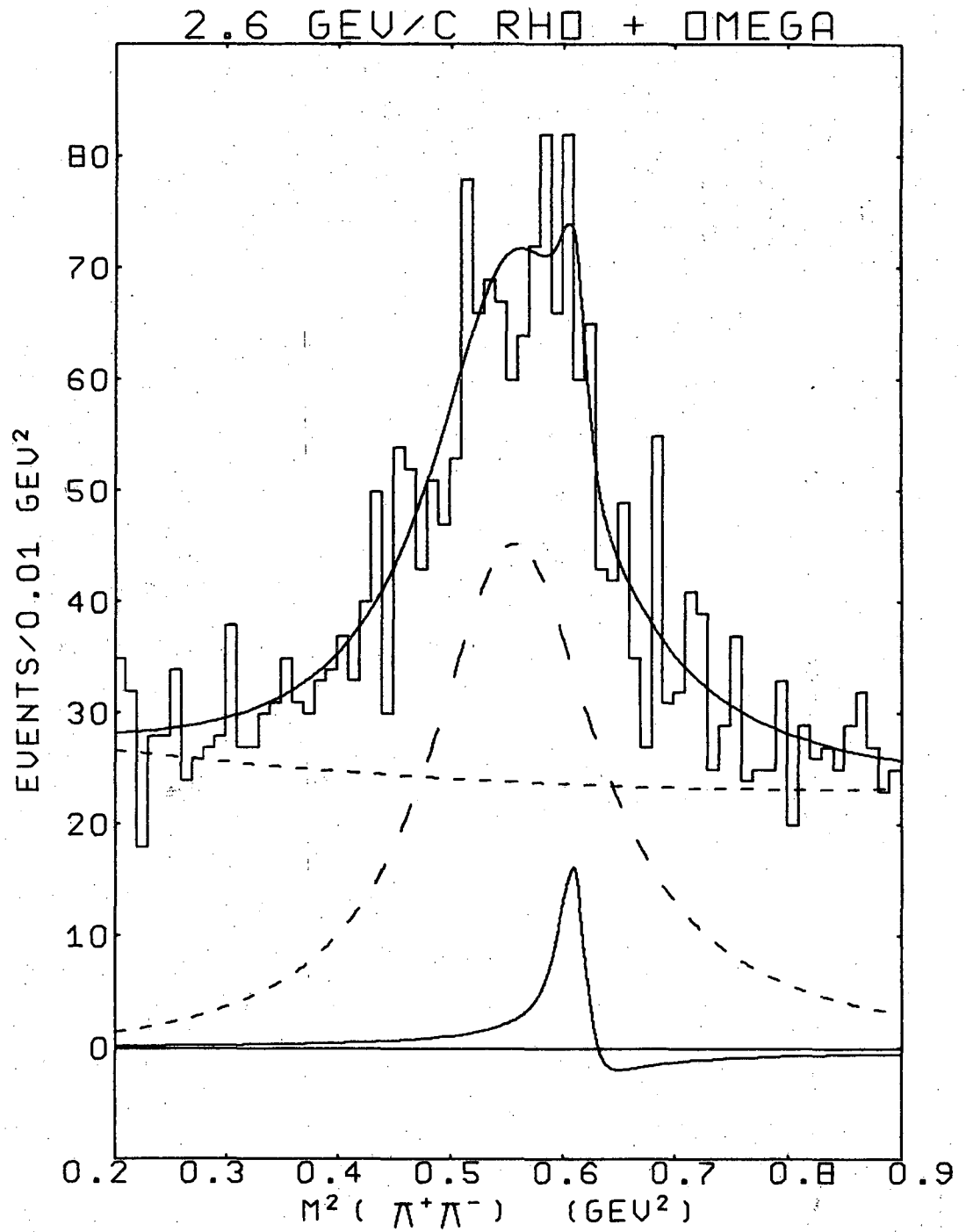
Fig. 13

2.1 GEV/C RHO + OMEGA



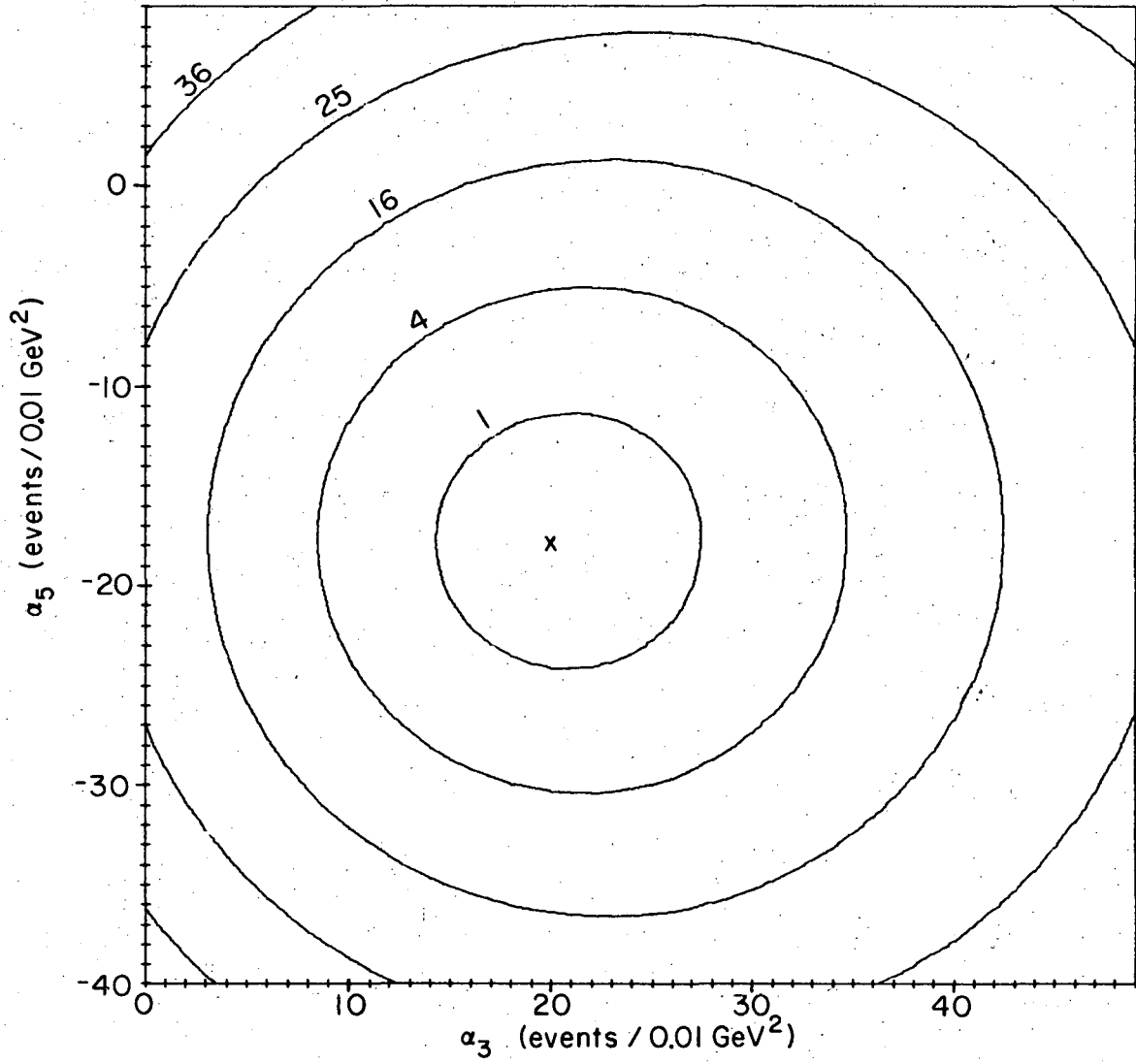
XBL 691-67

Fig. 14



XBL 691-68

Fig. 15



XBL 691-69

Fig. 16



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