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Physics of Non-Diffusive Turbulent Transport of Momentum and the Origins of Spontaneous Rotation in Tokamaks

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Recent results in the theory of turbulent momentum transport and the origins of intrinsic rotation are summarized. Special attention is focused on aspects of momentum transport critical to intrinsic rotation, namely the residual stress and the edge toroidal flow velocity pinch. Novel results include a systematic decomposition of the physical processes which drive intrinsic rotation, a calculation of the critical external torque necessary to hold the plasma stationary against the intrinsic residual stress, a simple model of net velocity scaling which recovers the salient features of the experimental trends, and the elucidation of the impact of the particle flux on the net toroidal velocity pinch. Specific suggestions for future experiments are offered.

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I. INTRODUCTION

The needs for understanding of, and predictive capacity for, both the off-diagonal flux of toroidal angular momentum and the origins of spontaneous or intrinsic rotation are now well established and accepted. Momentum transport has long been a subject of interest. Historically, the trend $\chi_\phi \sim \chi_i$ was predicted theoretically[1] and observed in pioneering experimental studies[2]. Subsequent observations of departures of χ_ϕ/χ_i from unity suggested the possibility of off-diagonal contributions to the momentum flux. This perception was re-inforced by several more detailed studies of the momentum flux[3], including dynamic modulation experiments[4]. Fluctuation studies also have indicated a link between sheared $\mathbf{E} \times \mathbf{B}$ flows and the parallel Reynolds stress[5]. In a related vein, the phenomena of spontaneous or intrinsic rotation is observed in nearly all tokamaks. In L-mode, the observed trends indicate that intrinsic rotation is strongly correlated with scrape-off-layer (SOL) asymmetry-induced flows[6]. H-mode plasmas display clearer empirical tendencies, namely that[7]:

1. rotation is typically co-current

2. the increment in central velocity Δv_ϕ at the LH mode transition scales with the increment in stored energy as $\Delta v_\phi \sim \Delta w/I_p$, with no observed dependence on ρ^* or ν^*. The Alfvenic Mach number at saturation scales as $M_A \sim \beta_N$

3. the offset value of v_ϕ in a co-to-counter torque scan matches the value of the intrinsic rotation. Moreover, the plasma can be held stationary against its tendency to rotate spontaneously by applying a torque in the counter-current direction[8].

Observations suggest that rotation is initiated at the edge and builds inward. Inversions at the L-H transition are possible. Intrinsic rotation is also possible in the core. In particular, values of χ_ϕ and the momentum pinch velocity V, inferred from perturbative experiments, cannot fit the measured $\langle v_\phi \rangle$ profiles of steady state plasma in JT-60U[9], and momentum transport bifurcations are observed in torque-free plasmas in TCV[10] and Alcator C-Mod[11].

The key physics quantity required to confront this body of observational evidence is the turbulent momentum flux. In general, the mean field momentum flux driven by electrostatic turbulence is given by (see, for example, Ref. [12] for a discussion of neoclassical transport processes)

$$\Pi_{r,\phi} = \langle n \rangle \langle \widetilde{v}_r \widetilde{v}_\phi \rangle + \langle \widetilde{v}_r \widetilde{n} \rangle \langle v_\phi \rangle + \langle \widetilde{n} \widetilde{v}_r \widetilde{v}_\phi \rangle \, . \tag{1}$$

Here the first term is the toroidal Reynolds stress and the second is the convective flux, hereafter neglected, unless otherwise noted. The third term, $\langle \widetilde{n}\widetilde{v}_r\widetilde{v}_\phi \rangle$, represents the nonlinear flux (as opposed to quasilinear), driven by processes such as mode-mode coupling, turbulence spreading, etc. It is hereafter neglected as beyond the scope of this paper. However, given the strongly nonlinear processes at work in generating rotation, we comment that confronting the nonlinear flux may ultimately be required. The Reynolds stress may be further decomposed as[13]

$$\langle \widetilde{v}_r\widetilde{v}_\phi \rangle = -\chi_\phi \frac{\partial \langle v_\phi \rangle}{\partial r} + V \langle v_\phi \rangle + \Pi^R_{r,\phi}, \tag{2}$$

where χ_ϕ is the turbulent viscosity, V is the convective velocity (i.e. the velocity pinch) and $\Pi^R_{r,\phi}$ is the residual stress. Note that χ_ϕ and V have well-known analogues in the theory of the particle flux, while $\Pi^R_{r,\phi}$ does not. In this paper, we discuss the status of our current understanding of χ_ϕ, V and $\Pi^R_{r,\phi}$, and the physics of turbulent transport of toroidal momentum and intrinsic rotation. The critical issues which are defined by the body of phenomenology which we address are:

1. What is the general structure of the turbulent momentum flux and the physics of its constituents?

2. what is the origin of intrinsic rotation, i.e. how can a plasma self-accelerate from rest? How is this related to residual stress? Can we predict the external torque required to cancel the intrinsic (i.e. self-generated) rotation?

3. In the event that intrinsic rotation originates from the inward convection of a flow at the plasma boundary, what is the physics of the pinch and how is it related to the corresponding particle flux?

4. What is the physics of the Rice scaling? Given the correlation between Rice scaling and H-mode, what is the influence of the pedestal physics on intrinsic rotation?

In the remainder of this paper we report on progress toward answers to these questions. In Section II – which addresses issue 1. – we survey the basic constituents of the turbulent momentum flux and their underlying physics. In Section III – which addresses issue 2. – we discuss the physics of the residual stress, which is the most unusual and counter-intuitive element of the turbulent momentum flux, but also the piece most important to intrinsic rotation. In Section IV – which addresses issue 4. – we outline a simple model which captures many of the basic scaling trends for intrinsic rotation. In Section V – which addresses issue 3. – we elucidate the role of the particle

flux and its impact on the toroidal velocity pinch. Section VI consists of a summary, discussion of suggested experiments, and brief comments on possible future work.

II. SURVEY OF TURBULENT MOMENTUM FLUX PHYSICS

The turbulent viscosity χ_ϕ is now relatively well understood. As was realized 20 years ago[1], χ_ϕ is closely related to the ion thermal diffusivity χ_i for drift wave turbulence. Recent simulation[14] and theory [15] works have discovered that near ITG marginality, when transport is dominated by the resonant scattering of slightly suprathermal ions (with $s = \omega/k_\| v_{Thi} \sim 2$), then

$$\frac{\chi_\phi}{\chi_i} \approx \frac{\langle s^2 \rangle}{(1 + \langle s^2 \rangle/2 + \langle s^4 \rangle/2)} \tag{3}$$

where the average is to be taken over the mean distribution function. Here the analysis used to derive Eqn. (3) neglected toroidal coupling effects. The simulations reported in Ref. [14] retained these, however, and appeared to be consistent with the reduced scope of the model utilized in Ref. [15]. This reveals that in stiff profile regimes, the *intrinsic* Prandtl number $Pr \neq 1$, but rather $Pr \sim 0.2 \rightarrow 0.5$, due to the inherent difference between wave-particle auto-correlation times for $\widetilde{v}_\phi$ andf $\widetilde{T}_i$. Here, it is important to note that the intrinsic Prandtl number is defined by the ratio of the purely diffusive fluxes, and differs from the conventionally quoted 'raw' Prandtl number $Pr \sim |\Pi_{r,\phi}/\partial\langle v_\phi \rangle/\partial r|/|Q/\partial\langle T_i \rangle/\partial r|$, defined without regard to the presence of non-diffusive fluxes.

The past two years have witnessed intensive interest in and study of the convective momentum velocity. Recent detailed theoretical work on the momentum pinch is reported in Refs. [16–18]. In general, the toroidal pinch may be decomposed into a turbulent equipartition (TEP) and thermoelectric (TH) piece

$$V = V_{TEP} + V_{TH}. \tag{4}$$

The turbulent equipartition convection velocity is purely inward (corresponding to a pinch) and is robust and mode independent.

Like the TEP pinch for density, the origin of the TEP pinch is in the compressibility of the $\mathbf{E} \times \mathbf{B}$ velocity in toroidal geometry ($\nabla \cdot V_{E\times B} \neq 0$), so that magnetically weighted angular momentum $V_\| R/B^2$ (rather than simply $nV_\| R$) is locally conserved. Thus, it is no surprise that the TEP momentum and particle pinches are strongly correlated. The TEP pinch has been derived from detailed gyrokinetic analysis[16, 17] and general considerations of angular momen-

tum homogenization[18]. For a stationary profile in the absence of a residual stress and $k_\parallel$, $V_{TEP}/\chi_\phi \approx -B^2/Rd/dr(R/B^2) \approx -3/R$, according to the definition with respect to $\langle v_\phi \rangle$ in Eqn. (2).

Extending the definition in Ref. [16], for $k_\parallel = 0$, the thermoelectric velocity V_{TH} is given by

$$V_{TH} = -\left\langle \sum_{\mathbf{k}} \mathrm{Re} \frac{\widetilde{v}_r^* \left(4\omega_{di} + \frac{v_{Thi}^2}{\langle v_\phi \rangle} k_\parallel^{Tor} \right) \widetilde{T}_i}{-i\left(\omega - 4\omega_{di} + i\Delta\omega_T\right)} \right\rangle. \tag{5}$$

Possible double counting of effects of $k_\parallel$ here can be avoided by separating its contribution to residual stress and the thermal pinch based on their physical origins, i.e., the $\mathbf{E} \times \mathbf{B}$ flow shear effect versus the toroidal effect.

The long wavelength limit ($k_\perp \rho_i \ll 1$) of the momentum evolution equation is given by Eqn. (32) of Ref. [16]

$$\begin{aligned} -i(\omega - 4\omega_{di} + i\Delta\omega_T)\widetilde{nv_\parallel} = &-\widetilde{v}_r \frac{\partial}{\partial r}\left(n_0 \langle v_\phi \rangle\right) - in_0 \left(3\langle v_\phi \rangle \omega_{di} + v_{Thi}^2 k_\parallel\right) \frac{|e|\,\widetilde{\phi}}{T_i} \\ &- in_0 \left(4\langle v_\phi \rangle \omega_{di} + v_{Thi}^2 k_\parallel\right) \frac{\widetilde{T}_i}{T_i} - iv_{Thi}^2 k_\parallel \widetilde{n}. \end{aligned} \tag{6}$$

A set of fluid moment equations for subsonic mean flow ($\langle v_\phi \rangle \ll v_{Thi}$) are completed with the addition of the continuity equation and the ion temperature evolution equation which can be derived from Ref. [19] based on a conservative gyrokinetic equation [20] by taking $k_\perp \rho_i \ll 1$ limit.

$$-i(\omega - 2\omega_{di} + i\Delta\omega_T)\frac{\widetilde{n}}{n_o} = -i\omega_* \frac{|e|\,\widetilde{\phi}}{T_i} - 2i\omega_{di}\frac{|e|\,\widetilde{\phi}}{T_i} - 2i\omega_{di}\frac{\widetilde{T}_i}{T_i}, \tag{7}$$

$$-i(\omega - \frac{14}{3}\omega_{di} + i\Delta\omega_T)\frac{\widetilde{T}_i}{T_0} = -\frac{4}{3}i\omega_{di}\frac{\widetilde{n}}{n_o} - i\omega_{*Ti}\frac{|e|\,\widetilde{\phi}}{T_i} - \frac{4}{3}i\omega_{di}\frac{|e|\,\widetilde{\phi}}{T_i}. \tag{8}$$

These also agree with equations in Ref. [21] in the local limit, noting that the acoustic branch of ITG mode[22] was not considered there. Note that while the authors of Ref. [21] did not choose to decompose their results into TEP and thermoelectric contributions, a decomposition is possible and useful for physics understanding.

Using Eqn. (8) in the expression for V_{TH}, requires treating the product of two propagators, i.e.,

$$\frac{1}{\omega - 4\omega_{di} + i\Delta\omega_T} \cdot \frac{1}{\omega - \frac{14}{3}\omega_{di} + i\Delta\omega_T}.$$

This quantity can be rewritten as a sum of two propagators

$$\frac{1}{\omega - 4\omega_{di} + i\Delta\omega_T} \cdot \frac{1}{\omega - \frac{14}{3}\omega_{di} + i\Delta\omega_T} = -\frac{3}{2\omega_{di}}\left(\frac{1}{\omega - 4\omega_{di} + i\Delta\omega_T} - \frac{1}{\omega - \frac{14}{3}\omega_{di} + i\Delta\omega_T}\right).$$

Using this identity, we can write V_{TH} for the $k_\parallel \to 0$ limit as

$$V_{TH} = 6\left(C_\phi - C_i\right)\left(\frac{1}{T_i}\frac{\partial T_i}{\partial r} + \frac{4}{3R}\left(1 + \frac{T_i}{T_e}\right)\right) - 8\left(V_{TH}^{NA,\phi} - V_{TH}^{NA,i}\right), \tag{9a}$$

where C_ϕ and C_i have expressions similar to the quasilinear momentum diffusivity and ion thermal diffusivity, respectively,

$$C_\phi = \left\langle \sum_{\mathbf{k}} \mathrm{Re}\frac{|\widetilde{v}_r|^2}{-i(\omega - 4\omega_{di} + i\Delta\omega_T)} \right\rangle, \tag{9b}$$

$$C_i = \left\langle \sum_{\mathbf{k}} \mathrm{Re}\frac{|\widetilde{v}_r|^2}{-i(\omega - \frac{14}{3}\omega_{di} + i\Delta\omega_T)} \right\rangle. \tag{9c}$$

Here, $V_{TH}^{NA,\phi}$ and $V_{TH}^{NA,i}$ are defined as

$$V_{TH}^{NA,\phi} \equiv \left\langle \sum_{\mathbf{k}} \mathrm{Re}\frac{-i\omega_{di}\left(\widetilde{n}^{NA}/n_0\right)\widetilde{v}_r^*}{-i(\omega - 4\omega_{di} + i\Delta\omega_T)} \right\rangle, \tag{10a}$$

$$V_{TH}^{NA,i} \equiv \left\langle \sum_{\mathbf{k}} \mathrm{Re}\left(\frac{-i\omega_{di}\left(\widetilde{n}^{NA}/n_0\right)\widetilde{v}_r^*}{-i(\omega - \frac{14}{3}\omega_{di} + i\Delta\omega_T}\right) \right\rangle, \tag{10b}$$

come from the non-adiabatic electron density response,

$$\frac{\widetilde{n}^{NA}}{n_0} = \frac{\widetilde{n}}{n_0} - \frac{|e|\,\widetilde{\phi}}{T_e}. \tag{11}$$

While a limiting form of the pinch for pure-ITG was calculated in Ref. [21], using Eqns. (6) and (7) only, the assumptions of $k_\parallel = 0$ and the strong mode localization at the low field side midplane are not compatible. According to our classification of pinches, a recent numerical calculation[23] indicates that the $k_\parallel^{Tor}$ contribution to Eqn. (5) is significant for ITG modes. Since the ion temperature profile is well-known to be stiff, i.e. cannot be significantly perturbed way above its marginality product, any scaling trend of thermopinch based on the fluid equations (without kinetic corrections) must be examined carefully since near marginal stability, drift-ITG modes can take on both a resonant and non-resonant character. Fluid descriptions can be useful, but higher moments must be retained.

The third element in the momentum flux is the residual stress, $\Pi_{r,\phi}^R$[18]. The residual stress is defined as that part of the Reynolds stress which is not directly proportional to either $\partial\langle v_\phi\rangle/\partial r$ or $\langle v_\phi\rangle$, i.e. the portion *other* than the diffusive and convective flux. Note that the residual stress is thus that part of $\langle\widetilde{v}_r\widetilde{v}_\phi\rangle$ which is independent of $\langle v_\phi\rangle$, but proportional to $\partial\langle n\rangle/\partial r$ and/or $\partial\langle T\rangle/\partial r$. The residual stress has no counterpart in the theory of the turbulent particle flux, since

momentum can obviously be exchanged between waves and particles, while density cannot. Note too, that the thermoelectric convective particle flux $\Gamma_n \sim V\left(\nabla\left\langle T\right\rangle\right)\left\langle n\right\rangle$, while the residual stress $S_\phi \sim \nabla\left\langle T\right\rangle$, etc but independent of $\left\langle v_\phi\right\rangle$. The residual stress defines an effective local internal toroidal momentum source

$$\frac{\partial\langle P_\phi\rangle}{\partial t} = S_{\phi,internal} = -\frac{\partial}{\partial r}\left(\langle n\rangle \Pi^R_{r,\phi}\right), \tag{12}$$

and so is crucial to the formation of intrinsic rotation profiles. The physics of the residual stress is discussed at length in the next section.

III. PHYSICS OF THE TURBULENT RESIDUAL (RADIATION) STRESS

The residual stress $\Pi^R_{r,\phi}$ is that part of the Reynolds stress $\langle\widetilde{v}_r\widetilde{v}_\phi\rangle$ which remains after turbulent diffusion and convection are subtracted. Its existence is a necessary consequence of wave-particle momentum exchange, which is enforced by outgoing wave boundary conditions even in a purely fluid theory. Physically, the residual stress $\Pi^R_{r,\phi} = \Pi\left(\nabla T_i, \nabla T_e, \nabla P_i, \nabla P_e, \nabla n \dots\right)$ converts part of the driving heat flux Q_i or Q_e to a net toroidal flow. Observe that the residual stress is the *only* way to spin-up a plasma from rest, in general,

$$\begin{aligned}\partial_t \int_0^a \left\langle P_\phi\right\rangle &= -nm\left[\chi_\phi \frac{\partial\left\langle v_\phi\right\rangle}{\partial r} + V\left\langle v_\phi\right\rangle + \Pi^R_{r,\phi}\right]\Bigg|_0^a \\ &\cong -nm\left[-\chi_\phi \frac{\partial\left\langle v_\phi\right\rangle}{\partial r} + V\left\langle v_\phi\right\rangle + \Pi^R_{r,\phi}\right]\Bigg|_{r=a},\end{aligned} \tag{13a}$$

for $\Pi\left(0\right) = 0$. Thus for $\left\langle v_\phi\right\rangle = 0, \left\langle v_\phi\right\rangle' = 0$, we have

$$\partial_t \int_0^a \langle P_\phi\rangle dr \cong -nm\Pi^R_{r,\phi}\left(a\right), \tag{13b}$$

so the radially integrated momentum drive is set by the pressure, density and temperature gradients at the plasma edge, acting through the residual stress. Note that a pinch alone cannot spin-up the plasma from rest, but instead requires some "seed" toroidal flow at the separatrix to initiate rotation, i.e. it requires $\left\langle v_\phi\left(a\right)\right\rangle \neq 0$. Of course, the critical pinch for the determination of the rate of change of the total momentum is that which acts *at* $r = a$. The separatrix boundary condition on the flow is also critical. The total stress corresponds to a net momentum *flux*, which along with the boundary condition on $\left\langle v_\phi\right\rangle$, determines the profile. In particular, for the relevant prototypical case where $\left\langle v_\phi\left(a\right)\right\rangle = 0$ (corresponding to a no-slip boundary, enforced by strong neutral drag),

we have for zero net momentum flux (corresponding to an intrinsic rotation solution)

$$\langle v_\phi(r)\rangle = -\int_r^a dr' \frac{\Pi^R_{r,\phi}(r')}{\chi_\phi(r')}, \tag{14}$$

so that $\Pi^R_{r,\phi} < 0$ corresponds to co-rotation while $\Pi^R_{r,\phi} > 0$ corresponds to counter-rotation. Note that *either* sign of $\Pi^R_{r,\phi}$ can generate a flow. Of course, $\Pi^R_{r,\phi}(r)$ can change sign in radius, and so produce internal flow reversals. The sign dependence of $\Pi^R_{r,\phi}$ should be contrasted to that for convection, where $V > 0$ is unfavorable for core profiles peaked on axis, while $V < 0$ is favorable. Thus, we see that $\Pi^R_{r,\phi}$ is conceptually distinct from a pinch or other convective effect.

The micro-physics of the residual stress is governed by resonant and non-resonant turbulent transport acting in the presence of broken parallel reflection symmetry (i.e. $k_\parallel$ symmetry breaking). The calculation of $\Pi^R_{r,\phi}$, in the resonant limit is discussed in the literature[15]. Here we focus on the non-resonant or "wave" contribution. Intuitively, this is the most appealing way to envision the origin of intrinsic rotation, namely as a consequence of the modulation of an anisotropic quasi-particle pressure. Taking the turbulent χ_ϕ momentum diffusivity as already determined and ignoring the pinch here, for simplicity, we see that the mean flow $\langle v_\phi\rangle$ then satisfies

$$\partial_t\langle v_\phi\rangle - \partial_r\chi_\phi\partial_r\langle v_\phi\rangle = -\partial_r\Pi^{wave}_{r,\phi}, \tag{15a}$$

where

$$\Pi^{wave}_{r,\phi} = \int d\mathbf{k} v_{gr} k_\parallel N. \tag{15b}$$

Here, $\Pi^{wave}_{r,\phi}$ is the net radial flux of parallel wave momentum $k_\parallel N$. Note that this calculation ignores the distinction between toroidal and parallel momentum, and so neglects contributions from the flux of perpendicular wave dynamics projected onto the toroidal direction. Note that a complete treatment of this issue will involve analysis of both toroidal and poloidal rotation, along with the calculation of both parallel and perpendicular wave momentum fluxes and Reynolds stress. At present, this is beyond the scope of possibilities. Indeed, it is first necessary to better understand perpendicular stresses and poloidal rotation, and only then to proceed to the full coupled analysis. χ_ϕ is simply the ambient turbulent diffusion of toroidal velocity. The quasi-particle population density is just $N(\mathbf{x}, \mathbf{k}, t)$, which obeys the standard wave-kinetic equation, i.e. Eqn. (28a) and Eqn. (30a) of Ref. [24]. Defining $S_\parallel = \delta\langle v_\phi\rangle'$, the modulation in toroidal velocity shear, we have

$$\partial_t S_\parallel - \partial_r\chi_\phi\partial_r S_\parallel = -\partial_r^2 \int d\mathbf{k} v_{gr} k_\parallel \delta N. \tag{16}$$

The RHS effectively accounts for the quasi-particle induced residual stress. Formulation of the problem as one of a modulational interaction is useful for clarifying the dynamics of flow shear amplification. Note that the edge boundary condition discussed above guarantees that flow shear amplification leads to net flow amplification. Linearizing the wave kinetic equation then yields the population response

$$\delta N = \tau_{c,mod}\left[k_\theta V_E' \frac{\partial \langle N\rangle}{\partial k_r} - v_{gr}\frac{\partial \langle N\rangle}{\partial r}\right]. \tag{17}$$

Note δN is calculated in the spirit of a Chapman-Enskog expansion for the population of wave packets. Here V_E' is the electric field shear modulation and $\tau_{c,mode}$ is the δN response correlation time. Thus

$$\Pi_{r,\phi}^{wave} = \int d\mathbf{k} k_\parallel v_{gr}\tau_c\left\{k_\theta \frac{\partial \langle N\rangle}{\partial k_r}\langle V_E\rangle' - v_{gr}\frac{\partial \langle N\rangle}{\partial r}\right\}. \tag{18a}$$

Note that this result assumes $\gamma\left(+k_\parallel\right) \approx \gamma\left(-k_\parallel\right)$, so symmetry breaking via directional dependence of growth rate, as in the parallel shear flow instability[1, 25], is not significant. This is discussed further in Ref. [15]. Note that in general, the parallel shear flow is not a particularly relevant free energy source. If a net external torque T^{ext} modulation was retained, the condition for a stationary state in the presence of the wave stress given by Eqn. (18a) is easily shown to be

$$T^{ext} = \int d\mathbf{k} k_\parallel v_{gr}\tau_c\left\{k_\theta \frac{\partial \langle N\rangle}{\partial k_r}\langle V_E\rangle' - v_{gr}\frac{\partial \langle N\rangle}{\partial r}\right\}_a - nm\chi_\phi \frac{\partial \langle v_\phi(a)\rangle}{\partial r}. \tag{18b}$$

Several observations are in order here. First note that the net residual stress is driven by the quasi-particle population gradients in both k_r and r. The k_r gradient $\partial\langle N\rangle/\partial k_r$, induces a stress via shearing when $k_\theta \partial v_{gr}/\partial k_r \neq 0$, so that the net k_r-space flow is compressible. Note that for drift waves, $k_\theta \partial v_{gr}/\partial k_r \cong -2k_\theta^2\rho_s^2 v_*/(1+k_\perp^2\rho_s^2)^2$, so the integrated contribution to the stress is even in k_θ and k_r, and exhibits some mode dependence via v_{*e}. We expect this trend to be generic. The r-gradient $\partial\langle N\rangle/\partial r$ induces a radiative diffusive inward flux of wave momentum, which may be either co or counter direction, depending on the sign of $k_\parallel$. The radiative diffusion flux $\sim -D_r\partial\langle P_\parallel\rangle_w/\partial r$, where $\langle P_\parallel\rangle_w$ is the wave parallel momentum density and $D_r \sim v_{gr}^2\tau_c$ is the quanta diffusivity. Note $D_r \sim D_{GB}$. The detailed physics of these processes is discussed further in Ref. [15].

Second, before proceeding to calculate $\partial\langle v_\phi(a)\rangle/\partial r$-the edge rotation gradient, we note that

$$\langle V_E\rangle' = \frac{B_\theta}{|\mathbf{B}|}\frac{\partial\langle v_\phi\rangle}{\partial r} + \frac{B_T}{|\mathbf{B}|}\langle V_E\rangle_0', \tag{19}$$

i.e. the net electric field shear is the sum of the contributions due to toroidal rotation and the other pieces, denoted by $\langle V_E\rangle_0'$. The latter includes both diamagnetic (i.e. ∇P_i-driven) velocity shear

and poloidal velocity shear. Of course this means that in the absence of $\langle V_E\rangle_0'$, toroidal velocity shear can feed back on itself, as in a modulational instability. To see this, note that since $S_\parallel$ and δN satisfy Eqns. (16, 17) and since Eqn. (19) implies $\langle V_E\rangle' = (B_\theta/|\mathbf{B}|)\, S_\parallel + (B_T/|\mathbf{B}|)\, \langle V_E\rangle_0'$, then in the limit where other drives of Π^R vanish, i.e. $\langle V_E\rangle_0' \to 0$, $\partial\langle N\rangle/\partial r \to 0$, Eqn. (16) reduces to just:

$$(\partial_t - \partial_r \chi_\phi \partial_r)\, \widehat{S}_\parallel = -\partial_r^2 \left\{ \int d\mathbf{k} v_{gr} k_\parallel \tau_{c,mod} k_\theta \frac{\partial \langle N\rangle}{\partial k_r} \widehat{S}_\parallel \right\}.$$

Hence, we see that the growth rate of a shear modulation of the parallel flow with radial wave number q is just

$$\gamma_q = q_r^2 \left\{ \left[\int d\mathbf{k} v_{gr} k_\parallel \tau_{c,mod,q_r} k_\theta \frac{\partial \langle N\rangle}{\partial k_r} \right] - \chi_\phi \right\}.$$

For standard drift waves, this may be re-written as:

$$\gamma_q = -q_r^2 \chi_\phi^{eff}$$

where

$$\chi_\phi^{eff} = \chi_\phi - 2V_* \int d\mathbf{k} k_\parallel \left(\frac{k_\theta^2 \rho_s^2}{\left(1 + k_\perp^2 \rho_s^2\right)^2} \right) \tau_{c,mod,q_r} k_r \frac{\partial \langle N\rangle}{\partial k_r}$$

Hence we readily see that:

1. the effect of the parallel flow shear induced modulation of the residual stress is to augment or renormalize χ_ϕ. Clearly, $\chi_\phi^{eff} > \chi_\phi$ and $\chi_\phi^{eff} < \chi_\phi$ are both *both* possible.

2. $\chi_\phi^{eff} < \chi_\phi$ is clearly symptomatic of the modulational *growth* of *instability* of the test shear. This is not surprising, since it is well known that modulational instability of shear flows is a sort of "negative viscosity" phenomenon. This process is also symptomatic of the generation of toroidal zonal flows.

The toroidal zonal flows discussed above have been observed in gyrokinetic particle simulation[26, 27]. More generally, this result suggests that any intrinsic rotation feeds back on itself via its contribution to electric field shearing, and so renormalizes the momentum diffusivity χ_ϕ. To see this, observe that plugging Eqn. (19) into Eqn. (18b) and re-writing gives a modified diffusivity. We refer to this renormalized diffusivity as $\chi_{\phi,R}$. Thus, the edge gradient is given by

$$\frac{\partial \langle v_\phi(a)\rangle}{\partial r} = \left[T_{ext} - \left\{ \left(\int d\mathbf{k} k_\parallel v_{gr} \tau_c k_\theta \frac{\partial \langle N\rangle}{\partial k_r} \right) \langle V_E\rangle_0' + D_{rad} \frac{\partial \langle P_\parallel\rangle_w}{\partial r} \right\}_a \right] / nm\chi_{\phi,R}(a) \quad (20a)$$

where

$$nm\chi_{\phi,R}(a) = nm\chi_\phi(a) - \left(\int d\mathbf{k} k_\parallel v_{gr} \tau_c k_\theta \frac{\partial \langle N \rangle}{\partial k_r} \right)_a \tag{20b}$$

is the 'renormalized' χ_ϕ which includes self-induced rotation feedback via $\langle V_E \rangle'$. Note that the *sign* of the χ_ϕ renormalization is determined by the product of the group velocity v_{gr}, the spectral population gradient $\partial \langle N \rangle / \partial k_r$, (which is usually negative) and the spectrally weighted $k_\parallel$. Observe that the correction to χ_ϕ can be positive and so it is at least conceivable that the observed χ_ϕ – deduced, say, from momentum perturbation experiments – may exceed the observed χ_i, $\chi_\phi > \chi_i$ which has been observed in JT-60U perturbation experiments[28].

Third, observe that Eqn. (20a) defines an effective critical torque which zeroes the edge velocity gradient, i.e. T^{ext}_{crit} for $\partial \langle V_\theta \rangle / \partial r|_a \to 0$. This may be thought of as defining a critical torque which exactly cancels the residual stress-driven intrinsic rotation[15]. Here, the critical torque is

$$T^{ext}_{crit} = \left\{ \int d\mathbf{k} \left(k_\parallel v_{gr} \tau_c k_\theta \frac{\partial \langle N \rangle}{\partial k_r} \right) \langle V_E \rangle'_0 + D_{rad} \frac{\partial \langle P_\parallel \rangle_w}{\partial r} \right\}_a . \tag{21}$$

Note that the critical torque is determined by $\langle V_E \rangle'_0$ (i.e. the electric field shear due to diamagnetic and poloidal rotation), the mode propagation velocity (in v_{gr}), the turbulence spectrum (in $\partial \langle N \rangle / \partial k_r$), the wave momentum density profile $\langle P_\parallel \rangle_w$ and D_{rad}, τ_c, etc. Of course, the critical torque defines the off-set in the linear plot of $\partial \langle v_\phi \rangle / \partial r|_a$ vs. T^{ext}. Interestingly, it is *renormalized* χ_ϕ – i.e. $\chi_{\phi,eff}$ – which sets the *slope* of this linear relation. Thus, the feed-back loop physics of intrinsic rotation enters more than just the off-set! Finally, we should recall that if the edge rotation velocity is finite,

$$\left. \frac{\partial \langle v_\phi \rangle}{\partial r} \right|_a = \frac{-1}{nm\chi_{\phi,eff}} \left\{ T^{ext} - \Pi^R_{r,\phi}|_a - V \langle v_\phi \rangle |_a \right\} . \tag{22}$$

In this case, the edge pinch velocity also enters the determination of $\partial \langle v_\phi \rangle / \partial r|_a$. Interestingly *only* the edge momentum pinch is relevant to intrinsic rotation. We speculate here that SOL physics in general, and SOL flow effects in particular[6], couple to core intrinsic rotation via the edge momentum pinch. The TEP momentum pinch, discussed in reference[16], is surely operative at the edge. Analysis of other possible contributions requires a study of the regime with collisionless fluid ion and dissipative/collisional electron dynamics. In particular, it would be interesting to see if a momentum analogue of the familiar ion mixing mode density pinch[29] exists. This is discussed further in Section V. In closing this section, we remark that coupling of intrinsic rotation to $\langle v_\phi(a) \rangle$ should also manifest itself as a sensitivity of the critical torque to SOL asymmetry – i.e. T^{ext}_{crit} should differ between single null and double null operation.

Virtually all of the results in this discussion are sensitive to spectrally averaged $k_\parallel$, i.e. $\langle k_\parallel \rangle$. One promising example, although not unique, is to follow Eqns. (36a), (36b) of Ref. [15], and balance nonlinear decay with shearing to obtain

$$\langle k_\parallel \rangle = -\int d\mathbf{k} \frac{\partial k_\parallel}{\partial k_r} k_\theta \langle V_E \rangle' \frac{\langle N \rangle}{\gamma_{NL,k}}, \tag{23a}$$

where

$$\langle k_\parallel \rangle = \int d\mathbf{k} k_\parallel \langle N \rangle. \tag{23b}$$

Here $\gamma_{NL,k}$ is the nonlinear decorrelation rate (i.e. inverse mode lifetime) for wave-vector $\mathbf{k}$. $\partial k_\parallel / \partial k_r \neq 0$ requires magnetic shear. This description is equivalent to that developed in real space, in which the shift of the spectrum off the resonant surface induced by the electric field shear sets the mean $k_\parallel$[30–33]. Note that Eqn. (23a) suggests a close link between radial electric field shear and the residual stress contribution to the momentum flux. This link has been verified by gyrokinetic simulations performed in Refs. [26, 27] (see Figs. 5 and 6).

IV. SIMPLE MODEL FOR INTRINSIC ROTATION SCALINGS

It's interesting to note that Equation (20a) effectively states that $\partial \langle v_\phi \rangle / \partial r|_a$ – and thus the net intrinsic rotation – will increase with $\langle V_E \rangle_0'$. Since $\langle V_E \rangle_0' = \partial_r (\partial \langle P \rangle / \partial r / neB_0) - \partial_r (\langle v_\theta \rangle B_0) / |\mathbf{B}|$ increases with edge pressure gradient, one direct prediction of this theory is a correlation between edge pressure gradient and intrinsic rotation velocity. This is qualitatively suggestive of the $\Delta \langle v_\phi \rangle \sim \Delta W_p / I_p$ scaling proposed by Rice[7], but now expressed in terms of more physical, local gradient quantities. One can go further and develop a transport model which evolves the:

(i.) toroidal momentum profile, in terms of χ_ϕ, V and $\Pi^R_{r,\phi}$ acting along with the external torques,

(ii.) density profile, in terms of D, V_n and fueling,

(iii.) ion temperature profile, in terms of χ and heating,

(iv.) fluctuation intensity, evolved by simple $\mathbf{E} \times \mathbf{B}$ shear-induced quenching[34].

This model represents a generalized Hinton model[35]. The model may be solved numerically, and also analytically, assuming a piecewise linear profile structure. For simplicity we apply a no-slip boundary condition so that $v_\phi(a) = 0$. Results indicate that the central rotation velocity is

determined primarily by the pedestal velocity, and that the latter scales as[30, 36]

$$\frac{\Delta\langle v_\phi\rangle}{v_{Thi}} \sim \left(\frac{\Delta r_c}{a}\right)\left(\frac{\Delta_{ped}}{a}\right) \sim \rho_*^\alpha \left(\frac{\Delta_{ped}}{a}\right). \tag{24}$$

Here Δ_{ped} is the pedestal width and Δr_c is the turbulence correlation length. Thus, $\alpha \sim 1$ corresponds to the Gyro-Bohm edge turbulence while $\alpha \sim 0$ corresponds to Bohm. The pedestal width is proportional to the pedestal pressure, i.e. $\Delta_{ped} \sim P_{ped}$, so $\Delta\langle v_\phi\rangle \sim P_{ped} \sim \Delta W_p$, the increment in the stored energy, as in the Rice scaling. More interestingly, we note that if:

(i.) the edge turbulence exhibits Bohm scaling, so $\Delta r_c/a \sim (\rho_*)^0 \sim 1$.

(ii.) we assume the Snyder[37] empirical pedestal width scaling $\Delta_{ped}/a \sim \beta_p^{1/2}$ which recovers the I_p dependence of the Rice scaling,

we then recover $\Delta\langle v_\phi\rangle/v_{Thi} \sim \beta_p^{1/2}$ which is effectively equivalent to the Rice scaling $\Delta v_\phi \sim \Delta W_p/I_p$[7, 38]. Interestingly, the unfavorable current scaling of intrinsic rotation appears as a consequence of the unfavorable current scaling of the pedestal width. This seems plausible, since otherwise transport scalings with current are nearly universally favorable. Note that in this scenario, intrinsic rotation is strongly tied to pedestal physics, which is also suggested by the experimental results. The absence of ρ_* scaling of intrinsic rotation velocity[7] appears as a consequence of Bohm scaling of the pedestal turbulence. The persistence of this unfavorable trend into the regime of ITER parameters is far from certain.

V. ROLE OF PARTICLE FLUX IN FLOW EVOLUTION

While the conserved angular momentum density is a natural quantity of theoretical interest and is what is probed in perturbative momentum transport experiments[4], the flow profile is of great practical interest, because its magnitude and radial profile influence the stability of resistive wall modes (RWM), turbulence-driven transport and the L→H power threshold. Similarly, we are interested in discriminating momentum transported by the parallel Reynolds stress and that originating from particle fluxes. This decomposition is particularly illustrative since it provides insight into the role of non-adiabatic electrons in determining the flow profile. The effects of non-adiabatic electrons on the momentum flux have not been addressed in prior published work. Recall that the radial flux of parallel flow is given by the Reynolds stress, $\Pi_{REY} \equiv \langle \widetilde{v}_r^* \widetilde{u}_\parallel \rangle$ which satisfies

$$n_0 R_0 \Pi_{REY} \cong \Pi_{ang} - \langle v_\phi \rangle R_0 \Gamma_{ptl}. \tag{25}$$

It's generally safer to calculate the particle radial flux from drift wave turbulence using the non-adiabatic electron response

$$\Gamma_{ptl} = \left\langle \sum_{\mathbf{k}} \mathrm{Re}(\widetilde{v}_r^* \widetilde{n}_e) \right\rangle = \left\langle \sum_{\mathbf{k}} \mathrm{Re}(\widetilde{v}_r^* \widetilde{n}^{NA}) \right\rangle . \tag{26}$$

Before delving into details of the calculation, some general remarks are in order. For electron drift waves (including trapped electron modes and collisional drift waves, but *not* ETG modes), to be linearly unstable, net particle flux, including diffusion and particle pinch, should be outward. Since the last term in Eqn. (25) has a multiplier $\langle v_\phi \rangle$, this outward flux of particles will manifest itself as an inward pinch of toroidal flow velocity[39]. This pinch will generally add to the TEP pinch and thermoelectric pinch of flow which has been calculated in the previous papers and Section II of this work. As shown in the Appendix, $|V_{TH}| \sim \mathcal{O}\left(\frac{\omega_{di}\gamma}{\mathrm{Re}(\omega)^2}\right)|V_{TEP}| << |V_{TEP}|$ for electron drift waves with $\mathrm{Re}(\omega) \propto \omega_{*e}$, and $\omega_{di}, \gamma < \mathrm{Re}(\omega)$. Therefore, the total convective pinch of parallel velocity, mostly consisting of the TEP pinch and the particle flux, from electron drift wave turbulence will almost certainly be inward. We gain useful insight from the previous works on particle flux. For ITG, the evaluation of the thermoelectric pinch is complicated by the strong sensitivity of this term to the linear dispersion relation. Thus, we will leave a quantitative evaluation of this term for ITG modes to a future analysis.

There exist at least three relevant asymptotic regimes classifying electron drift waves. In the order of decreasing collisionality, these are

(i.) Collisional drift wave: thermal electrons are in the so-called semi-collisional regime which satisfies $\nu_e > k_\parallel v_{The} > \omega \sim \omega_{*e}$, but $k_\parallel^2 v_{Te}^2 > \nu_{ei}\omega_{*e}$, so that they (diffuse and) thermalize along the magnetic field line faster than one wave period. Magnetic trapping of electrons plays no role due to the long time scale associated with it.

(ii.) Dissipative trapped electron (DTEM) mode: $\nu_{*e} \equiv (\nu_e/\varepsilon)/\omega_{be} < 1$, but $\nu_{eff} = \nu_e/\varepsilon > \omega, \omega_{*e} > \omega_{de}$. Therefore trapped electrons suffer collisions. Their non-adiabatic response decreases with $\omega_*/(\nu_e/\varepsilon)$, and associated particle flux is small.

(iii.) Collisionless trapped electron (CTEM) mode: $\nu_{*e} < 1$, and $\omega, \omega_{*e} \gtrsim \omega_{de} > \nu_e/\varepsilon$, so that collisional effects are negligible, but trapped electron procession can resonate with electron drift waves to destablize it.

For DTEM, the particle flux is approximately given by

$$\Gamma_{ptl} = -\left\langle \sum_{\mathbf{k}} \varepsilon^{1/2} |\widetilde{v}_r|^2 \frac{1 + \frac{3}{2}\eta_e - \mathrm{Re}(\omega)/\omega_{*e}}{(\nu_e/\varepsilon)} \right\rangle \frac{\partial n_0}{\partial r}, \tag{27}$$

Since it's expected that the saturated fluctuation spectrum peaks at a relatively low $(k_\perp \rho_i)^2$, the electron temperature gradient driven particle flux will dominate the residual density gradient driven particle flux (due to finite $(k_\perp \rho_i)^2$). This outward particle flux will be seen as an inward electron temperature-gradient driven convective flux of parallel velocity. For CTEM, the trapped electron resonance between precession drift-wave is a dominate excitation mechanism. For this,

$$\frac{\delta n^{NA}}{n_0} = i2\sqrt{\pi\varepsilon} \left(\frac{\omega}{\overline{\omega}_{de} G}\right)^{3/2} e^{-\omega/\overline{\omega}_{de} G} \left[1 - \frac{\omega_{*e}}{\omega}\left\{1 + \eta_e \left(\left(\frac{\omega}{\overline{\omega}_{de} G}\right) - \frac{3}{2}\right)\right\}\right] \frac{|e|\,\widetilde{\phi}}{T_e}, \tag{28}$$

(see Ref. [40]). Once again, ∇T_e-driven particle flux dominates a residual ∇n-driven particle flux due to FLR-induced down-shift of DW frequency. These ∇T_e-driven inward pinch of parallel velocity expected for both CTEM and DTEM is an intriguing result since it must be ions which carry the momentum according to the recoil (last) term of Eqn. (25). However, the flux really depends on the trapped electron related quantities due to the quasi-neutrality constraint!

For a plasma in which ITG is the dominant microturbulence but modified by non-adiabatic electrons, the direction of particle flux depends on the collisionality and η_e. It is well-known that the net particle flux can be inward if electron collisionality ν_{*e} is either very low or very high, and η_e is high enough. For the semi-collisional passing-electron-modified ITG mode, better known as the ion-mixing mode[29], the electrons are in the collisionality regime which corresponds to the collisional electron drift waves, i.e. $\nu_{*e} > 1$, $\nu_e > k_\parallel v_{The} > \omega, \omega_{*e}$, but $k_\parallel^2 v_{The}^2 > \omega\nu_{ei}$. While this regime is no longer relevant to large present-day tokamaks' core turbulence, it's still applicable to some tokamak edge turbulence, characterized by high collisionality. For instance $\nu_* \sim 10$ in some C-Mod edge plasmas[41]. This mode is potentially of high theoretical interest since intrinsic rotation (in particular, those in C-Mod[6]) seems to initiate at the very edge and propagate inward. Therefore, any viable theory for intrinsic rotation should include not only a rotation build-up mechanism at the edge (such as the residual stress[15, 30]), but also an inward pinch mechanism, in particular, at the edge where rotation develops. Indeed these two mechanisms can be mutually re-inforcing otherwise it's very difficult to explain core intrinsic rotation. With this in mind, a quantitative assessment is necessary for a possibility that the ion-mixing mode driven inward particle pinch can manifest itself as an outward flow pinch and reduce or even reverse the inward TEP inward pinch of flow. Extending a pioneering work by Coppi and Spight [29], Lee and

Diamond [42], have obtained a more precise expression for electron response from the Braginskii equations. The semicollisional passing electron density perturbation is

$$\frac{\widetilde{n}^{NA}}{n_0} = -i\frac{\omega_{*e}\nu_e}{k_\parallel^2 v_{Te}^2}\left(\frac{0.51}{\widehat{\chi}_e}\right)\left[\widehat{\chi}_e + (1+\alpha_T)^2 - \frac{3}{2}(1+\alpha_T)\eta_e\right]\frac{|e|\,\widetilde{\phi}}{T_e}, \tag{29}$$

where $\widehat{\chi}_e = 1.61$, $\alpha_T = 0.71$. Consequently, the particle flux is given by

$$\Gamma_{ptl}^{ion\ mixing} = -1.02\frac{[\widehat{\chi}_e + (1+\alpha_T)^2]}{\widehat{\chi}_e}\left(1 - \frac{\eta_e}{\eta_{crit}}\right)\left\langle \sum_{\mathbf{k}} \frac{\nu_e\,|\widetilde{v}_r|^2}{k_\parallel^2 v_{TE}^2}\right\rangle \frac{\partial n_0}{\partial r}. \tag{30}$$

Therefore, an inward particle flux is expected for $\eta_e > \eta_e^{crit} = 1.77$.

For less collisional plasmas with $\nu_{*e} < 1$, magnetically trapped electrons play an important role in determining radial particle flux from ITG-mode. For dissipative trapped electrons with $\nu_e/\varepsilon > \omega, \omega_{*e}$, Eqn. (29) still applies. It yields an outward flux since $\mathrm{Re}\,(\omega/\omega_{*e}) < 0$ for ITG modes. For collisionless trapped electrons (with $\omega, \omega_{*e} > \omega_{de} > \nu_e/\varepsilon$) however, the precession-drift wave resonance is no longer possible since ITG modes typically propagate in the ion diamagnetic direction. However, there exists non-resonant reactive contribution to non-adiabatic electron response. Since it depends sensitively on the linear dispersion relation, we don't present it here. It's well-known that for the collisionless trapped electrons, net particle flux can be inward[43–45] for high enough η_e values. Particle TEP inward pinch should contribute in this regime where the second adiabatic invariant exists [46–48]. While particle TEP pinch is in general comparable to momentum TEP pinch in magnitude, they are not identical and have different scalings with respect to magnetic shear. Therefore, with CTEM-dominated turbulence, we expect a partial cancellation between momentum TEP pinch and particle TEP pinch.

We speculate that an ensuing weak flow pinch in collisionless core plasma may not be a serious concern regarding confinement improvement. We expect that by having sufficient residual stress and flow pinch in the outer plasmas extending to the last closed flux surface, significant confinement improvement can be achieved relying on the flow shear in that region.

VI. SUMMARY AND DISCUSSION

In this paper, we have reported on recent progress on the theory of turbulent momentum transport and the origins of spontaneous rotation in tokamaks. The principal results of this paper are:

1. elucidation of the decomposition of the total momentum flux into diffusive ($\sim \chi_\phi$), pinch ($\sim V$) and residual stress components. These originate from the Reynolds stress (for χ_ϕ, a

portion of V and residual stress) and convective (for a portion of V) fluxes. The physics of each component is discussed.

2. the observation that generation of intrinsic rotation requires either

   (a) a non-zero value of the residual stress at the plasma boundary. In this case coupling of SOL flows to core rotation is not necessary.

   (b) or, a non-zero value of the product $V \langle v_\phi \rangle$ at the boundary, along with $V < 0$ (an inward velocity pinch). In this case, $\langle v_\phi (a) \rangle$ is very likely determined by SOL flows.

3. calculation of the residual stress via a mesoscale, modulational approach which captures both shearing and wave momentum transport effects. The intrinsic wave radiation stress which is also the residual stress, is shown to be proportional to $\langle V_E \rangle'$ and to $\partial \left\langle P_\| \right\rangle / \partial r$.

4. the result of 3. is used to calculate the external torque required to maintain the edge velocity gradient $\partial \langle v_\phi (a) \rangle / \partial r$ at a fixed value. This is essentially equivalent to the calculation of the torque required to cancel the residual stress-driven intrinsic rotation (i.e. $\partial \langle v_\phi (a) \rangle / \partial r \rightarrow 0$ for exact cancellation). This cancellation torque is inversely proportional to a dressed χ_ϕ, which includes the feedback via the $\langle v_\phi \rangle'$ piece of $\langle V_E \rangle'$. The modification of χ_ϕ can be large enough to affect interpretation of experimentally determined V/χ_ϕ ratios.

5. demonstration that inclusion of non-adiabatic electron effects generates a 'recoil contribution' to the turbulent velocity pinch via the coupling between flow and particle transport which is inherent to the transport of (conserved) angular momentum. We calculate this effect for collisional, DTEM and CTEM drift waves, and ITG turbulence. In most cases, an inward flux of toroidal velocity results.

6. we show that the Rice scaling of intrinsic rotation velocity can be recovered by a simple model which assumes only that:

   (a) intrinsic rotation is linked to the L→H transition

   (b) the edge turbulence follows Bohm *scaling* (i.e. radial correlation length $\Delta r_0 \sim (a)^1 (\rho_*)^0$)

   (c) the pedestal width scales as $\Delta_{ped}/a \sim \beta_p^{1/2}$.

Note most of the results listed above are concerned with the generation or *acceleration* of intrinsic rotation. Optimization of total angular momentum content and rotation profile control so as to supress RWMs requires actual profile calculations which, in some sense 'match' the edge or pedestal layer (controlled primarily by residual stress) with the core (controlled primarily by diffusion and the convective pinch). These calculations are beyond the scope of this paper and will be addressed in a future publication.

Ongoing and future work will focus on studies of electron heat transport driven regimes[49], electromagnetic coupling and saturation[50], alternative symmetry breaking mechanisms (especially polarization stresses[51] and GAM shearing), coupling to poloidal rotation effects, and SOL-core interaction. Understanding the edge pinch of momentum and its interaction with the edge rotation velocity driven by SOL flows is a particularly important near-term goal. Finally we also plan to apply the theory to the interesting TCV internal momentum transport bifurcations[10].

The theoretical and computational investigations summarized in this paper suggest several challenges which necessitate further experimental work in order to formulate an effective response. These include, but are not limited to:

1. Residual stress physics;
   (a) exploring the relationship between the Rice scaling and the cancellation torque's dependence on energy content W and plasma current I_p
   (b) identifying $\langle v_\phi \rangle$ profile corrugations induced by zonal flows and characterization of the degree of $\langle v_\phi \rangle$ profile "choppiness".
   (c) assessment of residual stress effects in the core, primarily in high ∇P regimes with and without ITB's.
2. Boundary effects and SOL flow-core interaction;
   (a) studying the evolution of intrinsic rotation during slow transitions
   (b) exploring the relation between intrinsic rotation velocity and local edge quantities which control the residual stress
   (c) comparison of single null and double null rotation in L and H mode
   (d) comparisons of rotation on low and high neutral opacity regimes

(e) performing experiments which explore the viscous stress of SOL flows on core plasma rotation

(f) studies of poloidal rotation with the aim of determining the degree of departure from neoclassical values.

3. Core transport physics relevant to rotation;

   (a) comparisons between momentum and density pinch velocity

   (b) studies of intrinsic Prandtl number in stiff profile regimes

   (c) studies of intrinsic rotation in electron-dominated plasmas – a topic which is highly ITER relevant!

   (d) studies of the comparative stiffness of ion, electron and toroidal momentum profiles

4. Basic studies;

   (a) studies of intrinsic azimuthal rotation[52] in linear basic experiments, where detailed measurements of fluctuation induced Reynolds stresses, etc. are readily available

Clearly, there is no lack of interesting work to be done on the subject of intrinsic rotation!

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Appendix: Estimation of thermoelectric pinch for TEM modes

In this appendix we provide an explicit quasilinear estimation for the magnitude of the thermoelectric pinch for TEMs. In order to evaluate the thermoelectric pinch given by Eqn. (9a) it will be necessary to compute the difference between $V_{TH}^{NA,\phi}$ and $V_{TH}^{NA,i}$. This can be facilitated by

rewriting this difference in the form:

$$V_{TH}^{NA,\phi} - V_{TH}^{NA,i} = \left\langle \mathrm{Re} \sum_{\mathbf{k}} \left(\frac{\omega_{di}}{(\omega - 4\omega_{di})} - \frac{\omega_{di}}{\left(\omega - \frac{14}{3}\omega_{di}\right)} \right) \widetilde{v}_r^* \widetilde{n}^{NA} \right\rangle$$
$$\approx C_2^{TEM} \Gamma_{ptl} + C_1^{TEM} \Gamma_{\mathrm{Im}}, \tag{A1}$$

where $\Gamma_{ptl} \equiv \left\langle \mathrm{Re} \sum_{\mathbf{k}} \widetilde{n}^{NA} \widetilde{v}_r^* \right\rangle$, $\Gamma_{\mathrm{Im}} \equiv \left\langle \mathrm{Im} \sum_{\mathbf{k}} \widetilde{n}^{NA} \widetilde{v}_r^* \right\rangle$,

$$C_1^{TEM} \equiv |\omega_{di}|^2 \left\{ \frac{1}{\left(\mathrm{Re}\,(\omega) + 4\,|\omega_{di}|\right)^2} - \frac{1}{\left(\mathrm{Re}\,(\omega) + \frac{14}{3}\,|\omega_{di}|\right)^2} \right\},$$

$$C_2^{TEM} \equiv |\omega_{di}|^2 \left\{ \frac{1}{\mathrm{Re}\,(\omega) + \frac{14}{3}\,|\omega_{di}|} - \frac{1}{(\mathrm{Re}\omega) + 4\,|\omega_{di}|} \right\}.$$

Here, we have also assumed that there's a dominant $k_\perp$ which contributes to this expression, as well as the local approximation.

Utilizing Eqn. (A1), Eqn. (9a) can be rewritten as:

$$V_{TH} = 6\left(C_\phi^{QL} - C_i^{QL}\right)\left(\frac{1}{T_i}\frac{\partial T_i}{\partial r} + \frac{4}{3R}\left(1 + \frac{T_i}{T_e}\right)\right) - 8\left(C_2^{TEM}\Gamma_{ptl}/n_0 + \frac{\mathrm{Im}\,(\omega)}{|\omega_{di}|}C_1^{TEM}\Gamma_{\mathrm{Im}}/n_0\right). \tag{A2}$$

The coefficients C_1^{TEM} and C_2^{TEM} have been evaluated numerically via the use of a simple linear model for TEMs based on Eqns. (7) and (8). This system yields a dispersion relation which is quadratic in complex ω. We have only taken a root for which $\mathrm{Re}\,(\omega) > 0$ for TEM. Their magnitudes are bounded by $\left|C_2^{TEM}\right| < 0.036$ and $\left|C_1^{TEM}\right| < 0.016$ for $R/L_n > 3$ and $R/L_{Ti} < 3$ (i.e. ITG stable case). From this estimate, it is clear that the second term in Eqn. (A2) is negligible in comparison to the momentum transported by particle fluxes. Figures 1-4 contain plots showing in more detail the behavior of C_2^{TEM} and C_1^{TEM} as the density and ion temperature gradients are varied, such that the magnitude of this term for a variety of parameter regimes is made clear.

Similarly, the first term in Eqn. (A2) can be estimated via rewriting the difference of the coefficients as

$$C_\phi - C_i = \left(\frac{\mathrm{Im}\,(\omega)}{|\omega_{di}|}\right) C_1^{TEM} \lesssim \frac{\gamma}{|\omega_{di}|}\,(0.016)\,, \tag{A3}$$

so that the magnitude of V_{TH} can be estimated as

$$V_{TH} \lesssim 6 \times 0.016 \times 2 \times \frac{8}{3R} \ll \frac{3}{R} = V_{TEP}. \tag{A4}$$

Here we have used the fact that for TEM (i.e., stable ITG), $\left|\frac{1}{T_i}\frac{\partial T_i}{\partial r}\right| \lesssim \frac{8}{3R}$. Hence, the thermoelectric contribution to the flow pinch can be seen to be negligible in comparison to the TEP portion for TEMs.

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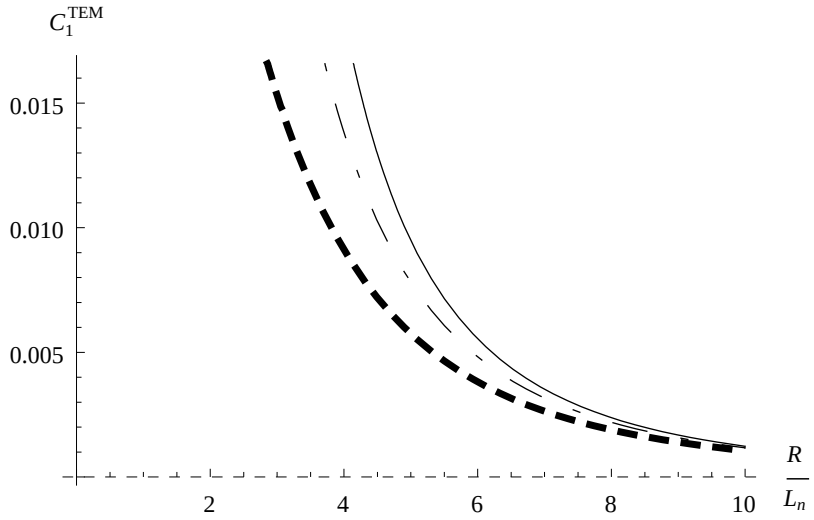

Figure 1: Plot of C_1^{TEM} as a function of R/L_n. The dashed line corresponds to $R/L_{Ti} = 0$, the dash-dotted line to $R/L_{Ti} = 2$ and the solid line to $R/L_{Ti} = 3$.

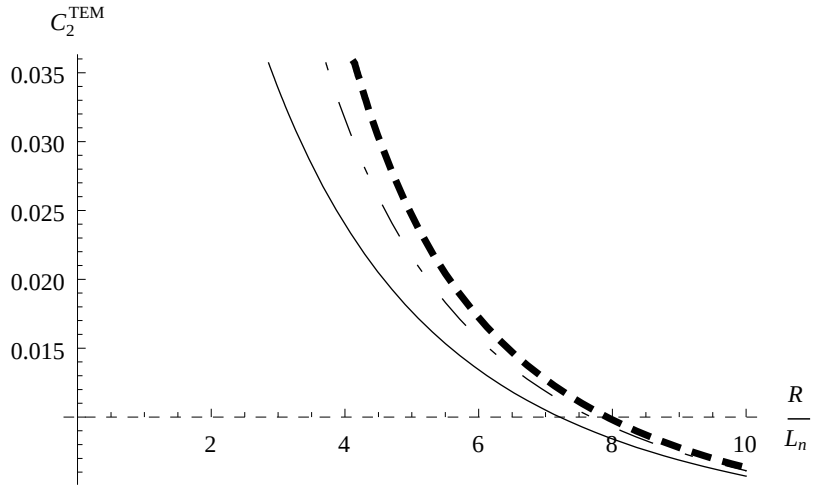

Figure 2: Plot of C_2^{TEM} as a function of R/L_n. The solid line corresponds to $R/L_{Ti} = 0$, the dash-dotted line to $R/L_{Ti} = 2$ and the dashed line to $R/L_{Ti} = 3$.

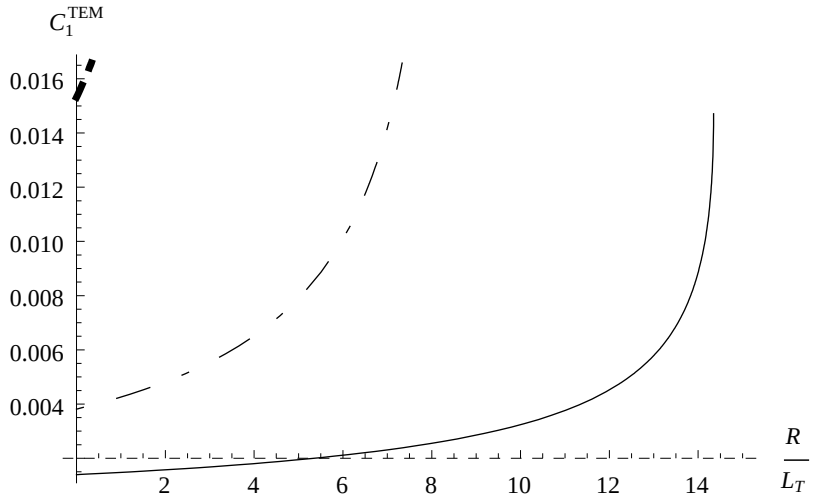

Figure 3: Plot of C_1^{TEM} as a function of R/L_{Ti}. The solid line corresponds to $R/L_n = 9$, the dash-dotted line to $R/L_n = 6$ and the dashed line to $R/L_n = 3$.

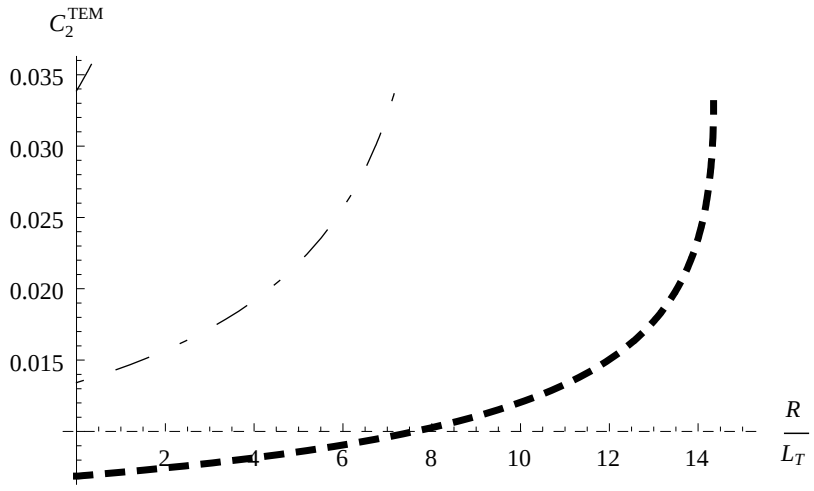

Figure 4: Plot of C_2^{TEM} as a function of R/L_{Ti}. The dashed line corresponds to $R/L_n = 9$, the dash-dotted line to $R/L_n = 6$ and the solid line to $R/L_n = 3$.

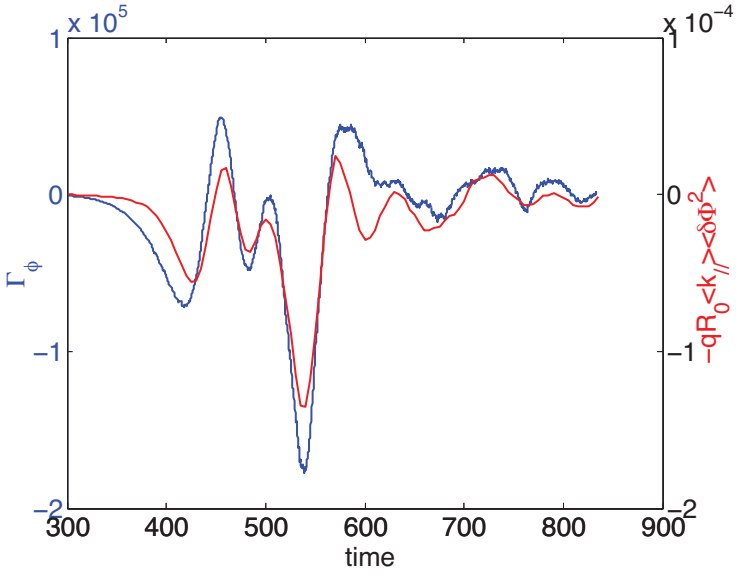

Figure 5: Time evolution of momentum flux (blue) and the spectrally averaged parallel wave number (red).

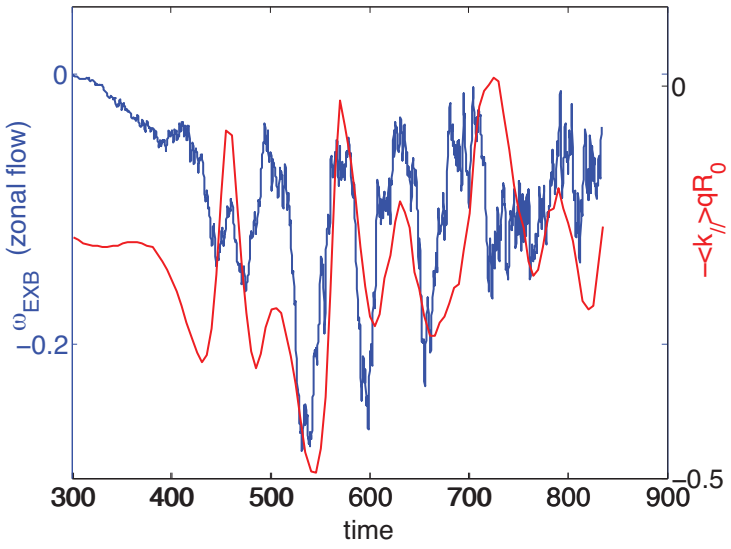

Figure 6: Time evolution of $\mathbf{E} \times \mathbf{B}$ shear rate (blue) and the spectrally averaged parallel wave number (red).