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Consolidation Settlement in Aquifers Caused by Pumping

Hugo A. Loáiciga, F.ASCE[1]

Abstract: Equations are derived to calculate one-dimensional (vertical) consolidation settlement in aquifers caused by groundwater extraction. The settlement equations capture the effect that the decline in pore-water pressure caused by groundwater extraction has on increased vertical effective stress. The settlement equations for single-layered confined or unconfined aquifers and multiple-layered aquifers are derived by linking the increase in vertical effective stress to the reduction of the void ratio using the reconstructed field consolidation curve of aquifer sediments. This paper's approach blends groundwater hydraulics with the classical theory of one-dimensional consolidation widely used in geotechnical engineering. Closed-form consolidation settlement equations are presented for single-layer, homogeneous, isotropic confined aquifers with steady-state or transient groundwater flow and for single-layer, homogeneous, isotropic, unconfined aquifers under steady-state flow. The consolidation equations for consolidated settlement in heterogeneous, anisotropic, single-layer or multilayer aquifers are expressed as the numerical integration of the vertical strain induced by groundwater pumping. The numerical-integration settlement equations require the implementation of a groundwater simulation model to calculate the pore pressure decline within aquifer layers, followed by the calculation of the increase in vertical effective stress and the reduction in pore volume. One numerical example confirms the accuracy of this paper's approach to aquifer consolidation by comparing with the solution obtained with the three-dimensional poroelastic theory. **DOI: 10.1061/(ASCE)GT.1943-5606.0000836.**

CE Database subject headings: Aquifers; Pore water; Water pressure; Stress; Pumps; Settlement.

Author keywords: Confined aquifer; Unconfined aquifer; Pore-water pressure; Effective stress; Hydraulic head; Drawdown; Consolidation settlement.

Introduction and Problem Statement

Groundwater extraction is known to cause consolidation of compressible water-bearing formations (aquifers). This topic has received attention in the technical literature [Galloway and Burbey (2011) provide a recent review]. Many authors use the term land subsidence to describe pumping-related consolidation settlement affecting relatively large areas, in some cases having a regional extent [for case studies see Ortega-Guerrero et al. (1999), Zektser et al. (2005), and Osmanoglu et al. (2011)]. The mechanism behind pumping-related consolidation is in some respects similar to that which causes consolidation of compressible soils when they are subjected to vertical loads. The cause of consolidation is the increase of the vertical effective stress and the ensuing reduction of pore volume. Groundwater extraction reduces pore-water pressure and increases the effective stress, which, in turn, reduces the volume of voids. On the other hand, there are substantive differences between the classical one-dimensional (1D) consolidation theory developed by Terzaghi in the 1920s (Terzaghi 1925) and the theory that describes pumping-related consolidation in soil strata as presented herein. One difference lies in the cause and nature of pore-pressure change and the increase in vertical effective stress in consolidated soils. Specifically, the Terzaghi theory of consolidation assumes that a sudden vertical load applied to a soil stratum induces an instantaneous increase in vertical total stress in the soils beneath the application area. The rise in total stress produces an immediate increase in pore pressure (i.e., the excess pressure above the pressure present prior to loading) equal in magnitude to the change in total stress. Thereafter, the excess pore pressure caused by loading dissipates over time by vertical groundwater flow through the soil column. The increase in vertical total stress by loading is transferred to soil particles as the excess pore pressure dissipates, thus raising the vertical effective stress and effecting (primary) consolidation settlement until the excess pore pressure has vanished, at which point the increase in vertical effective stress equals the initial increase in total stress caused by loading. In the case of groundwater pumping, on the other hand, there is neither applied external vertical load nor the creation of instantaneous excess pore pressure. Groundwater is removed from storage from the beginning of pumping, which lowers the pore pressure continually from the initial value existing prior to pumping. The decline in pore pressure is accompanied by an increase in vertical effective stress. Pore-pressure decline and effective stress rise continue with persistent pumping, possibly reaching a steady state (i.e., time equilibrium), if hydrogeologic conditions allow the equalization of groundwater extraction with induced groundwater recharge and reduced discharge [see a pertinent discussion of the pertinent mechanism of induced recharge and reduced discharge in Heath (1987) and Lohman (1989)]. Primary consolidation ceases in pumped aquifers upon reaching equilibrium of the pore pressure and achieving a steady-state vertical effective stress.

Another difference between consolidation by vertical loading and that caused by groundwater extraction in single-layer aquifers is the nature of the flow regime governing each instance. The Terzaghi consolidation theory proposes reduction of pore volume by removal of pore water through vertical flow of groundwater through soil, either single drainage or double drainage through the top and or bottom of a soil stratum, following the loading of the soil column. Groundwater extraction, on the other hand, produces predominantly

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horizontal radial flow toward a fully penetrating pumping well in single-layer confined and unconfined aquifers in which the lateral extent of groundwater flow is much larger than the initial saturated thickness of a pumped aquifer. Predominantly vertical flow occurs mainly in low-hydraulic conductivity aquitards separating permeable strata depleted by groundwater extraction (Helm 1976).

Fig. 1 illustrates the decrease in pore pressure in a confined aquifer 15.2-m thick pumped at a constant rate of 4.6 m^3/minute. The aquifer's hydraulic conductivity and storage coefficient equal 9.81 m/min and 0.0642, respectively. The decline in pressure was measured in observation wells placed at distances $r = 30.5$ m and $r = 122$ m from the pumping well. Fig. 1 shows how the decline in pressure is much smaller with increasing distance from the pumping well. Also, the pressure change tends to level off with advancing time.

This paper presents analytical and numerical equations for the calculation of (1D) primary consolidation of confined aquifers, unconfined aquifers, and multilayer aquifers affected by groundwater pumping. A key objective of this work is to merge the principles of groundwater hydraulics of pumped aquifers with standard methodology for the determination of primary consolidation of compressible sediments used in geotechnical engineering. Several authors have applied numerical methods to model land subsidence based on three-dimensional (3D) consolidation (or poroelastic) theory (Gambolati et al. 2000). The 3D consolidation theory explicitly couples aquifer deformation with groundwater hydraulics. The 3D approach, however, is infrequently used. One reason for the infrequent use is the limitation associated with the linear elastic, homogeneous, and isotropic assumptions imposed on the aquifer matrix [see Kramer (1996), for a discussion of stress deformation in elastic media see Ingebritsen et al. (2007), and for a modern presentation of the poroelastic equations originally see Biot (1956)]. These assumptions complicate the specification of elastic-theory parameters in realistic aquifer settings. The complexities inherent to the solution of the 3D consolidation equations may also contribute to explain its limited use. Gambolati et al. (2000) showed that the approach of first solving the groundwater flow equations, followed by the calculation of aquifer deformation, produced nearly identical land subsidence as that obtained from the solution of the coupled 3D poroelastic equations. Their example dealt with an elaborate hydromechanical database and numerical modeling of subsidence of normally consolidated sediments underlying the city of Venice, Italy.

This paper focuses on the practical problem of calculating 1D consolidation settlement considering the stress history of soils captured by their reconstructed consolidation curve commonly used in geotechnical engineering. Our work shows that coupling is achieved between groundwater hydraulics and aquifer deformation by incorporating the reconstructed consolidation characteristics of a soil into the groundwater-pumping driven consolidation equations. One example demonstrates the effectiveness of this paper's approach in predicting vertical consolidation when compared with 3D consolidation predictions.

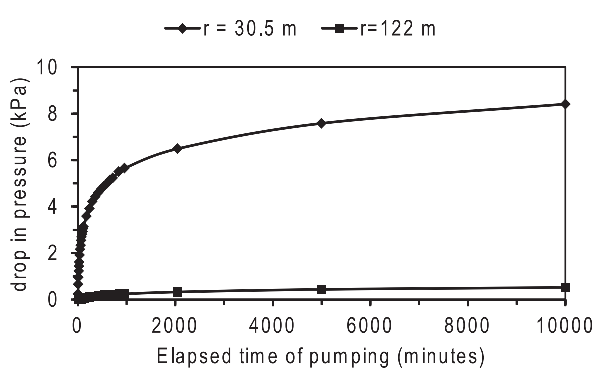

Fig. 1. Decline in pore pressure in a confined aquifer in Sioux Flats (Iowa) at distances 30.5 and 122 m from a well pumping 4.6 m^3/min; data from Loáiciga and Hudak (2003)

Steady-State (Ultimate) Consolidation Settlement of Confined Aquifers

Fig. 2 shows a homogeneous, isotropic, confined aquifer with hydraulic conductivity K, saturated thickness b, storage coefficient S, and saturated unit weight γ_{sat}, subjected to a constant pumping rate Q, whose constant hydraulic head prior to the initiation of pumping is everywhere equal to h_0 in the aquifer. Pumping starts at time $t = 0$, thereafter inducing a hydraulic head $h(r,t)$ and drawdown $s(r,t) = h_0 - h(r,t)$ at distance r from the pumping well at time $t > 0$. In Appendix I, the change in vertical effective stress at an elevation z, $\Delta\sigma'(r,t) \equiv \sigma'(r,z,t) - \sigma'(r,z,0)$, is related to the change in pore pressure, $\Delta P(r,t) \equiv P(r,z,t) - P(r,z,0)$, by the following expression:

$$\Delta\sigma'(r,t) = -\Delta P(r,t) = \gamma_w\, s(r,t) = \frac{\gamma_w Q}{4\pi K b} W(u) \qquad r,t > 0;\ 0 \le z \le b \tag{1}$$

where γ_w = unit weight of water $\cong$ 9.81 kN/m^3; and the well function $W(u)$ is

$$W\left(u = \frac{r^2 S}{4tKb}\right) = -\ln(u\gamma) - \sum_{m=1}^{\infty} \frac{(-1)^m u^m}{m\,(m!)} = -Ei(-u) \tag{2}$$

in which $\Upsilon = 1.78107\ldots$ is the Euler exponential constant, and Ei = exponential integral (Loáiciga 2009). The well function in Eq. (2) tends to infinity when $r \to 0$. This produces physically unrealizable large changes in the vertical effective stress and pore pressure when using Eq. (1) near the borehole. Therefore, the distance from the well at which consolidation settlement is calculated must be $r > 0$. Eq. (2) becomes infinity when $t \to \infty$. It must be used with finite time only.

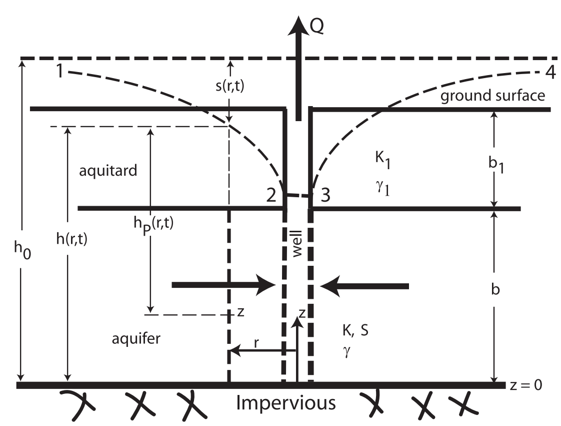

Fig. 2. Geometry and variables in a confined aquifer; groundwater flow is radial toward the well pumped at a constant rate Q

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The (1D) consolidation settlement in a confined aquifer at a distance r from the pumping well and at time t, $d(r,t)$, is the integral of the vertical strain ϵ over the saturated thickness of the aquifer. The aquifer is assumed to have a void ratio e_0 at time $t=0$ over its saturated thickness b, and has a void ratio $e(r,z,t)$ at distance r from the pumping well, at time $t>0$, and elevation z. Thus

$$d(r,t) = \int_0^b \epsilon(r,z,t)\,dz = \int_0^b \frac{e_0 - e(r,z,t)}{1+e_0}\,dz \tag{3}$$

Eq. (3) is the basis for the method proposed in the following sections to determine consolidation settlement under various stress histories in a soil for various aquifer systems.

Steady-State Consolidation Settlement in Normally Consolidated Aquifers

In this instance, the initial vertical effective stress $[\sigma'(r,z,0)]$ is approximately equal to the preconsolidation stress $[\sigma'_p(r,z)]$, $\sigma'(r,z,0) \cong \sigma'_p(r,z)$, the latter a known quantity. The steady-state vertical effective stress $[\sigma'_f(r,z,t_{fr})]$ is the value of the vertical effective stress that prevails at a distance r from the pumping well and at elevation z once the pressure field has reached a steady state at time t_{fr}, and it exceeds the preconsolidation stress throughout the saturated thickness b. The reconstructed field consolidation curve is adopted in this work to describe the relationship between void ratio e and $\log_{10}\sigma'$. A sample consolidation curve is shown in Fig. 3 for the soil composed of microfossils and diatoms underlying Mexico City (the Mexico City clay).

The reduction in the void ratio in normally consolidated soil occurs along the virgin curve of the consolidation curve with compression index $C_c = -\Delta e/\Delta\log_{10}\sigma'$. Appendix I proves that the ultimate consolidation settlement $d_c(r)$ corresponding to the steady-state vertical effective stress $\sigma'_f(r,z,t_{fr})$ is

$$d_c(r) = \frac{C_c}{(1+e_0)\ln(10)} \frac{\gamma_w Q W(u_{fr})}{4\pi Kb} \int_0^b \frac{dz}{\sigma'(r,z,t_{fr})} \qquad r>0 \tag{4}$$

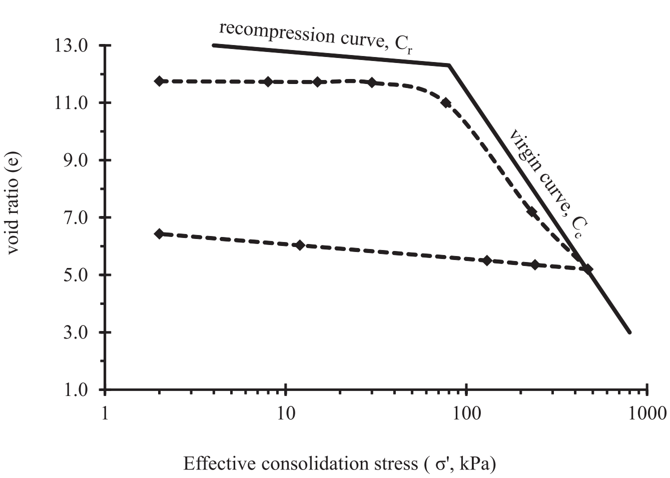

Fig. 3. Reconstructed (solid line) and laboratory-derived (dashed line) consolidation curves for the soil composed of microfossils and diatoms underlying Mexico City; data from Rutledge (1944)

in which the dimensionless variable u_{fr} corresponds to time t_{fr} at which the steady-state vertical effective stress $\sigma'(r,z,t_{fr})$ is reached at a distance r from the well

$$u_{fr} = \frac{r^2 S}{4\, t_{fr}\, Kb} \tag{5}$$

The time t_{fr} in Eq. (4) must be sufficiently long for the vertical effective stress to reach a steady-state value $\sigma'(r,z,t_{fr})$. When time t_{fr} is relatively long and the distance to the well r is relatively small, u_{fr} becomes relatively small. For example, setting $u_{fr} = 0.001$, the time t_{fr} is solved for in Eq. (5)

$$t_{fr} = \frac{250\, r^2\, S}{K\, b} \tag{6}$$

Other values for u_{fr} and t_{fr} could be used, as required by the behavior of pore pressure as a function of time observed in pumping tests.

Appendix I shows that, on integration of Eq. (4), the ultimate, or steady-state, consolidation settlement $d_c(r)$ becomes

$$d_c(r) = \frac{C_c}{(1+e_0)\ln(10)} \frac{\gamma_w Q W(u_{fr})}{4\pi Kb} \frac{1}{\gamma'} \ln\left(\frac{\alpha}{\alpha - \gamma' b}\right) \qquad r>0 \tag{7}$$

The coefficients α and γ' in Eq. (7) are defined as follows:

$$\alpha = \gamma_1\, b_1 + \gamma_{sat}\, b - \gamma_w \left[h_0 - \frac{Q}{4\pi Kb} W(u_{fr})\right] \tag{8}$$

$$\gamma' = \gamma_{sat} - \gamma_w \tag{9}$$

Steady-State Consolidation Settlement in Overconsolidated Aquifers with Stresses in the Recompression Curve

In this instance, the steady-state vertical effective stress is less than the preconsolidation stress, $\sigma'(r,z,0) < \sigma'(r,z,t_{fr}) < \sigma'_p(r,z)$, throughout the saturated thickness b, and therefore the consolidation settlement is governed by the recompression index $C_r = (-\Delta e/\Delta\log_{10}\sigma')$ along the recompression curve. It is proven in Appendix II that the ultimate consolidation settlement is given by

$$d_c(r) = \frac{C_r}{(1+e_0)\ln(10)} \frac{\gamma_w Q W(u_{fr})}{4\pi Kb} \frac{1}{\gamma'} \ln\left(\frac{\alpha}{\alpha - \gamma' b}\right) \qquad r>0 \tag{10}$$

The coefficients α and γ' are as defined in Eqs. (8) and (9), respectively.

Steady-State Consolidation Settlement in Overconsolidated Aquifers with Stresses Encompassing the Recompression and Virgin Curves

In this case, the preconsolidation stress is larger than the initial effective stress but smaller than the final effective stress, $\sigma'(r,z,0) < \sigma'_p(r,z) < \sigma'(r,z,t_{fr})$. It is assumed that the (known) overconsolidation margin $\sigma'_m(r) = \sigma'_p(r,z) - \sigma'(r,z,0)$ is constant with elevation, according to common practice (Coduto et al. 2011). The ultimate consolidation settlement is obtained by integrating the reconstructed consolidation curve. Appendix III proves the

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following expression for the ultimate consolidation settlement (where $r > 0$)

$$d_c(r) = \frac{1}{(1+e_0)\ln(10)}\left\{\frac{C_r\sigma'_m(r)}{\gamma'}\ln\left(\frac{\alpha_0}{\alpha_0-\gamma' b}\right) + \frac{C_c}{\gamma'}\left[\frac{\gamma_w\, Q\, W(u_{fr})}{4\pi Kb} - \sigma'_m(r)\right]\ln\left(\frac{\alpha}{\alpha-\gamma' b}\right)\right\} \quad (11)$$

The coefficients α and γ' are given in Eqs. (8) and (9), respectively, and the coefficient α_0 is

$$\alpha_0 = \sigma'_m(r) + \gamma_1\, b_1 + \gamma_{sat}\, b - \gamma_w h_0 \quad (12)$$

Alternative Approach for Determining the Steady-State Consolidation Settlement in Confined Aquifers

An alternative approach to calculating the ultimate consolidation settlement in confined aquifers is to assume that pore pressure has reached a steady state everywhere in the aquifer without regard for the temporal evolution of pore pressure caused by pumping. This allows solution of the steady-state equation for confined aquifer flow, from which the steady-state vertical effective stress $[\sigma'(r,z)]$ and consolidation settlement $[d_c(r)]$ are determined. The first step in this approach is to derive the steady-state (time-independent) drawdown at a distance r from the pumping well, $s(r) = h_0 - h(r)$. This is accomplished by integrating the groundwater flow equation in a confined aquifer, leading to the following expression for the drawdown (Thiem 1906):

$$s(r) = \frac{Q}{2\pi Kb}\ln\left(\frac{R}{r}\right) \quad (13)$$

in which R = radius of influence beyond which the drawdown is zero under the steady-state flow toward the well (Fig. 2). R can be estimated from field observations in pumped aquifers. Several empirical equations for R have been proposed for the radius of influence in confined and unconfined aquifers [see Bear (1979) for a summary of equations].

The consolidation settlement for normally consolidated and overconsolidated soils is derived in a manner analogous to that presented in Appendixes I–III. The results are as follows.

Normally Consolidated Soil

When

$$\sigma'(r,z) > \sigma'_p(r,z) \cong \sigma'(r,z,0),$$
$$d_c(r) = \frac{C_c\,\gamma_w\, s(r)}{(1+e_0)\ln(10)}\frac{1}{\gamma'}\ln\left(\frac{\alpha}{\alpha-\gamma' b}\right) \qquad r>0 \quad (14)$$

in which

$$\alpha = \gamma_1\, b_1 + \gamma_{sat}\, b - \gamma_w[h_0 - s(r)] \quad (15)$$

and γ' is given by Eq. (9). The drawdown $s(r)$ in Eq. (14) is obtained from the steady-state drawdown Eq. (13).

Overconsolidated Soil

$$\sigma'(r,z,0) < \sigma'(r,z) < \sigma'_p(r,z),$$
$$d_c(r) = \frac{C_r\,\gamma_w\, s(r)}{(1+e_0)\ln(10)}\frac{1}{\gamma'}\ln\left(\frac{\alpha}{\alpha-\gamma' b}\right) \qquad r>0 \quad (16)$$

with coefficients α and γ' as defined by Eqs. (15) and (9), respectively.

Overconsolidated Soil

$$\sigma'(r,z,0) < \sigma'_p(r,z) < \sigma'(r,z),$$
$$d_c(r) = \frac{1}{(1+e_0)\ln(10)}\left\{\frac{C_r\sigma'_m(r)}{\gamma'}\ln\left(\frac{\alpha_0}{\alpha_0-\gamma' b}\right) + \frac{C_c}{\gamma'}\left[\gamma_w s(r) - \sigma'_m(r)\right]\ln\left(\frac{\alpha}{\alpha-\gamma' b}\right)\right\} \quad (17)$$

The coefficients α and γ' are the same as those in Eqs. (15) and (9), respectively. The coefficient α_0 is that in Eq. (12).

Consolidation Settlement in Confined Aquifers in the Transient Period

Groundwater extraction lowers the pore pressure and raises the effective stress continually. Each level of effective stress reached over time is associated with an amount of consolidation settlement. It is assumed that a combination of the field effective stress and void ratio corresponds to an equal combination of the effective consolidation stress and void ratio found in the reconstructed consolidation curve. This being the case, the methods that led to the equations for ultimate consolidation settlement are applicable to derive the equations for calculating settlement prior to reaching steady-state vertical effective stress. Letting $d(r,t)$ denote the consolidation settlement at a radial distance r from a pumping well at time $t < t_{fr}$, and $\sigma'(r,z,t)$ represent the vertical effective stress at elevation z in the confined aquifer at the same r, t, the following equations present the consolidation settlements for normally consolidated and overconsolidated conditions before reaching a steady state

Normally Consolidated Soil

When

$$\sigma'(r,z,t) > \sigma'_p(r,z) \cong \sigma'(r,z,0),$$
$$d(r,t) = \frac{C_c}{(1+e_0)\ln(10)}\frac{\gamma_w\, Q\, W(u)}{4\pi Kb}\frac{1}{\gamma'}\ln\left(\frac{\alpha}{\alpha-\gamma' b}\right) \qquad r>0 \quad (18)$$

The coefficient α equals

$$\alpha = \gamma_1 b_1 + \gamma_{sat}\, b - \gamma_w\left[h_0 - \frac{Q}{4\pi Kb}W(u)\right] \quad (19)$$

and $\gamma' = \gamma_{sat} - \gamma_w$. The dimensionless variable $u = r^2 S/(4t\, Kb)$.

Overconsolidated Soil

When

$$\sigma'(r,z,0) < \sigma'(r,z,t) < \sigma'_p(r,z),$$
$$d(r,t) = \frac{C_r}{(1+e_0)\ln(10)}\frac{\gamma_w Q\, W(u)}{4\pi Kb}\frac{1}{\gamma'}\ln\left(\frac{\alpha}{\alpha-\gamma' b}\right) \qquad r>0 \quad (20)$$

with coefficients α and γ' defined by Eqs. (19) and (9), respectively.

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Overconsolidated Soil

$\sigma'(r,z,0) < \sigma'_p(r,z) < \sigma'(r,z,t)$,

When

$$d(r,t) = \frac{1}{(1+e_0)\ln(10)}\left\{\frac{C_r\sigma'_m(r)}{\gamma'}\ln\left(\frac{\alpha_0}{\alpha_0-\gamma' b}\right) + \frac{C_c}{\gamma'}\left[\frac{\gamma_w Q W(u)}{4\pi K b} - \sigma'_m(r)\right]\ln\left(\frac{\alpha}{\alpha-\gamma' b}\right)\right\} \qquad r>0 \tag{21}$$

The coefficients α and γ' are given in Eqs. (19) and (9), respectively, and the coefficient α_0 was defined in Eq. (12).

Geometry of the Consolidation Settlement: Superposition

Under conditions of radial flow induced by one pumping well in a homogeneous and isotropic aquifer of large lateral extent, the shape of the loci of points with equal consolidation settlement $d(r,t)$ for fixed time t is a circle or radius r centered at the well. The field of ultimate consolidation settlement is a series of concentric circles, each exhibiting increasing consolidation settlement as the radial distance to the pumping well diminishes. The principle of superposition applies whenever there are $n>1$ wells pumping the same aquifer each with a rate Q_i and located at coordinates x_i, y_i, $i = 1, 2, 3, \ldots, n$. Denote the radial distance from a location where the consolidation settlement is quantified to the ith well by r_i. Let the consolidation settlement caused at the location of interest by the ith well be $d(r_i,t)$. By superposition of individual effects, the total consolidation settlement at the location of interest $[d_0(t)]$ is

$$d_0(t) = \sum_{i=1}^{n} d(r_i,t) \tag{22}$$

The individual consolidation settlements $d(r_i,t)$ are given by equations corresponding to the soil stress history, be it normally consolidated or overconsolidated, as previously presented.

Average Degree of Consolidation

The average degree of consolidation, $U_t(r)$, is defined as the ratio of consolidation settlement at time t and distance r from the pumping well, $d(r,t)$, divided by the ultimate consolidation settlement at the same distance, $d_c(r)$ (Holtz and Kovacs 1981)

$$U_t(r) = \frac{d(r,t)}{d_c(r)} \tag{23}$$

The consolidation settlement $d(r,t)$ and the ultimate consolidation settlement $d_c(r)$ appearing in Eq. (23) have been presented in several aforementioned equations, each corresponding to a specific case of a soil's stress history and effective stresses induced by pumping.

Consolidation Settlement of Unconfined Aquifers: Transient Case

Small Drawdown

Fig. 4 shows the geometry and variables involved in consolidation analysis of a homogeneous, isotropic, unconfined aquifer. In general, transient, unconfined groundwater flow is such that the hydraulic head, a distance r from a pumping well at a fixed time $t(t>0)$, $h(r,z,t)$, varies with elevation z within the saturated thickness of an aquifer, as shown in Fig. 4. The drawdown of the hydraulic head, $s(r,z,t)$, is defined as the initial, undisturbed, saturated thickness (h_0) minus the hydraulic head at elevation z and time $t>0$, or $s(r,z,t) = h_0 - h(r,z,t)$. The upper boundary of the saturated thickness defines the phreatic surface whose elevation is denoted by $h_w(r,t)$, which outlines the cone of depression (1–2 and 3–4 in Fig. 4) caused by pumping. In Fig. 4, the drawdown of the phreatic surface, $s(r,h_w,t) = h_0 - h_w(r,t)$, is less than the drawdown at a lower elevation z $[s(r,z,t)]$ within the saturated thickness (Segment C-C′). This creates downward groundwater flow. Groundwater flow is, however, predominantly horizontal toward the well.

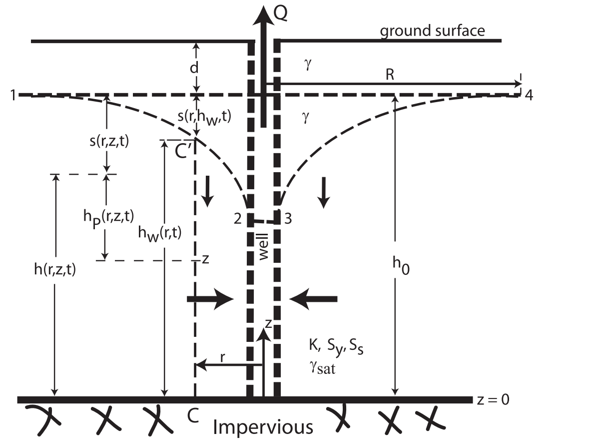

Fig. 4. Geometry and variables in an unconfined aquifer subject to a constant rate of pumping Q

In Fig. 4, the change in vertical total stress at elevation z, at distance r from the well, and at time $t>0$ when below the phreatic surface: $0 \le z \le h_w(r,t)$ is

$$\Delta\sigma \equiv \sigma(r,z,t) - \sigma(r,z,0) = (\gamma - \gamma_{sat})s(r,h_w,t) \tag{24}$$

and when above the phreatic surface, and below the initial saturated thickness: $h_w(r,t) \le z \le h_0$ (setting the pore pressure equal to zero in the vadose zone)

$$\Delta\sigma = (\gamma - \gamma_{sat})(h_0 - z) \tag{25}$$

where γ and γ_{sat} = unit weights of the aquifer above and below the phreatic surface, respectively. The change in vertical effective stress $[\Delta\sigma' = \sigma'(r,z,t) - \sigma'(r,z,0)]$ is related to the change in pore pressure $[\Delta P \equiv P(r,z,t) - P(r,z,0)]$ as follows:

$$\begin{aligned}\Delta\sigma' &= -\Delta P - (\gamma_{sat} - \gamma)s(r,h_w,t)\\ &= \gamma_w\, s(r,z,t) - (\gamma_{sat} - \gamma)s(r,h_w,t)\end{aligned} \tag{26}$$

in the interval $0 \le z \le h_w(r,t)$ (below the phreatic surface), and

$$\Delta\sigma' = -\Delta P - (\gamma_{sat} - \gamma)(h_0 - z) = (h_0 - z)\,[\gamma - (\gamma_{sat} - \gamma_w)] \tag{27}$$

in the interval $h_w(r,t) \le z \le h_0$ (above the phreatic surface, but below the initial saturated thickness).

The changes in vertical effective stress and pore pressure are a function of elevation within the unconfined aquifer. Also, the magnitude of the change in effective stress does not equal the magnitude

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of the change in pore pressure. From Fig. 4, the vertical effective stress at time $t = 0$, $\sigma'(r, z, 0)$, can be shown to be

$$\sigma'(r, z, 0) = \gamma d \ + \ (\gamma_{sat} \ - \ \gamma_w)(h_0 - z) \tag{28}$$

The vertical effective stress at time $t > 0$, $\sigma'(r, z, t)$, is given by the following equation:

$$\begin{aligned}\sigma'(r, z, t) = \gamma d \ - [(\gamma_{sat} - \gamma)\, s(r, h_w, t)] \\ + (\gamma_{sat} - \gamma_w)(h_0 - z) + \gamma_w\, s(r, z, t)\end{aligned} \tag{29}$$

in the interval $0 \le z \le \ h_w(r, t)$ (below the phreatic surface), and

$$\sigma'(r, z, t) = \ \gamma d \ + \ \gamma \ (h_0 - z) \tag{30}$$

in the interval $h_w(r, t) \le z \le \ h_0$ (above the phreatic surface, but below the initial saturated thickness).

There are not known analytical solutions for the drawdowns $s(r, z, t)$ and $s(r, h_w, t)$ in the unconfined aquifers. Neuman (1975) reported an analytically derived drawdown $s(r, z, t)$ for the case when the drawdown on the phreatic surface is much smaller than the thickness of the phreatic surface, $s(r, z, t) \ll h_w(r, t)$. With this simplification, the change in total vertical stress approaches zero, that is, $\Delta\sigma \cong 0$. Furthermore, the change in vertical effective stress becomes

$$\begin{aligned}\Delta\sigma' \equiv \sigma'(r, z, t) - \sigma'(r, z, 0) \\ = -\Delta P = -[P(r, z, t) - P(r, z, 0)] \cong \gamma_w\, s(r, z, t)\end{aligned} \tag{31}$$

Under the simplification that the drawdown is small, the vertical effective stress at time $t > 0$ becomes

$$\sigma'(r, z, t) = \ \gamma d \ + (\gamma_{sat} - \gamma_w)(h_0 - z) + \gamma_w\, s(r, z, t) \tag{32}$$

The consolidation settlement at time t, $d(r, t)$, in normally consolidated soils, where $\sigma'_p(r, z) \ \cong \ \sigma'(r, z, 0) < \sigma'(r, z, t)$, is approximated by the following equation:

$$d(r, t) \cong \frac{C_c\, \gamma_w}{(1 + e_0) \ln(10)} \int_0^{h_0} \frac{s(r, z, t)}{\sigma'(r, z, t)} dz \tag{33}$$

The vertical effective stress in the denominator of Eq. (33) is expressed by Eq. (32). The available analytical solution for the drawdown $s(r, z, t)$ (Neuman 1975) does not allow a closed-form integration of Eq. (33). Therefore, the consolidation settlement $d(r, t)$ must be calculated numerically. To this end, the soil stratum beneath the initial undisturbed hydraulic head h_0 is divided into n nonoverlapping sublayers of suitable thickness Δz_i each, where the sum of the thicknesses add up to h_0. The drawdown $s(r, z, t)$ and vertical effective stress $\sigma'(r, z, t)$ are evaluated at the midelevation z_i of each sublayer. The integral in Eq. (33) is approximated numerically by the following equation:

$$d(r, t) \cong \frac{C_c\, \gamma_w}{(1 + e_0) \ln(10)} \sum_{i=1}^{n} \frac{s(r, z_i, t)}{\sigma'(r, z_i, t)} \Delta z_i \tag{34}$$

When the soil is overconsolidated and $\sigma'(r, z, 0) < \sigma'(r, z, t) < \sigma'_p(r, z)$, the consolidation settlement is approximated as follows:

$$d(r, t) \cong \frac{C_r\, \gamma_w}{(1 + e_0) \ln(10)} \sum_{i=1}^{n} \frac{s(r, z_i, t)}{\sigma'(r, z_i, t)} \Delta z_i \tag{35}$$

For overconsolidated soil where $\sigma'(r, z, 0) < \sigma'_p(r, z) < \ \sigma'(r, z, t)$, the consolidation settlement is approximated by integrating numerically along the recompression and virgin curves

$$\begin{aligned}d(r, t) \cong \frac{1}{(1 + e_0)\ \ln(10)} \\ \times \sum_{i=1}^{n} \left[\frac{C_r\, \sigma'_m(r)}{\sigma'_p(r, z_i)}\ \Delta z_i + \frac{C_c\ [\gamma_w\, s(r, z_i,\ t) - \sigma'_m(r)]}{\sigma'(r, z_i, t)} \Delta z_i \right]\end{aligned} \tag{36}$$

The overconsolidation margin $\sigma'_m(r) = \sigma'_p(r, z) - \sigma'(r, z, 0)$ does not vary with elevation, by assumption, and must be a known quantity. The preconsolidation stress with depth $\sigma'_p(r, z_i) = \sigma'_m + \sigma'(r, z_i, 0)$, where the initial vertical effective stress is given by Eq. (28).

Large Drawdown

The calculation of a large-drawdown consolidation settlement in unconfined aquifers requires numerical integration of the vertical strain over the initially saturated thickness h_0 of the aquifer, analogous to the case of small-drawdown consolidation previously presented for homogeneous and isotropic aquifers. The major variant that emerges in the large-drawdown case is that the pore-water pressure, hydraulic head, and drawdown that occur at a distance r from the well and time $t > 0$ must be calculated by numerical groundwater flow simulation. Once the pore pressure has been determined by numerical simulation, one can determine the vertical effective stress and consolidation settlement. In this instance, the calculation of the consolidation settlement is a subcase of the general method, which is subsequently presented. The method to calculate large-drawdown consolidation settlement in an unconfined aquifer is also subsequently discussed.

Consolidation Settlement of Unconfined Aquifers: Steady-State Case

Analytical expressions for the drawdown that lead to closed-form integration of the vertical strain, and thus, the consolidation settlement, are possible in the case of homogeneous, isotropic, unconfined aquifers that exhibit steady-flow. This simplification is attainable provided that groundwater flow is nearly horizontal toward the pumping well (the so-called Dupuit assumption). The solution for the steady-state drawdown $s(r) = h_0 - h(r)$ at a distance r from the pumping well involves the radius of influence, R, previously elaborated on when discussing steady-state confined flow. Referring to Fig. 4, R is the distance from the pumping well beyond which the drawdown vanishes. Integration of the equation of radial groundwater flow in an unconfined aquifer leads to the following expression for the steady-state drawdown $s(r)$ (Dupuit 1863):

$$s(r) = h_0 - h(r) = h_0 - \sqrt{h_0^2 - \frac{Q}{\pi K} \ln\left(\frac{R}{r}\right)} \tag{37}$$

The drawdown $s(r)$ and hydraulic head $h(r)$ are constant for a fixed distance r. The radius of influence, R, can be estimated from field observations in pumped aquifers. Several empirical equations have been proposed to estimate the influence radius, R (Bear 1979). One such equation is as follows:

$$R = 1.9 \sqrt{\frac{h_0\, K\, t_f}{S_y}} \tag{38}$$

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where R and h_0 are in meters, K is in meters per unit time, where the units of time are equals to those of t_f, which is the time it takes to achieve a steady state; and S_y = specific yield of the unconfined aquifer.

The equations for consolidation settlement corresponding to steady-state unconfined flow are derived by integrating the vertical strain over the saturated portion of the aquifer and over the zone comprised within the initial saturated thickness and phreatic surface, as follows:

$$d(r) = \int_0^{h_w(r)} \epsilon(r,z)\,dz + \int_{h_w(r)}^{h_0} \epsilon(r,z)\,dz \qquad (39)$$

The implementation of Eq. (39) requires knowledge of the vertical effective stress. From the aquifer characteristics shown in Fig. 4, the following expressions are found for the initial vertical effective stress $\sigma'(r,z,0)$ and for the steady-state vertical final stress $\sigma'(r,z)$ in each of the consolidation zones.

In the interval $0 \le h(r) \le h_w(r)$ (below the phreatic surface)

$$\sigma'(r,z,0) = \gamma\,d + (\gamma_{sat} - \gamma_w)(h_0 - z) \qquad (40)$$

$$\sigma'(r,z) = \gamma\,d + \gamma\,(h_0 - z) \qquad (41)$$

In the interval $h_w(r) \le h(r) \le h_0$ (above the phreatic surface, but below the initial saturated thickness) the initial vertical effective stress is expressed by Eq. (40). The steady-state vertical effective stress is

$$\sigma'(r,z) = d\,\gamma + [\gamma - (\gamma_{sat} - \gamma_w)]\,s(r) + (\gamma_{sat} - \gamma_w)(h_0 - z) \qquad (42)$$

Eqs. (39)–(42) are combined in proper fashion to yield the steady-state (ultimate) consolidation settlement for the various stress histories of a soil.

Normally Consolidated Soil

When

$$\sigma'(r,z) > \sigma'_p(r,z) \cong \sigma'(r,z,0):$$

$$d_r = \frac{C_c}{(1+e_0)\ln(10)} \left\{ \frac{\gamma^*}{\gamma} \left[s(r) - d\ln\left(\frac{\eta}{d}\right) \right] + \frac{\gamma^*}{\gamma'} s(r) \ln\left(\frac{\xi}{\eta\;\gamma}\right) \right\} \qquad (43)$$

The following equations define parameters appearing in Eq. (43):

$$\gamma^* = \gamma - (\gamma_{sat} - \gamma_w) \qquad (44)$$

$$\gamma' = \gamma_{sat} - \gamma_w \qquad (45)$$

$$\xi = \gamma\,d + \gamma^* s(r) + \gamma'\,h_0 \qquad (46)$$

$$\eta = d + s(r) \qquad (47)$$

in which the steady-state drawdown is calculated with Eq. (37).

Overconsolidated Soil

When

$$\sigma'(r,z,0) < \sigma'(r,z) < \sigma'_p(r,z):$$

$$d_r = \frac{C_r}{(1+e_0)\ln(10)} \left\{ \frac{\gamma^*}{\gamma} \left[s(r) - d\ln\left(\frac{\eta}{d}\right) \right] + \frac{\gamma^*}{\gamma'} s(r) \ln\left(\frac{\xi}{\eta\;\gamma}\right) \right\} \qquad (48)$$

Overconsolidated Soil

When

$$\sigma'(r,z,0) < \sigma'_p(r,z) < \sigma'(r,z):$$

$$d_r = \phi_r \frac{\sigma'_m}{\gamma'} \left[\ln\left(\frac{\nu}{\omega}\right) \right] + \phi_c \left[\frac{\gamma^*}{\gamma} s(r) - \frac{\omega^*}{\gamma} \ln\left(\frac{\eta}{d}\right) + \left(\frac{\gamma^* s(r) - \sigma'_m}{\gamma'} \right) \ln\left(\frac{\xi}{\eta\;\gamma}\right) \right] \qquad (49)$$

where γ^* was defined in Eq. (44). The following definitions apply in Eq. (49):

$$\phi_r = \frac{C_r}{(1+e_0)\ln(10)} \qquad (50)$$

$$\phi_c = \frac{C_c}{(1+e_0)\ln(10)} \qquad (51)$$

$$\nu = \gamma\,d + \gamma' h_0 + \sigma'_m \qquad (52)$$

$$\omega = \gamma\,d + \sigma'_m \qquad (53)$$

$$\omega^* = \gamma^* d + \sigma'_m \qquad (54)$$

Consolidation Settlement in Layered, Heterogeneous, and Anisotropic Aquifers

This section covers the calculation of (1D) consolidation settlement in (1) single-layer unconfined aquifers where the drawdown is large and (2) multilayered aquifer systems comprised of heterogeneous and anisotropic confined and unconfined strata and aquitards (all three types of formation are generically referred to as layers). The numerical equations presented in this section do not differentiate between the single-layer unconfined and multilayered aquifer cases, because the former is a subcase of the latter. In a heterogeneous and anisotropic aquifer system, one must, in most instances, resort to numerical simulation of the groundwater flow regime caused by pumping to obtain the drawdowns and the corresponding pore pressure everywhere in the aquifer layers. There are public-domain and commercial numerical groundwater flow models available for this purpose.

Fig. 5 shows a two-layer aquifer system of practical importance. The situation portrayed in Fig. 5 is that of a permeable, coarse-grained, semiconfined aquifer (Layer 1) of low compressibility overlain by a low-permeability, fined-grained, (unconfined) aquitard (Layer 2) of high compressibility. The semiconfined aquifer is pumped by a well screened through its saturated thickness at rate Q. Layer 1 has saturated thickness b_1, whereas Layer 2 has initial saturated thickness b_2. The initial hydraulic heads in both layers equals h_0. The hydraulic conductivity, storage coefficient, and saturated unit weight of the semiconfined Layer 1 are denoted by K_1, S_1, and γ_{1sat}, respectively. The hydraulic conductivity, storage coefficient, specific yield, saturated unit weight, and unsaturated unit weight of the unconfined Layer 2 are K_2, S_2, S_{2y}, γ_{2sat}, and γ_2, respectively. The drawdown on the phreatic surface of the unconfined Layer 2 at distance r from the well and at time t, denoted by $s(r, h_w t)$ in Fig. 5, is smaller than the drawdown at the bottom of Layer 2 at the same distance r and time t, or $s_2(r, b_1, t)$, causing downward flow in the aquitard. The drawdown $s_2(r, b_1, t)$ at the bottom of the unconfined Layer 2 equals the drawdown $s_1(r, t)$ in the

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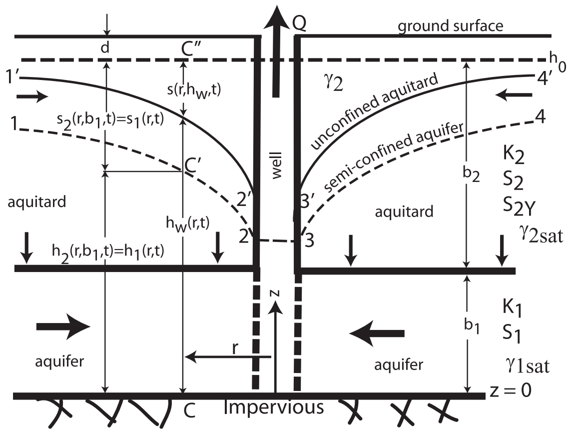

Fig. 5. Two-layer aquifer system composed of a low-compressibility, semiconfined aquifer, overlain by a (unconfined) high-compressibility aquitard

semiconfined Layer 1 at the same distance and time because of the hydraulic connection on the boundary between the two layers. The drawdown and the hydraulic head $h_1(r,t)$ in the semiconfined Layer 1 are constant over its thickness b_1.

The well, which is screened over the saturated thickness of the semiconfined layer, starts pumping and dewatering the aquifer system, as shown in Fig. 5. After some time of pumping, at some arbitrary time $t>0$ prior to reaching a steady state, two distinct cones of depression form. The trace 1′-2′-3′-4′ depicts the cone of depression in the aquitard, whereas traces 1 and 2 and 3–5 represent that in the semiconfined layer. The more permeable, semiconfined, layer sustains a more rapid drop in the hydraulic head than the overlying aquitard, producing downward flow toward the semiconfined layer. The drop in the hydraulic head and pore water pressure in the compressible aquitard drives consolidation in the aquifer system.

Any layer j ($j = 1,2,3,\ldots,N$) of a multilayer aquifer system may be heterogeneous, and therefore the indexes C_c and C_r, the overconsolidation margin σ'_m, the initial void ratio e_0, the unit weight, and all hydraulic properties may vary within the layer. The jth layer is divided into n_j nonoverlapping sublayers of suitable thicknesses Δz_{ij}, $i = 1,2,3,\ldots,n_j$, that add up to its initial saturated thickness b_j, or

$$\sum_{i=1}^{n_j} \Delta z_{ij} = b_j \quad j = 1,2,3,\ldots,N \tag{55}$$

where z_{ij} = midelevation of the ith sublayer in the jth layer. Fig. 6 shows a two-layer system in which each layer is split into two sublayers. The initial effective stresses $\sigma'(r,z_{ij},0)$, the preconsolidation stress $\sigma'_p(r,z_{ij}) = \sigma'(r,z_{ij},0) + \sigma'_m(r,z_{ij})$, and vertical total stress at time $t>0$, $\sigma(r,z_{ij},t)$, at the midelevation z_{ij} are derivable from the characteristics of the aquifer in standard fashion.

The pore pressure $P(r,z_{ij},t)$ is obtained from numerical simulation of the groundwater flow regime. The vertical effective stress $\sigma'(r,z_{ij},t)$ is calculated as the total vertical stress $\sigma(r,z_{ij},t)$ minus the pore pressure $P(r,z_{ij},t)$, the latter set equal to zero whenever a layer has been dewatered at that location. Mathematically

$$\sigma'\big(r,z_{ij},t\big) = \sigma\big(r,z_{ij},t\big) - P\big(r,z_{ij},t\big)$$
$$i = 1,2,3,\ldots,n_j;\ j = 1,2,3,\ldots,N \tag{56}$$

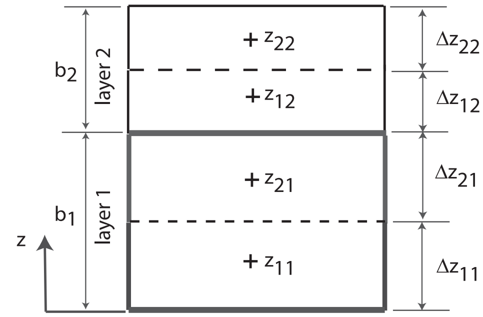

Fig. 6. Two-layer system in which each layer is subdivided into two sublayers; Layer 1 and Layer 2 thicknesses are b_1 and b_2, respectively

Using Fig. 5 as an example, let z_{12} be an Elevation 1 in Layer 2, such that $z_{12} = h_1(r,t)$, that is, an elevation on the cone of depression of the semiconfined layer a distance r from the well at time t (Point C' on Fig. 5). The vertical effective stresses at time $t=0$ and $t>0$ are

$$\sigma'(r,z_{12},0) = \sigma(r,z_{12},0) - P(r,z_{12},0) = d\,\gamma_2 + s_1(r,t)(\gamma_{2sat} - \gamma_w) \tag{57}$$

$$\sigma'(r,z_{12},t>0) = [d + s(r,h_w,t)]\,\gamma_2 + [s_1(r,t) - s(r,h_w,t)](\gamma_{2sat} - \gamma_w) \tag{58}$$

The change (rise) in effective vertical stress is then

$$\Delta\sigma'(r,z_{12},t>0) = [\gamma_2 - (\gamma_{2sat} - \gamma_w)]s(r,h_w,t) \tag{59}$$

The consolidation settlement of the jth layer at distance r from the pumping well at time $t>0$ is derived using the relationships between the compression and recompression indexes and the change in void ratio caused by a change in vertical effective stress, that is, $C_c = -\Delta e/\Delta\log_{10}\sigma'$ and $C_r = -\Delta e/\Delta\log_{10}\sigma'$. The vertical strain is then integrated numerically over the initial saturated thickness b_j of the jth layer, complying with the stress history of the soil, as reflected by its reconstructed consolidation curve. The following numerical approximations are obtained for the consolidation settlement in the jth layer.

Normally Consolidated Soil

When

$$\sigma'(r,z,t) > \sigma'_p(r,z) \cong \sigma'(r,z,0),$$
$$d_j(r,t) = \sum_{i=1}^{n_j} \frac{C_{ci}}{(1+e_{0i})} \Delta z_{ij} \log_{10}\left[\frac{\sigma'\big(r,z_{ij},t\big)}{\sigma'\big(r,z_{ij},0\big)}\right], \quad j = 1,2,3,\ldots,N \tag{60}$$

Overconsolidated Soil

When

$$\sigma'(r,z,0) < \sigma'(r,z,t) < \sigma'_p(r,z),$$
$$d_j(r,t) = \sum_{i=1}^{n_j} \frac{C_{ri}}{(1+e_{0i})} \Delta z_{ij} \log_{10}\left[\frac{\sigma'\big(r,z_{ij},t\big)}{\sigma'\big(r,z_{ij},0\big)}\right], \quad j = 1,2,3,\ldots,N \tag{61}$$

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Overconsolidated Soil

$\sigma'(r,z,0) < \sigma'_p(r,z) < \sigma'(r,z,t)$,

When

$$d_j(r,t) = \sum_{i=1}^{n_j}\left\{\frac{C_{ri}}{(1+e_{0i})}\Delta z_{ij}\ \log_{10}\left[\frac{\sigma'_p(r,z_{ij})}{\sigma'(r,z_{ij},0)}\right] + \frac{C_{ci}}{(1+e_{0i})}\Delta z_{ij}\ \log_{10}\left[\frac{\sigma'(r,z_{ij},t)}{\sigma'_p(r,z_{ij})}\right]\right\},\quad j = 1,2,3,\ldots,N \qquad (62)$$

The consolidation settlement of the N-layer aquifer system equals the sum of the individual-layer settlements

$$d(r,t) = \sum_{j=1}^{N} d_j(r,t) \qquad (63)$$

Eqs. (60)–(63) are of general applicability. They apply to single ($N = 1$) or multilayer aquifers ($N > 1$), confined or unconfined, or to mixtures of confined, unconfined layers, or aquitard layers, which may be heterogeneous and anisotropic. The steady-state equivalents of Eqs. (60)–(63) are essentially the same, except that the time t of elapsed pumping must be as long as necessary to achieve a steady state of the groundwater flow regime in the aquifer system.

Comparison Example

Brief Overview of Three-Dimensional Consolidation Theory

Prior to comparing the calculation of consolidation settlement using the methods presented in this work with the consolidation settlement calculated with the 3D consolidation (or poroelastic) equations in a test case, it is worthwhile to synthesize the latter equations.

The 3D consolidation equations consist of three formulas for deformation equilibrium and one for groundwater flow. The deformation of the aquifer matrix caused by groundwater extraction and concomitant drop in pore pressure and rise in effective stress has three components, which in rectangular Cartesian coordinates x, y, and z are denoted by u_x, u_y, and u_z, respectively. The vector of deformations, whose components are u_x, u_y, and u_z, is denoted by $\underline{u}$. These deformations are incremental, that is, caused by changing effective stress. The form of the 3D consolidation (poroelastic) equations have, as unknown variables, the deformations of the aquifer matrix, u_x, u_y, and u_z, and the (incremental) pore pressure $\widehat{P}$. The incremental pore pressure is negative when there is groundwater extraction. These variables are, in general, functions of x, y, and z and time t. The aquifer matrix's deformation-related coefficients are Poisson's ratio (v) and the elastic modulus (E), also called Young's modulus. β_w represents the compressibility of water, approximately equal to $5 \times 10^{-10}\ \text{Pa}^{-1}$ (De Marsily 1986). The aquifer's hydraulic conductivity and porosity are denoted by K and ϕ, respectively. The unit weight of water and the saturated aquifer matrix are γ_w and γ_{sat}, respectively. For an arbitrary scalar variable, such as the incremental pore pressure $\widehat{P}$, the gradient and Laplacian of the scalar variable are denoted by $\nabla\widehat{P}$ and $\nabla^2\widehat{P}$, respectively. For an arbitrary vector, such as $\underline{u}$, the divergence and Laplacian of the vector are denoted by $\nabla \cdot \underline{u}$ and $\nabla^2\,\underline{u}$, respectively. Arfken (1985) provides details on the divergence, gradient, and Laplacian operators. The incremental effective stress ($\widehat{\sigma}'$) is related to the incremental total stress ($\widehat{\sigma}$) and the incremental pore pressure ($\widehat{P}$) by the equation

$$\widehat{\sigma}' = \widehat{\sigma} - \alpha_\beta \times\ \widehat{P} \qquad (64)$$

The (Biot) coefficient α_β is a function of the volumetric compressibility of the aquifer matrix [$\beta_b = 3(1-2v)/E$] and the compressibility of the solids (β_s)

$$\alpha_\beta = 1 - \frac{\beta_s}{\beta_b} \qquad (65)$$

where the coefficient α_β is nearly equal to 1 in compressible aquifer matrices. The 3D consolidation equations subsequently presented assumes $\alpha_\beta = 1$, and therefore $\widehat{\sigma}' = \widehat{\sigma} - \widehat{P}$.

With this preamble, the 3D consolidation equations in rectangular Cartesian coordinates in a linear elastic, homogeneous, and isotropic aquifer are as follows (Gambolati et al. 2000; Timoshenko and Goodier 1970).

Three equations of deformation equilibrium

$$\frac{E}{2(1-2v)(1+v)} \times \nabla\left(\nabla \cdot \underline{u}\right) + \frac{E}{2(1+v)} \times \nabla^2\,\underline{u} = \nabla\widehat{P} \qquad (66)$$

One flow equation

$$K\ \nabla^2\widehat{P} = \gamma_w\,\phi\,\beta_w\frac{\partial\widehat{P}}{\partial t} + \gamma_w\frac{\partial\nabla \cdot \underline{u}}{\partial t} \qquad (67)$$

Eqs. (66) and (67) are supplemented with boundary conditions and initial conditions on the incremental deformations and incremental pore pressure to define a field problem amenable to numerical solution. The initial conditions are typically zero deformations and incremental pressure throughout the aquifer. The boundary conditions are usually zero deformations and incremental pore pressure a distance sufficiently long from a pumping well or wells. Other boundary conditions are possible. At the well location, the pumping rate equals the integral of the specific discharge toward the well given by Darcy's law over the screened portion of the well.

Parameter Specification in Steady-State One-Dimensional and Three-Dimensional Consolidation in A Confined Aquifer

The aquifer type used in this example is confined with steady-state groundwater flow, relying on Eqs. (9), (13), (14), and (15) to calculate 1D consolidation settlement in normally consolidated clay. Steady-state or ultimate consolidation settlement is being compared in this example. The simple geometry of this test case makes it easy to parameterize the 1D equations [Eqs. (9), (13), (14), and (15)] developed in this work and the 3D equations of poroelasticity [Eqs. (66) and (67)]. In this manner, the roles of flow and deformation processes involved in the two alternative models (1D and 3D) takes preeminence in the comparison, avoiding confounding issues that might otherwise arise from complex geometry and heterogeneous aquifer properties. Fig. 2 shows the geometry of the confined-aquifer case with constant pumping rate Q. The aquifer matrix corresponds to highly compressible, normally consolidated, clay whose reconstructed consolidated curve is shown in Fig. 3. The compression index $C_c = 9.3$. The initial void ratio $e_0 = 12.3$. The elastic modulus and Poisson's ratio were based on values given in ASCE (1994) for very soft saturated clay: $E\ \cong 1{,}000$ kPa $v = 0.45$. The saturated unit weight of the aquifer and the unit weight of the aquitard are $\gamma_{\text{sat}} = 20\ \text{kN/m}^3$ and $\gamma_1 = 17\ \text{kN/m}^3$. The aquifer and

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aquitard thicknesses equal $b = 30$ m and $b_1 = 35$ m. The radius of influence and the initial hydraulic head are $R = 500$ m and $h_0 = 65$ m. The pumping rate was set equal to $Q = 20\ \mathrm{m^3/day}$, and the hydraulic conductivity was set equal to $K = 0.02$ m/day.

Three-Dimensional Consolidation Equations in Polar Cylindrical Coordinates

Polar cylindrical coordinates are introduced to describe the 3D consolidation equations to take advantage of the radial nature of the flow regime and the benefit of symmetry in the resulting pressure and deformation fields. Polar cylindrical coordinates r, θ, and z are related to the rectangular Cartesian coordinates x, y, and z by $r = (x^2+y^2)^{1/2}$ and $\theta = \tan^{-1}(y/x)$, expressed in radians, whereas z is the vertical Cartesian coordinate in both systems of coordinates. The coordinate $r \geq 0, 0 \leq \theta \leq 2\pi$, and $-\infty \leq z \leq \infty$. The 3D consolidation [Eqs. (66) and (67)] in rectangular Cartesian coordinates are rewritten as follows in polar cylindrical coordinates.

Deformation equilibrium along the r-coordinate

$$\frac{E}{2(1-2v)(1+v)} \times \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial\,(r\,u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right] + \frac{E}{2(1+v)} \times \left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right) = \frac{\partial \widehat{P}}{\partial r} \qquad (68)$$

Deformation equilibrium along the θ-coordinate

$$\frac{E}{2(1-2v)(1+v)} \times \frac{1}{r}\frac{\partial}{\partial \theta}\left[\frac{1}{r}\frac{\partial\,(r\,u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right] + \frac{E}{2(1+v)} \times \left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right) = \frac{1}{r}\frac{\partial \widehat{P}}{\partial \theta} \qquad (69)$$

Deformation equilibrium along the z-coordinate

$$\frac{E}{2(1-2v)(1+v)} \times \frac{\partial}{\partial z}\left[\frac{1}{r}\frac{\partial\,(r\,u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right] + \frac{E}{2(1+v)} \times \nabla^2 u_z = \frac{\partial \widehat{P}}{\partial z} \qquad (70)$$

Groundwater flow equation

$$K\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \widehat{P}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \widehat{P}}{\partial \theta^2} + \frac{\partial^2 \widehat{P}}{\partial z^2}\right] = \gamma_w\,\phi\,\beta_w\frac{\partial \widehat{P}}{\partial t} + \gamma_w\frac{\partial}{\partial t}\left[\frac{1}{r}\frac{\partial\,(r\,u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}\right] \qquad (71)$$

The right side of Eq. (71) equals zero when the flow is in a steady state, as is the case in this example. The deformations and incremental pressure depend on r, z, and t, but not on θ, because of the symmetry of the flow regime toward the well and the assumed homogeneity and isotropy of the aquifer. The terms $\nabla^2 u_r$, $\nabla^2 u_\theta$, and $\nabla^2 u_z$ in Eqs. (68), (69), and (70), respectively, denote the Laplacian of the components of matrix deformation along the r-, θ-, and z-coordinates, respectively. The Laplacian of a scalar φ (where φ may equal u_x, u_y, u_z, or $\widehat{P}$) in Eqs. (68)–(70) expressed in polar cylindrical coordinates is written as follows:

$$\nabla^2\varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2} + \frac{\partial^2 \varphi}{\partial z^2} \qquad (72)$$

Results of One-Dimensional and Three-Dimensional Consolidation

Eqs. (9), (13), (14), and (15) provided the 1D consolidation settlement $d_c(r)$. Eqs. (68)–(71) (or, more precisely, their steady-state versions) were solved numerically to obtain the aquifer matrix deformations and pore-pressure field. The vertical deformation (u_z) at the upper boundary of the confined aquifer represents the 3D consolidation settlement (or land subsidence) at distance r from the well upon reaching steady-state flow. Fig. 7 shows the steady-state 1D consolidation and 3D consolidation results.

In Fig. 7, the 1D and 3D consolidation settlements are in general agreement. However, the analytical (1D) results tend to overestimate settlement in the vicinity of the well's borehole and up to about 150 m from the well. The near-well borehole overstimation is caused by the behavior of the well function near the pumping well, a topic previously discussed. The 1D consolidation settlement underestimates land subsidence beyond 150 m, a trend that is accentuated with distance away from the well. The 1D consolidation settlement is zero at the radius of influence (500 m). This is a mathematical boundary condition imposed by the chosen radius of influence, whose estimation is shrouded with uncertainty. The 3D consolidation settlement calculations indicate that consolidation occurs beyond the assumed radius of influence in the 1D method. The overall good resemblance found between the 1D and 3D consolidation results is an encouraging sign of the predictive skill of the 1D model developed herein. Future tests of this paper's methods shall be forthcoming.

Summary and Conclusions

Equations to calculate 1D consolidation settlement were derived for single-layer confined and unconfined aquifers subjected to groundwater extraction. The consolidation-settlement equations cover the transient and steady-state case flow regimes. The single-layer aquifer cases allowed closed-form expression of the consolidation settlement equations under simplifying assumptions, and these equations are presented in this paper. This work also addressed the case of 1D consolidation settlement in multilayer aquifer systems undergoing pumping in which individual layers may be heterogeneous and anisotropic. The multilayer aquifer case requires calculation of the pore-water pressure by numerical simulation of groundwater flow, in either transient or steady-state groundwater flow regimes. Once the pore-water pressure is determined, the vertical

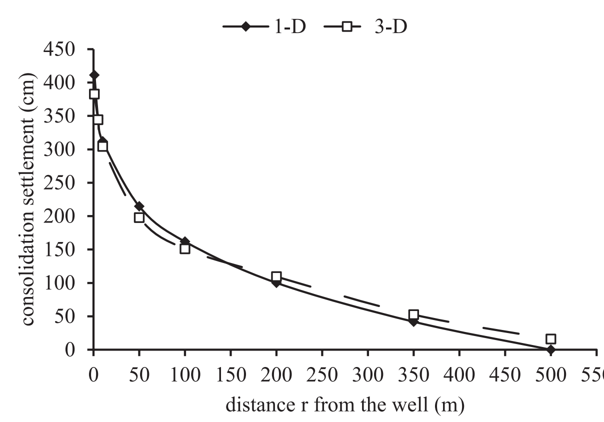

Fig. 7. Steady-state (vertical) consolidation settlement obtained with the one-dimensional and three-dimensional consolidation models

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effective stress is calculable according to standard geotechnical procedure. The equations of this paper can then be implemented in a straightforward manner to calculate the consolidation settlement at any time and place in an aquifer.

An example compared the consolidation settlement obtained in a confined aquifer with steady-state groundwater flow using the 1D consolidation formulas developed in this paper and by numerically solving the 3D consolidation equations of poroelasticity. The calculated 1D and 3D consolidation settlement showed overall good similarity from the well's borehole radius to the radius on influence.

The main contribution of this paper is to have linked the principles of groundwater hydraulics to standard methodology for calculating 1D consolidation settlement based on the reconstructed consolidation curve for soils. The latter curve captures the stress history of aquifer sediments. The resulting consolidation settlement equations, either in closed-form or in numerical form, are relatively simple and easy to implement in single-layer or multilayer aquifer systems.

Appendix I. Proof of Eqs. (1), (4), and (7)

Eqs. (1), (4), and (7) are proven in Appendix I. To prove Eq. (1), first notice that the initial vertical effective stress in the confined aquifer of Fig. 2 is

$$\sigma'(r,z,0) = b_1\,\gamma_1 + b\,\gamma_{sat} - h_0\,\gamma_w - \gamma' z \tag{73}$$

The vertical effective stress at time $t > 0$ equals

$$\sigma'(r,z,t) = b_1\,\gamma_1 + b\,\gamma_{sat} - [h_0 - s(r,t)]\;\gamma_w - \gamma' z \tag{74}$$

in which $s(r,t)$ is the drawdown $s(r,t) = h_0 - h(r,t)$. From Eqs. (73) and (74), the change in vertical effective stress is

$$\Delta\sigma'(r,z,t) = \sigma'(r,z,t) - \sigma'(r,z,0) = \gamma_w\, s(r,t) \tag{75}$$

The pore-water pressures at times $t = 0$ and $t > 0$ are

$$P(r,z,0) = (h_0 - z)\;\gamma_w \tag{76}$$

$$P(r,z,t) = [h_0 - s(r,t) - z]\gamma_w \tag{77}$$

Therefore, the change in pore pressure equals

$$\begin{aligned}\Delta P(r,z,t) = P(r,z,t) - P(r,z,0) \;&=\; -\gamma_w\; s(r,t)\\ &=\; -\Delta\sigma'(r,z,t)\end{aligned} \tag{78}$$

The drawdown $s(r,t)$ in an homogeneous, isotropic, spatially boundless (read, mathematically infinite) subject to a constant pumping rate Q and characteristics as shown in Fig. 2 was found by Theis (1935) and reviewed with details by Loáiciga (2009). The result is

$$s(r,t) = \frac{Q}{4\pi Kb}W(u) \quad r,t>0;\; 0 \le z \le b \tag{79}$$

Eqs. (75), (78), and (79) prove Eq. (1).

To prove Eq. (4), notice that the compression coefficient C_c in the reconstructed consolidation curve of Fig. 3 is related to the changes in void ratio and vertical effective stress as follows:

$$C_c = -\frac{de}{d\log_{10}(\sigma')} = -\ln(10)\frac{de}{d\ln(\sigma')} = -\ln(10)\;\sigma'\frac{de}{d\sigma'} \tag{80}$$

Therefore, from Eqs. (75), (79), and (80)

$$e_0 - e \;=\; \frac{C_c\,\Delta\sigma'}{\ln(10)\,\sigma'} = \frac{C_c}{\ln(10)\;\sigma'}\frac{\gamma_w\,Q}{4\pi\,Kb}W(u) \tag{81}$$

The proof of Eq. (4) is completed by the following expression:

$$\begin{aligned}d_c(r) &= \int_0^b \epsilon(r,z,t_{fr})dz = \int_0^b \frac{e_0 - e(r,z,t_{fr})}{1+e_0}\,dz\\ &= \frac{C_c}{(1+e_0)\;\ln(10)}\frac{\gamma_w\,Q}{4\pi\,Kb}W(u_{fr})\int_0^b \frac{dz}{\sigma'(r,z,t_{fr})}\end{aligned} \tag{82}$$

in which $e(r,z,t_{fr})$ equals the steady-state void ratio.

To prove Eq. (7), notice that the steady-state drawdown is given by

$$s(r,t_{fr}) = \;s(r) = \frac{Q}{4\pi\,Kb}W(u_{fr}) \tag{83}$$

Next, write the vertical effective stress at time t_{fr}, as follows [Eq. (74)]:

$$\sigma'(r,z,t_{fr}) = \alpha - \gamma' z \tag{84}$$

where α was defined in Eq. (8) and $\gamma' = \gamma_{sat} - \gamma_w$. Integrate the last term on the right side of Eq. (82)

$$\int_0^b \frac{dz}{\sigma'(r,z,t_{fr})} = \int_0^b \frac{dz}{\alpha - \gamma' z} = \frac{1}{\gamma'}\ln\left(\frac{\alpha}{a - \gamma' b}\right) \tag{85}$$

Substitution of Eq. (85) in Eq. (82) completes the proof of Eq. (7).

Appendix II. Proof of Eq. (10)

The purpose of Appendix II is to prove Eq. (10), the equation for consolidation settlement according to the recompression portion of the reconstructed consolidation curve. This proof proceeds analogous to that used to prove Eq. (7) in Appendix I pertaining to the consolidation settlement following the virgin branch of the reconstructed consolidation curve. The only change to be made is to replace C_c in Eqs. (80)–(82) with the coefficient C_r.

Appendix III. Proof of Eq. (11)

Eq. (11) is proven in Appendix III. The proof is analogous to those used to prove the consolidation settlement equations for virgin compression (Appendix I) and recompression (Appendix II), except that Appendix III concerns consolidation settlement along the recompression and virgin branches of the reconstructed consolidation curve. Notice that the initial, preconsolidation, and final vertical effective stresses have magnitudes such that $\sigma'(r,z,0) < \sigma'_p(r,z) < \sigma'(r,z,t_{fr})$. The vertical stress change along the virgin branch of the reconstructed consolidation curve is $\sigma'(r,z,t_{fr}) - \sigma'_p(r,z)$, whereas the vertical stress change $\sigma'_p(r,z) - \sigma'(r,z,0) = \sigma'_m(r)$ takes place along the recompression branch of the consolidation curve, where $\sigma'_m(r)$ denotes the overconsolidation margin. The effective stress change along the virgin curve can be rewritten as $\sigma'(r,z,t_{fr}) - \sigma'(r,z,0) - \sigma'_m(r)$. The consolidation settlement

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involving recompression and virgin compression can be written as the integral of the vertical strains through recompression and virgin compression

$$d_c(r) = \int_0^b \frac{e_0 - e_p(r,z)}{(1+e_0)} dz + \int_0^b \frac{e_p(r,z) - e(r,z,t_{fr})}{(1+e_0)} dz \quad (86)$$

in which $e_p(r,z)$ = void ratio corresponding to the preconsolidation stress σ'_p. The first integral on the right side of Eq. (86) represents recompression. The change in vertical effective stress corresponding to recompression equals the overconsolidation margin. Therefore, the consolidation settlement by recompression becomes

$$\int_0^b \frac{e_0 - e_p(r,z)}{(1+e_0)} dz = \frac{C_r \, \sigma'_m}{(1+e_0)\ \ln(10)} \int_0^b \frac{dz}{\sigma'_m(r) + \sigma'(r,z,0)} \quad (87)$$

The denominator of the integrand on the right side of Eq. (87), which equals the preconsolidation stress, is expressed as follows:

$$\sigma'_m(r) + \sigma'(r,z,0) = \sigma'_m(r) + b_1\gamma_1 + b\gamma_{sat} - h_0\gamma_w - \gamma' z = \alpha_0 - \gamma' z \quad (88)$$

where α_0 and γ' are defined by Eqs. (12) and (9), respectively.

The change in vertical effective stress through virgin compression can be written as follows:

$$\sigma'(r,z,t_{fr}) - \sigma'_p(r,z) = \sigma'(r,z,t_{fr}) - \sigma'(r,z,0) - \sigma'_m(r) = \frac{\gamma_w \, Q}{4\pi \, Kb} W(u_{fr}) - \sigma'_m(r) \quad (89)$$

The second integral on the right side of Eq. (86) represents virgin compression. The consolidation settlement by virgin compression is expressed by the following equation:

$$\int_0^b \frac{e_p(r,z) - e(r,z,t_{fr})}{(1+\ e_0)} dz = \frac{C_c}{(1+e_0)\ln(10)} \left(\frac{\gamma_w \, Q}{4\,\pi\, Kb} W(u_{fr}) - \sigma'_m \right) \int_0^b \frac{dz}{\sigma'(r,z,t_{fr})} \quad (90)$$

The denominator of the integrand on the right side of Eq. (90) is the vertical effective stress at time t_{fr}, already presented in Eq. (84). The integrals on the right sides of Eqs. (87) and (90) are performed in a manner analogous to the integral in Eq. (85). The sum of Eq. (87) and (90), upon integration, equals Eq. (11), thus completing its proof.

Notation

The following symbols are used in this paper:

b = thickness of a confined aquifer;
b_j = thickness of the *j*th layer in a layered aquifer;
b_1 = thickness of a confined layer, or thickness of the first (bottom) layer in a layered aquifer;
b_2 = thickness of an aquitard, or thickness of the second (next-to- bottom) layer in a layered aquifer;
C_c = compression index of the virgin curve in a reconstructed field consolidation curve;
C_{ci} = compression index of soil in the *i*th sublayer of the *j*th layer in a layered aquifer;
C_r = recompression index in a reconstructed field consolidation curve;
C_{ri} = recompression index of soil in the *i*th sublayer of the *j*th layer in a layered aquifer;
d = thickness of the vadoze zone in an unconfined aquifer;
$d(r,t)$ = ultimate consolidation settlement evaluated at distance r from the pumping well and time t;
$d(r_i,t)$ = consolidation settlement caused by the *i*th well at time t at a location of interest placed at a distance r_i from the *i*th well;
$d_c(r)$ = ultimate consolidation settlement evaluated at distance r from the pumping well;
$d_j(r,t)$ = consolidation settlement in the *j*th layer of a layered aquifer evaluated at distance r from the pumping well and time t;
$d_0(t)$ = consolidation settlement at time t at a location 0 resulting from the superposition of individual consolidation settlements caused by several pumping wells;
E = elastic (Young's) modulus;
$Ei()$ = exponential integral with arbitrary argument;
$e(r,z,t)$ = void ratio evaluated at distance r from the pumping well, elevation z, and time t;
$e(r,z,t_{fr})$ = void ratio evaluated at distance r from the pumping well, at depth z, and at time t_{fr};
$e_p(r,z)$ = void ratio corresponding to the preconsolidation stress σ'_p;
e_{0i} = initial void ratio in the *i*th sublayer of the *j*th layer in a layered aquifer;
h = hydraulic head;
$h(r)$ = hydraulic head at distance r from the pumping well;
$h(r,t)$ = hydraulic head evaluated at distance r from the pumping well and time t;
$h(r,z,t)$ = hydraulic head evaluated at distance r from the pumping well, elevation z, and time t;
$h_p(r,z,t)$ = pressure head evaluated at distance r from the pumping well, elevation z, and time t;
h_w = elevation of the phreatic surface in an unconfined aquifer;
$h_w(r)$ = elevation of the phreatic surface evaluated at distance r from the pumping well;
$h_w(r,t)$ = elevation of the phreatic surface evaluated at distance r from the pumping well and time t;
h_0 = initial hydraulic head;
$h_1(r,t)$ = hydraulic head in a confined aquifer evaluated at distance r from the pumping well and time t;
$h_2(r,b_1,t)$ = hydraulic head in an aquitard evaluated at distance r from the pumping well, elevation b_1, and time t;
K = saturated hydraulic conductivity;
K_1 = saturated hydraulic conductivity of the first (bottom) layer in a layered aquifer;
K_2 = saturated hydraulic conductivity of the second (next-to-bottom) layer in a layered aquifer;
$\ln()$ = natural logarithm of an arbitrary argument;

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N = number of wells in a layered aquifer;
n = number of pumping wells in an aquifer, or the number of sublayers in an unconfined aquifer;
n_j = number of sublayers in layer j of a layered aquifer;
P = pore pressure;
$\widehat{P}$ = increment of pore pressure (positive or negative);
$P(r,z,t)$ = pore pressure evaluated at distance r from the pumping well, elevation z, and time t;
$P(r,z,0)$ = pore pressure evaluated at distance r from the pumping well, elevation z, and time 0;
Q = pumping rate in a well;
R = radius of influence of a pumping well;
r = distance from the pumping well to a point of interest within the aquifer, or the radial polar coordinate;
S = storage coefficient;
S_s = specific storage coefficient in an unconfined layer;
S_Y = specific yield in an unconfined aquitard;
S_1 = storage coefficient in a confined layer (Layer 1);
S_2 = storage coefficient in a confining aquitard (Layer 2);
S_{2Y} = specific yield in a confining aquitard (Layer 2);
$s(r)$ = drawdown of the hydraulic head evaluated at distance r from the pumping well;
$s(r,h_w,t)$ = drawdown of the elevation of the phreatic surface evaluated at distance r from the pumping well, elevation h_w, and time t;
$s(r,t)$ = drawdown of the hydraulic head evaluated at distance r from the pumping well and time t;
$s(r,z,t)$ = drawdown of the hydraulic head evaluated at distance r from the pumping well, elevation z, and time t;
$s(r,z_i,t)$ = drawdown of the hydraulic head evaluated at distance r from the pumping well, elevation z_i, and time t, in which z_i denotes the elevation of the midpoint in the ith layer in a layered aquifer;
$s_1(r,t)$ = drawdown of the hydraulic head in a confined layer (Layer 1) evaluated at distance r and time t;
$s_2(r,b_1,t)$ = drawdown of the hydraulic head in a confining aquitard layer (Layer 2) evaluated at distance r from the pumping well, elevation b_1, and time t;
t_f = time it a takes for drawdown to reach a steady state in an unconfined aquifer, and used in Eq. (38);
t_{fr} = long time after which the confined aquifer (Theis 1935) drawdown approaches a steady state;
$U_t(r)$ = average degree of consolidation at time t and distance r from the pumping well;
u = argument of the well function $W(u)$;
$\underline{u}$ = deformation vector whose components are u_x, u_y, and u_z;
u_{fr} = argument of the well function evaluated at distance r from the well and at time t_{fr};
u_r, u_θ = incremental deformation along the polar cylindrical coordinates r and θ, respectively;
u_x, u_y, u_z = incremental deformations of the aquifer matrix along the axes x, y, and z, respectively;
v = Poisson's ratio;
$W(u)$ = well function evaluated at u;
x = Cartesian coordinate on the horizontal plane;
y = Cartesian coordinate on the horizontal plane;
z = Cartesian elevation coordinate (vertical);
z_i = elevation of the midpoint in the ith sublayer in an unconfined aquifer;
z_{ij} = elevation of the midpoint of the ith sublayer in the jth layer of a layered aquifer;
α = coefficient defined in Eq. (8);
α_β = Biot's (1956) coefficient defined in Eq. (65);
α_0 = coefficient defined in Eq. (12);
β_b = volumetric compressibility of the aquifer matrix;
β_s = compressibility of solids;
β_w = compressibility of water;
γ = moist unit weight of an unconfined porous matrix;
γ' = buoyant unit weight = $\gamma_{sat} - \gamma_w$;
$\gamma*$ = modified unit weight defined in Eq. (44);
γ_{sat} = saturated unit weight of a porous matrix;
γ_w = unit weight of water;
γ_1 = saturated unit weight of a confining aquitard;
γ_{1sat} = saturated unit weight of a confined layer (Layer 1);
γ_{2sat} = saturated unit weight of a confining layer (Layer 2);
$\epsilon(r,z)$ = vertical strain evaluated at distance r from the pumping well, at elevation z;
$\epsilon(r,z,t)$ = vertical strain evaluated at distance r from the pumping well, at elevation z, and time t;
ΔP = change in pore pressure;
Δz_i = thickness of the ith sublayer of an unconfined aquifer;
Δz_{ij} = thickness of the ith sublayer in the jth layer of a layered aquifer;
$\nabla\widehat{P}$ = gradient of the increment of pore pressure;
$\nabla \cdot \underline{u}$ = divergence of the deformation vector $\underline{u}$;
$\nabla(\nabla \cdot \underline{u})$ = gradient of the divergence of the deformation vector $\underline{u}$;
$\nabla^2\widehat{P}$ = Laplacian of the increment of pore pressure;
$\nabla^2 \underline{u}$ = Laplacion of the deformation vector $\underline{u}$;
η = coefficient defined in Eq. (47);
θ = polar angular coordinate;
φ = arbitrary scalar variable;
ν = coefficient defined in Eq. (52);
ξ = coefficient defined in Eq. (46);
σ = total vertical stress;
σ' = effective vertical stress;
$\widehat{\sigma}'$ = incremental effective stress;
$\sigma(r,z,t)$ = total vertical stress evaluated at distance r from the pumping well, elevation z, and time t;
$\sigma(r,z,0)$ = total vertical stress evaluated at distance r from the pumping well, elevation z, and time 0;
σ'_m = preconsolidation margin;
$\sigma'_m(r)$ = preconsolidation margin evaluated at distance r from the pumping well;
$\sigma'_m(r,z)$ = preconsolidation margin evaluated at a distance r from the pumping well and depth z;
σ'_p = preconsolidation effective stress;
$\sigma'_p(r,z)$ = preconsolidation effective stress evaluated at distance r from the pumping well and depth z;
$\sigma'_p(r,z_i)$ = preconsolidation effective stress evaluated at distance r from the pumping well and depth z_i at the center of the ith sublayer of an unconfined aquifer;
$\sigma'_p(r,z_{ij})$ = preconsolidation effective stress evaluated at distance r from the pumping well and depth z_i at the center of the ith sublayer of the jth layer of a layered aquifer;

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$\sigma'(r, z, t)$ = vertical effective vertical stress evaluated at distance r from the pumping well, elevation z, and time t;
$\sigma'(r, z, t_{fr})$ = total vertical stress evaluated at distance r from the pumping well, elevation z, and time t_{fr};
$\sigma'(r, z_i, t)$ = vertical effective vertical stress evaluated at distance r from the pumping well, elevation z_i, and time t;
$\sigma'(r, z, 0)$ = vertical effective vertical stress evaluated at distance r from the pumping well, elevation z, and time 0;
$\sigma'(r, z_i, 0)$ = vertical effective vertical stress evaluated at distance r from the pumping well, elevation z_i, and time 0;
$\sigma'(r, z_{ij}, t)$ = vertical effective stress evaluated at distance r from the pumping well, at elevation z_{ij}, at the center of the ith sublayer of the jth layer of a layered aquifer, and at time t;
$\sigma'(r, z_{ij}, 0)$ = vertical effective stress evaluated at distance r from the pumping well, at elevation z_{ij}, at the center of the ith sublayer of the jth layer of a layered aquifer, and at time 0;
ϕ = porosity of an aquifer;
ϕ_c = coefficient defined in Eq. (51);
ϕ_r = coefficient defined in Eq. (50);
ω = coefficient defined in Eq. (53); and
ω^* = coefficient defined in Eq. (54).

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