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Reinventing discovery learning: a field-wide research program

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Abstract Whereas some educational designers believe that students should learn new concepts through explorative problem solving within dedicated environments that constrain key parameters of their search and then support their progressive appropriation of empowering disciplinary forms, others are critical of the ultimate efficacy of this discovery-based pedagogical philosophy, citing an inherent structural challenge of students constructing historically achieved conceptual structures from their ingenuous notions. This special issue presents six educational research projects that, while adhering to principles of discovery-based learning, are motivated by complementary philosophical stances and theoretical constructs. The editorial introduction frames the set of projects as collectively exemplifying the viability and breadth of discovery-based learning, even as these projects: (a) put to work a span of design heuristics, such as productive failure, surfacing implicit know-how, playing epistemic games, problem posing, or participatory simulation activities; (b) vary in their target content and skills, including building electric circuits, solving algebra problems, driving safely in traffic jams, and performing martial-arts maneuvers; and (c) employ different media, such as interactive computer-based modules for constructing models of scientific phenomena or mathematical problem situations, networked classroom collective "video games," and intercorporeal master–student training practices. The authors of these papers consider the potential generativity of their design heuristics across domains and contexts.

Keywords Attitude · Epistemic forms and games · Explorative practice · Problem posing · Productive failure · Situated intermediary learning objectives

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“[If someone says to you] ‘I struggled but still did not discover,’ do not believe him” [Talmud Megila 6b], because the struggle in and of itself is a great discovery, a great find indeed. (Rabbi Menachem Mendel of Kotzk, 1787–1859)

Introduction

Scholars of education have long considered the enduring research problem of designing effective pedagogical practices for STEM teaching and learning. When we sort the wealth of solutions proposed for this general problem, we might discern a range of pedagogical practices subtended by two untenable extreme positions. One hypothetical practice would be to leave students to their own devices, trusting that with sufficient time, nutrition, and encouragement they will re-invent the entire cultural legacy, from Aristotle to Einstein. The equally naïve, authoritarian antipode of this laissez-faire proposition would be to offer our civilization’s cultural legacy for students’ individual passive consumption in the form of hermetic, “teacher-proof” oral presentations, texts, and audio-visual media. Between these contradistinctive polar positions, we find a plethora of proposed frameworks for designing activities by which students will best learn curricular content. Each of these frameworks can be characterized as nurturing from underlying philosophical and theoretical assumptions regarding human epistemology, the ontology of STEM concepts, and how individual cognition develops and thrives in naturalistic and sociocultural ecologies.

Some educational researchers design constrained environments that include a task as well as means of working on the task but no particular method or instructions on how exactly to utilize these means so as to accomplish the task. Students are presented with the environment, given some minimal orientation to its various utilities, and tasked to satisfy some objective, such as transforming the current state or organization of a situation along particular parameters toward a goal state (e.g., assembling a circuit to switch on a light) or so as to determine certain information that is embedded in the situation (e.g., modeling a pirate story so as to locate a hidden treasure). In the course of attempting to engage the provided utilities so as to satisfy the task objective, pragmatic roadblocks or dilemmas emerge for the students that defy their know-how. The hope is that students draw on their existing knowledge, figure out how to use the available tools so as to extend and reorganize their knowledge, and thus through adjustment and practice become skilled in solving a new class of problems pertaining to the goal lesson content. This class of problems becomes named, represented, generalized, situated in the greater curricular structure, and cast further into additional situations. That is, what we call conceptual learning is the expansion of one’s capacity to use the cultural forms of a discipline in perceiving and treating a broader class of situations with greater skill and nuance.

The teacher’s role, per this paradigm, is to facilitate students’ engagement in the problem-solving activities in ways that both optimize for individual content learning and, in so doing, foster their development more broadly of various academic habits of mind, epistemic dispositions, cognitive routines, and social aptitudes, such as reflection, modeling, inference making, help seeking, perseverance (Kapur 2014a, b), collaboration (Scardamalia and Bereiter 2014), metaphorical reasoning (Presmeg 1992), productive argumentation (Asterhan and Schwarz 2009), meta-representational competence (diSessa and Sherin 2000), mapping (Afamasaga-Fuata’i 2009), and meta-cognitive self-regulation in solving open-ended challenging problems (Schoenfeld 1985). The teacher’s targeted intervention may take the form of offering students feedback on their actions, products, and multimodal utterance so that they reach beyond their current capacity by adopting task-specific elements of expert perspectives (Newell and Ranganathan 2010; Newman et al. 1989; Shvarts and Abrahamson 2018; Sfard 2002). For example, a teacher may support students in reasoning through apparent impediments to task completion; guide students to perceive task-relevant aspects of a situation as they pertain to the available tools and vis-à-vis the task goals (Palatnik and Koichu 2015); and steer the students' situated actions and revoice their expressed reasoning so as to elicit their understandings and shape and generalize these into forms aligned with disciplinary practice (Bartolini Bussi and Mariotti 2008; Flood and Abrahamson 2015; O'Connor and Michaels 1996). In all this, the teacher cultivates a classroom epistemic climate respectful and inclusive of diverse subjective levels and forms of linguistic, inscriptional, and embodied participation and contribution to collective mathematical reasoning (Feucht 2010; Gutiérrez 2013).

This general approach to the design of learning is often tagged as "constructivist," because it engineers into practice Piaget's principle of genetic epistemology by which learning is the individual's incremental and iterative construction of adaptive skill (Kamii and DeClark 1985; Piaget 1968); or "guided re-invention," by way of emulating implications of realistic mathematics education, the Dutch didactics (Freudenthal 1968, 1983, 1991; Gravemeijer 1999). More generally, these designs seek to foster an understanding of STEM concepts that goes beyond procedural fluency by way of interrogating historical instruments (Diénès1971; Meira 1998; Skemp 1976), reconsidering and reassembling them as personal construction material (Blikstein 2008; Chase and Abrahamson 2015; diSessa 2000; Papert 1980; Wilensky and Reisman 2006). These ideas, which hark back to Aristotelian empiricism, have more recent roots in the Enlightenment (Froebel 2005; Rousseau 1979), pragmatism (Dewey 1944), modern educational reform (Montessori 1967), radical constructivism (von Glasersfeld 1987), and systemic, emergent, or enactivist perspectives (Abrahamson and Sánchez-García 2016; Barab et al. 1999; Davis and Sumara 2008; Greeno 1998; Simmt and Kieren 2015) already hinted both in Piaget (1970) and Vygotsky (1965).

Nathan (2012) has characterized this broad reform-oriented educational approach as "progressive formalization." By way of contradistinction, he characterized a diametrically opposed approach as "formalisms first." Per that view, students learn STEM content best when they first develop fluency in enacting standard solution procedures: students are shown normative algorithms; they apply these algorithms to a set of problems; and only eventually contextualize and exercise these logico–mathematical procedures in the form of specific concrete "application" situations. This latter approach, which often cites findings from empirical experimentation conducted by educational psychologists and cognitive scientists (e.g., Brown et al. 2009; Koedinger et al. 2008; Schwartz and Bransford 1998; Sloutsky et al. 2005; Uttal et al. 1997), has led to vitriolic critiques of instructional methodology inspired by the constructivist world view (Kirschner et al. 2006; Kirschner and van Merriënboer 2013; Klahr 2010). And whereas these reproaches have been variously rebutted (Goldstone and Sakamoto 2003; Kapur 2016; Nathan 2012), researchers have still to weigh tradeoffs, determine sweet spots, and perhaps rechart the battle field between these pedagogical antipodes (Rosen et al. 2016, in press).

Likely, the field of STEM educational research will keep attempting to adjudicate on these butting pedagogical frameworks, with each camp intermittently lashing out empirical salvos. Then again, one might step back to sketch a bigger picture that harvests, subsumes, dissolves, and reconfigures these opposing vectors by means of exemplifying prospects of their conciliatory implementation, drawing on the best of each. In that vein, Abrahamson has proposed a sociocultural view of student discovery in mathematics learning as a theoretical foundation for his heuristic framework, embodied design. What the student discovers through engaging in embodied-design activities is not a formal solution procedure per se. Rather, the student first devises and articulates a mathematically correct, yet qualitative and informal solution that draws on naive perceptual judgment or sensorimotor coordination. Later, when the teacher introduces disciplinary frames of reference into the learning environment at an appropriate timing, such as measurement instruments or representation formats for problem analysis, the student discovers how to utilize these artifacts so as to corroborate and enhance their naïve solution in accord with their interpretation of the new discursive task. In particular, the student figures out how to perceive a mathematical model of a situation as bearing the same meaning as their own naïve inference for that situation, or how to engage the artifacts so as to serve the same function as their own strategy for operating the situation (Abrahamson 2009a, b, 2012a, b, 2014, 2015; Abrahamson and Trninic 2015).

Whereas research evaluating the merits of embodied design often requires dedicated environments for sensorimotor interaction (Abrahamson and Lindgren 2014), other empirical investigations of discovery-based learning focus more specifically on evaluating for benefits of enabling students to struggle in a problem space before teaching them new solution algorithm. Coining the term "productive failure," Kapur (2008) has demonstrated the pedagogical advantage of instructional sequences wherein students are frustrated by the incapacity of their conceptual reach before learning more powerful techniques. Through several quasi-experimental and controlled experimental studies, Kapur has demonstrated how engaging students in solving problems that require concepts they have not learnt yet can be productive, provided students are able to generate multiple representations and solutions even if these solutions are incorrect or sub-optimal. In other words, their initial problem-solving failure activates relevant prior knowledge, helps students notice critical features, and prepares them to learn from subsequent instruction (Kapur, 2010, 2011, 2012, 2014a, b, 2013; Kapur and Bielaczyc 2012).

We are now at a curious point in the history of end-user educational technology, where pre-K students are consulting Siri on all matters existential as pragmatic before asking their siblings or parents. As such, children are looking to technology to personify omniscient expertise. But teachers, whether organic or technologically embedded, are more than knowledge banks. If we choose discovery learning as a desirable educational agenda, what might it take to embed effective facilitation flowcharts into silicon (Abdullah et al. 2017)? This special issue offers some directions of thought.

As artificial intelligence populates our electronic devices, these educational platforms will only be as effective as are the pedagogical principles guiding their software engineering. We appear to be at an unprecedented Archimedean point, where we might leverage technology to wield a digital educational revolution. Learning scientists should acknowledge their purview and mandate to inform the design of these interactive media at the fingertips of a billion eager minds.

Overview of contributions to the special issue

Chase and Abrahamson (2018) investigate students' conceptual development in a mathematical domain by analyzing their manual operations and multimodal utterance as they construct representations to solve contextualized problems. The authors demonstrate the implicit knowledge students bring to bear in approaching the problems as well as the emergence of new heuristics for managing complex modeling tasks. The authors have previously characterized these heuristics as “situated intermediary learning objectives” (SILOs) with potential to generalize beyond the activity context (Chase and Abrahamson 2015). In the current paper, the authors argue that, in fact, these SILOs are not siloed. Rather, the set of situated construction heuristics coalesces into a systemic problem-solving schema by constraining each other’s implementation. The paper presents case studies to make evident how, in the course of repairing their models to accord with a problem’s source information, one SILO may cast an implementation constraint on another SILO. Discovery-based learning, per Chase and Abrahamson, is the incremental and iterative assembly of an interconnected set of construction heuristics. A concept is tight systemic know-how for approaching problems pertaining to a domain. This work, which draws on the notion of subjective transparency (Meira 1998, 2002), bears implications for the design and facilitation of technologically enabled interactive learning environments.

Looking critically at discovery-based learning environments, Wilkerson et al. (2018) take on the pedagogical problem of these environments potentially bearing differential effect across student cohorts. In particular, the authors analyze implicit challenges in some 5th-grade students’ attempts to participate productively in modeling-based science inquiry activities that the authors developed and implemented. The authors draw on the epistemic forms/epistemic games framework (Collins and Ferguson 1993) to explore alignments between, on the one hand, how students made sense of the visual displays they were building and how, on the other hand, the designers had hoped they would approach the situation. Although those students strategized the technological production of an animated explanation, per the assigned task, some focused on building a coherent sequence of *what* each scene portrays (the rhetorical form of the available medium) came at the expense of attending to *how* each scene evolved into the next one, namely dynamics in the phenomenon under inquiry (the pedagogical objective of the lesson). This learning challenge was alleviated when the activity facilitators discerned on-the-fly the product/process distinction and were able to re-direct the students’ attention. The article thus orients us to important dimensions of student cognition and teacher practice in modeling-based science activities.

Complementing literature on the widely researched benefits of problem solving, Kapur (2018) turns attention to the often-neglected benefits of engaging students in problem-*posing* (Getzels 1979). The thesis is that problem posing may afford greater opportunities to discover and more flexibly assemble critical features of the underlying structure than problem *solving*, which in turn may better prepare students to transfer what they have learned in subsequent instruction. Kapur randomly assigned students to one of two conditions: (a) problem-posing with solution generation, where they generated problems and solutions to a novel situation; or (b) problem-posing without solution generation, where they generated only problems. Findings reveal that problem-posing with solution-generation prior to instruction resulted in significantly better conceptual knowledge, without any significant difference in procedural knowledge and transfer. These findings are intriguing because, on the one hand, solution generation is critical for the development of conceptual knowledge, and consequently, transfer. On the other hand, generating problems was even more critical to transfer even if it somewhat compromised the development of conceptual knowledge. We are used to thinking of a simple linear relationship between conceptual knowledge and transfer: What helps conceptual gain should also aid transfer. These findings illustrate a somewhat more complex dynamic in discovery learning environments between conceptual knowledge growth and its transferability.

Roll et al. (2018) demonstrate the complex interaction between guidance in discovery environments and student attributes, both in the short and long terms, and investigate whether the benefits of guidance persist after it is eventually removed. Roll et al. assigned students to either a Non-Directive or a Directive condition as they engaged in an interactive physics simulation environment. In the Non-Directive condition, participants received a set of goals to focus their inquiry on, in addition to implicit support built into the simulation. In the Directive condition, students additionally received detailed directions and task breakdown for their inquiry. Findings showed that gains in knowledge were not always commensurate with gains in attitudinal growth. In the short-term, although directive support improved knowledge gains for the Higher Knowledge group, it suppressed their attitudinal growth. In contrast, directive support did not result in knowledge gains for the Higher PoCC (perceptions of competence and control) group, but it helped with their attitudinal growth. In the long-term, when directive support was removed, only the effect on attitudes persisted. While ideally designers of discovery learning would want both knowledge and attitudinal growth, this work shows how supporting the former may in fact adversely affect the latter.

Levy et al. (2018) propose and evaluate a heuristic design framework for building activities that foster youth development of complexity perspectives on natural and social phenomena through engaging in the enactment of goal-oriented participatory simulation. The framework, which balances affordances for exploration with technologically embedded constraints on the scope and parameters of accessible information, was instantiated for this study in the form of a multi-player simulation of driving in dense traffic. Therein the teacher assigned role was to manage the activity and reflective discourse but not directly delineate the targeted insights (i.e., the learning objectives). Empirical data from a pilot implementation with high-school students suggest the potential effectiveness of this general approach: Participants' actions and utterances evidence their understanding of reciprocal relations between the dynamics of multiple particulate agents and emergent aggregate phenomena that, in turn, impact the agents' physical circumstances. In and of itself, the activity could serve an important social function of educating drivers to appreciate that civil maneuvering on the road may actually be in their best personal interests of both safety and efficiency.

Finally, Trninic (2018) queries the field's implicit characterization of discovery-based learning as inherently differentiated from repetitive practice exercises. His thesis is situated in a discussion of traditional pedagogical practices in disciplines focused on the development of motor-action competence, namely the martial arts. Therein, experts directly instruct novices to enact specific forms. Yet in the course of practicing these forms, the students become aware of implicit principles that organize and empower the enactment. Trninic presents excerpts from Tai Chi masters' testimonies that appear to integrate elements from both constructivist and instructivist pedagogical approaches. The term *explorative practice* is proposed to capture the essence and function of pedagogical exercises that combine a high degree of guidance with a high expectation of discovery.

Taken as a whole, this collection of papers presents a balanced view on enduring themes and tensions in the field's ongoing quest to advise the design and facilitation of environments that offer students opportunities to develop understanding and competence as well as to learn how to learn.

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