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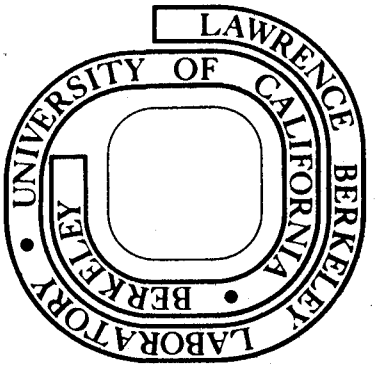

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THE CLASSICAL S-MATRIX FOR COULOMB EXCITATION*

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ABSTRACT

An expression of the classical S-matrix for Coulomb excitation is derived and directly evaluated without resorting to stationary phase integration methods. The results obtained are in quantitative agreement with other quantum mechanical and semi-classical predictions. This and the simplicity of the method suggest the feasibility of extending it to cases where other methods are not easily applied.

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The electromagnetic interaction that appears in the collision of a deformed target nucleus with a nuclear projectile excites rotational states of the target. Until recently only light projectile ions were available and partial wave, coupled channel, quantum mechanical calculations of this Coulomb excitation process were feasible.

With the advent of heavier projectiles exact quantum mechanical calculations have become impractical. However, the shorter de Broglie wavelength of heavy ions brings the problem closer to the realm of Classical Mechanics, and in recent years the Uniform Semi-classical Approximation (USCA), developed mainly by W. Miller,[1-3] was applied to this problem[4,5] with results that compare very well with the predictions of the semi-classical method of Winther and de Boer.[6]

Miller derives an integral expression for the classical S-matrix which he refers to as the *initial value representation of the S-matrix.*[2] The USCA method consists in evaluating this integral expression by the stationary phase approximation.

In this initial value representation, the Coulomb excitation of a two-dimensional rotor by a backscattered projectile has an S-matrix given by:

$$S_{I_2 \leftarrow 0} = \frac{1}{2\pi} \int_0^{2\pi} d\beta_1 \left[\frac{\partial \beta_2(\beta_1)}{\partial \beta_1} \right]^{\frac{1}{2}} \exp\left(i \left\{ \phi_2(\beta_1) + \bar{\beta}_2(\beta_1) [I_2(\beta_1) - I_2] \right\} \right) \tag{1}$$

where the final spins I_2 are $I_2 = 0, 2, 4, \ldots$ (in units of $\hbar$). Here β_1 is the initial orientation angle between the rotor axis and the beam axis, and $\bar{\beta}_2(\beta_1)$ is the final angle *shift* which is defined as

$$\bar{\beta}_2(\beta_1) = \beta_2(\beta_1) - \frac{mR_2(\beta_1)}{P_2(\beta_1)} \frac{\hbar I_2(\beta_1)}{\ell} \tag{2}$$

where $\beta_2(\beta_1)$ is the orientation angle, $I_2(\beta_1)$ is the angular momentum of the rotor (in units of $\hbar$), $R_2(\beta_1)$ is the projectile-target distance, and $P_2(\beta_1)$ is the relative momentum of the projectile in the center-of-mass system, all evaluated at a point along the trajectory such that the interaction has already taken place (the spin I has taken a constant value). The reduced mass of the system and the moment-of-inertia of the rotor are indicated by m and $\mathscr{I}$ respectively.

Finally, $\phi_2(\beta_1)$ in Eq. (1) is the classical phase in units of $\hbar$ evaluated along the trajectory that has an initial orientation β_1 and a final one β_2.

$$\phi_2(\beta_1) = -\frac{1}{\hbar}\int_{t(\beta_1)}^{t(\beta_2)} [R(t)\,\dot{P}(t) + \beta(t)\,\hbar\,\dot{I}(t)]\,dt \qquad (3)$$

where all variables were defined above.

The reason for taking the angle shift $\bar{\beta}_2$ and not the angle variable β_2 in the expression for the S-matrix is that Eq. (1) must be independent of the point in the asymptotic region where we terminate the trajectory. This requirement is satisfied by an expression such as Eq. (2). Wong and Marcus[7] give a more general relationship which can also be used in this case.

A real deformed nucleus must be considered as a three-dimensional rotor. The final spin takes the values $I_2 = 0.5, 2.5, 4.5, \ldots$ since to the quantum mechanical spin I there corresponds a classical spin $I + \frac{1}{2}$.[4] From geometrical considerations[4,9] the different orientations β_1 must be weighted by the factor $(8\pi \sin\beta_1)^{\frac{1}{2}}$ with $0 \leq \beta_1 \leq \pi$.

From Eq. (1) and preceding considerations we obtain the following expression for the Coulomb excitation S-matrix of a deformed nucleus by

a backscattered projectile

$$S_{I_2 \leftarrow 0} = \frac{1}{\sqrt{\pi}} \int_0^{\pi} d\beta_1 \left[\sin\beta_1 \frac{\partial \bar{\beta}_2(\beta_1)}{\partial \beta_2} \right]^{\frac{1}{2}} \exp\left(i \left\{ \phi_2(\beta_1) + \bar{\beta}_2(\beta_1) [I_2(\beta_1) - I_2] \right\} \right) \tag{4}$$

where all magnitudes retain their previous definition and $I_2 = 0.5, 2.5, 4.5, \ldots$.

This integral is not difficult to integrate even for massive targets and projectiles. In the case of ^{40}Ar on ^{238}U at 170 MeV laboratory energy shown in Fig. 1, it was sufficient to take a partition of the interval $[0,\pi]$ into 100 points to evaluate the excitation probabilities defined by

$$P_{(I)} = |S_{I \leftarrow 0}|^2 \tag{5}$$

with good accuracy. We compared our results with the Winther-de Boer[6] predictions since no quantum mechanical codes are feasible in this case. The energy levels were taken from the rotational model with constant moment of inertia but it would be very simple to allow for variable moment of inertia by taking the appropriate expression for the classical Hamiltonian used to calculate the trajectories. Equation (4) is useful even at energies near the Coulomb barrier where the onset of the nuclear potential makes the Winther-de Boer method no longer valid. The nuclear potential can be easily included in the classical equations of motion used to obtain the variables in the integrand of Eq. (4).

The fact that the stationary phase approximation is not used makes expression (4) more accurate where that approximation breaks down. Figure 2 and Table I show such a case; in this example the exponent in Eq. (4) does not change very rapidly and the stationary

phase approximation is not so good. The direct evaluation of the classical S-matrix gives much better agreement with the exact quantum-mechanical results than the USCA for this case.

There are also cases where more than one internal degree of freedom must be considered, such as in rotational-vibrational excitation. For these cases the uniform approximation to the classical S-matrix is more complicated[11] and the multi-dimensional search for the stationary points may be as difficult from a numerical point of view[10] as the direct evaluation of the classical S-matrix.

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TABLE I. Comparison of the ratios of the normalized excitation probabilities predicted by the Integral Expression (4) and by the USCA method to the Quantum-Mechanical coupled channel results of the AROSA code[8] in the case depicted in Fig. 2.

SPIN	INTEGRAL/Q.M.	USCA/Q.M.
0	1.06	0.97
2	0.90	1.06
4	0.99	0.90
6	1.13	0.61
8	1.15	0.29

FIGURE CAPTIONS

Fig. 1. Calculation of Coulomb excitation probabilities to excite members of the rotational ground band in ^{238}U for back-scattering of ^{40}Ar at E_{lab} = 170 MeV. The quadrupole moment of ^{238}U is taken to be 11.12 eb^2. The results given by the Winther-de Boer code are indicated by dark circles joined by full lines. The classical S-matrix values are indicated by open circles joined by dashed lines. The energy levels of ^{238}U are taken from the rotational model.

Fig. 2. Calculation of Coulomb excitation probabilities to excite the rotational band in ^{152}Sm for backscattering of ^{8}Be at E_{lab} = 30 MeV. The Quantum Mechanical calculations were performed using the coupled channel code AROSA.[8] The quadrupole moment of ^{152}Sm is taken to be 5.85 eb^2. Quantum Mechanical results are shown as dark circles joined by full lines, USCA results by open squares joined by dotted lines and integral expression (4) values are indicated by open circles joined by dashed lines. The energy levels of ^{152}Sm are taken from the rotational model for all calculations.

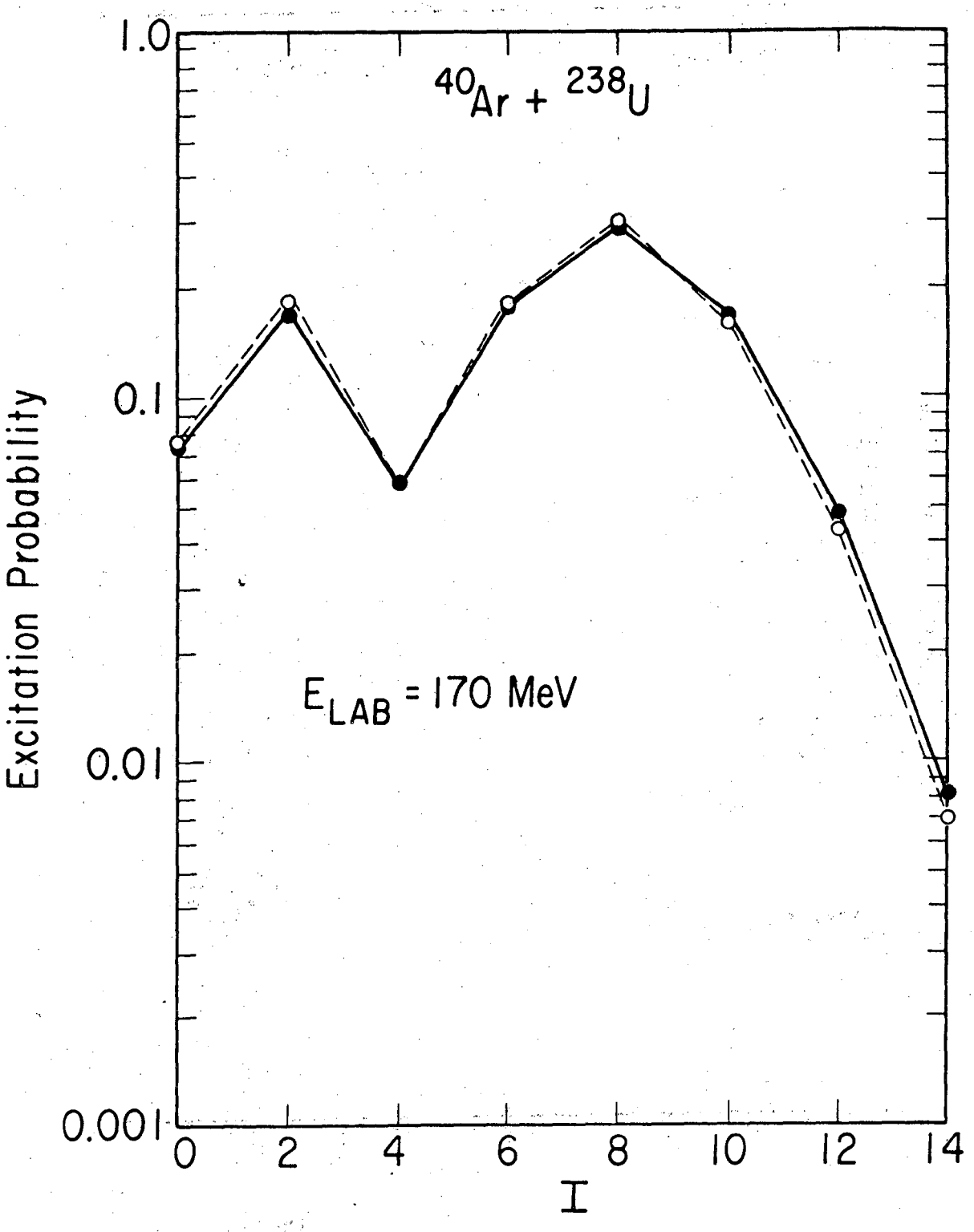

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Fig. 1

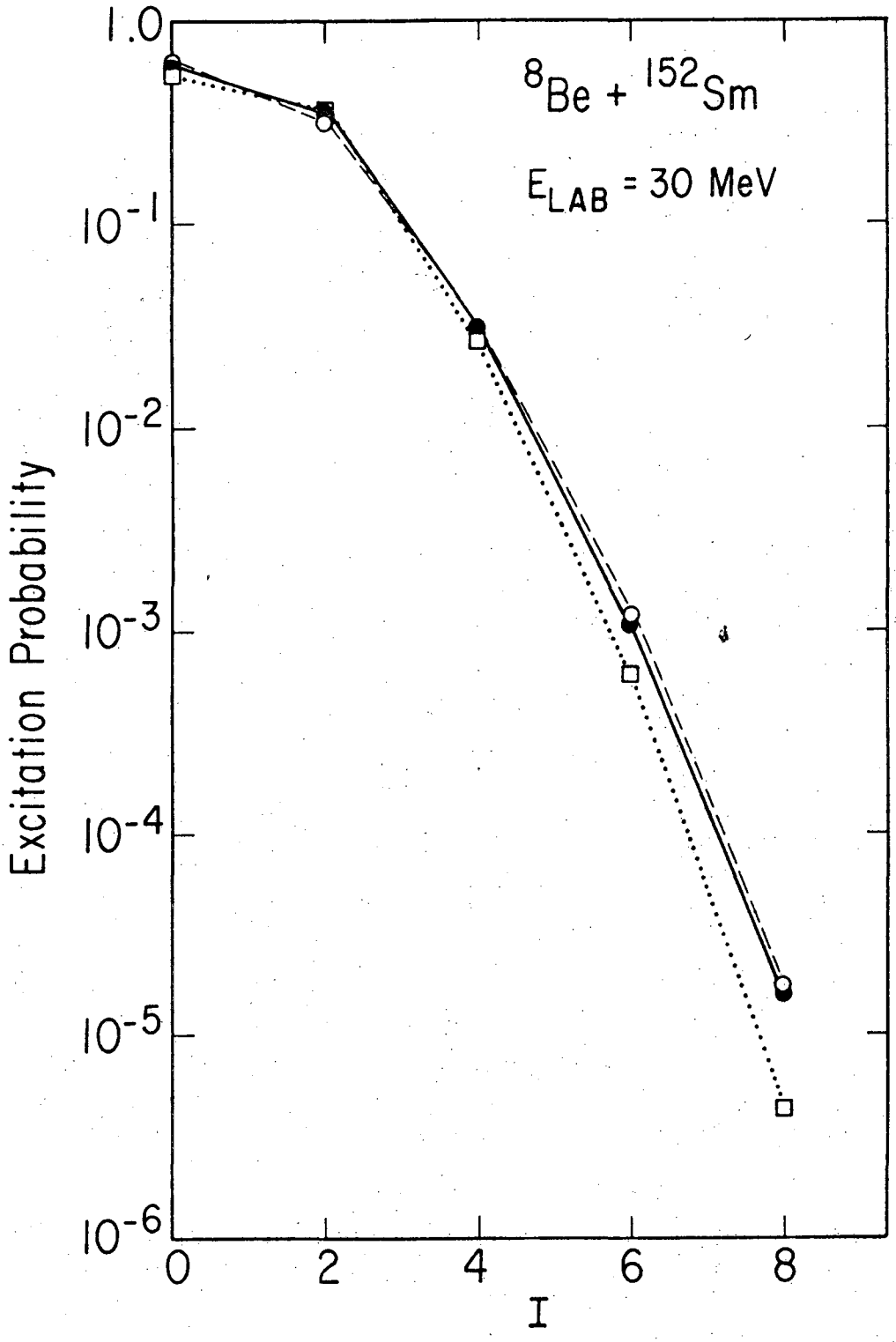
1.0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Excitation Probability
$^{8}Be + ^{152}Sm$
E_{LAB} = 30 MeV
0
2
4
6
8
I
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Fig. 2

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