Title

Forces in a Thin Multipole-magnet Coil

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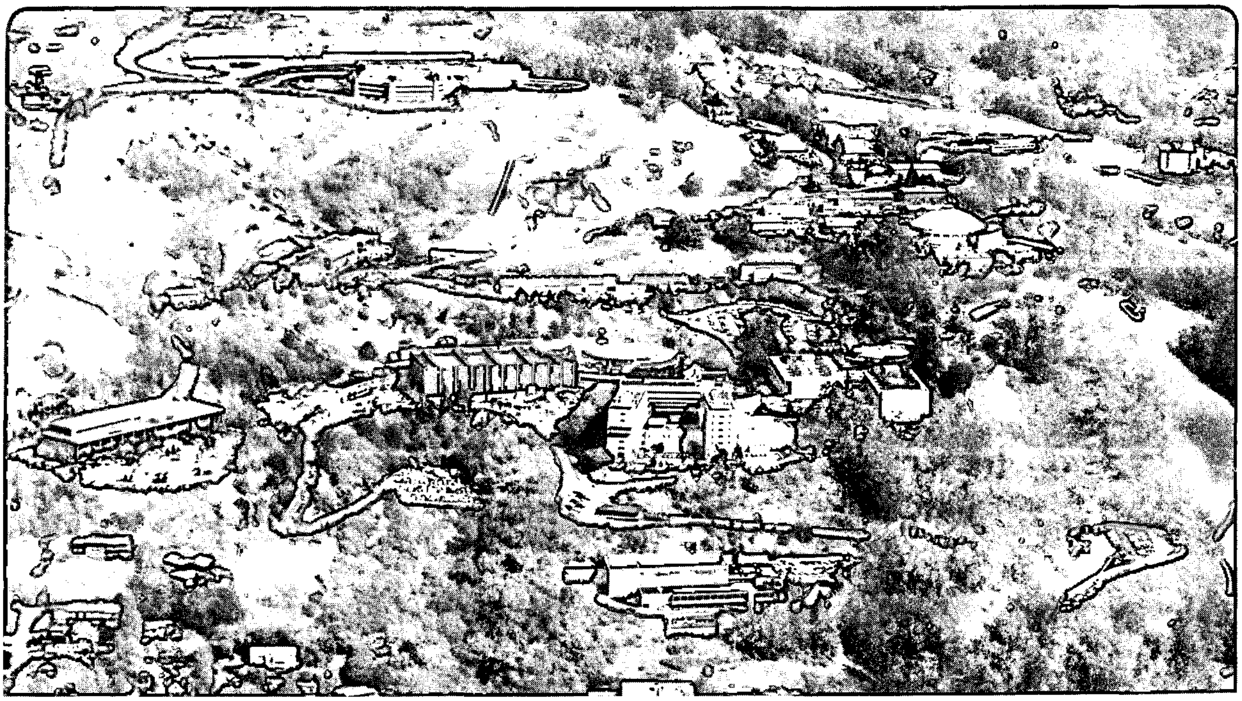

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PROGRAM — PROJECT — JOB PEP II RING MAGNET DEVELOPMENT			
Analysis			
TITLE Forces in a thin multipole-magnet coil			

A rev., 11-3-80
OBSOLETE: See M5597, 10-13-80

For a thin circular coil of the usual "multipole magnet" type the fields just inside the coil are (Eng. Note M5251)

$$\begin{vmatrix} B_{r1} \\ B_{\theta 1} \end{vmatrix} = -\frac{\mu_0 J_0}{2}\left[1+\left(\frac{a}{b}\right)^{2m}\right]\begin{vmatrix} \sin m\theta \\ \cos m\theta \end{vmatrix}$$

$$\begin{vmatrix} B_{r2} \\ B_{\theta 2} \end{vmatrix} = -\frac{\mu_0 J_0}{2}\begin{vmatrix} 1+\left(\frac{a}{b}\right)^{2m} \\ -1+\left(\frac{a}{b}\right)^{2m} \end{vmatrix}\begin{vmatrix} \sin m\theta \\ \cos m\theta \end{vmatrix}$$

The vector average of the two fields is

$$\begin{vmatrix} \bar{B}_r \\ \bar{B}_\theta \end{vmatrix} = -\frac{\mu_0 J_0}{2}\begin{vmatrix} 1+\left(\frac{a}{b}\right)^{2m} \\ \left(\frac{a}{b}\right)^{2m} \end{vmatrix}\begin{vmatrix} \sin m\theta \\ \cos m\theta \end{vmatrix}$$

The electromagnetic body force is

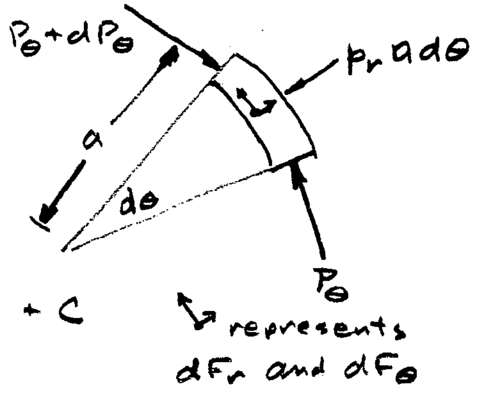
$$dF_r = -\bar{B}_\theta\, dI, \quad dF_\theta = +\bar{B}_r\, dI$$

and $dI = J_0 a \cos m\theta\, d\theta$, so

$$dF_r = +\frac{\mu_0 J_0^2 a}{2}\left(\frac{a}{b}\right)^{2m}\cos^2 m\theta\, d\theta$$

$$dF_\theta = -\frac{\mu_0 J_0^2 a}{2}\left[1+\left(\frac{a}{b}\right)^{2m}\right]\sin m\theta \cos m\theta\, d\theta$$

For a coil that is free to slip within a surrounding structure the equilibrium requirements are



$$dP_\theta = dF_\theta$$

$$p_r a\, d\theta = P_\theta\, d\theta + dF_r$$

Upon integrating P_θ we get

$$P_\theta = -\frac{\mu_0 J_0 a}{2}\left[1+\left(\frac{a}{b}\right)^{2m}\right]\frac{1}{2m}\sin^2 m\theta + C$$

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When we adjust C to make $P_\theta = 0$ at the poles where $\theta = \pi/2m$ we get

$$P_\theta = \frac{\mu_0 J_0^2}{2} a \left[1 + \left(\frac{a}{b}\right)^{2m}\right] \frac{1}{2m} \cos^2 m\theta$$

The radial pressure equation then becomes

$$p_r\, a\, d\theta = \frac{\mu_0 J_0^2}{2} a \left[1 + \left(\frac{a}{b}\right)^{2m}\right] \frac{1}{2m} \cos^2 m\theta \, d\theta$$

$$+ \frac{\mu_0 J_0^2}{2} a \left(\frac{a}{b}\right)^{2m} \cos^2 m\theta \, d\theta$$

or $$p_r = \frac{\mu_0 J_0^2}{2} \frac{1}{2m} \left[1 + (1+2m)\left(\frac{a}{b}\right)^{2m}\right] \cos^2 m\theta$$

Alternatively it is sometimes most convenient to express the pressure in terms of the field in the aperture referred to the coil radius.

The magnitude of the field vector just inside the coil is

$$|B_{a,in}| = \frac{\mu_0 J_0}{2} \left[1 + \left(\frac{a}{b}\right)^{2m}\right]$$

which leads to

$$\frac{\mu_0 J_0^2}{2} = \frac{|B_{a,in}|^2}{2\mu_0} \frac{4}{\left[1 + \left(\frac{a}{b}\right)^{2m}\right]^2}$$

As a second alternative, the field in the aperture is often expressed as the gradient $d|B|/dr$ for a quadrupole magnet $(m=2)$, $d^2|B|/dr^2$ for a sextupole magnet $(m=3)$, and so forth — or as $d^{m-1}|B|/dr^{(m-1)}$ for a 2m-pole magnet, and it is convent to express the pressure in such terms

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The appropriate derivatives can be expressed

$$\frac{d^{m-1}|B|}{dr^{m-1}} = \frac{\mu_0 J_0}{2}\,\frac{1}{a^{m-1}}\,(m-1)!\,\left[1+\left(\frac{a}{b}\right)^{2m}\right]$$

provided we take $0! \equiv 1$, which my HP45 gives.

So

$$\frac{\mu_0 J_0^2}{2} = \left[\frac{d^{m-1}|B|}{dr^{m-1}}\right]\frac{2a^{2(m-1)}}{\mu_0\,\{(m-1)!]^2}\,\left[1+\left(\frac{a}{b}\right)^{2m}\right]^{-2}$$

Units

B, $\mu_0 J_0$, have units of teslas

$\frac{B^2}{\mu_0}$, μJ_0^2, p have units of newtons/meter^2

F, P have units of newtons/meter

J_0 has units of amperes

a, b, r have units of meters

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