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THE LAW OF ONE PRICE, BORDERS AND PURCHASING POWER PARITY

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Abstract

Conventional wisdom claims that the Law of One Price (LOP) fails in commodity markets, commodity borders are wide and Purchasing Power Parity (PPP) fails. But the evidence supporting those claims comes primarily from retail markets where price differentials do not represent risk-free profits. As we show, prices from a wide range of auction markets strongly support the LOP, reject wide borders and do not reject PPP. In addition, recognizing the difference between retail and auction markets helps explain several puzzles associated with exchange rates. The Keynesian paradigm dominates macroeconomics. We question that dominance for two reasons: (1) by reviving PPP we reject Liquidity Preference and support Loanable funds and (2) we reject the standard Keynesian assumption that commodity markets clear slowly and asset markets clear rapidly. Whether commodity or asset, retail markets clear slowly. Whether commodity or asset, auction markets clear rapidly.

Key words: exchange rates; arbitrage; trade; LOP; PPP; borders; transaction costs; retail; wholesale; auction; forward; correspondence rules.

JEL Classifications: B41, F31, F41.

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1. Introduction.

Conventional wisdom claims that borders are wide and rejects the LOP and PPP. But the evidence supporting that position uses primarily retail prices. We find that the LOP holds and that borders are not wide in auction commodity markets. Our evidence also suggests that PPP probably holds in auction markets.

The conventional wisdom regarding the LOP is inconsistent. It accepts the theory in assets markets and rejects it in commodity markets. That position implies that traders in asset markets respond quickly to risk-free profits while traders in commodity markets ignore such profits, which is highly unlikely and inconsistent with basic economic assumptions of profit or wealth maximization.

The key to understanding the conventional wisdom is recognizing the difference between markets. Whether asset or commodity, all retail markets adjust slowly. Whether asset or commodity, all auction markets clear relatively rapidly. In addition, at retail, all goods are non-traded and arbitrage is impossible. No one buys shoes at Marks and Spencer in London and sells them to Macys in New York. On the other hand, commodities like diesel fuel and corn from auction markets are widely traded and, as we show, arbitrage is effective.

Wholesale markets lie between retail and auction markets. Trade is extensive in wholesale markets but arbitrage is not because one cannot hedge risk in wholesale markets like one can in auction markets like those found on the Chicago Board of Trade. The difference between these markets plays an important role in testing the LOP and PPP, and in evaluating borders.

Correspondence or semantic rules link theoretical terms like "price" to things we can observe. Without such rules, testing is impossible. The distinction between retail, wholesale and auction, plays a key role in the correspondence rules used to test the LOP and PPP and to evaluate borders.

2. Correspondence Rules.

Correspondence rules link theories to the world we see.[1] The following reviews the formal logic behind the critical role of correspondence rules. Let $\mathbf{a}\rightarrow\mathbf{b}$ represent "if **a** then **b**". $\mathbf{a}\rightarrow\mathbf{b}$ denies that **a** is "true" and that **b** is "not true", *i.e.*, $\text{n}(\mathbf{a}^\wedge\text{n}\mathbf{b})$, which in turn implies that either **a** is not true *or* **b** is true, *i.e.*, $\text{n}\mathbf{a}^\vee\mathbf{b}$. The relevant point is that $\mathbf{a}\rightarrow\mathbf{b}$ is "true" when **a** is "false" *regardless* of whether **b** is "true" or "not true".

Note that, in the following, **T**, **C**, **p** and **q** are all statements. A classic example for $\mathbf{a}\rightarrow\mathbf{b}$ would be "If all swans are white then this swan is white."

Let **T** represent the LOP, PPP or any theory and **C** the correspondence rules. To be empirically meaningful, some of the terms in **T** *must* be linked through correspondence rules to things we can measure. A Scholastic claim that 100 angels can fit on the head of a pin is an example of a "theory" that is not empirically meaningful because there is no way to measure the number of angels.

One way to express the logical structure of testing theories is as follows: $\mathbf{T}\rightarrow\mathbf{C}\rightarrow(\mathbf{p}\rightarrow\mathbf{q})$ where $\mathbf{p}\rightarrow\mathbf{q}$ represents a statement about some testable implication of combining **T** and **S.** Note that rejecting $\mathbf{p}\rightarrow\mathbf{q}$ does *not,* by itself, reject **T**. If **C** is "false", then $\mathbf{T}\rightarrow[\mathbf{C}\rightarrow(\mathbf{p}\rightarrow\mathbf{q})]$ is "true" because $[\mathbf{C}\rightarrow(\mathbf{p}\rightarrow\mathbf{q})]$ is true and the evidence does not reject **T**. The important point is that the

[1] The relevant literature uses different terms for "correspondence rules" that mean the same thing. For example, Hempel (1966, 72-75) uses "bridge principles". See Winther (2016) for a survey of the relevant literature.

confidence we have in rejecting **T** when **(p→q)** is false, can be no greater than the confidence we have in **C**.[2]

In the context of this paper, we can express the logical structure for the LOP as **LOP→C→(p→q)** where **LOP** is the theory of the LOP described below and (**p→q**) might be "If the LOP holds then price differentials should be stationary and half-lives short."

Articles using retail price differentials routinely claim to reject the LOP and often claim to reject effective arbitrage. But retail price differentials cannot reject the LOP or effective arbitrage because they do not represent potential risk-free profits.

Theories constrain correspondence rules. Take the law of gravity as an example. Dropping a feather and an iron ball from the top of the leaning tower of Pisa does not reject the law because it requires a vacuum. Dropping an iron ball on the moon where it does not accelerate at 32 feet per second does not reject the law because the rate of acceleration depends on mass.

Appropriate correspondence rules for testing the LOP and PPP depend on their definitions. Our confidence in any rejection can be no stronger than the confidence we have in the correspondence rules.

3. Definitions.

For simplicity, the definitions discussed here ignore information and transaction costs. We discuss their effects later.

[2] (**T^ C)→(p→q)** is the alternative way to express the logical structure. The conclusion is the same. The falsity of (**p→q)** does not, by itself, reject **T**. To do that, we must accept **C** as true. A false **p→q** implies that *either* **T** *or* **C**, but not necessarily both, is false. A false **C** meets that condition, which leaves T unrejected.

3.1. LOP.

For the LOP **T**, we use the definition found in most economic dictionaries and encyclopedias and that is implicit in the literature that rejects the LOP. That definition includes the mechanism that produces equality between prices, arbitrage. The following from *The Penguin Dictionary of Economics* (1998, 241) is a fairly typical definition of the LOP:

> The law, articulated by Jevons, stating that 'In the same open market, at any moment, there cannot be two prices for the same kind of article. The reason is that, if they did exist, arbitrage should occur until the prices converge.

The New Palgrave Dictionary of Economics (2008) does not have a separate heading for the LOP. It is discussed on page 189 under the heading of Arbitrage.

> The assertion that two perfect substitutes (for example, two shares of stock in the same company) must trade at the same price is an implication of no arbitrage that goes under the name of the law of one price.

This definition implies that the **C** for the LOP must include a condition that observed price differentials represent potential risk-free profits.

Dictionaries and encyclopedias specializing in economics clearly define arbitrage as a "risk-free" transaction.[3] For example, *The New Palgrave Dictionary of Economics* (2008, 188) begins the discussion of arbitrage as follows: An arbitrage opportunity is an investment strategy that guarantees a positive payoff in some contingency with no possibility of a negative payoff….". *Wikipedia* (25 May 2010) says the following:

> When used by academics, an arbitrage is a transaction that involves no negative cash flow at any probabilistic or temporal state and a positive cash flow in at least one state; in simple terms, it is a risk-free profit.

[3] In this context "risk-free" refers to certainty regarding prices. There is always the risk that a contract might not be fulfilled.

Let $P_k(t)$ represent the domestic price of some good *k*, $P^*(t)$ the foreign price of the same good and $S(t)$ the domestic price of foreign exchange. For a price differential to reject the LOP, it *must* represent a risk-free profit. Retail price differentials do not represent risk-free profits.[4]

Tests of the LOP in both financial and commodity markets often just ask whether or not $log[P_k(t)]$ equals $log[P^*(t)]$ or $log[P^*(t)S(t)]$. Tests for unit roots and estimates of half-lives often use eq. (1) where z(t) equals $log[P_k(t)/P^*(t)]$ or $log[P_k(t)/P^*(t)S(t)]$.

$$\Delta z(t) = \alpha + \beta z(t\text{-}1) + \gamma_1 \Delta z(t\text{-}k) + e(t) \qquad (1)$$

The augmented Dicky-Fuller test for a unit root uses β and estimates of half-lives for deviations from the LOP often use $log0.5/log(|1.0 - \beta|)$.

But arbitrage is more complicated in commodity than in asset markets. One important complication is that commodity arbitrage normally involves time in transit while financial arbitrage does not. Someone buying a financial asset like a bond in New York and selling it in London as part of an arbitrage transaction does not need to deliver the bond itself. All that is required is a change in ownership.

Someone buying wheat in New York for delivery in Rotterdam as part of an arbitrage transaction must physically deliver the wheat. Shipping wheat from New York to Rotterdam involves transport costs and takes time. As a result, commodity arbitrage, unlike financial arbitrage, normally involves time in transit which creates additional costs.

As a result of those costs, we need to revise the standard test condition for commodity arbitrage and the LOP. Let the U.S. export wheat to Rotterdam. Accounting for time in transit, $log[P_k(t)/P^*(t)S(t]$ becomes $log[P_k(t)/P^*(t\text{-} \tau)S(t\text{-}\tau)]$ where $P_k(t)$ is an export price, τ is the time in

[4] To the best of our knowledge, no one has used identical wholesale prices to test the LOP.

transit and $P^*(t-\tau)S(t-\tau)$ is a forward import price denominated in the domestic currency. With this month the most recent month, if t is August then $t-1$ is September.

Forward commodity prices are not easy to find. Fortunately, as Kearns (2007) points out, in commodity markets, forward prices are unbiased estimates of future spot prices. That allows us to use future commodity spot prices as proxies for forward prices.

Transport costs require another revision to the test equation. Let $T(t)$ represent the transport costs. Most research, like ours, deals with such costs by simply adding $T(t)$ to the price differential, $z(t)$. $z(t)$ becomes $log\{[P_k(t)+T(t)]/P^*(t)S(t)\}$ or $log\{[P_k(t)+T(t)]/P^*(t-\tau)S(t-\tau)\}$.

As is now widely recognized, those costs create econometric problems in the form of nonlinearities. See for example Obstfeld and Taylor (1997). For price differentials below $T(t)$ and above $-T(t)$ there is no incentive to close differentials in the form of risk-free profits. As a result, even when arbitrage is effective, price differentials are likely to be martingales between two elastic thresholds. We leave the analysis of the effects of those thresholds to those with a comparative advantage in econometrics. We concentrate on getting the economics right.

3.2. PPP.

There are several versions of PPP. The monetary version implies that the appropriate prices are the ones used to convert nominal money balances into real money balances. The utility version says that domestic money should buy the same amount of utility abroad as at home. By PPP we mean the arbitrage version based on the LOP and described in almost every macro textbook. The **C** for that **T** requires that price differentials represent risk-free profits because the theory is based on the LOP which assumes effective arbitrage.

Let $P(t)$ represent a domestic basket of goods made up of $P_k(t)$ with $P^*(t)$ a foreign basket of identical $P^*(t)$. The $P_k(t)$ and $P^*(t)$ must be consistent with the LOP and the baskets must have the same weights. Given these constraints, PPP claims that the domestic price of foreign exchange $S(t)$ should equal $P(t)/P^*(t)$.

Articles claiming to reject PPP use wholesale or retail prices, where price differentials do *not* represent potential risk-free profits and, therefore, do *not* reject PPP. Unfortunately, the $P(t)$ and $P^*(t)$ needed to test PPP properly do not yet exist.

4. LOP Literature Review.

We review the LOP, Borders and PPP literature separately, beginning with the LOP. For each topic, we consider the evidence first from retail commodity markets, then wholesale markets and finally auction markets.

4.1. Retail.

To reduce size, we restrict this review to articles that use retail prices for roughly "identical" products. Such articles claim to either reject the LOP or find little support for it, often with half-lives of a year of more. Most also claim to reject effective arbitrage. But retail price differentials do not represent risk-free profits, so they do not reject effective arbitrage or the LOP.

Note that these claims of rejection are not restricted to retail markets. They are categorical. As a result, they have misled most economists into believing that commodity arbitrage and the LOP fail in *all* commodity markets when they hold in auction markets.

Relevant articles include Parsley and Wei (1996) who use U.S. quarterly retail prices and find half-lives that range from 12 to 20 months for tradeables and 45 months for services.[5]

Asplund and Friberg (2001) use catalog prices for identical goods in duty-free outlets such as on Scandinavian ferries and report half-lives of 24 months. Without accounting for time in transit or nonlinearities, they reject the LOP for all outlets.

Haskel and Wolf (2001) use annual prices from IKEA in different European countries, but do not report half-lives. They account for nonlinearities, but not time in transit. They conclude that substantial violations of the LOP are pervasive.

Lutz (2004) uses biannual new car prices in Europe without accounting for time in transit. When he does not account for nonlinearities, he rejects the LOP. When he accounts for nonlinearities, he finds some support in the form of relatively short half-lives of six months.

Finally, Goldberg and Verboven (2005) also use new car prices in Europe without accounting for nonlinearities or time in transit. They report half-lives from 16 to 19 months, which we would interpret as a rejection of the LOP.

Based largely on these articles, the conventional wisdom is that arbitrage and the LOP fail in commodity markets. Unfortunately, these articles misrepresent their results. They apply only to retail markets and they reject neither the LOP nor effective commodity arbitrage because their price differentials do not represent risk-free profits.

Using auction markets where price differentials represent potential risk-free profits, we measure half-lives in days, not years.

4.2. Wholesale.

[5] Although many prices in Parsley and Wei (1996) are not for identical products, we include them in this list because some are for identical products and they are widely cited as evidence that the LOP fails. Although they do not explicitly reject the LOP, most readers would probably interpret their half-lives for what they call tradeables as a rejection.

Some early work, e.g. Isard (1977) and Richardson (1978), uses wholesale prices to test the LOP, but the prices are not for identical products. In spite of that, they report some long-run support for the LOP. We are not aware of any articles that use identical wholesale prices to test the LOP.

4.3. Auction.

Although generally ignored, several earlier articles use auction prices to evaluate the LOP. They support it. But the available data prevent tests that match the quality of tests in financial markets. Obtaining that quality of data for commodities will require a large investment.

We begin with Protopapadakis and Stoll (1983), a landmark article that uses weekly prices.[6] It is, we believe, the first article to use auction prices and account for time in transit.[7] It considers arbitrage where arbitragers buy at near future dates and sell at more distant future dates. Without accounting for nonlinearities, Protopapadakis and Stoll find that for 9 of the 13 commodities the average deviation from the LOP is small. But they do not report half-lives. They note that the variance around the LOP declines with maturity. That pattern is consistent with the propositions on costs in Alchian (1959).

Protopapadakis and Stoll (1986) extend their 1983 article by including expectations and providing a measure of the speed of adjustment. They find that it takes just 10 weeks to eliminate 90% of the deviation from the LOP. But they do not report half-lives. For a comparison with retail prices, Parsley and Wei report that it takes four to five *quarters* to eliminate only 50% of the deviation.

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[6] We do not use their prices for tin and rubber because they may not be for identical products.
[7] Coleman (2009a, 2009b) model the effects of time in transit.

Goodwin *et al* (1990) use monthly auction prices for 17 narrowly defined agricultural products. Both U.S. and foreign prices are in U.S. dollars. They account for time in transit, which increases support for the LOP. Results are also sensitive to transport costs and interest rates. They accept the LOP for 15 of the 17 products, but do not report half-lives or account for nonlinearities.

Goodwin (1992), applies multivariate cointegration tests to five international auction wheat markets where transport costs are available on a monthly basis.[8] When his tests ignore transportation costs, they reject long-run LOP. When they include those costs, they support long-run LOP. He does not account for time in transit, report half-lives or account for nonlinearities.

Michael et al (1994), using the same data as Goodwin et al (1990), demonstrate the importance of transaction costs and thresholds in testing for cointegration, but do not report half-lives.

Pippenger and Phillips (2008) describe four pitfalls that create the general, but mistaken, belief that the LOP fails: (1) the use of retail prices, (2) omitting transaction costs, (3) ignoring time in transit and (4) not using identical products. Using auction wheat prices between the U.S. and Japan or Rotterdam, they show how each pitfall reduces cointegration. The most important pitfall is the use of retail prices. They do not report half-lives or consider the role of thresholds.

Conventional wisdom ignores the literature supporting the LOP. Articles like Parsley and Wei (1996) and Asplund and Friberg (2001) never cite any of the earlier auction literature. On the other hand, Pippenger and Phillips (2008) extensively cite the literature rejecting the LOP.

5. Borders.

[8] Using cointegration and a broader range of products like wheat, wool and zinc rather than "identical" products like Dark Northern Spring wheat, 14% protein, Vataja (2000) also finds support for long-run LOP.

The LOP literature deals with the overall effectiveness of the LOP. The Borders literature compares the effectiveness within and between national borders, e.g., within Canada and within U.S. versus between the U.S. and Canada. Although the conclusion applies only to retail markets, it is expressed as categorical. As a result, most macroeconomists believe that borders in general are wide. We show below that, for auction markets, borders are hardly noticeable.

5.1. Retail.

The seminal article in the Borders literature is Engel and Rogers (1996). Using retail prices and auction exchange rates, they find that the price variation between two cities in two different countries is much larger than for two equidistant cities in the same country.

Several supporting articles using retail prices and auction exchange rates followed.[9] They include: Parsley and Wei (1996, 2001), Engel and Rogers (2001a, 2001b), Cheung and Lai (2006) and Horvath *et al* (2008). Some articles claim their wide borders reject effective arbitrage and the LOP, but they do not because their price differentials do not represent risk-free profits.

5.2. Auction.

As far as we aware, no earlier article uses wholesale or auction prices to evaluate the width of borders. As Section 7 below shows, with auction prices borders essentially disappear.

6. PPP.

The conventional wisdom is that, unless there is substantial inflation, PPP fails. Rogoff's widely cited review article, Rogoff (1996), strongly rejects PPP under stable monetary

[9] Using similar retail data, at least three articles reject wide borders. They are Morshed (2003, 2007) and Gorodnichenko and Tesar (2009).

conditions. That rejection is based largely on earlier reported long half-lives for deviations from PPP of three to five years, most of which use retail prices.

6.1. Retail.

Later research using retail prices supports his conclusion. Using TAR, Obstfeld and Taylor (1997) find average half-lives for deviations from PPP of about eight months.[10] But Smallwood (2008), who also accounts for nonlinearities, finds half-lives for CPIs that range from four years to infinity. Kunkler and MacDonald (2015), who consider the bias to half-lives from aggregation report average half-lives of about 1.5 years. More recently, Nagayasu (2021), who considers causality and spillovers, but ignores time in transit and thresholds, reports average speeds of adjustment that suggest half-lives of about 1.5 years.

6.2. Wholesale.

We would expect wholesale prices to produce more support for the LOP and PPP than retail prices because, while there is no trade between retail markets, there is substantial trade between wholesale markets. The evidence supports this expectation.[11]

Early users of wholesale prices to test PPP include Isard (1977) and Krugman (1978). Both find a little support for PPP, but do not report half-lives. Since then, almost all the relevant research has used CPIs. Kouretas (1997) and Michael et al (1997) are exceptions.[12] Without accounting for time in transit or thresholds, Kouretas applies cointegration tests to real exchange rates using both CPIs and WPIs over identical intervals. He cannot reject a unit root for CPIs but

[10] Lo and Zivot (2001) also estimate nonlinear adjustment to cointegration, but do not report half-lives.
[11] While inappropriate correspondence rules cannot reject a theory, they can support it.
[12] Kargbo (2009) also compares CPIs and WPIs, but his results are inconsistent. Presumably because he does not use the same intervals for CPIs and WPIs.

can reject using WPIs. Using wholesale prices and ESTAR, Michael et al show that recognizing nonlinearities is important for testing PPP.

We now turn to tests using our auction prices.

7. Our Auction Prices.

We do not consider the non-linearities created by thresholds. We concentrate on resolving the problems created by inappropriate correspondence rules. Once those problems are resolved, accounting for thresholds should further increase the support for the LOP and PPP.

We begin with a description of our data, which is more extensive than elsewhere. Our auction silver and grain and prices have been used before to test the LOP, but most of our prices cover longer intervals than used earlier. In addition, we use several identical petroleum products, *e.g.*, daily regular gasoline prices, for the first time. Table 1 describes our data: shipping rates, grains, silver and petroleum products, as well as acronyms, ports and sources.[13]

TABLE 1
PRODUCTS, PORTS AND SOURCES

Product	Acronym/Ports	Source
No. 2 Dark Northern Spring Wheat,14% Protein	DNS Gulf, Japan, Pacific, Rotterdam	*World Wheat (Grain) Statistics.* Monthly
No. 2 Western White Wheat	WW Pacific, Japan	*World Wheat (Grain) Statistics.* Monthly
No. 2 Hard Winter Wheat, 13% Protein	HW13 Gulf, Japan	*World Wheat Grain) Statistics.* Monthly
No 2. Hard Winter Wheat, Ordinary	HWO Gulf, Pacific	*World Wheat (Grain) Statistics,* Monthly
No. 2 Western White Wheat	WW Pacific, Japan	*World Wheat (Grain) Statistics.* Monthly
No. 2 Yellow Corn	YC Gulf, Rotterdam	UN Conference on Trade and Development, *Handbook of Statistics.* Monthly
Shipping rates for	Gulf, Rotterdam, Pacific,	*World Wheat (Grain) Statistics.* Monthly

[13] Shipping rates are one-month forward rates.

wheat	Japan	
No. 2 Diesel Fuel	Gulf, New York, Los Angeles, Rotterdam, Singapore	EAI Energy Spot Prices. Daily
Fuel Oil	Gulf, New York, Los Angeles, Rotterdam, Singapore	EAI Energy Spot Prices. Daily
Gasoline, Regular	Gulf, New York, Los Angeles, Rotterdam and Singapore	EAI Energy Spot Prices. Daily
Jet Fuel	Gulf, New York, Los Angeles, Rotterdam, Singapore	EAI Energy Spot Prices. Daily
Silver	U.S., U.K.	Protopapadakis and Stoll (1986)† Weekly

Original Sources: † Samuel Montague.

The first part describes our grain prices.[14] They are monthly averages of daily prices where both the U.S and foreign price are in U.S. dollars so that exchange rates are not needed. Some advantages of these data are: (1) they are for 'identical' products, *e.g*., No. 2 Dark Northern Spring wheat, 14% protein, (2) matching freight rates are available for wheat but not between U.S, ports, (3) for Rotterdam wheat prices cover an unusually large number of years, about 30, (4) export prices are free on board (FOB) and import prices include certificates, insurance and freight (CIF). (For Japan, prices do not include insurance.) and (5) we can be reasonably sure about the direction of trade, which allows us to account for time in transit. Although auction grain prices have been used before to evaluate the LOP, in most cases, ours cover a substantially longer interval.

The second part describes our daily auction prices for petroleum products and weekly prices for silver.[15] This is the first time anyone has used daily auction prices to evaluate the LOP and Borders. Like our grain prices, both U.S. and foreign petroleum prices are in U.S. dollars. Silver prices are in dollars and sterling.

[14] Some prices are missing. When only one month is missing, we replace it with the previous month. When two months in a row are missing, we replace the first month with the preceding month and the second month with the following month. This should not introduce spurious structure into differences. We do the same with daily petroleum prices.

[15] We do not use the rubber or tin prices from Protopapadakis and Stoll (1986) because we suspect that they are not for identical products. Unfortunately, their other prices appear to have been lost.

As shown below, daily prices provide stronger support for the LOP than monthly prices. Other things equal, the higher data frequency the shorter half-lives.

7.1. Monthly Grain Prices.

Our monthly wheat price differentials for Japan end in 1981:10 because starting in 1982 several months of Japanese data are missing. Then in the 1990s Japan erected non-tariff barriers to wheat imports that created artificial price differentials. See Fukuda *et al* (2004). Our monthly corn price differentials end in mid-1998 because about then Europe began imposing import restrictions on genetically modified food and almost all U.S. corn is genetically modified.

We begin with unit root tests for grain price differentials. We use two different differentials: (1) a $z(t)$ without transportation costs that ignores time in transit denoted $logP_k(t) - logP^*(t)S(t)$ and (2) a $z(t)$ that includes transportation costs and accounts for time in transit denoted $log[P_k(t) +T(t)] - logP^*(t-1)S(t-1)$ where $T(t)$ is our transport costs. The dollar price in the import port is lagged one month to account for time in transit from U.S. to foreign ports.

7.1.1. Unit Roots: We use RATS dfunit.src and ppunit.src without the trend option. Our rejection of a unit root therefore implies true stationarity, not just trend stationary. We use both the Augmented Dicky-Fuller (ADF) and Phillips-Perron (PP) unit root tests because the PP test is less sensitive to lag length. We use the same number of lags for our unit root tests as we do for our half-lives in Tables 3 and 4.[16]

Table 2 reports our unit root tests for monthly $logP_k(t) - logP^*(t)S(t)\}$ and $log[P_k(t)+T(t)] - logP^*(t-1)S(t-1)$. For example, for DNS between Gulf ports and Rotterdam, -9.1 is the ADF test

[16] Our estimates for half-lives search eq. (1) for an appropriate γ_i.

for *logP*$_k$*(t) - logP*(t)S(t)}* and -9.0 the PP test while -6.7 is the ADF test for *log[P*$_k$*(t)+T(t)] - logP*(t-1)S(t-1)* and -9.6 the PP test.

TABLE 2*
Unit Root Tests for Both Grain Price Differentials

Product	Ports	Interval	T Stats.	Product	Ports	Interval	T Stats.
DNS	Gulf-Rott	1971:7-2001:12	-9.1 -9.0 -6.7 -9.6	HW13	Gulf-Japan	1973:4-1981:10	-5.4 -6.7 -6.7 -6.7
DNS	Gulf-PP	1973:4-1986:4	-7.4 -5.5	HWO	Gulf-PP	1974:1-1986:4	-3.6 -4.6
DNS	PP-Japan	1973:4-1981:10	-6.9 -7.0 -6.2 -6.2	WW	PP-Japan	1973:4-1981:10	-4.3 -4.3 -4.9 -5.0
DNS	Gulf-Japan	1973:4-1981:10	-5.5 -5.5 -5.8 -5.8	Corn	Gulf-Rott	1985:12-1998:6	-6.7 -6.8 -8.3 -8.3
DNS	PP-Japan	1973:4-1981:10	-6.9 -7.0 -6.2 -6.2				

* Significant at 5%, -2.9. Significant at 1%, -3.5.

All grain price differentials are stationary. In every case we can reject a unit root at the 1% level or better.

7.1.2. Half-Lives: We use eq. (1) and $log0.5/log(|1.0 - \beta|)$ to estimate half-lives in months for the price differential *z(t)*.

$$\Delta z(t) = \alpha + \beta z(t\text{-}1) + \gamma_1 \Delta z(t\text{-}k) + e(t) \quad (1)$$

We use the Cumulated Periodogram Test to test for correlated *e(t)* and the Breusch-Godfrey SC Test to test for correlation between Δ*z(t-k)* and *e(t)*. For the SC test we report a significance level. For the Cumulated Periodogram we report the maximum gap divided by the 5% significance level for that gap. If that statistic is 1.0, we can reject correlation at 5%. If less than 1.0, we can reject at better than 5%.

Table 3 reports our results for *logP*$_k$*(t) - logP*(t)S(t)}*. As an example, for DNS between Gulf ports and Rotterdam, the lag is zero, the Breusch-Godfrey SC Test is 0.20, the Cumulated Periodogram test is 0.52 and the half-life is 2.1 months.

TABLE 3

Half Lives Measured in Months for Grains: *logP*$_k$*(t) - logP*(t)S(t)*

Grain	Ports	Dates	Lag/Tests	Half-Life	Grain	Ports	Dates	Lag/Tests	Half-Life
DNS	Gulf-Rott	71:07-01:12	0 0.20 0.52	2.1	HWO	Gulf-PP	74:1-86:4	1 0.15 0.48	2.5
DNS	Gulf-Japan	73:4-81:10	0 0.11 0.38	1.4	HW13	Gulf-Japan	73:4-81:10	0 0.66 0.81	1.4
DNS	PP-Japan	73:4-81:10	0 0.22 0.89	1.2	WW	PP-Japan	73:4-81:10	0 0.38 0.74	2.3
DNS	Gulf-PP	73:4-86:4	3 0.33 0.82	1.0	Corn	Gulf-Rott	85:12-98:6	0 0.82 0.55	1.0
								Mean Excluding Gulf-PP	1.6

These are much shorter half-lives than found using retail prices which are often measured in years. The half-lives in Table 4, which use *log[P*$_k$*(t)+T(t)] - logP*(t-1)S(t-1),* are even shorter.

TABLE 4

Half Lives Measured in Months for Grains: *logP*$_k$*(t)+T(t)] - logP*(t-1)S(t-1)*

Grain	Ports	Dates	Lag/Tests	Half-Life	Grain	Ports	Dates	Lag/ Tests0	Half-Life
DNS	Gulf-Rott	71:07-01:12	3 0.87 0.41	1.4	HW13	Gulf-Japan	73:4-86:4	0 0.38 0.65	1.1
DNS	Gulf-Japan	73:4-86:4	0 0.31 0.75	1.3	WW	PP-Japan	73:4-86:4	0 0.37 0.57	2.0
DNS	PP-Japan	73:4-81:10	0 0.35 0.81	1.6	Corn	Gulf-Rott	85:12-98:6	0 0.10 0.44	0.7
								Mean	1.35

Including time in transit and shipping costs reduces half-lives. On average, excluding DNS Gulf-PP and HWO Gulf-PP where we do not have shipping costs, average half-lives in Table 3 are 1.6 months while in Table 4 they are 1.35 months. That is a reduction of about 18%.

While our half-lives for grain price differentials strongly support the LOP, our weekly silver and daily petroleum prices provide even more support. We measure their half-lives in *days not months.*

7.2. Silver.

Our silver prices between London and New York were first used by Protopapadakis and Stoll (1986), but not to evaluate Borders or estimate half-lives. We do not have transport costs or know the direction of trade. Using the same tests as in Table 2, these price differentials are clearly stationary. The ADF is -10.1 and the PP is -14.6. The half-life is just 0.8 of a five-day week, or 4 days.

7.3. Petroleum.

No one has used daily petroleum prices before to estimate borders and half-lives. As with wheat prices, both domestic and foreign prices are in U.S. dollars. The petroleum price differentials in Table 5 are stationary and their half-lives in Table 6 are short.

Table 5
Unit Root Tests for Petroleum Price Differentials

Ports	Products	Lag	T Statistics*	Ports	Products	Lags	T Statistic
Gulf/NY	Diesel	4	-3.2 -9.8	LA/Gulf	Jet Fuel	1	-7.3 -9.2
Gulf/NY	Fuel Oil	2	-8.0 -9.1	Rott/Gulf	Fuel Oil	1	-7.7 -8.0
Gulf/NY	Gasoline	1	-13.3 -14.9	Rott/Gulf	Gasoline	3	-11.8 15.0
Gulf/NY	Jet Fuel	3	-5.8 -11.9	Rott/Gulf	Jet Fuel	2	-6.6 -9.7
LA\NY	Diesel	0	-10.1 -10.3	Sing/Gulf	Fuel Oil	0	-6.8 -9.5
LA/NY	Fuel Oil	0	-6.9 -7.1	Sing/Gulf	Gasoline	3	-4.5 -15.6
LA/NY	Gasoline	1	-10.4 -10.3	Sing/Gulf	Jet Fuel	6	-3.7 -6.7
LA/NY	Jet Fuel	3	-5.4 -11.1	Rott/LA	Fuel Oil	1	-10.3 -10.3
Rott/NY	Fuel Oil	1	-9.3 -9.5	LA/Rott	Gasoline	2	-9.0 -8.9
Rott/NY	Gasoline	2	-12.3 -14.8	Rott/LA	Jet Fuel	1	-7.5 -9.1
Rott/NY	Jet Fuel	3	-9.9 -11.5	Sing/LA	Fuel Oil	0	-10.3 -10.3

Sing/NY	Fuel Oil	0	-6.1 -6.3	Sing/LA	Gasoline	0	-7.9 -8.2
Sing/NY	Gasoline	3	-7.1 -19.4	Sing/LA	Jet Fuel	3	-3.9 -7.6
Sing/NY	Jet Fuel	3	-4.9 -8.2	Sing/Rott	Fuel Oil	0	-6.5 -6.5
LA/Gulf	Diesel	1	-5.9 -9.2	Sing/Rott	Gasoline	1	-7.1 -7.6
Gulf/LA	Fuel Oil	0	-6.8 -6.9	Sing/Rott	Jet Fuel	5	-3.8 -7.7
Gulf/LA	Gasoline	0	-11.0 -11.1				

*5% significance level is -2.9. 1% is -3.4.

7.3.1: Unit Roots: As in Table 2, Table 5 uses both the Augmented Dicky-Fuller and Phillips-Perron tests for unit roots without an option for trend. The price differential is $logP_k(t)$ - $logP^*(t)S(t)$ because we do not have transport costs or know the direction of trade. We use the lag for the corresponding half-life in Table 6. To save space in Table 5, intervals are shown in Table 6.

Except for diesel fuel between Gulf ports and New York, both tests reject a unit root at beyond 1%. For that differential, ADF rejects at 5% while PP does so at beyond 1%. All price differentials are stationary, not just trend stationary.

7.3.2. Half-Lives: We estimate *daily* half-lives for petroleum price differentials in the same way we estimate monthly half-lives. Table 6 reports daily half-lives.

TABLE 6

Half Lives in Days for Daily Petroleum Price Differentials

Ports	Product	Interval	Lag/ Test Stat	Half-Life in Days	Ports	Products	Interval	Lag/ Test Stat.	Half Life In Days
Gulf/ NY	Diesel	95:10:2- 07:1:31	4 0.45 0.88	9.2	LA/ Gulf	Jet Fuel	90:12:3- 07:1:31	1 0.66 0.60	14.2
Gulf/ NY	Fuel Oil	93:7:16- 07:1:31	2 0.10 0.61	20.7	Rott/ Gulf	Fuel Oil	93:9:14- 07:1:31	1 0.17 0.62	19.2
Gulf/ NY	Gasoline	86:6:2- 07:1:31	1 0.11 0.79	8.7	Rott/ Gulf	Gasoline	87:4:1- 07:1:31	3 0.61 0.55	8.8
Gulf/ NY	Jet Fuel	90:12:3- 07:1:31	3 0.16 0.62	9.8	Rott/ Gulf	Jet Fuel	90:12:3- 07:1:31	2 0.84 0.57	10.0
LA/ NY	Diesel	95:10:2- 07:1:31	0 0.19 0.82	11.5	Sing/ Gulf	Fuel Oil	93:7:16- 07:1:31	2 0.21 0.39	29.2
NY- LA	Fuel Oil	93:10:24- 07:1:31	0 0.39 0.77	24.8	Sing/ Gulf	Gasoline	92:8:3- 07:1:31	3 0.14 0.69	5.4
LA/ NY	Gasoline	86:6:2- 07:1:31	1 0.11 0.50	18.0	Sing/ Gulf	Jet Fuel	90:12:3- 07:1:31	6 0.36 0.99	12.7
LA/	Jet Fuel	90:12:3-	3	13.6	Rott/	Fuel Oil	93:10:24-	1	21.4

NY		07:1:31	0.40 0.62		LA		07:1:31	0.39 0.60	
Rott/ NY	Fuel Oil	93:7:16- 07:1:31	1 0.18 0.52	13.5	Rott/ LA	Gasoline	87:4:1- 07:1:31	2 0.11 0.51	21.5
Rott/ NY	Gasoline	87:4:1- 07:1:31	2 0.10 0.84	8.7	Rott/ LA	Jet Fuel	90:12:3- 07:1:31	1 0.45 0.91	16.8
Rott/ NY	Jet Fuel	90:12:3- 07:1:31	3 0.51 0.43	11.4	Sing/ LA	Fuel Oil	93:10:24- 07:1:31	0 0.31 0.75	11.1
Sing/ NY	Fuel Oil	93:7:16- 07:1:31	0 0.32 0.92	34.1	Sing/ LA	Gasoline	92:8:3- 07:1:31	0 0.52 0.99	21.2
Sing/ NY	Gasoline	92:8:3- 07:1:31	5 0.43 0.64	5.9	Sing/ LA	Jet Fuel	90:12:3- 07:1:31	3 0.71 0.51	21.3
Sing/ NY	Jet Fuel	90:12:3- 07:1:31	4 0.42 0.07	11.0	Sing/ Rott	Fuel Oil	93:11:1- 07:1:31	0 0.13 0.53	28.3
LA/ Gulf	Diesel	95:10:2- 07:1:31	1 0.44 0.65	10.9	Sing/ Rott	Gasoline	92:8:3- 07:1:31	1 0.29 0.33	23.2
LA/ Gulf	Fuel Oil	93:10:29- 07:1:31	0 0.70 0.59	25.8	Sing/ Rott	Jet Fuel	90:12:3- 07:1:31	5 0.86 0.52	21.2
LA/ Gulf	Gasoline	86:6:2- 07:1:31	0 0.69 0.94	16.0				Mean	16.3

The average half-life in Table 6 is 16.3 days. But even this short half-life overstates half-lives for petroleum prices for four reasons: (1) we do not know the direction of trade, (2) we do not have transportation costs, (3) we ignore thresholds and (4) even higher frequency data, *e.g.*, tick data, should produce still shorter half-lives.

Conventional wisdom categorically rejects the LOP because it fails in retail markets where there is no trade and arbitrage is impossible. *But arbitrage and the LOP work in auction markets where there is trade and arbitrage is possible.*

8. Half-Lives and Frequency.

Although it is fairly well known that low frequency data, *e.g.,* monthly, quarterly or annual, biases half-lives upward, we are not aware of any relevant examples. In Table 7, we use our daily price differentials to illustrate how moving from monthly, to weekly and then to daily data reduces half-lives. Daily data in Table 7 are the same as used in Table 6. Weekly data are Wednesdays. Monthly data are monthly averages of our daily data.

TABLE 7

Half Lives in Days: Monthly, Weekly and Daily Data

Ports	Product	Month	Lag/ Test	H.L. Days	Week	Lag/ Test.	H.L Days	Day	Lag Test	H.L. Days
NY/Gulf	Diesel	1995:10-2007:1	0 0.11	35.7	1995:10:4-2007:1:31	2 0.37	14.5	1995:10:2-2007:1:31	4 0.45	9.2
NY/Gulf	Fuel Oil	1993:11-2007:1	0 0.85	55.6	1993:11:3-2007:1:31	0 0.45	28.7	1993:11:1-2007:1:31	2 0.38	20.7
NY/Gulf	Gasoline	1998:6-2007:1	0 0.57	38.9	1986:6:4-2007:1:31	1 0.86	16.1	1986:6:2-1/31/2007	1 0.11	8.7
NY/Gulf	Jet Fuel	1990:12-2007:1	0 0.57	31.8	1990:12:5-2007:1:31	0 0.87	13.7	1990:12:3 2007:1:31	3 0.40	9.8
NY/LA	Diesel	1995:10-2007:1	0 0.51	30.2	1995:10:4-2007:1:31	0 0.64	15.9	1995:10:2-2007:1:31	0 0.19	11.5
NY/LA	Fuel Oil	1993:11-2007:1	0 0.76	58.4	1993:11:3-2007:1:31	0 0.86	32.0	1993:11:1-2007:1:31	0 0.39	24.8
NY/LA	Gasoline	1986:6-2007:1	0 0.16	39.5	1986:6:4-2007:1:31	4 0.42	22.7	1986:6:2-2007:1:31	1 0.11	18.0
NY/LA	Jet Fuel	1990:12-2007:1	0 0.19	32.0	1990:12:5-2007:1:31	1 0.22	17.3	1990:12:3-2007:1:31	3 0.40	13.6
NY/Rott	Fuel Oil	1993:11-2007:1	1 0.92	30.6	1993:11:3-2007:1:31	1 0.67	18.0	1993:11:1-2007:1:31	1 0.18	11.5
NY/Rott	Gasoline	1987:4-2007:1	0 0.22	34.9	1987:4:1-2007:1:31	1 0.31	16.4	1987:4:1-2007:1:31	2 0.10	8.7
NY/Rott	Jet Fuel	1990:12-2007:1	0 0.14	26.1	1990:12:5-2007:1:31	0 0.90	18.5	1990:12:3-1/31/2007	3 0.51	11.4
NY/Sing	Fuel Oil	1993:11-2007:1	2 0.23	76.7	1993:11:3-2007:1:31	0 0.38	41.7	1993:11:1-2007:1:31	0 0.35	33.5
NY/Sing	Gasoline	1992:8-2007:1	0 0.48	43.1	1986:6:4-2007:1:31	1 0.52	22.9	1992:8:3-1/31/2007	5 0.42	5.9
NY/Sing	Jet Fuel	1990:12-2007:1	2 0.28	64.9	1990:12:5-2007:1:31	0 0.83	29.5	1990:12:3-2007:1:31	5 0.11	11.7
Gulf-LA	Diesel	1995:10-2007:1	0 0.92	27.6	1995:10:4-2007:1:31	0/ 0.14	16.4	1995:10:2-2007:1:31	1 0.44	10.9
Gulf-LA	Fuel Oil	1993:11-2007:1	0 .35	72.7	1986:6:4-2007:1:31	0 0.52	33.4	1993:11:1-2007:1:31	0 0.70	25.8
Gulf-LA	Gasoline	1986:6-2007:1	0 0.32	35.4	6/4/1986-1/31/2007	0 0.60	20.6	1986:6:2-2007:1:31	0 0.69	16.0
Gulf-LA	Jet Fuel	1990:12-2007:1	0 0.62	36.2	1990:12:5-2007:1:31	1 0.74	19.4	1990:12:3-2007:1:31	1 0.66	14.2
Gulf-Rott	Fuel Oil	1993:11-2007:1	1 0.16	88.1	1993:11:3-2007:1:31	1 0.55	33.3	1993:11:1-2007:1:31	1/ 0.21	19.2
Gulf-Rott	Gasoline	1987:4-2007:1	0 0.40	31.5	1987:4:1-2007:1:31	1 0.36	15.7	1987:4:1-2007:1:31	3/ 0.61	8.8
Gulf-Rott	Jet Fuel	1990:12-2007:1	0 0.4	28.0	1990:12:5-2007:1:31	0 0.91	16.3	1990:12:3-2007:1:31	2 0.84	10.0
Gulf-Sing	Fuel Oil	1993:8-2007:1	0 0.85	72.1	8/4/1993 1/31/2007	0 0.52	39.0	1993:11:1-2007:1:31	0 0.39	29.2
Gulf-Sing	Gasoline	1992:8-2007:1	0 0.39	47.4	1992:8:5-2007:1:31	1 0.96	22.2	1992:8:3-2007:1:31	3/ 0.14	5.4

Gulf-Sing	Jet Fuel	1990:12-2007:1	3 0.70	100.6	1990:12:5-2007:1:31	1 0.12	34.2	1990:12:3-2007:1:31	6 0.36	12,7
LA-Rott	Fuel Oil	1993:11-2007:1	0 0.55	80.6	1993:11:3-2007:1:31	1 0.69	27.6	1993:11:1-2007:1:31	1 0.39	21.4
LA-Rott	Gasoline	1987:4-2007:1	0 0.65	49.6	4/1/1987-2007:1:31	0 0.20	28.8	1987:4:1-2007:1:31	4 0.24	21.5
LA-Rott	Jet Fuel	1990:12-2007:1	0 0.81	39.0	1990:12:5-2007:1:31	0 0.19	24.1	1990:12:3-2007:1:31	1 0.45	16.8
LA-Sing	Fuel Oil	1993:11-2007:1	2 0.55	31.4	1993:11:3-2007:1:31	1 0.50	17.7	1993:11:1-2007:1:31	0 0.32	11.1
LA-Sing	Gasoline	1992:8-2007:1	0 0.92	45.9	1992:8:5-2007:1:31	0 0.13	25.9	1992:8:3-2007:1:31	0 0.35	21.2
LA-Sing	Jet Fuel	1990:12-2007:1	0 0.41	66.6	1990:12:5-2007:1:31	2 0.43	40.0	1990:12:3-2007:1:31	3 0.71	21.3
Rott-Sing	Fuel Oil	1993:8-2007:1	0 0.85	81.6	1993:11:3-2007:1:31	1 0.99	43.2	1993:11:1-2007:1:31	3 0.13	28.3
Rott-Sing	Gasoline	1992:8-2007:1	0 0.71	62.7	1992:8:5-2007:1:31	0 0.23	35.5	1992:8:3-2007:1:31	1 0.29	23.2
Rott-Sing	Jet Fuel	1990:12-2007:1	0 0.67	83.8	1990:12:5-2007:1:31	2 0.65	40.7	1990:12:3-2007:1:31	5 0.86	21.2
Mean				50.9			25.5			16.4

Using weekly rather than monthly data reduces our petroleum half-lives by about 50%, from an average of about 50 days for monthly data to an average of about 25 days for weekly data. Using daily rather than weekly data reduces them by about another third from 25.5 to 16.3 days.

The grain prices in Tables 3 and 4 are monthly averages of daily prices.[17] If using daily instead of monthly data had the same effect for grain half-lives as it does for petroleum half-lives, the average half-life in Table 3 would be about 13 days. For Table 4, the average half-life would be about 9 days.

We conjecture that, if we had commodity tick data like Akram et al (2008) have for Covered Interest Parity, we would measure half-lives in hours not days.

8. Borders.

Retail price differentials produce wide borders. With auction price differentials, borders disappear. A simple comparison of petroleum half-lives between New York and Gulf ports

[17] We have tried to obtain the daily data, but without success.

versus from New York to Rotterdam and Gulf ports to Rotterdam rejects wide borders. Wide borders imply that the average half-life from New York and Gulf ports to Rotterdam in Table 6 should be much larger than the average half-life between New York and Gulf ports. In spite of the fact that New York and Gulf ports are closer to each other than either is to Rotterdam, the average half-life between New York and Gulf ports is 13 days while the average half-life between those ports and Rotterdam is only 12 days. That difference in favor of Rotterdam is hard to reconcile with a wide border.

The Borders literature compares the variances of $\Delta[logP_k(t) - logP^*(t)]$ where both retail prices are for similar goods in the same country, to variances of $[logP_k(t) - logP^*(t)S(t)]$ where both prices are for similar goods but $P_k(t)$ are domestic and $P^*(t)$ are foreign. Exchange rates, $S(t)$, are the domestic price of foreign exchange and are from auction markets. They use this measure of volatility because in levels international retail price differentials may not be stationary. If not, comparing border width in levels would be comparing levels of infinity.

Research supporting wide borders regresses volatility against distance and a border dummy. Both distance and the border dummy are positive and significant.

We run the same regression using half-lives for auction price differentials. The first line in Table 8 reports our results from regressing the half-lives from Tables 3 and 6 against a border dummy and two different measures of distance: the Great Circle Route used by Engel and Rogers (1996) and by boat. In both cases the $-^2$ is negative, distance is insignificant and the border dummy is negative and insignificant.

TABLE 8

Half Lives in Days versus Distance and a Border Dummy*

Half-Lives	Intercept	Distance	Border	$-^2$	Half-Lives	Intercept	Distance	Border	$-^2$
Tables	29.3	-00004[G]	-4.9	-0.03	Tables	25.3	0.0007[B]	-9.6	-0.02

3&6	(9.05)	(0.0012)	(12.4)		3 & 6	(7.67)	(0.0011)	(12.3)	
Excluding Gulf-LA, NY-LA & Gulf-PP	26.3 (8.68)	0.0013^G (0.0017)	-9.8 (13.7)	-0.01	Excluding Gulf-LA, NY-LA & Gulf-PP	14.6 (5.29)	0.0031^B (0.0017)	-11.5 (12.6)	0.008

* Standard errors in parentheses. ^{G} Great Circle Route. ^{B} By boat.

We suspect that the price differentials between the East and West coasts create a problem. Using distance by boat produces a distance of about 5,000 miles, which seems too far. Using the great circle route ignores the low cost of moving petroleum products by super tankers. In addition, oil does not move from one coast to the other as much as it moves from Texas, Oklahoma and other inland locations to the two coasts. To try to account for these possible problems, the second line of Table 8 excludes price differentials between the two coasts.

For distance by boat, that adjustment produces a distance measure that is significant at 10% and an -^{2} that is small, but not negative. But the border dummy remains negative and not significant. There is no evidence of a wide border in Table 8. In fact, there is no evidence of a border.

The Borders literature creates wide borders out of thin air by mixing retail price differentials with auction exchange rates. Let $logP_k(t)$ represent some U.S. retail price, $P^*(t)$ a similar retail price either in the U.S or abroad and $S(t)$ an auction price of the foreign currency. Engel and Rogers (1996) and others who claim to find wide borders compare the variance of $\Delta[\log P_k(t)- logP^*(t)]$ where both prices are in different cities in the same country to the variance of $\Delta[\log P_k(t)- logP^*(t)S(t)]$ where $P_k(t)$ is domestic and $P^*(t)$ foreign. (For relative price levels, $P(t)$ and $P^*(t)$ are CPIs.) They conclude that borders are wide because the variance of $\Delta[\log P_k(t)- logP^*(t)S(t)]$ using similar products is much larger than the variance of $\Delta[\log P_k(t)- logP^*(t)]$

and the variance of $\Delta[\log P(t) - \log P^*(t)S(t)]$ using CPIs is much larger than the variance of $\Delta[\log P(t) - \log P^*(t)]$.

PPP and the LOP imply the opposite result. With PPP, changes in $S(t)$ should offset changes in $P(t)/P^*(t)$ caused by inflation, *reducing* the volatility of $\Delta[\log P_k(t) - \log P^*(t)S(t)]$ relative to $\Delta[\log P_k(t) - \log P^*(t)]$. According to the LOP, arbitrage equates $\log P_k(t)$ and $\log P^*(t)S(t)$, *reducing* the variance of $\Delta[\log P_k(t) - \log P^*(t)S(t)]$ relative to $\Delta[\log P_k(t) - \log P^*(t)]$.

Table 9 uses prices and exchange rates from Engel and Rogers (19960) and our silver data to show how a combination of retail prices and auction exchange rates creates an illusion of wide borders.[18]

TABLE 9

Variances for $\Delta[\log P_k(t) - \log P^*(t)S(t)]$ and $\Delta[\log P_k(t) - \log P^*(t)]$

Variance	Silver	CPI	Food at Home	Food Away from Home	Alcoholic Beverages	Shelter	Fuel and Utilities
$\Delta[\log P_k(t) - \log P^*(t)S(t)]$	0.00015	0.00147	0.00026	0.00019	0.00031	0.00029	0.00292
$\Delta[\log P_k(t) - \log P^*(t)]$	0.00062	0.00005	0.00016	0.00009	0.00020	0.00020	0.00288
Ratio	0.24	29.4	1.6	2.1	1.6	1.45	1.01

The line labeled $\Delta[\log P_k(t) - \log P^*(t)S(t)]$ reports the variances for that price differential starting with silver. The line below reports the variances for $\Delta[\log P_k(t) - \log P^*(t)]$. The bottom line reports the ratio of $\Delta[\log P_k(t) - \log P^*(t)S(t)]$ over $\Delta[\log P_k(t) - \log P^*(t)]$.

For silver, where both prices and exchange rates are auction, including the exchange rate substantially reduces variance. Using CPIs and auction exchange rates, including the exchange rate increases variance by 29 times. When combined with retail commodity prices, including auction exchange rates in Table 9 never reduces variance and often increases it substantially.

[18] We did not cherry pick here or anywhere else. The CPIs and the first 5 of the 10 product groups were chosen before any variances in Table 9 were estimated. It is a sad comment on research in macroeconomics that any such disclaimer is needed.

Wide borders are a statistical artifact created by mixing retail prices and auction exchange rates.

9. PPP.

By now it should be obvious that combinations of retail prices and auction exchange rates do not reject PPP. The variances for relative CPIs in Table 9 illustrates how that combination artificially increases the variance of real exchange rates. Our strong support for the LOP using auction prices provides strong indirect support for PPP. But we will have to wait for auction price levels before we can test PPP appropriately. *Until then, we should restore absolute PPP to not rejected.*

10. Puzzles.

The Keynesian paradigm assumes sticky (retail) commodity prices and flexible (auction) asset prices where exchange rates are assumed to be asset prices. That Keynesian combination of retail commodity prices and auction asset prices creates several exchange rate related puzzles. We consider five: (1) PPP works during inflation but not in normal times, (2) PPP may work in the long run but not the short run, (3) the long half-lives for deviations from PPP, (4) the apparent excessive volatility of exchange rates and (5) the exchange rate disconnect.

10.1. Inflation versus Normal.

Frenkel (1981) is an important source of the belief that Purchasing Power Parity works during inflation but fails in normal times. Using wholesale and cost of living price indexes, he compares the performance of relative price levels during the inflationary 1920s to their performance during the "normal" 1970s. His results for wholesale and cost of living indexes are similar. During “normal” times -^{2}s are close to zero and coefficients estimated imprecisely. During inflation, -^{2}s are substantial and coefficients estimated with precision. He concludes that PPP worked during the inflationary 1920s, but failed during the more normal 1970s.

Davutyan and Pippenger (1985) point out that his results are a statistical illusion due to thresholds. A simple example makes their point. Suppose relative price levels are essentially constant during normal times and exchange rates never exceed what would be reasonable thresholds created by transaction costs. PPP always holds, but -^{2}*s* are close to zero and regression coefficients imprecise because there is no link between relative price indexes and exchange rates.

Now consider the case where relative price levels and exchange rates both rise due to inflation and exchange rates often exceed reasonable thresholds. -^{2}*s* are larger and coefficients more precise, but PPP works better during normal times than during inflation. In the presence of thresholds, we must interpret regression results carefully.[19]

This puzzle is the result of mixing retail prices with auction exchange rates in the context of thresholds. In normal times, relative CPI volatility is small due to sticky retail prices and any links to auction exchange rates are, at best, very weak. Wholesale prices are less sticky but empirically they do only slightly better in normal times.

As inflation increases, retail and wholesale prices become more flexible. In hyperinflations like those in the 1920s, retail prices approach the flexibility of auction prices in “normal” times. -^{2}s increase as both prices and exchange rates respond to inflation.

With auction prices, the difference between inflationary and normal times should largely disappear. With or without inflation, auction prices are very flexible and closely linked to

[19] Davutyan and Pippenger (1985) suggest looking at what is not explained rather than what is explained.

exchange rates. Problems with $-^2$ largely disappear with auction prices and with them most of the apparent distinction between inflationary and normal times.

10.2. Long Run versus Short Run.

The evidence clearly rejects relative CPIs as explanations for short-run exchange rates. But there is some support for them as a long-run explanation. In addition to Sarno and Taylor (2002b) and Taylor (2006), see the articles cited above regarding the long-run versus short-run debate.

The explanation for this puzzle is essentially the same as for Inflation versus Normal. Replace "Inflation" with "long run" and "Normal" with "short run". In the short run, relative CPIs fail because sticky retail prices, the absence of arbitrage and time in transit disconnect spot exchange rates from relative CPIs. In the long run, retail prices become more flexible and weak economic links strengthen, producing more long-run support for relative CPIs.

With auction prices, the difference between short run and long run should largely disappear. In all runs, auction prices are flexible and thresholds narrow because information and transaction costs per dollar traded are relatively low.[20]

10.3. Long Half-Lives for PPP.

Obstfeld and Rogoff (2000) and Sarno (1996) list long half-lives for deviations from PPP, usually in the form of relative CPIs, as one of the major puzzles in international macroeconomics. Half-lives are long when research follows the Keynesian paradigm that combines flexible auction exchange rates with sticky retail prices.

[20] Without restrictions, appealing to information and transaction costs can explain anything, which means they explain nothing. We assume that such costs behave like other costs. More precisely, they behave like the postulates on costs in Alchian (1959).

The short half-lives for deviations from the *auction* LOP imply that PPP should hold even in the short-run using auction prices. Auction prices are far more flexible and thresholds are much narrower because information and transaction costs per dollar traded are much smaller where commodities are traded by the shipload rather than by the pound.

The relevant auction data do not yet exist. But PPP is based on the LOP and there are auction prices for a variety of commodities. As shown above, monthly auction grain prices and daily auction petroleum prices indicate that the LOP holds even in the short run, *e.g*., monthly. This evidence provides strong indirect support for short-run PPP.

10.4. Excessive Volatility.

As is well known, the volatility of exchange rates is much larger than the volatility of relative CPIs.[21] This difference in volatility is the primary evidence behind the mistaken belief that exchange-rate volatility is "excessive". The explanation for this puzzle is similar to the one for the three previous puzzles. Exchange rates are from volatile auction markets while CPIs use sticky retail prices.

Although we currently do not have baskets of auction prices appropriate for PPP, there are individual spot auction prices, which can give us some insight into auction PPP. At least it compares auction to auction.

Using weekly spot data from U.S. and Canadian grain markets, Bui and Pippenger (1990) report that the volatility of exchange rates implied by the volatility of relative prices, *i.e.,* the variance of $\Delta log[P_k(t)\backslash P^*(t)]$, is slightly *greater* than the volatility of actual spot exchange rates, *i.e*., the variance of $\Delta logS(t)$.

[21] We are unaware of any articles comparing the volatility of relative wholesale price indexes to the volatility of exchange rates.

Using auction exchange rates and our auction prices for silver, the variance of $\Delta\log[P_k(t)\backslash P^*(t)]$ is 0.00062 while the variance of $\Delta\log S(t)$ is 0.00048. Again, the volatility of exchange rates is less than that implied by the LOP.[22]

As shown in Table 9, excessive volatility appears to be a statistical artifact created by mixing retail prices with auction exchange rates. When we finally collect the appropriate data, comparing the volatility of exchange rates to the volatility of relative baskets of auction prices with the same weights should finally dispel any lingering belief in excessive volatility.

10.5. Exchange-Rate Disconnect.

The exchange-rate disconnect refers to the lack of any clear link between exchange rates and economic fundamentals. It is another one of the six major puzzles in Obstfeld and Rogoff (2000). Our results for the LOP suggest a strong link between exchange rates and auction prices. If the LOP holds for auction commodity markets in general like it does for grains, silver and petroleum products then there should be a strong link between relative auction price levels and exchange rates.

11. Implications for Macroeconomics.

The Keynesian paradigm dominates macroeconomics. Two of its important assumptions are: (1) exchange rates are asset prices and (2) asset markets clear relatively quickly while commodity markets clear relatively slowly.

We reject both assumptions. By supporting PPP, our results reject the idea that exchange rates are asset prices.[23] By pointing out that all *retail* markets, commodity and asset, clear

[22] We would expect the variance of relative auction prices to be greater than the variance of exchange rates. In a basket of goods, changes in relative prices tend to average out. So, what we find with the LOP is consistent with PPP.

[23] The asset approach to exchange rates is essentially an extension of Liquidity Preference to an open economy. Therefore, by rejecting the asset approach, we are indirectly rejecting Liquidity Preference. The only two attempts to

relatively slowly while all *auction* markets clear relatively rapidly, we reject the Keynesian assumption that asset markets clear rapidly and commodity markets clear slowly.

12. Summary and Conclusion.

Articles using retail price differentials claim that borders are wide and reject both the LOP and PPP. But those rejections are invalid because retail price differentials do not represent potential risk-free profits. In addition, retail markets are highly segmented because there is no arbitrage or even trade. In auction markets, borders hardly exist, the LOP holds with half-lives measured in days and PPP probably works even in the short run. Until we are able to test PPP with the appropriate auction prices, *PPP should be restored to not rejected.*

The distinction between retail and auction markets also helps resolve five puzzles associated with exchange rates. All five are the result of using auction exchange rates and retail prices. Using auction exchange rates and auction prices should resolve all five.

Finally, we raise some serious macroeconomic questions. The Keynesian paradigm dominates macroeconomics. Two of its important assumptions are: (1) exchange rates are asset prices and (2) asset markets clear relatively quickly while commodity markets clear relatively slowly.

We reject both assumptions. By supporting PPP, our results reject the idea that exchange rates are asset prices. By pointing out that all *retail* markets, commodity and asset, clear relatively slowly while all *auction* markets clear relatively rapidly, we reject the Keynesian assumption that asset markets clear rapidly and commodity markets clear slowly.

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evaluate Liquidity Preference and Loanable Funds head to head of which we are aware, Pippenger (2003 and 2007), find that the weight of evidence favors Loanable Funds.

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