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A THEORETICAL APPROACH TO THE PROBLEM OF PARTIAL EQUILIBRATION IN HEAVY ION REACTIONS*

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Abstract: The cross sections and the angular distributions of fragments emitted in heavy ion reactions are calculated on the basis of a diffusion mechanism associated with the mass (charge) asymmetry of a short-lived intermediate complex. The calculated quantities are compared with experimental results.

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Many heavy ion reactions recently studied in our group [1-5] seem to indicate various stages of relaxation in the collective degrees of freedom excited in the process. In particular, a large fraction of the total cross section is associated with the production of fragments which, i) have fully relaxed kinetic energy spectra; ii) portray a mass (charge) distribution not wholly consistent with statistical equilibrium; iii) have an angular distribution which is forward peaked and whose forward peaking depends upon the difference in atomic number between the fragment and the projectile.

These general features appear to be consistent with the following qualitative picture: i) promptly after the initial collision, friction brings the two nuclei into rigid contact with each other, while the initial kinetic energy is dissipated into the internal degrees of freedom; a rotating intermediate complex of well defined mass asymmetry is formed; ii) a diffusion process, comparable in rate to, or slower than, the rotation of the intermediate complex, leads to the exchange of particles between the two touching fragments, thus generating a time-dependent distribution in the asymmetry of the intermediate complex; iii) the complex decays with a time constant comparable to or shorter than the rotational period.

We are presenting here an attempt to translate such a picture to a more quantitative basis by sketching the model in terms of the dominant experimental features without accounting for details which are still very uncertain. The central feature of the model is the diffusion of the intermediate complex along the asymmetry degree of freedom. We use the Master Equation approach to describe the time-dependent population $\phi_Z(t)$ of systems whose asymmetry is characterized by the atomic number Z of one of the fragments:

$$\dot{\phi}_Z = \sum_{Z'} (\Lambda_{ZZ'} \phi_{Z'} - \Lambda_{Z'Z} \phi_Z) \qquad (1)$$

where $\dot{\phi}_Z$ represents the time derivative of ϕ_Z and $\Lambda_{ZZ'}$, $\Lambda_{Z'Z}$ are the macroscopic transition probabilities between the systems whose asymmetries are described by Z and Z'.

The macroscopic transition probabilities can be written in terms of the microscopic transition probabilities and of the level densities of the macroscopic states:

$$\Lambda_{zz'} = \lambda_{zz'}\, \rho_z \, ; \; \Lambda_{z'z} = \lambda_{z'z}\, \rho_{z'} \, ; \quad \lambda_{zz'} = \lambda_{z'z} \, . \tag{2}$$

The symmetry of the λ's stems from microscopic reversibility. The quantities ρ_z represent the level densities of the intermediate complexes. These level densities can be written down in terms of the potential energy of the intermediate complex V_z measured with respect to the rotating ground state,

$$\rho_z = \rho(E - V_z) \tag{3}$$

where E is the energy of the system also measured with respect to the rotating ground state and $\rho(x)$ is the functional form for the level density.

Since $\Lambda_{zz'}$ must be of the order of $\frac{V_F}{D}$ (where D is a typical linear size of the system and V_F is the Fermi velocity of the nucleons), the $\lambda_{zz'}$ decrease as the level densities increase. For lack of better knowledge, we asrume:

$$\lambda_{zz'} = \frac{\lambda_0}{[\rho_z \rho_{z'}]^{1/2}} \, . \tag{4}$$

The level densities can be expanded as follows:

$$\rho(E - V_z) = \rho(E)\, e^{-V_z/T} \tag{5}$$

where

$$T^{-1} = \left. \frac{d \ln \rho(x)}{dx} \right|_{x = E - V_z} \tag{6}$$

and the master equation can be rewritten as:

$$\dot{\phi}_z = \lambda_0 \sum_{z'} e^{(V_z + V_{z'})/2T} \left[\phi_{z'} e^{-V_z/T} - \phi_z e^{-V_{z'}/T} \right] . \tag{7}$$

This equation is certainly correct in the long time limit when $\dot{\phi}_z = 0$. A sufficient condition which satisfies $\dot{\phi}_z = 0$ is that each term of the sum is zero. This gives:

$$\frac{\phi_{Z'}}{\phi_Z} = \frac{\rho_{Z'}}{\rho_Z} = e^{-V_{Z'}/T} \Big/ e^{-V_Z/T} \tag{8}$$

which expresses the equilibrium condition.

An important assumption in the present model is that the sum in (1) and (7) is extended only to nearest Z's. In other words, we assume that there is no correlation among transferred particles. A key quantity in the model, and a very uncertain one, is the potential energy of the intermediate complex V_Z. While one can imagine that the gross shape of the complex should be that of two touching fragments, its detailed shape is dependent in a complicated way upon the diffusion process itself. We assume that, at least for the rather light systems which we have considered, the shape of the intermediate complex actually corresponds to a configuration of two touching spheres.

The angular momentum can in principle affect the shape of the intermediate complex and, consequently, the value of the potential energy. We write the potential energy as follows:

$$V(x_i) = V_{LD}(x_i) - V_{LD}(x_i^0) + \frac{\ell^2\hbar^2}{2}\left[\frac{1}{\mathcal{J}(x_i)} - \frac{1}{\mathcal{J}(x_i^0)}\right] \tag{9}$$

where the x_i is the set of coordinates chosen to describe the shape of the system; x_i^0 are the corresponding values for the ground state; V_{LD} is the liquid drop energy and $\mathcal{J}$ is the moment of inertia.

The diffusion constant λ_0 should also depend on the shape of the intermediate complex. We have considered it to be proportional to the interaction area of the two touching spheres, which, in turn, is proportional to their reduced radius:

$$\lambda_0 = 2\pi\kappa \frac{R_1 R_2}{R_1 + R_2} \tag{10}$$

where R_1 and R_2 are the radii of the two touching spheres.

The local temperature T is calculated by means of the expression $T = \sqrt{\frac{E-V_z}{a}}$ where $a = A/8\ \mathrm{MeV}^{-1}$ and A is the mass number of the intermediate complex. An example of the potential energies of the system Ag + Ar for various values of angular momentum is shown in fig. 1(a).

A calculation of the populations ϕ_z at various times after the beginning of diffusion is shown in fig. 1(b). The calculation of the cross sections and angular distributions of the fragments requires that at least some essential part of the dynamics associated with the reaction be taken into account. We assume that, after collision, target and projectile slip one over the other by an angle θ_s proportional to the initial tangential velocity at the time of collision. As the slippage takes place, the kinetic energy is dissipated. After the slippage, the system rotates with a rigid moment of inertia. Furthermore, we assume that, after a given contact time t, the fragment is emitted at an angle θ with a new $Z = Z_{exit}$. This information allows one to calculate the set of impact parameters b which satisfy the above conditions. The final cross section is given by:

$$\frac{d\sigma_z}{d\Omega} = \int_0^\infty \frac{dt}{\tau}\ e^{-t/\tau} \left\{ \sum_b \phi_z(b,t) \frac{b\ P(b)}{\left| \sin\theta\ \frac{d\theta}{db} \right|} \right\} \tag{11}$$

where τ is the mean life of the intermediate complex and the sum contained in the integrand is carried over all the values of b which, after a time t, lead to the emission of a fragment of atomic number Z at a center-of-mass angle θ.

The quantity P(b) or $P(\ell)$ at a given bombarding energy is another quantity that cannot be determined on theoretical grounds as yet. It represents the probability $0 \leq P \leq 1$ that a given ℓ-wave will lead to a reaction of the kind described in this paper. It seems safe to assume that $P(\ell)$ will be substantially different from zero for rather large ℓ-waves, since the low ℓ-waves are certainly associated with the formation of the compound nucleus and thus responsible at least for the cross section of the evaporation residue. On the other hand, the highest ℓ-waves are associated with direct or quasi-elastic cross sections and therefore are not expected to be involved in the relaxed cross section studied here to any great extent. It is possible to construct a fairly satisfactory expression for $P(\ell)$ from experiment.

In the present paper we have tried to reproduce the experimental data obtained for the reaction Ag + 288 MeV Ar. The function $P(\ell)$ which we have used has a lower bound ℓ_{min} defined by the evaporation residue cross section of 670 mb [6] and an upper bound defined by the sum of the evaporation residue cross section and the relaxed cross section, which is ~ 1700 mb. Furthermore, the edges of such a rectangular function have been rounded off gently. At the same time we have made the rather bold assumption that no sizable amount of fission is actually present in the cross section. We assume that it is not present for low ℓ-waves because of the high fission barrier. We also assume that it is not present for high ℓ-waves because, though angular momentum lowers the fission barrier [7], the compound nucleus is not formed.

In fig. 2 the calculated angular distributions of the fragments are plotted for atomic numbers from $Z = 6$ to $Z = 32$. The fragments with $Z < 18$ have a forward peaked angular distribution, the forward peaking being more enhanced for atomic numbers close to the projectile. The fragments with $Z > 18$ show angular distributions which quickly become symmetric about 90° as the atomic number is farther removed from $Z = 18$. In fact, for atomic numbers close to symmetric division, the angular distribution becomes indistinguishable from $1/\sin\theta$. This result suggests that, even when particles are emitted with a $1/\sin\theta$ distribution, one should not draw the immediate conclusion that a compound nucleus has been formed.

A rather strange angular distribution peaking at ~ 75° is observed for $Z = 18$. This anomolous angular distribution is due to the fact that at $t = 0$ all the ϕ_Z are equal to zero with the exception of ϕ_{18} which is unity. While we have not investigated this feature in detail as yet, it may possibly contain an explanation for the angular distributions observed for the reactions induced by Kr[8,9] and Cu[10] ions on heavy targets.

In fig. 3 the comparison between experiment and theory is shown. It is important to notice that no normalization has been performed. The shapes of the angular distributions are reproduced with remarkable accuracy, while the absolute values are reproduced within better than 50%. Noticeable deviations are visible for Z = 6. On the other hand, no allowance for shell effects has been made in the evaluation of the ridge line potential, nor has the effect of secondary decay been accounted for.

A more extensive analysis of the available experimental data is in progress. An attempt is also being made to treat the thermalization of the kinetic energy and the particle diffusion occurring prior to the thermalization by means of a generalized theory. While this work was in progress, we learned that a diffusion mechanism has also been postulated by W. Norenberg in order to interpret the quasi-elastic part of the heavy ion cross section.

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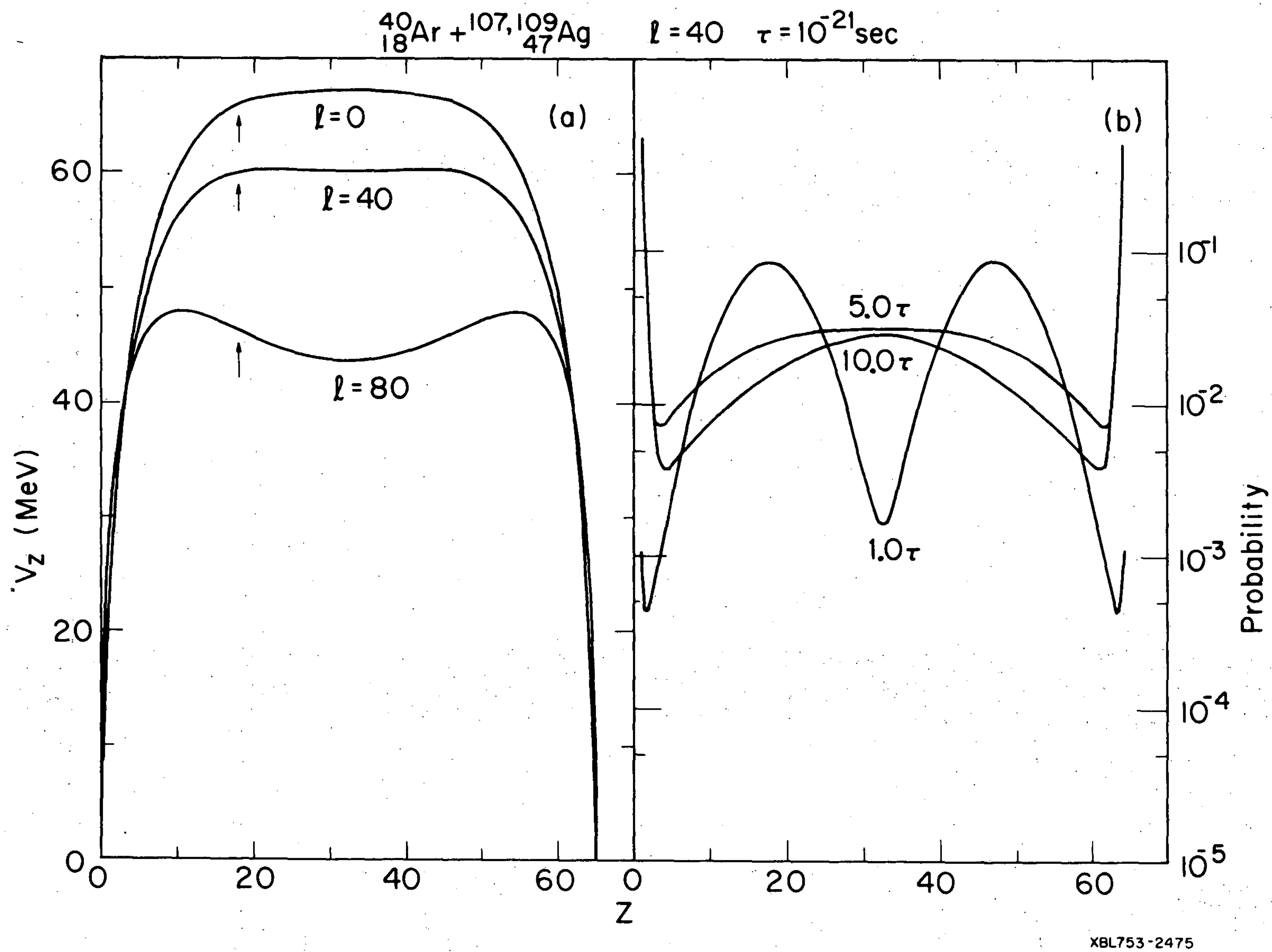
FIGURE CAPTIONS

Fig. 1. (a) Potential energy of the intermediate complex as a function of Z for three ℓ-waves. (b) The probabilities $\phi_Z(t)$ as a function of Z at three different times for the $\ell = 40$ collision. For short times the distribution resembles a Gaussian as expected in a random walk problem. For longer times, the drift is more visible.

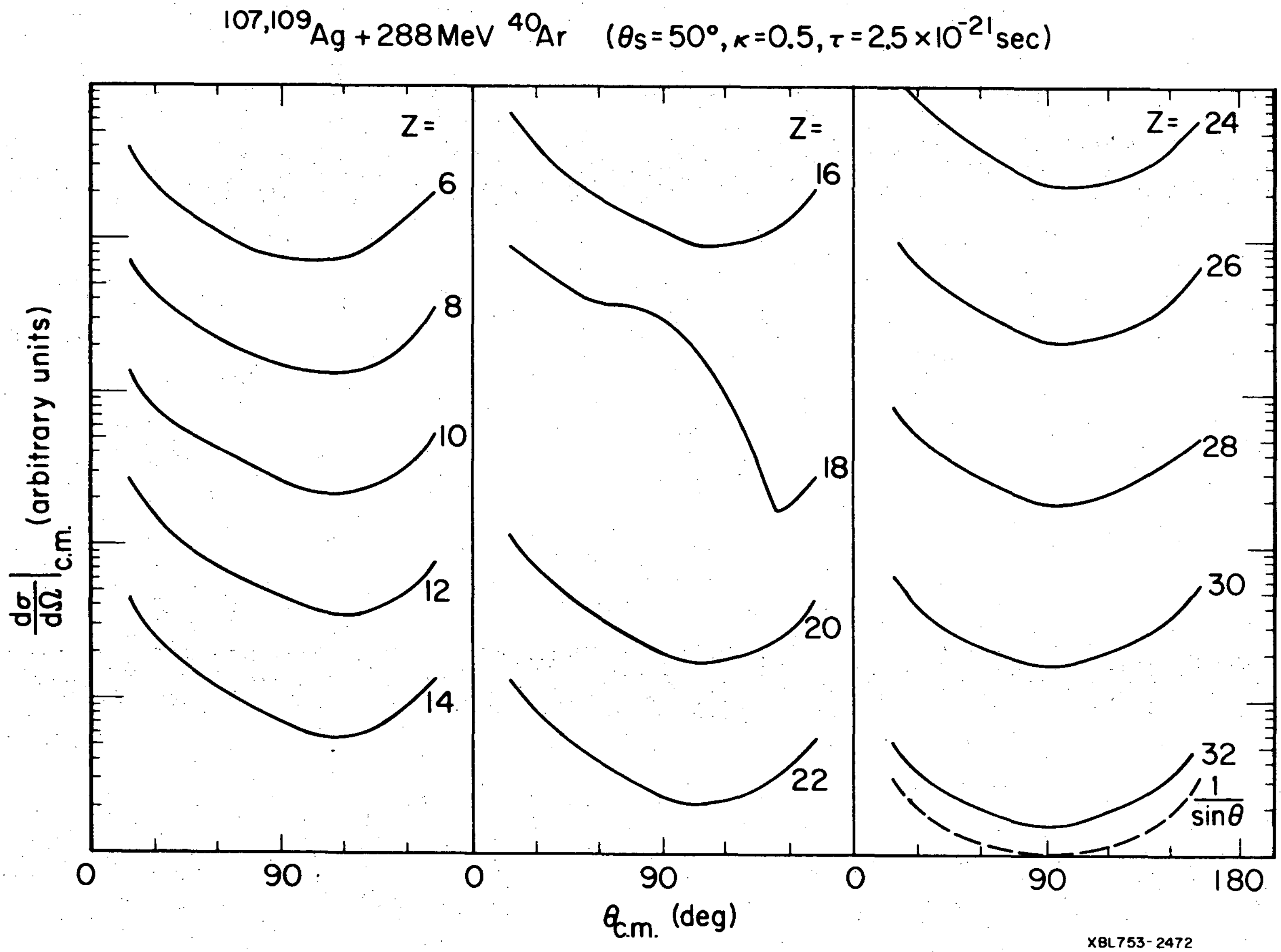
Fig. 2. Calculated center-of-mass angular distribution for various exit channels using the model described in the text. The main feature is the non-negligible forward peaking which decreases as the Z of the exit particle is further removed from the Z of projectile. The angular distributions approach $1/\sin\theta$ for exit channels far removed from the projectile.

Fig. 3. Absolute comparison between the experimental and the theoretical angular distributions using the model described in the text. The values of the constants used are given in upper right hand corner of the figure. The constant κ has the dimensions $fm^{-1}\ 10^{21}\ sec^{-1}$.



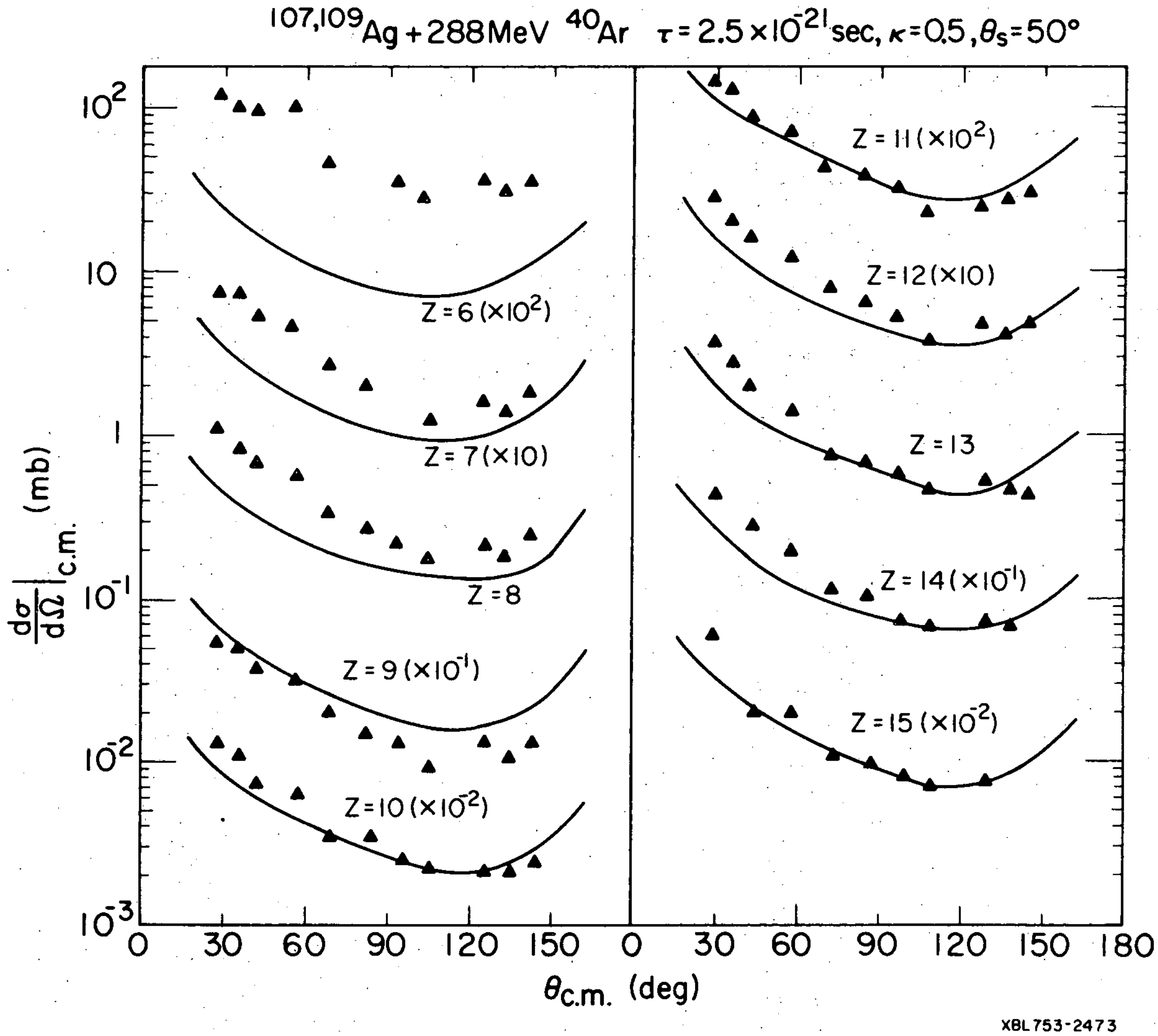
$^{40}_{18}Ar + ^{107,109}_{47}Ag$ $\ell = 40$ $\tau = 10^{-21}$ sec
(a)
$\ell = 0$
$\ell = 40$
$\ell = 80$
V_Z (MeV)
(b)
5.0τ
10.0τ
1.0τ
Probability
Z
XBL753-2475

Fig. 1



$^{107,109}Ag + 288\,MeV\ ^{40}Ar$ ($\theta s = 50°, \kappa = 0.5, \tau = 2.5 \times 10^{-21}$ sec)
$\frac{d\sigma}{d\Omega}\Big|_{c.m.}$ (arbitrary units)
Z =
6
8
10
12
14
Z =
16
18
20
22
Z =
24
26
28
30
32
$\frac{1}{\sin\theta}$
0
90
0
90
0
90
180
$\theta_{c.m.}$ (deg)
XBL753-2472

Fig. 2



^{107,109}Ag + 288 MeV ^{40}Ar τ = 2.5 × 10^{-21} sec, κ = 0.5, θ_s = 50°
$\frac{d\sigma}{d\Omega}\Big|_{c.m.}$ (mb)
$\theta_{c.m.}$ (deg)
Z = 6 (×10^2)
Z = 7 (×10)
Z = 8
Z = 9 (×10^{-1})
Z = 10 (×10^{-2})
Z = 11 (×10^2)
Z = 12 (×10)
Z = 13
Z = 14 (×10^{-1})
Z = 15 (×10^{-2})
XBL 753-2473

Fig. 3

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