July 10, 2004 LBNL-41481, UCB-PTH-98/14, RU-98-05
hep-th/9803051

M Theory on $AdS_p \times S^{11-p}$ and Superconformal Field Theories

Ofer Aharony[1a], Yaron Oz[2b], Zheng Yin[2c]

[1]*Department of Physics and Astronomy*
Rutgers University, Piscataway, NJ 08855-0849, U.S.A.
and
Institute for Theoretical Physics
University of California, Santa Barbara, CA 93106, U.S.A.

[2]*Department of Physics, University of California at Berkeley*
Berkeley, CA 94720-7300, U.S.A.
and
Theoretical Physics Group, Mail Stop 50A–5101
Ernest Orlando Lawrence Berkeley National Laboratory,
1 Cyclotron Road, Berkeley, CA 94720, U.S.A.

email: [a] oferah@physics.rutgers.edu, [b] yaronoz@thsrv.lbl.gov, [c] zyin@thsrv.lbl.gov.

Abstract

We study the large N limit of the interacting superconformal field theories associated with N M5 branes or M2 branes using the recently proposed relation between these theories and M theory on AdS spaces. We first analyze the spectrum of chiral operators of the 6d $(0,2)$ theory associated with M5 branes in flat space, and find full agreement with earlier results obtained using its DLCQ description as quantum mechanics on a moduli space of instantons. We then perform a similar analysis for the D_N type 6d $(0,2)$ theories associated with M5 branes at an R^5/Z_2 singularity, and for the 3d $\mathcal{N}=8$ superconformal field theories associated with M2 branes in flat space and at an R^8/Z_2 singularity respectively. Little is known about these three theories, and our study yields for the first time their spectrum of chiral operators (in the large N limit).

1 Introduction

A duality between a certain limit of some superconformal field theories (SCFTs) in d dimensions and string or M theory compactified on spaces of the form $AdS_{d+1} \times W$ has recently been proposed in [1] (see also [2–8]). Here W is a compact manifold which in the maximally supersymmetric cases is a sphere. A precise correspondence between the supergravity limit on the AdS_{d+1} side and an appropriate large N limit on the conformal field theory side has been formulated in [9, 10]. According to [10] the correlation functions in the conformal field theory, which has as its spacetime M_d, the boundary at infinity of AdS_{d+1}, can be calculated systematically from the dependence of the supergravity action on the asymptotic behaviour of its fields at the boundary M_d. In particular, one can deduce the scaling dimensions of operators in the conformal field theory from the masses of particles in string theory (or M theory). Using this correspondence, the dimensions of chiral fields in four dimensional $\mathcal{N} = 4$ SYM were matched with the masses of Kaluza-Klein states on $AdS_5 \times S^5$. Note that for chiral primary fields (which are in short representations of the superconformal algebra) the dimension is determined in terms of the R-symmetry representation, and it cannot receive any corrections (see [11, 12] for details). Related works which appeared recently are [13–27].

In this paper we will study the proposed duality for several SCFTs. They are all realized as the low energy theories on the branes of M theory. The first theory is the six dimensional $(0, 2)$ superconformal field theory on the worldvolume of N parallel M5 branes. This theory has a Matrix-like DLCQ description as a quantum mechanics on the moduli space of instantons [28, 29]. Using this description chiral primary operators in the theory were identified with compact cohomologies of the resolved moduli space of instantons, and their spectrum was computed in [30]. We will discuss also the $(0, 2)$ D_k theories which arise for 5-branes at R^5/Z_2 singularities. These theories also have a DLCQ description [28], but their spectrum of operators has not been computed until now. The final theory we study is the three dimensional superconformal field theory on the worldvolume of N parallel and coincident M2 branes. This is the strong coupling (infrared) limit of 3d $\mathcal{N} = 8$ U(N) gauge theories. Little is known about this theory and it does not have a matrix description.

In [1] it was proposed that the SCFT of N M5 branes is dual to M theory on $AdS_7 \times S^4$, while the SCFT on N M2 branes is dual to M theory on $AdS_4 \times S^7$. Our aim is to study these relations between eleven dimensional supergravity and M theory on the AdS spaces and the large N limit of the corresponding superconformal field theories. In the next section we will briefly review the precise correspondence between the supergravity and

SCFTs. In section 3 we will consider the $(0,2)$ theory on N M5 branes, as well as the D_N $(0,2)$ theory. We will obtain the dimensions of the chiral operators of these theories from the masses of Kaluza-Klein modes of supergravity on $AdS_7 \times S^4$, and find agreement with the spectrum predicted using DLCQ. In section 4 we will consider the three dimensional SCFT corresponding to the low energy theory of N M2 branes. Using the masses of the Kaluza-Klein modes of supergravity on $AdS_4 \times S^7$ we compute the spectrum of chiral operators of this theory at large N. Section 5 is devoted to a discussion.

2 SCFT/AdS correspondence

In the following we will briefly review the SCFT/AdS correspondence proposed in [9, 10]. The boundary M_d of AdS_{d+1} is a d-dimensional Minkowski space with points at infinity added. The symmetry group of AdS_{d+1} is $SO(d,2)$. It is also the conformal group on M_d. The proposed duality relates string theory (or M theory) on AdS_{d+1} to the large N limit of some SCFTs on its boundary M_d. In the Euclidean version the boundary is S^d. Consider for simplicity the maximally supersymmetric case, so that the internal space is also a sphere. Let ϕ be a scalar field on AdS_{d+1} and ϕ_0 its restriction to the boundary S^d (defined appropriately for massive fields in [10]). According to the SCFT/AdS correspondence ϕ_0 couples to a conformal field $\mathcal{O}$ on the boundary via $\int_{S^d} \phi_0 \mathcal{O}$. The proposed relation between the generating functional $\langle exp \int_{S^d} \phi_0 \mathcal{O} \rangle_{SCFT}$ of the SCFT on the boundary and the AdS_{d+1} theory is [10]

$$\langle exp \int_{S^d} \phi_0 \mathcal{O} \rangle_{SCFT} = Z_s(\phi_0) \ , \tag{2.1}$$

where $Z_s(\phi_0)$ is the supergravity (string/M theory) partition function computed with boundary condition $\phi \sim \phi_0$ at infinity.

When ϕ has mass m the corresponding operator $\mathcal{O}$ has conformal dimension Δ given by

$$m^2 = \Delta(\Delta - d) \ . \tag{2.2}$$

Irrelevant, marginal and relevant operators of the boundary theory correspond to massive, massless and "tachyonic" modes in the supergravity theory. If a p-form C on AdS is coupled to a $d-p$ form operator $\mathcal{O}$ on the boundary, then the relation between the mass of C and the conformal dimension of $\mathcal{O}$ is given by

$$m^2 = (\Delta + p)(\Delta + p - d) \ . \tag{2.3}$$

The value of m^2 in this formula refers to the eigenvalue of the Laplace operator on the AdS space. In the supergravity literature, the values that are usually quoted for p-forms

are the eigenvalues $\tilde{m}^2$ of the appropriate Maxwell-like operators. The relation of these to the dimension is given by

$$\tilde{m}^2 = (\Delta - p)(\Delta + p - d) \ . \tag{2.4}$$

Some of these chiral fields are universal. There is always a massless graviton in the AdS, which couples to the stress energy tensor of the SCFT (of dimension $\Delta = d$). If the internal space W has continuous rotational symmetry, there are also massless vector fields in its adjoint representation, coupling to the R symmetry currents of the SCFT (of dimension $\Delta = d - 1$).

3 $(0, 2)$ SCFTs in Six Dimensions

Consider M theory on $AdS_7 \times S^4$ with a 4-form flux of N quanta on S^4, and with the "radii" of the AdS_7 and S^4 being $R_{AdS} = 2R_{S^4} = 2l_p(\pi N)^{1/3}$. Eleven dimensional supergravity is applicable at energies much smaller than the Planck scale $1/l_p$. For large N this includes the energy range of the KK modes, whose mass is of the order of $1/R_{AdS}$ (and we will measure it in these units below). The bosonic symmetry of this compactification of eleven dimensional supergravity is $SO(6, 2) \times SO(5)$.

In [1] it was proposed that the $(0, 2)$ conformal theory, which is the decoupled intrinsic theory on N parallel M5 branes*, is dual to M theory on the above background in some appropriate sense. The $SO(6, 2)$ part of the symmetry of the supergravity theory is the conformal group of the SCFT, which can be thought of as living on the boundary of the AdS space. The $SO(5)$ part of the symmetry corresponds to the R symmetry of the superconformal theory.

The Kaluza-Klein excitations of supergravity, in the maximally supersymmetric cases, all fall into small representations of supersymmetry (since they contain no spins larger than 2). Thus, their mass formula is protected from quantum and string/M theory corrections. According to the proposal in [10], they couple to chiral fields of the SCFT on the boundary, whose scaling dimensions are similarly protected from quantum corrections. The spectrum of the Kaluza-Klein harmonics of supergravity on $AdS_7 \times S^4$ was analyzed in [31]. There are three families of scalar excitations. Two families contain states with only positive m^2 and correspond only to irrelevant operators. One family contains also states with negative and zero m^2. They fall into the k-th order symmetric traceless representation of $SO(5)$

*See [11] for a discussion and references concerning these SCFTs.

with unit multiplicity. Their masses are given by [†]

$$m^2 = 4k(k-3), \qquad k = 2, 3, \dots \ . \tag{3.1}$$

The field corresponding to $k = 1$ also appears in the supergravity, and this is the singleton which may be gauged away except at the boundary of the AdS space and decouples from all other operators. In the field theory we can identify it with the decoupled free center of mass motion. This will be true in all the constructions of this type, and in the rest of the paper we will only discuss the interacting fields.

Using (2.2), the dimensions of the corresponding operators in the SCFT are

$$dim(\mathcal{O}) = \{2k, \quad k = 2, 3, \dots\} \ . \tag{3.2}$$

These are precisely the dimensions of the chiral primary operators found in [30], which parameterize the space of flat directions $(R^5)^N/S_N$ in a "gauge" invariant way[‡]. Thus, this can be viewed as a test of the conjecture of [1]. In the matrix description of the $(0,2)$ conformal theory these operators correspond to compact cohomology elements of the resolved moduli space of instantons, localized at the origin [30]. Note that in [30] only those chiral fields whose scalars are in totally symmetric traceless representations of $SO(5)$ were analyzed, but the duality suggests that these are the only chiral fields that have finite dimensions for large N. For large N we find that the field with $k = 2$ is the only relevant scalar deformation of the SCFT. This deformation breaks supersymmetry completely, and it would be interesting to analyze which non-supersymmetric field theory it leads to in the infrared.

There is one family of vector bosons that contains also massless states

$$\tilde{m}^2 = 4(k^2 - 1), \qquad k = 1, 2, \dots \ . \tag{3.3}$$

Using (2.4), the dimensions of the corresponding 1-form operators in the SCFT are

$$dim(\mathcal{O}) = \{2k + 3, \quad k = 1, 2, \dots\} \ . \tag{3.4}$$

The massless vector at $k = 1$ in (3.3) corresponds to the dimension five R symmetry current.

[†]The overall coefficient of the mass formula depends on a parameter e giving the scale of the internal manifold W and used in the relation between the 4-form field strength and the totally antisymmetric tensor. Here it is determined by matching the mass formula and (2.2).

[‡]Note that for finite N the number of these fields is larger than the dimension of the moduli space, so these fields are not all independent on the moduli space.

In general, chiral fields corresponding to all the towers of Kaluza-Klein harmonics are related to the scalar operators of (3.2) by the superconformal algebra, as discussed in the four dimensional case in [32]. Each value of k gives rise (at least for large enough k) to one field in each tower of KK states, with an $SO(5)$ representation that is determined by the representation of the scalar field. In particular, the R symmetry currents and the stress energy tensor sit in the same superconformal representation as the scalar field with $k = 2$ mentioned above. The highest component (in the θ expansion) of these superconformal multiplets gives a series of scalar operators whose dimension starts at 12. The lowest one of these operators couples to the trace of the graviton in spacetime, as discussed in [25][§]. In a superfield notation, the tower of scalars (3.2) corresponds to θ^0 terms in the multiplets, θ^2 terms lead to a vector and a self-dual 3-form, θ^4 terms lead to a graviton, a scalar and a 2-form, θ^6 terms lead again to a vector and a self-dual 3-form, and the θ^8 terms lead again to a scalar. By θ^i terms here we mean fields which can be reached by acting with i SUSY generators on the scalars of (3.2). The terms with an odd number of θ's include spinors and gravitinos. For low values of k, some of the terms with a higher number of θ's are descendants of the terms with a lower number of θ's [32], but for large k all the fields in the multiplet are independent.

A simple generalization of this construction gives the large N limit of the D_N $(0, 2)$ SCFTs, which correspond to the low energy theories of N M5-branes coinciding at an R^5/Z_2 orientifold singularity [33, 34]. In the original theory of the 5-branes, the Z_2 acts by a reflection of the 5 directions transverse to the 5-branes, and also by changing the sign of the 3-form field C of eleven dimensional supergravity. Taking the near horizon limit as in [1], we find that the Z_2 acts by a total inversion of the 5 Cartesian coordinates embedding S^4 in R^5 around the center of the sphere. It has no fixed points so that the resulting manifold is completely smooth (orbifolds in string theory were discussed in a similar context in [18, 24]). In the supergravity solution, all we need to do therefore is to identify the fields on one side of the sphere with the fields at the antipodal points, with also a sign change for the C-field. For large N, when the sphere is large and the antipodal points are far away from each other, we expect to still be able to trust the supergravity solution after this identification. The identification projects out half of the spherical harmonics on the S^4. For scalars, only those with even k in (3.2) remain. As before, the rest of the chiral operator spectrum is determined by the superconformal symmetry. The correlation functions of the remaining operators will be the same as they were, with corresponding operators, in the A_{2N} case (to leading order in $1/N$). Note that these theories also have a DLCQ description [28] as a quantum mechanics on the moduli

[§]Similar operators with dimension $\Delta = 2d$ exist also for $d = 3$ and $d = 4$.

space of D_N instantons, but it is difficult to use it to compute the spectrum since there is no obvious resolution of the singularities in the moduli space (unlike the case discussed in [30]). Note also that the orientifold carries a 5-brane charge of $(-\frac{1}{2})$, but this becomes negligible in the large N limit where we can trust the supergravity solution.

4 Three Dimensional $\mathcal{N} = 8$ SCFTs

Consider now M theory on $AdS_4 \times S^7$ with a 7-form flux of N quanta on S^7. The "radii" of AdS_4 and S^7 are given by $2R_{AdS} = R_{S^7} = l_p(32\pi^2 N)^{1/6}$. Eleven dimensional supergravity is applicable for energies of the order of $1/R_{AdS}$ if N is large. The bosonic symmetries of this compactification are $SO(3,2) \times SO(8)$.

In [1] it was proposed that the conformal theory on N parallel M2 branes on the boundary on AdS_4 is dual to M theory on the above background*. The $SO(3,2)$ part of the symmetry of the supergravity theory is the conformal group of the SCFT on the boundary. The $SO(8)$ part of the symmetry corresponds to the R symmetry of the boundary SCFT. From the point of view of type IIA string theory, this SCFT is the infrared (strong coupling) limit of the 3d $\mathcal{N} = 8$ $U(N)$ gauge theory on N coincident D2-branes.

As before, we study the correspondence between the Kaluza-Klein excitations of supergravity and the chiral fields of the SCFT. The spectrum of the Kaluza-Klein harmonics of eleven dimensional supergravity on $AdS_4 \times S^7$ was analyzed in [35, 36]. There are three families of scalar excitations and two families of pseudoscalar excitations. Three of them contain states with only positive m^2 and correspond to irrelevant operators. One family contains also states with negative and zero m^2 with masses given by †

$$m^2 = \frac{1}{4}\left((k-2)(k-4)-8\right) = \frac{1}{4}k(k-6), \qquad k = 2, 3, ... \tag{4.1}$$

They fall into the k-th order symmetric traceless representation of $SO(8)$ with unit multiplicity.

Using (2.2), the scaling dimensions of the corresponding chiral operators in the SCFT are

$$dim(\mathcal{O}) = \{\frac{k}{2}, \quad k = 2, 3, ...\} \ . \tag{4.2}$$

*See [11] for a discussion and references related to this conformal field theory.

†Note that to match our formulas with the conventions of [35, 36], there is (besides the overall normalization of the mass as before) also a shift in m^2, which is shifted relative to the Laplacian in [35, 36].

As before, we can identify these operators with the natural gauge invariant coordinates on the moduli space of these theories, which is $(R^8)^N/S_N$. Regarding this theory as the IR limit of the 3D $\mathcal{N} = 8$ SYM theory, some of these operators may be identified with operators of the form $\mathrm{tr}(X^{i_1}X^{i_2}...X^{i_k})$, where the X^i are the scalar fields in the vector multiplet. However, in the gauge theory there are only 7 scalar fields, and the additional field arises from dualizing the vector field, which one cannot do explicitly in the non-Abelian case. For $k = 2 \ldots 5$ these are relevant operators in the conformal theory, and for $k = 6$ they are marginal.

As before, chiral operators corresponding to the other towers of Kaluza-Klein harmonics are related to those from this family by the superconformal symmetry. Again, we can identify the operators of (4.2) with θ^0 components of the small superconformal multiplet, and then the θ^2 terms include a pseudoscalar and a vector field, the θ^4 terms include a graviton, a scalar and an axial vector, the θ^6 terms include a pseudoscalar and a vector field, and the θ^8 terms give a scalar. The odd θ terms give spinors and gravitinos.

In this case, unlike the previous case, there is one other family of pseudoscalar excitations which also contains states with negative and zero m^2, corresponding to relevant and marginal operators (respectively) in the SCFT. The masses of this family are given by

$$m^2 = \frac{1}{4}\left((k-1)(k+1) - 8\right), \qquad k = 1, 2, \ldots \,. \tag{4.3}$$

The k'th state transforms in a representation of $SO(8)$ corresponding to the product of a $\mathbf{35}_c$ with $(k-1)$ $\mathbf{8}_v$'s (in a symmetric traceless way). Using (2.2) the dimensions of the corresponding operators in the SCFT are

$$dim(\mathcal{O}) = \{\frac{k+3}{2}, \quad k = 1, 2, ...\} \,. \tag{4.4}$$

For instance, for $k = 1$ we have 35 pseudoscalar relevant operators of dimension 2. In the UV SYM which flows to this SCFT, we can identify these operators with a product of two fermions times $k-1$ scalars, as in [10, 32]. The deformation of the SCFT by any of the relevant scalar operators breaks the supersymmetry completely.

As in the M5 branes case, we identify one family of vector bosons that also contains massless states. The masses of this family are given by

$$\tilde{m}^2 = \frac{1}{4}(k^2 - 1), \qquad k = 1, 2, \ldots \,. \tag{4.5}$$

Using (2.4), the dimensions of the corresponding 1-form operators in the SCFT are

$$dim(\mathcal{O}) = \{\frac{k+3}{2}, \quad k = 1, 2, ...\} \,. \tag{4.6}$$

The massless vector at $k = 1$ in (4.5) corresponds to the dimension two R symmetry current.

As earlier, we can also put the M2 branes at an R^8/Z_2 singularity (which this time is just an orbifold point). Again we find that the resulting 3D $\mathcal{N} = 8$ SCFT is a truncation of the theory discussed above. Only half of its chiral operators remain, the even k elements of (4.2) and the operators related to them by superconformal symmetry.

5 Discussion

In this note we computed the spectrum of chiral operators in the large N limit of several series of supersymmetric conformal field theories with 16 supercharges using the conjecture of [1]. In general, on the supergravity side the only operators of low m^2 are the KK modes, which fall into small representations. Thus, the conjecture of [1] implies that only chiral operators of the SCFT have dimensions that do not grow with N in the large N limit. This is quite surprising from the conformal field theory point of view, as is the similar statement for the large g^2N limit of the 4D $\mathcal{N} = 4$ SYM theories. In the supergravity approximation we cannot reliably study non-chiral operators at all. In principle, in the full M theory such operators could be analyzed. We would expect the m^2 of such states to be of the order of M_p^2, which translates into operators of dimension $N^{1/6}$ in the six dimensional case and $N^{1/3}$ in the three dimensional case. However, it is difficult to access such operators from the M theory side, since there is not yet any known non-perturbative definition for M theory on constant curvature spaces (it is not clear how to generalize the DLCQ formulation of Matrix theory [37] to this case).

In addition to the conjecture about the large N behavior of the superconformal field theories in [1], there was also a conjecture about the agreement of the finite N theories with the appropriate string/M theory backgrounds. However, it is not clear how to check this (very strong) conjecture. One obvious property of the finite N theories is that their spectrum of ("single-particle") chiral primary operators is finite, namely the series of operators described above are truncated at $k = N$. This corresponds to a dimension of order N, which in turn corresponds to an m^2 that is much larger than the Planck scale, so it is not clear how to see this in the string/M theory side. Presumably, in order to check this conjecture in some way, one would need to compute higher order corrections in $1/N$ (say, to 3-point functions). In the case of the 6D theories, such computation are in principle accessible on the SCFT side through their DLCQ construction [30], but it is not clear how to compute such quantum corrections on the M theory side.

Acknowledgements

We would like to thank K. Bardakci and S. Kachru for discussions. O.A. would like to thank J. M. Maldacena and the other participants of the "Duality in String Theory" workshop at the ITP for useful discussions. The work of Y.O. and Z.Y. was supported in part by NSF grant PHY-951497 and DOE grant DE-AC03-76SF00098. The work of O.A. was supported in part by the National Science Foundation under grant no. PHY94-07194, and by the DOE under grant no. #DE-FG02-96ER40559.

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