Analytical study of high altitude turbulence wide-field wavefront sensing: impact on the design and reconstruction quality of future solar AO systems

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ABSTRACT

The European Solar Telescope is a 4-m planned facility designed to have high spatial resolution capabilities to understand the mechanisms of magnetic coupling in the chromosphere and the photosphere. It will feature both a conventional and a multi-conjugate adaptive optics (AO) of similar complexity than the systems for night-time Extremely Large Telescopes. A particularity of solar AO is that it uses the solar granulation as a reference; therefore the wavefront sensing is performed using correlations on images with a field of view of ~10”. A sensor collecting such a wide field of view averages wavefront information from different sky directions, affecting the sensing of high altitude turbulence, the sampling of which does not depend anymore on the size of the subapertures only, but rather on the size of the projection of the extended field of view. Understanding this effect is crucial for the design of future solar facilities, i.e. to choose the adequate height of the DMs on MCAO systems, and also to predict the quality of the reconstruction that such system would be able to achieve. For that reason, we have studied wide field sensing and found the analytical equations that describe the process, in order to use this information as an input to improved designs of solar AO systems.

Keywords: adaptive optics, reconstruction algorithms, solar adaptive optics, wide field sensing

1. INTRODUCTION

One of the challenges of solar AO is that the wavefront sensor has to work on extended and low contrast objects such as sunspots or solar granulation[1]. A correlating Shack-Hartmann is used to sense the wavefront. The FOV has to be large enough to contain structure for the correlation algorithm to work robustly, but not too large, to avoid averaging of wavefront information from the upper layers of the atmosphere. Usually a FOV of 8-10 arcsec is used. With such FOV, the anisoplanatism affects the measurements of the correlating SHWFS, averaging the wavefront information over the field of view and thus decreasing the sensitivity to wavefront distortions introduced at large heights above the telescope aperture. For low elevation observations, the increased line-of-sight distance to the turbulent layers leads to a wider wavefront area to be averaged for a given FOV. We observe two limiting cases (see Figure 1): when the telescope diameter is smaller than the projection of the FOV on the upper layer, and when it is much larger than it. In the first case, the tip-tilt information is acquired from a surface bigger than the telescope diameter, but it is corrected on the telescope aperture, giving rise to an error, for small diameters, larger than the error when only the tip-tilt is corrected.

As the diameter of the aperture approaches the projection of the FOV on the layer, the error approaches that of a system where only the tip-tilt is corrected, and at a certain point it starts to be smaller, because the system starts to be able to correct more modes. When the diameter of the telescope is much larger than the projection of the FOV on the upper layer, the extended field has an effect on the fitting error, which will be limited by the size of the projection, as can be seen in Figure 2.

The contribution of this anisoplanatism to the AO measurements must be taken into account in solar AO performance evaluation[2]. This was never done before the starting of AO studies for the 4-m class solar telescopes. So far no analytical formulas included the wavefront sensor anisoplanatism error in the error budget or in the error model used to perform the simulation. Therefore, the only way to estimate its effect was numerically, by including an end-to-end model of the wide field cross-correlation WFS in the simulations.

We have developed an analytical formula that follows the behavior of the performance which can now be included in the error budget of the system in order to better evaluate solar MCAO systems. With this expression, and knowing the atmospheric profile of the site, we can i.e. foreseen whether it is worth it to place a mirror at a certain height or if the error due to the undetected turbulence at that given height will be too large. This will help us to better design and understand AO solar systems.

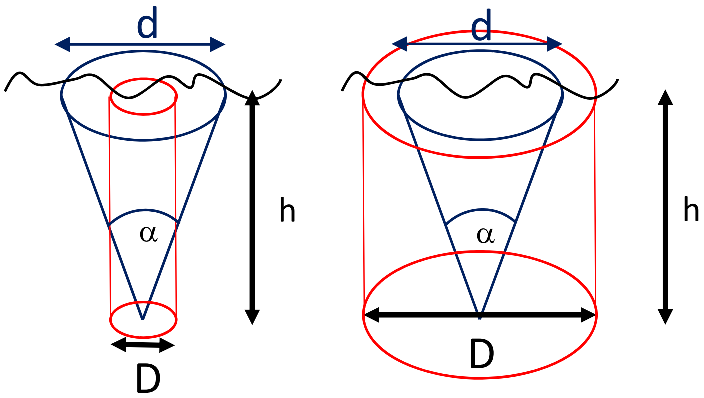

Figure 1: limiting cases for the sensing of the high altitude turbulence: left, telescope diameter much smaller than the projection of the FOV; right, telescope diameter much larger than the projection of the FOV

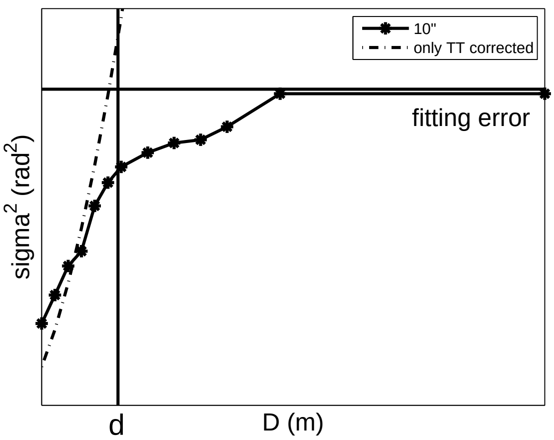

Figure 2: error variance for Kolmogorov statistics

2. ANALYTICAL STUDY OF THE STREHL DEGRADATION DUE TO UNDETECTED HIGH ALTITUDE TURBULENCE

In this section we explain the analytical expression of the Strehl degradation due to undetected high altitude turbulence. The error variance of the fitting when having an atmosphere with several layers is the addition of the error variance of the fitting at each layer, calculated considering the r_0^i at each particular layer:

$$\boldsymbol{\sigma^2_{fitting} = \sum_{i=layers} \sigma^2_i\left(r^i_0\right)} \tag{1}$$

being $\sigma^2_i\left(r^i_0\right)$ the error variance of the fitting generalized for a WFWFS

$$\boldsymbol{\sigma^2_i\left(r^i_0\right) = cte_1 \left(\frac{D^i_1}{r^i_0}\right)^{5/3} \left(\frac{D^i_2}{D^i_1}\right) - cte_2 \left(\frac{D^i_2}{r^i_0}\right)^{5/3}} \tag{2}$$

where cte_1 and cte_2 depend on the reconstruction method, the atmospheric model and the influence function (being 0.121 and 0.043 respectively using FrIM3D[3] with Kolmogorov),

$$D^i_1 = d^i_{layer} + d_0$$

$$\boldsymbol{D^i_2 = \begin{cases} D + d_0 & D < D_{lim} \\ D_{lim} & else \end{cases}} \tag{3}$$

and

$$d^i_{layer} = h^i_{layer} \cdot \tan\theta_{fov} / \sin\phi$$

is the diameter of the projection of the WFWFS FOV on the high-altitude layer. The height of the layer is h^i_{layer}, θ_{fov} is

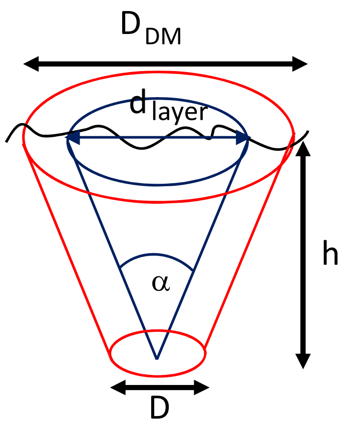

Figure 3: diameter of the metapupil due to the FOV of the WFWFS when the DM is at a certain height.

the WFWFS FOV, ϕ is the elevation of the telescope and d_0 is the subaperture size.

To calculate D_{lim} we have to take into account that the error variance regime changes when the diameter of the metapupil at the height of the mirror, $D_{DM} = D + d_{DM} + d_0$, (see Figure 3) is two times the diameter of the projection of the FOV of the WFWFS, to have the necessary sampling. The limit D_{lim}, to have the adequate sampling, occurs when:

$$\boldsymbol{D_{DM} > 2 * d^i_{layer} + d_0} \tag{4}$$

$$D_{lim} = 2 * d^i_{layer} - d_{DM} \quad \text{with,}$$

$$\boldsymbol{d_{DM} = h_{DM} \cdot \tan\theta_{fov} / \sin\phi,} \tag{5}$$

being the diameter of the projection of the WFWFS FOV on the high-altitude DM at height h_{DM}. For $D < D_{lim}$ the projection of the WFWFS FOV is too large compared to the diameter of the telescope and the turbulence cannot be sampled adequately. Since the turbulence cannot be sampled, it cannot be corrected. When $D > D_{lim}$, the sampling is enough to sense and correct some turbulence, but the fitting error will be primarily dominated by the size of the projection of the WFWFS FOV.

3. SINGLE-CONJUGATE ADAPTIVE OPTICS

We first study the Single Conjugate AO (SCAO) case. In this case the correction is done with only one mirror, at the pupil of the telescope, and therefore $d_{DM} = 0$. In Figure 4 we have plotted the Strehl depending on the telescope diameter for two different WFWFS FOVs, 10” and 20”. The lines in the plot are the curves calculated from the analytical equation, and the marks are the values given by the numerical simulations done with FrIM. The size of the subapertures, d_0, is set to 4 cm because we are interested in studying the effect of the WFWFS FOV, and therefore we want to correct as good as theoretically possible.

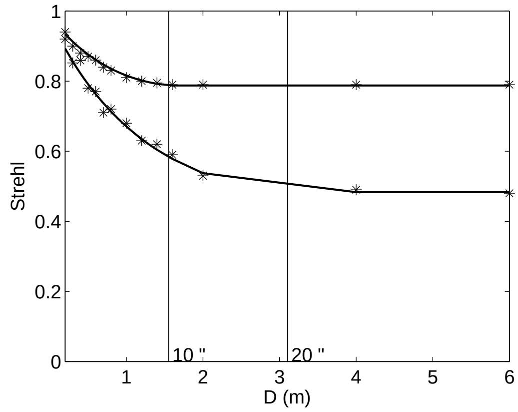

Figure 4: analytical (solid line) vs numerical (marks) Strehl for different WFWFS FOV, when the DM is at the pupil (SCAO). The atmosphere has only 1 turbulent layer at 8 km, the r_0 is 50 cm at zenith and the elevation 30 deg.

We can see that the Strehl is more degraded for the case with a FOV of 20”. Also that the results of the numerical simulations agree with the theoretical predictions.

We have analysed also how the error due to undetected turbulence changes with the height of the turbulent layer. With this we can predict from which height we will not be able to correct the turbulence, or also the minimum r_0 at each height that can be corrected with a given system. In Figure 5 we have plotted the results for the EST. We see that with a FOV of 8” the minimum r_0 at 20 km to have a degradation of the Strehl smaller than 0.7 would be 40 cm, but if the FOV is 15” then we would need an r_0 larger than 70 cm.

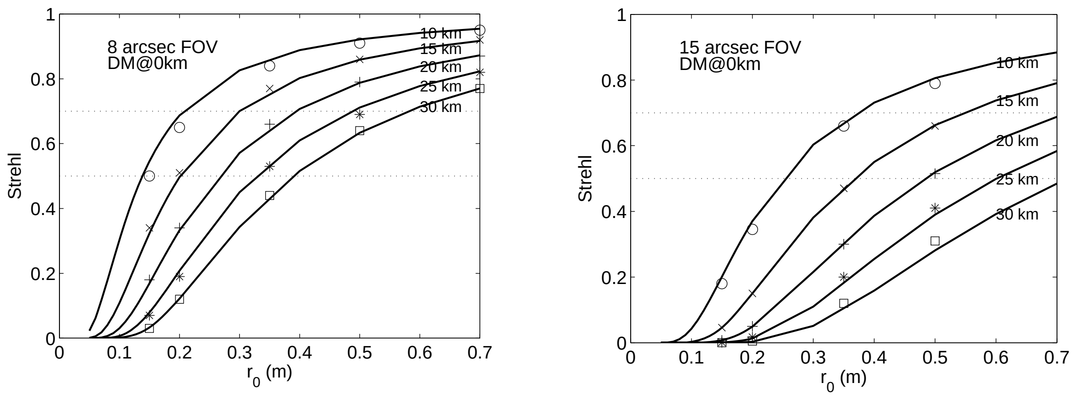

Figure 5: effect of the FOV of the WFWFS, the height of the layer and the r_0 on the performance of the system, and comparison of the theoretically predicted Strehl and the numerical result of the simulations. The lines are the predicted theoretical performance and the marks the numerical results from FrIM.

4. MULTI-CONJUGATE ADAPTIVE OPTICS

In the Multi Conjugate AO (MCAO) case the correction is done with placing the mirror at the plane conjugated to the height of the layer, and therefore $d_{DM} = d_{layer}$. In Figure 6 we have plotted the Strehl depending on the telescope diameter for a WFWFS FOVs of 10". The lines in the plot are the curves calculated from the analytical equation, and the marks are the values given by the numerical simulations done with FrIM. The size of the subapertures, d_0, is set to 8 cm. Again we can see that the results of the numerical simulations agree with the theoretical predictions.

We have also analyzed for this case how the error due to undetected turbulence changes with the height of the turbulent layer, and we have compared it to the case of having the DM at the pupil. In Figure 7 we have plotted the Strehl on-axis for the EST case. We see that when the DM is at the pupil, the minimum r_0 at 20 km to have a degradation of the Strehl smaller than 0.7 would be 50 cm, but if the DM is conjugated at the height of the layer, then we would need an r_0 larger than 42 cm. This is due to the extension of the FOV of the WFWFS. If the WFS would not have an extended FOV, the correction on-axis would be the same in one case or the other, and the only advantage of having the DM conjugated at the height of the pupil would be the correction of the Strehl in a larger field of view. Therefore, MCAO is clearly better than SCAO even for correcting on-axis in the case of solar telescopes.

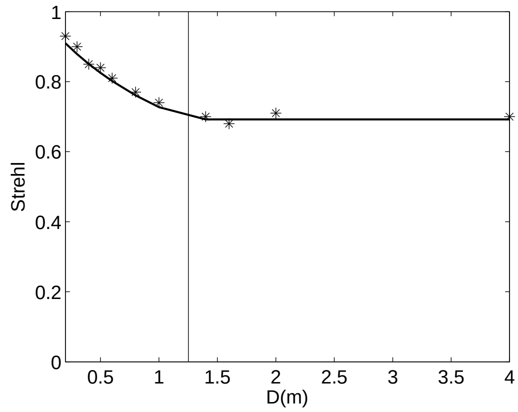

Figure 6: analytical (solid line) vs numerical (marks) Strehl when the DM is at the layer. The atmosphere has only 1 turbulent layer at 25 km, the r_0 is 50 cm at zenith and the elevation 0 deg.

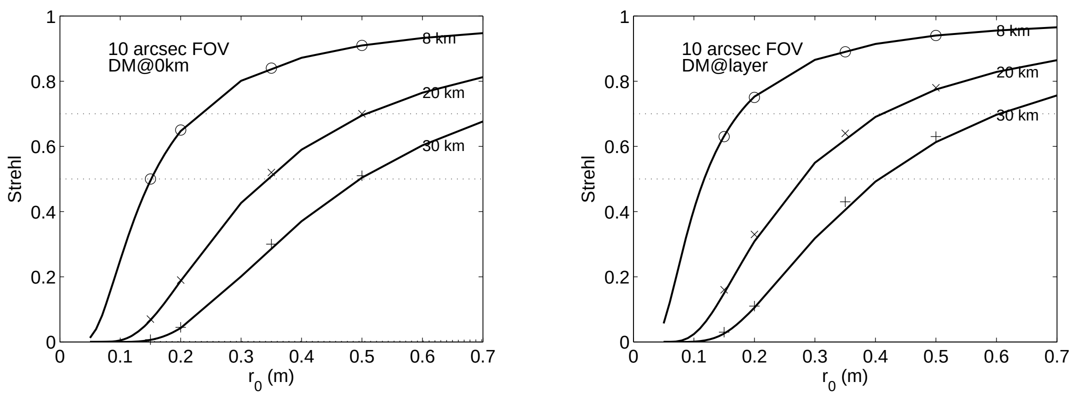

Figure 7: effect of the height of the layer and the r_0 on the performance of the system and the correction at the pupil (left) vs the correction at the layer (right). Comparison of the theoretically predicted Strehl and the numerical result of the simulations. The lines are the predicted theoretical performance and the marks the numerical results from FrIM.

5. ERROR BUDGET INCLUDING GENERALIZED FITTING

With the equations for the generalized fitting described in Section 2, we can estimate the Strehl ratio including the fitting, temporal delay, WFS measuring and bandwidth errors. As a reminder, we describe the equations used for each particular error source:

- The temporal delay error, in case no temporal prediction is made, can be estimated using the following expression:

$$\sigma_{delay}^2 = 0.962\left(\tau/\tau_0\right)^{5/3} \qquad \textbf{Equation 6}$$

where τ is the delay and τ_0 the coherence time of the atmospheric turbulence.

- The WFS measuring error can be estimated with the following equation:

$$\sigma_{WFS}^2 = \frac{5m^2}{4n_r^2 \cdot contrast^2 \cdot SNR^2} \qquad \textbf{Equation 7}$$

where *m* is the width of the reference subimage autocorrelation function in pixels, n_r is the subimage size in pixels, SNR is the signal-to-noise ratio, and the contrast is the is the measured contrast of the observed solar scene.

- The bandwidth error is due to the limited correcting bandwidth of the AO system. The bandwidth error is proportional to the ratio between the frequency of the turbulence, quantified by the Greenwood frequency f_G , and the bandwidth f_S of the AO system:

$$\sigma_{BW}^2 = \left(\frac{f_G}{f_S}\right)^{5/3} \qquad \textbf{Equation 8}$$

In the special case of a single turbulent layer moving at a speed *v*, the Greenwood frequency f_G can be written:

$$f_g = 0.427\frac{v}{r_0} \qquad \textbf{Equation 9}$$

In general, closed loop bandwidths in excess of 100 Hz are required for solar AO systems.

The parameters used to evaluate all these errors are written on Table 1.

λ (nm)	τ_0 (ms)	τ (ms)	SNR	contrast	m (pix)	n_r (pix)	f_S (kHz)	f_G (Hz)
550	2	1	223	3%	4	15	1	213

Table 1: EST AO system parameters

The total error is:

$$\sigma^2 = \sigma_{fitting}^2 + \sigma_{delay}^2 + \sigma_{WFS}^2 + \sigma_{BW}^2 \qquad \textbf{Equation 10}$$

The atmospheric parameters used for the simulations are an integrated r_0 of 15 cm (very conservative), and 2 layers. We consider two cases:

- low elevation case: layer @ pupil r_0=16 cm, altitude layer @ 25 km with r_0=50 cm
- high elevation case: layer @ pupil r_0=16 cm, altitude layer @ 15 km with r_0=50 cm

we use 2 DMs, one at the pupil with d_0=8 cm, and one at the altitude layer with d_0=30 cm. And the Strehl, comparing with the results simulated with FrIM3D[3]:

	Strehl (theoretical)	Strehl(simulated)
DM@25 km	0.40	0.41
DM@15 km	0.50	0.50

Table 2: EST AO Strehl, FrIM3D simulations vs theoretical result with the error budget

An homogeneous Strehl higher than 40% over the 1 arcmin FoV can be obtained even for low elevations and despite the intrinsic anisoplanatism of the WFWFS.

6. CONCLUSIONS AND FUTURE WORK

In this paper we have presented the equations that describe the effect of high altitude turbulence correlation on the performance of a solar AO system. We have shown that for telescope diameters larger than the projection of the FOV of the WFWFS on the upper layers, like the EST, what limits the performance of the system is the diameter of the projection of the FOV of the WFWFS on the layers, rather than the size of the subapertures. The reduction in the Strehl for low elevations is an effect of the generalized fitting error. Therefore, we need to consider this parameter, and not only the r_0, when designing a MCAO system for a solar telescope, and when predicting its performance.

Finally we have elaborated a complete error budget including the generalized fitting error. We have compared the Strehl obtained with the error budget with the numerical results and have found a very good agreement among both. We have calculated the total error for two very conservative atmospheric profiles (with r_0= 15 cm, which corresponds to a not very good day at the OT), corresponding to two situations (having the telescope pointing low and pointing towards zenith) and have found that an homogeneous Strehl higher than 40% over the 1 arcmin FoV can be obtained for the EST even for low elevations and despite the intrinsic anisoplanatism of the WFWFS.

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