Quantum Dynamics Via Planck-Scale-Stepped Action-Carrying "Graph Paths"

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Abstract

A divergence-free, parameter-free, path-based discrete-time quantum dynamics is designed to not only enlarge the achievements of general relativity and the standard particle model, by *approximations* at spacetime scales far above Planck scale while far below Hubble scale, but to allow tackling of hitherto inaccessible questions. "Path space" is *larger than* and *precursor to* Hilbert-space basis. The wave-function-propagating paths are action-carrying *structured graphs*—cubic and quartic structured vertices connected by structured "fermionic" or "bosonic" "particle" *and* "nonparticle" arcs. A Planck-scale path step determines the gravitational constant while controlling *all* graph structure. The basis of the theory's (zero-rest-mass) elementary-particle Hilbert space (which includes neither gravitons nor scalar bosons) resides in *particle* arcs. *Nonparticle* arcs within a path are responsible for energy and rest mass.

• This work is supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U. S. Department of Energy under Contract DE-AC03-76SF00098.

Although *fundamental* status for Planck scale has been widely conjectured throughout the past century by both physicists and cosmologists, accommodation by *continuous* quantum dynamics has proved elusive. We here summarize a proposed dynamics based on Planck-scale-*stepped* (classical) action-carrying spacetime *paths* that in important respects resemble *Feynman graphs*. These "graph-paths", located within a discretized "path spacetime", define (by "connecting to") an elementary-particle Hilbert space attached to spacelike surfaces that constitute a tiny portion of path spacetime. (Spacing between successive wave-function surfaces is large on atomic scale although tiny on Hubble scale.) The wave-function propagator between successive wave-function surfaces (the "global S matrix") adapts the Dirac-Feynman rule (1) to the actions of path *portions*—open graphs that connect these surfaces. Path action depends *only* on the 3 (dimensionful) parameters $\hbar$, c and path step. The proposed approach promises not only to enlarge the achievements of general relativity and the standard particle model, through approximations at spacetime scales much larger than Planck scale while much smaller than Hubble scale, but to illuminate hitherto-inaccessible puzzles such as "measurement".

"Path spacetime", the product of a discrete 1-dimensional "age space" and a continuous 3-dimensional "boost space", is a portion of the interior of a forward lightcone-- the set of points whose Minkowski distance from lightcone vertex is a positive-integral multiple of a Planck-unit δ ($\delta \sim 10^{-43}$ sec). In other words, path spacetime comprises the set of (3-dimensional) hyperboloids whose "age" (since "big bang") is $\mathbf{N}\delta$, with $\mathbf{N} = 1, 2 \ldots$ ("Time arrow" is built into path spacetime.) "Wave function spacetime" will below be defined as a *subset* of these hyperboloids. "Homogeneity of 3-space" amounts to equivalence of point collections related by Lorentz transformations—an idea of Milne. (2)

The "local frame" of a spacetime point is the frame in which that point's positive-timelike 4-vector displacement from lightcone vertex is *purely* timelike. In local frame an *infinitesimal* spatial displacement $d\boldsymbol{x}$ on Hyperboloid $\mathbf{N}$ relates to an infinitesimal boost-space displacement $d\boldsymbol{\beta}$ by $d\boldsymbol{x} = \mathbf{N}\delta c\, d\boldsymbol{\beta}$. Boost space has curvature of order unity but deviation from flatness is small for spatial displacements small compared to $\mathbf{N}\delta c$ (small on Hubble scale). Although all statements in this paper recognize curvature of 3-space—i.e., finiteness of $\mathbf{N}$--physics ("here and now") usually deals with almost-flat spatial regions in the logarithmic age neighborhood of $\sim 10^{60}$ Planck units.

Any path is a set of *directed* and *labeled* loops of successive c-velocity steps in path spacetime, each step spanning an age interval $\pm\delta$ (forward or backward). The label is 6-valued—a product of 2- and 3-valued labels, a choice that doubles our covering of the Lorentz group while representing electric charge and energy. Outcomes supporting this label choice are chiral isospin, a 30° elementary Weinberg angle, 3 colors and 3 generations of fermions. (3) Closing of loops means 6 locally-conserved quantum numbers. However, only 2 (one of these being electric charge) are superselectedly carried exclusively by elementary particles.

Path-loop constraints tailored to the 3-dimensionality of space require any age-monotonic loop segment (of 2 or more steps) to be accompanied over the entire monotonic-segment extent by *exactly 3* other age-monotonic loop segments that are "almost parallel" in the sense that the *center* of such a path segment *quartet* is spatially straight (in boost space). Two quartet members move forward in age while the remaining two members move backward. Transverse spatial displacements between quartet members, specified by the path step and **N**, come in 2 patterns, respectively called "fermionic" and "bosonic", that are described in Reference (3). We shall refer to such a quartet of loop segments as a "structured arc" (or, more often, simply as "arc"). Any arc's spatially straight central axis has a local-frame velocity magnitude dependent only on **N**-- smaller than c by a fraction of order $\mathbf{N}^{-2}$.

"Structure of an arc" refers not only to the transverse spatial pattern of 4 path segments but to the quartet of 6-valued labels attached thereto. References (3) and (4) place on *label* patterns constraints (certain of which are relevant to what follows) required by consistency with Hilbert space and representation of energy, rest mass, chirality and photon coupling to electric charge. The total number of allowed arc label patterns thereby drops below 6^4 by roughly one order of magnitude.

"Structure of a vertex" refers to the spacetime pattern of connections between 3 or 4 path-segment quartets which there meet. At a cubic vertex 2 arcs are fermionic while 1 is bosonic. At a quartic vertex either all 4 arcs are bosonic or all 4 are fermionic. Reference (4) exhaustively catalogues vertex structures.

Despite vertex structure, vertex spacetime location enjoys precise meaning as the intersection between spatially straight central axes of those arcs joined by the vertex. Longitudinal arc orientations at a vertex constrain (structured-arc) transverse orientations. [(4)] (Vertex location functionally resembles the spacetime label shared by those local fields whose *product* constitutes one term of a field-theoretic polynomial Lagrangian density; arc transverse orientation functionally resembles a field's spin label.)

The action of a path is the *sum* of actions *separately* attached to arcs and vertices. Vertex actions are 0, ±1 in units $\pi\hbar/2$ chosen (see below) to conform to Planck's relation between energy and classical angular velocity ("frequency"). An arc carries Poincaré-invariant electromagnetic and gravitational "action at a distance" that is below prescribed through arc orientation and length, relative spacetime separations from other arcs and arc electric charge and energy. (Gravitational constant is implicit in the energy unit.) Path-loop labels define arc electric charge, while below-described "trivial" vertices perpetuate spatial straightness in "trivially-extended" arcs and discretely attach energy thereto.

"Electromagnetic vertices", defined by termination thereon of bosonic arcs called "photonic" that are identified by a *combination* of labeling *and* termination structure, [(4)] are unique in carrying *no* action. (Although itself actionless, an electromagnetic vertex "inserted" into a charged arc affects electromagnetic action in replacing the *single* charged arc by a *pair* of charged arcs that are *differently* oriented.)

Graph-path action at a distance is lifted from Wheeler and Feynman's ("W-F") fieldless representation of classical electromagnetism [(5)]. Action of an arc-step is a sum of Poincaré-invariant contributions from past and future "source" arc-steps. Denoting the spacetime locations of beginning and end of arc-step-s central axis by the 4-vector symbols $\boldsymbol{s}_{\mathrm{b}}$ and $\boldsymbol{s}_{\mathrm{e}}$, the s displacement in spacetime,

$$\boldsymbol{l}_{\mathrm{s}} \equiv \boldsymbol{s}_{\mathrm{e}} - \boldsymbol{s}_{\mathrm{b}}, \tag{1}$$

is a positive-timelike 4-vector. The backward lightcone with vertex at $\boldsymbol{s}_{\mathrm{e}}$ "intersects" another arc step within the same path if the *end* of the latter step is outside this lightcone and the *beginning* inside. Employing the symbol s^{r} to designate such a "retarded-source" arc step and the 4-vector symbol $\boldsymbol{s}^{\mathrm{r}}{}_{\mathrm{b}}$ to designate the spacetime-location of s^{r} *beginning*, we define the positive-timelike 4-vector,

$$\boldsymbol{x}_{\mathrm{s}}{}^{r} \equiv \boldsymbol{s}_{\mathrm{e}} - \boldsymbol{s}^{r}{}_{\mathrm{b}}. \tag{2}$$

Denoting arc-step electric charges by the symbols Q_s and Q_s^r, the W-F increment of action at a distance carried by arc-step s due to the retarded source-step s^r is

$$-\tfrac{1}{2}\, Q_s Q_s^r\, l_s \bullet l_s^r \,/\, \boldsymbol{x}_s^r \bullet l_s^r . \qquad (3)$$

In graph-path dynamics the symbol Q_s may take values 0, ±⅓, ±⅔, ±1 in units of order $\hbar^{1/2}$ which a separate paper will specify by "grand unification" of electromagnetic arc action with weak and strong vertex action.

The formula for an increment of "advanced" action is similar to (3) with the retarded superscript r replaced by an advanced superscript a. All 4-vectors in the advanced-action formula continue to be positive timelike, with

$$\boldsymbol{x}_s^a \equiv \boldsymbol{s}^a{}_{\mathrm{e}} - \boldsymbol{s}_{\mathrm{b}} . \qquad (4)$$

The W-F device works also for gravity. Gravitational action is given by a formula analogous to (3), where the product of electric charges is replaced by a (positive) Lorentz-invariant inner-product of below-defined energy-momentum 4-vectors that each belongs to a spatially-straight "trivially-extended arc"—a succession of "continuing" arcs separated by "trivial" vertices that is below called a "*t* " element of the graph. The aggregate of all *t* elements encompasses all arc steps without multiple counting.

Wave-function spacetime comprises the *subset* of path-spacetime hyperboloids of age $\mathbf{N}_{\mathrm{w}}$ (Mδ), where $\mathbf{N}_{\mathrm{w}}$ = 1,2, … and M is a huge integer enjoying a stepped-dynamics "fundamental" status like the scale interval spanned by inflation in standard cosmology. Eventually *number theory* needs to indicate a unique value for the integral ratio M between wave-function step and path step. (An example of how number theory can identify a special huge prime is provided by the Mersenne prime sequence, 2^2-1=3, 2^3-1=7, 2^7-1=127, 2^{127}-1.)

S-matrix interpretation for the wave-function step requires Mδ to be large on atomic scale, although tiny on Hubble scale, and characterizable as "setting the scale of measurement" (presumably, thereby, *below* the scale of "observer consciousness"). A value for M in the logarithmic neighborhood of 10^{38} (Mδ ~ 10^{-5} sec) appears satisfactory. (For a continuous-time *field* approximation where δ would serve as "ultraviolet-divergence cutoff", Mδ would provide "infrared cutoff".) Because the ($\mathbf{N}_{\mathrm{w}}$ = 1) *initial* wave function of the universe has age Mδ, a relation between M and the scale interval of cosmological inflation is plausible.

The "global S Matrix" connecting 2 successive wave functions (belonging to successive values of $\mathbf{N}_{\mathrm{w}}$) is built by adapting the Dirac-Feynman rule to the actions of *open-graph* portions of closed-graph paths—open graphs located within the (simply-connected) spacetime region between 2 successive wave-function hyperboloids. The "ends" of these open graphs are the *truncated arcs* cut by wave-function hyperboloids. Path constraints *forbid* graph vertices from locating *on* wave-function surfaces.

A zero-rest-mass elementary-particle (S-matrix) Hilbert space is *based* on the intersections between path-graph arcs and wave-function hyperboloids. Bosonic and fermionic *quartets* of labeled points in (3-dimensional) boost space, each (Planck-scale) structured quartet with a center and with a spatial orientation, provide coordinate basis for a Fock space: "Pauli symmetrized" functions of SL(2,C) group parameters represent zero-rest-mass vector bosons and spin-½ fermions with standard-model quantum numbers. (3) A conjugate basis labeled by group invariants (Casimirs) related to particle momentum and angular momentum in local frame, is provided by Fourier-like analysis of wave-function dependence on Lorentz parameters. (6) Bosonic "particle" arcs enjoy structures allowing "contact" with elementary vector bosons; the structures of fermionic particle arcs permit contact with elementary fermions.

Certain arcs, identifiable by label structure, are *forbidden* to cross wave-function hyperboloids. Those bosonic arcs in the "nonparticle" category are called "inertial" because, as explained below, they enable (through "trivial" vertices) the *momentum* content of graph paths. (Path momentum and wave-function momentum become approximately related by *stationary* action.) Nonparticle fermionic arcs are called "zbw" because of enabling (through quartic fermionic vertices, as discussed below) the zitterbewegung that underpins fermion rest mass.

Any arc (particle or nonparticle, bosonic or fermionic) may connect "continuously" at a "trivial" vertex to *another* identically-structured arc of the *same* longitudinal orientation. The members of such a "trivially-continuing" arc pair (both incoming, both outgoing or one incoming and the other outgoing) *differ* in transverse orientation—by 90° if bosonic and by 180° if fermionic. "Handedness" (left or right) of bosonic-arc (discrete) rotation at a trivial vertex is determined by arc structure. The action attached to *any* trivial vertex (bosonic or fermionic) is $-\pi\hbar/2$.

A nontrivial-vertex-bounded spatially-straight succession t of ($n_t + 1$) trivially-continuing arcs connected by n_t trivial vertices we call a "trivially-extended" arc. The age *difference* between t end points (ending age minus beginning) in δ units is another nonnegative integer N_t, while the positive-timelike 4-vector symbol $\boldsymbol{x}_t$ denotes the spacetime displacement between t central-axis endpoints. The 4-vector,

$$\boldsymbol{p}_t \equiv [n_t\pi\hbar/2(N_t\delta)^2]\, \boldsymbol{x}_t, \tag{5}$$

is the "energy-momentum" of the trivially-extended arc. In the local frame of t midpoint, the energy E_t is (almost) the rational number n_t/N_t times the Planck energy unit $\pi\hbar/2\delta$. For bosonic t, the frequency associated to E_t is the mean angular velocity of rotating transverse-orientation. Gravitational action, attached both to individual particle *and* to individual *nonparticle* trivially-extended arcs, is determined by the energy-momentum of such arcs, as defined by (5).

A trivial vertex is cubic if the continuing arc pair is fermionic and quartic if the continuing pair is bosonic. (A *quadratic* kinetic energy term within a field-theoretic Lagrangian density fulfills *some* of the functions of a trivial vertex.) At a cubic trivial vertex the single *other* arc is inertial while at a quartic trivial vertex *both* other arcs are inertial, each terminating at (*not* "continuing through") the trivial vertex. The unique set of labels identifying an inertial arc not only endows this arc with zero values for all 6 conserved quantum numbers but distinguishes it from all *particle* bosonic arcs. [3,4]

Although a path graph displays many features of a Feynman graph, the latter lacks inertial arcs that terminate in trivial vertices. (The energy-momentum attached to a Feynman-graph arc is an *extra* label that does not derive significance from the graph itself.) Nonetheless the wave-function-propagating continuous *paths* (*not* Feynman graphs) of standard (Hamiltonian) quantum dynamics [7] share the discretized graph-path feature of traversing a (coordinate-momentum) *phase space*. Our *essential* innovation is a path space with *nonparticle* "dimensions"--inertial arcs and zbw arcs—that are unrepresented in Hilbert space.

A standard-theory analog to inertial arcs is found in the *graviton* arcs of a Feynman graph. Although our Hilbert space contains no elementary gravitons it does include photons and inertial arcs share certain structural features with photonic arcs. [4] In a Feynman graph, photon arcs and graviton arcs enjoy parallel status.

How does an inertial arc resemble a graviton? (a) Inertial arcs that are trivially extended by trivial vertices (connecting 4 inertial arcs) themselves carry energy-momentum and associated gravitational action. (b) *Stationary* action, together with *nontrivial* vertices that alter longitudinal arc direction, is expected at large scales to select out "gravitationally-curved" paths. The notion that inertial arcs "generate" energy by "terminating on other arcs" implies a graviton-suggestive converse "path-bremstrahlung" notion: The larger the energy of a (stationary-action) "accelerating arc", the larger is such an arc's "emission rate" of inertial arcs. ("Emitted" inertial arcs, even though never manifested through particles, are not "locally reabsorbed" within the graph-path; "extendable" by intersections with other inertial arcs, they traverse "macroscopic" spatial intervals.)

Label-defined nonparticle zbw fermionic arcs (carrying zero values for the 2 superselected conserved quantum numbers carried by elementary particles), play a role at quartic fermionic vertices that parallels the participation of inertial arcs at quartic bosonic vertices. (*All* quartic *fermionic* vertices involve zbw arcs.) The analog of a trivial quartic vertex is a "zitterbewegung vertex" in which 2 zbw arcs terminate while two "continuing" fermionic arcs relate to each other by *opposite* longitudinal directions and "chirally-reversed" labels. Chirality reversal, defined in Reference (3), corresponds to **L**↔**R** interchange in the standard model (where the triplet of W bosons couples to **L** fermions). A zbw vertex resembles a standard-model fermion-Higgs Feynman-graph vertex—a zbw arc *pair* playing the role of a Higgs scalar. (Our Hilbert space does *not* include elementary scalar bosons.) There are however 9 different label structures for zbw arcs, even though zbw vertex actions are always $\pm\pi\hbar/2$.

Chirality reversal accompanies helicity reversal for an elementary fermion. Dirac's fermion wave function with nonzero rest mass is a direct sum of opposite-helicity, opposite-velocity zero-rest-mass (Majorana) wave functions—a composition often described as "zitterbewegung". Presuming the action of zbw vertices to generate fermion rest mass, graph-path dynamics implies calculability of the observed quark-lepton mass matrix (in Planck-energy units) without need for arbitrary parameters.

All vertices, apart from electromagnetic, zbw and quark-W vertices, enjoy a "Chan-Paton structure" yielding SU(N) symmetries for strong (N=3), weak (N=2) and GUT (N=5) action. [4] Rules for the *signs* of vertex actions imitate the (Yang-Mills) gauge symmetry of local field theory. Although *all* graph-path vertices share certain important structural features, an exact Planck-scale *supersymmetry* would clash with the *difference* in spacetime structure of fermionic and bosonic arcs.

All vertices exhibit CPT symmetry but a remarkable aspect of W-quark vertex structure is a *bosonic asymmetry* under inversion of path direction. In all other vertices bosonic arcs maintain equivalent postures under this inversion. Because path-direction inversion corresponds to a CP transformation, a special role may be anticipated for W-quark interaction in the breaking of CP symmetry. The *initial* (global) wave function is our theory's repository for "spontaneous symmetry breaking"—an issue to which no serious thought has yet been given.

This enterprise was engendered by extended 1983-84 discussions with V. Poénaru. Almost-daily conversations with Henry Stapp over the subsequent two decades contributed enormously to details of our proposal. Criticism from Jerry Finkelstein has been extremely valuable. The author would also like to acknowledge the impact on his thinking from the series of renormalization-group-based essays in *Physics Today* by Frank Wilczek. These essays suggested the postulate of universal magnitude for vertex (trivial or nontrivial) action. The *only* parameters of our theory are $\hbar$, c and δ. Contact between experimental data and the "fundamental" Planck-scale dynamical laws that we are proposing will require computations borrowing heavily from renormalization-group methods.

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