Title
Searching for E(5) behavior in nuclei

Permalink
https://escholarship.org/uc/item/59g0c4b3

Authors
Clark, R.M.
Cromaz, M.
Deleplanque, M.A.
et al.

Publication Date
2004

Searching for E(5) Behavior in Nuclei

R.M. Clark, M. Cromaz, M.A. Deleplanque, M. Descovich, R.M. Diamond, P. Fallon,
I.Y. Lee, A.O. Macchiavelli, H. Mahmud, E. Rodriguez–Vieitez, F.S. Stephens, D. Ward
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720

(March 23, 2004)

Abstract

The properties of even–even nuclei with 30≤Z≤82, A≥60 have been examined to find examples displaying the characteristics of E(5) critical–point behavior for the shape transition from a spherical vibrator to a triaxially soft rotor. On the basis of the known experimental state energies and E2 transition strengths, the best candidates that were identified are ^{102}Pd, ^{106,108}Cd, ^{124}Te, ^{128}Xe, and ^{134}Ba. The closest agreement between experimental data and the predictions of E(5) is for ^{128}Xe and for the previously suggested example of ^{134}Ba. It is proposed that ^{128}Xe may be a new example of a nucleus at the E(5) critical point.

PACS numbers: 21.60.-n; 21.10.-k

Notable benchmarks of collective nuclear behavior are the harmonic vibrator [1], the symmetrically deformed rotor [2], and the triaxially soft rotor [3]. They correspond to limits of the Interacting Boson Model (IBM) and an algebraic description of the nature of the transition between these limits has been developed in direct analogy with classical phase transitions [4]. Recently, it has been suggested that a useful approach is to find an analytic approximation of the critical point of the shape change as a new benchmark against which nuclear properties can be compared [5,6].

The critical–point description of the transition from a symmetrically deformed rotor to a spherically harmonic vibrator, denoted as X(5), involves the solution of the Bohr collective Hamiltonian with a potential that is decoupled into two components – an infinite square well potential in the quadrupole defomation parameter, β, and a harmonic potential well for the triaxiality deformation parameter, γ. This is an approximation of the 'true' potential found at the critical point of the shape change from IBM calculations [6]. Several examples of nuclei close to the X(5) critical point have been suggested including ^{150}Nd [7] and ^{152}Sm [8,9]. However, not all the predicted characteristics of X(5) are reproduced and the applicability of the description is still a topic of discussion (see, for example, [10]). Recently, a search was carried out to find other examples of nuclei which display the predicted characteristics of the X(5) critical–point description [11]. It was found that the best candidates for X(5) behavior are ^{126}Ba, ^{130}Ce, and the N=90 isotones of Nd, Sm, Gd, and Dy.

The purpose of this paper is to report the results of a search to find examples of nuclei near the critical–point of the transition from a triaxially soft rotor to a spherically harmonic vibrator, denoted as E(5). This description involves the solution of the Bohr Hamiltonian with an infinite square well potential depending only on β [5]. IBM calculations indicate that this is a fair approximation to the flat–bottomed potential that is calculated at the critical point of the shape transition from gamma–soft rotor to harmonic vibrator. IBM calculations have also been used to account for finite boson number effects [12–15] which alter the predictions of absolute values, and ratios, of both state energies and transition strengths. The transition strengths are also affected by the level of approximation regarding the quadrupole transition operator. Initial calculations used a first-order quadrupole operator [5] but including second–order terms alters the predictions significantly [16].

It is important to identify a set of observables which are characteristic of E(5) behavior and which do not change dramatically with the details of a given calculation. Table I presents a set of such robust observables. The Table includes the original predictions for E(5) [5], the results for E(5) using the second–order quadrupole operator [16], and the results of an IBM calculation at the critical–point of the shape transition for different boson numbers [15].

A few words of explanation are required to understand the Table. Eigenfunctions from the E(5) solution can be characterized by two quantum numbers (ξ,τ) related to zeros of Bessel functions as described in [5]. The ξ quantum number labels major families of E(5) levels, while τ labels the phonon-like levels within a given ξ family. The first excited 0^+ state is predicted to be in the ξ=2 family and is labelled in shorthand as 0^+_ξ. The second excited 0^+ state is predicted to belong to the three phonon-like multiplet of the ξ=1 family (that is, (ξ,τ) =(1,3)) and is labelled as 0^+_τ. All other states are labelled in more conventional notation (for example, 2^+_2 indicates the second 2^+ state).

The evolution of these 0^+ states can be traced from the limit of the spherical vibrator to the gamma–soft rotor. In the vibrator limit the 0^+_ξ state is a member of the two–phonon multiplet with a strongly allowed E2 branch to the 2^+_1 one–phonon level. In the gamma–soft limit this level has risen in excitation energy and the E2 transition to the 2^+_1 state is forbidden. In contrast, the 0^+_τ level preserves its three–phonon character across the entire shape change. For real nuclei, the degeneracies of the multiplet structure will only be imperfectly preserved due to mixing with other states but fingerprints of multiphonon excitations can be reasonably expected to survive [17].

The key observables presented in Table I will be used to identify candidate nuclei for E(5) behavior. The energy and transition rate predictions are parameter free except for overall scaling factors. The quantities are normalized to either the energy of the 2^+_1 state or to the B(E2;2^+_1 $\rightarrow 0^+_1$). Some of the key properties of the E(5) description can be summarized as follows:

1. The energy ratio E(4_1^+)/E(2_1^+) should be ≈2.20.
2. The B(E2;$4_1^+ \rightarrow 2_1^+$) value should be ≈1.5 times the B(E2;$2_1^+ \rightarrow 0_1^+$) value.
3. There should be two excited 0^+ states lying at approximately 3–4 times the energy of the 2_1^+ state.
4. The decay of the 0_τ^+ should reflect its multiphonon structure. There is an allowed E2 transition to the 2_2^+ level, but no allowed transition to the 2_1^+ level.
5. The decay of the 0_ξ^+ state should also be characteristic of E(5). There is an allowed transition to the 2_1^+ level with a strength of ≈0.5 the B(E2;$2_1^+ \rightarrow 0_1^+$) value.

The first two points reflect the fact that the E(5) behavior lies intermediate between that for the harmonic vibrator and γ–soft rotor. While the ordering of the 0_τ^+ and 0_ξ^+ states is sensitive to effects such as the number of bosons (see Table I), their decays are reflective of the E(5) symmetry properties.

If the E(5) description is to be taken as a benchmark for nuclear shape transitions, then it is important to find examples which follow the predicted behavior. The first suggested example was ^{134}Ba [12]. Other candidates that have been put forward include ^{102}Pd [14] and ^{104}Ru [13]. The purpose of this paper is to report a search for examples of nuclei that are candidates for E(5) critical–point behavior.

As a starting point the ENSDF data file [18] [1] was searched for examples of even–even nuclei, with $30 \leq Z \leq 82$, $A \geq 60$, with $2.00 < E(4_1^+)/E(2_1^+) \leq 2.40$. As pointed out by Mallmann [19] this ratio (and other similar ratios) are characteristic of different collective motions of the nucleus. The value expected for a harmonic vibrator is E(4_1^+)/E(2_1^+)=2.00 while that for a γ–soft rotor is E(4_1^+)/E(2_1^+)=2.50. The value of this ratio for an E(5) nucleus is predicted

---

[1] The ENSDF data file used in the search was last updated in December 2002. It does not necessarily include all published information up to that date since certain mass chains may not have been evaluated for several years.

to be 2.20. Over 70 nuclei were found within the search range of E(4_1^+)/E(2_1^+) values and they are presented in Table II.

The next criterion used in the search was to identify the subset of these nuclei which have their two lowest, firmly assigned, excited 0^+ states lying between 2.5 and 4.5 times the energy of the 2_1^+ level. The excitation energies, normalized to the energy of the 2_1^+ level, of the two lowest excited 0^+ states, if known in the candidate nuclei, are also shown in Table II. By applying this criterion the scope of the search is restricted in two important ways. First, only nuclei with relatively complete experimental level schemes up to an excitation energy of 4.5 times the energy of the 2_1^+ level are likely to pass this condition. Second, only the two lowest excited 0^+ states are being examined and, therefore, there is an implicit assumption that these two states can be mapped to the two lowest 0^+ excitations in the E(5) description. If other excited 0^+ levels (for example, based on intruder configurations) come lower than 2.5 times E(2_1^+), the candidate nucleus is excluded from the search. This may then eliminate potential candidates in which higher excited 0^+ states (still within the correct energy range) correspond to the 0_τ^+ and 0_ξ^+ levels.

By applying this criterion, there are only six candidate nuclei that remain. These are ^{102}Pd, ^{106,108}Cd, ^{124}Te, ^{128}Xe, and ^{134}Ba (see Table II). Figs. 1 and 2 compare available information on the state energies (normalized to the energy of the 2_1^+ states) and E2 transition strengths (normalized to the B(E2;$2_1^+ \rightarrow 0_1^+$) values) in each of these nuclei with the predictions of E(5) (using the higher–order quadrupole operator [16] to calculate E2 strengths).

Each of these candidate nuclei can now be examined in more detail:

^{102}Pd: It has been previously suggested that ^{102}Pd may be an example of an E(5) nucleus [14]. The normalized B(E2;$4_1^+ \rightarrow 2_1^+$) value is in good agreement with the predicted value. However, the normalized B(E2;$2_2^+ \rightarrow 2_1^+$) is approximately a factor of 3 too low. (As shall be seen, this anomalously low strength for the $2_2^+ \rightarrow 2_1^+$ transition occurs in all three E(5) candidate nuclei in the A$\sim$100–110 mass region). Moreover, while the normalized B(E2;$4_2^+ \rightarrow 2_2^+$) strength is in fair agreement with the E(5) prediction, the normalized B(E2;$4_2^+ \rightarrow 4_1^+$) strength has an upper limit which is a factor of 2 lower than expected. The two known excited 0^+ states have measured lifetimes. The 0_3^+ level decays to the 2_1^+ level with an E2 transition strength that is in good agreement with the expected strength for the $0_\xi^+ \rightarrow 2_1^+$ transition. It might then be reasonable to associate this 0_3^+ level with the 0_ξ^+ state of the E(5) description. However, the 0_2^+ level of ^{102}Pd cannot be the 0_τ^+ state. The experimental half–life of the 0_2^+ state is 14.5(4)ns implying non–collective E2 transition strengths to lower–lying states. As argued in reference [14] this 0_2^+ level is probably based on an intruder configuration and is outside of the E(5) model space.

^{106}Cd: For ^{106}Cd absolute E2 transition strengths are known for only a few transitions. The normalized B(E2;$4_1^+ \rightarrow 2_1^+$) value is close to that predicted from the E(5) picture. The normalized B(E2;$2_2^+ \rightarrow 2_1^+$) is approximately a factor of 3 too low. There is only experimental information on branching ratios for transitions from other relevant states in ^{106}Cd. For instance, B(E2;$4_2^+ \rightarrow 2_2^+$)/B(E2;$4_2^+ \rightarrow 2_1^+$)≈84 (see Fig. 1) indicating a strongly favored E2 branch from the 4_2^+ state to the 2_2^+ state which matches the predicted behavior. The 0_2^+ level has only one known E2 branch which is to the 2_1^+ level and it might, therefore, be associated with the 0_ξ^+ state in the E(5) picture. The energy of this level is a little lower than predicted (also the case in ^{102}Pd). For the 0_3^+ level B(E2;$0_3^+ \rightarrow 2_2^+$)/B(E2;$0_3^+ \rightarrow 2_1^+$)≈230, indicating that it might be associated with the 0_τ^+ state. The excitation energy is approximately correct (experimentally E(0_τ^+)/E(2_1^+)=3.39 compared to a predicted value of 3.59).

^{108}Cd: The decay scheme of ^{108}Cd is very similar to that of ^{106}Cd. Again, the normalized B(E2;$4_1^+ \rightarrow 2_1^+$) value is close to the predicted value while the normalized B(E2;$2_2^+ \rightarrow 2_1^+$) is too low (by a factor of ≈2). The ratio B(E2;$4_2^+ \rightarrow 2_2^+$)/B(E2;$4_2^+ \rightarrow 2_1^+$)≈12 indicates a favored E2 branch from the 4_2^+ level to the 2_2^+ level. The 0_3^+ state might be associated with the 0_τ^+ state – it has a favored E2 branch to the 2_2^+ level (B(E2;$0_3^+ \rightarrow 2_2^+$)/B(E2;$0_3^+ \rightarrow 2_1^+$)≥1000). The 0_2^+ level has only one known decay branch, which involves an E2 transition to the 2_1^+ level. It would then seem reasonable to associate this level with the 0_ξ^+ state. However, a recent study [20] has identified transitions above this level that appear to form an intruder band. The 0_2^+ level is then most likely based upon an intruder configuration, analogous to those known in the heavier Cd isotopes. It is possible that the 0_2^+ level in ^{106}Cd might also be based on a similar intruder configuration.

We now look at the candidates in the A∼120–130 region in more detail.

^{124}Te: The value of $E(4_1^+)/E(2_1^+) \simeq 2.07$ for ^{124}Te is significantly lower than the $E(4_1^+)/E(2_1^+)$ ratio for any of the other candidates that are being examined. An $E(4_1^+)/E(2_1^+)$ value so close to 2.0 would suggest that the nucleus is near to the vibrational limit. However, the 0_2^+ state, which would then be expected to exist at a similar energy to the 4_1^+ level as a member of the two–phonon multiplet, lies at a normalized energy of $E(0_2^+)/E(2_1^+) \simeq 2.75$. (The 0_2^+ level is approximately 400 keV higher than the 4_1^+ level). In Fig. 2 the level scheme and known E2 transition strengths [21] are shown in more detail. The normalized $B(E2;4_1^+ \rightarrow 2_1^+)$ and $B(E2;2_2^+ \rightarrow 2_1^+)$ transition strengths have large uncertainties but are consistent with the predictions of the E(5) model. For other relevant E2 transitions in ^{124}Te only branching ratios or lower limits are known, but these are also consistent with the expectations of E(5) behavior. For instance, $B(E2;4_2^+ \rightarrow 2_2^+)/B(E2;4_2^+ \rightarrow 2_1^+) \approx 5$, indicates a favored E2 decay to the 2_2^+ level. The 0_2^+ state has only one known E2 branch which decays into the 2_1^+ state with $B(E2;0_2^+ \rightarrow 2_1^+)/B(E2;2_1^+ \rightarrow 0_1^+) > 0.67$, suggesting that this state might be associated with the 0_ξ^+ state. The 0_3^+ level has only one known E2 branch which decays to the 2_2^+ level with $B(E2;0_3^+ \rightarrow 2_2^+)/B(E2;2_1^+ \rightarrow 0_1^+) > 1.76$. This suggests that it could be the 0_τ state.

^{128}Xe: For ^{128}Xe, a remarkably good agreement is found between the predictions of E(5) and the available experimental information. For the absolute measured E2 transition strengths, both the normalized $B(E2;4_1^+ \rightarrow 2_1^+)$ and $B(E2;6_1^+ \rightarrow 4_1^+)$ transition strengths are in perfect agreement (within errors) of the E(5) predictions. The normalized $B(E2;2_2^+ \rightarrow 2_1^+)$ and $B(E2;2_2^+ \rightarrow 0_1^+)$ also match well. For other relevant transitions only information on branching ratios is available. However, this also fits with the E(5) predictions. The ratio $B(E2;4_2^+ \rightarrow 2_2^+)/B(E2;4_2^+ \rightarrow 2_1^+) \approx 57$ (see Fig. 2) indicates a strongly favored E2 branch from the 4_2^+ state to the 2_2^+ state. The 0_2^+ level has $B(E2;0_2^+ \rightarrow 2_2^+)/B(E2;0_2^+ \rightarrow 2_1^+) \approx 14$, suggesting that it might be associated with the 0_τ^+ state, and the energy is in remarkable agreement if this is true (experimentally $E(0_\tau^+)/E(2_1^+) = 3.57$ compared to a predcited value of 3.59). The 0_3^+ level has only one known E2 branch which is to the 2_1^+ level and it could therefore be associated with the 0^+_ξ state even though its excitation energy is significantly higher than predicted. Overall, the available experimental information on relevant states and transitions in ^{128}Xe is in good agreement with the predictions of E(5).

^{134}**Ba:** The nucleus ^{134}Ba was the first proposed E(5) candidate [12]. A good agreement is found with the predictions of E(5) and the available experimental information. (With the exception of the 3^+_1 state which has a lower than predicted excitation energy and an anomalously low B(E2;$3^+_1 \rightarrow 2^+_2$) transition strength). The normalized B(E2;$4^+_1 \rightarrow 2^+_1$) and B(E2;$2^+_2 \rightarrow 2^+_1$) transition strengths agree with the predictions of the E(5) model. The ratio B(E2;$4^+_2 \rightarrow 2^+_2$)/B(E2;$4^+_2 \rightarrow 2^+_1$)$\approx$41 indicates a strongly favored E2 branch to the 2^+_2 level. The 0^+_2 level was associated with the 0_τ state since it has a favored E2 decay to the 2^+_2 level with B(E2;$0^+_2 \rightarrow 2^+_2$)/B(E2;$0^+_2 \rightarrow 2^+_1$)$\approx$27. Absolute E2 strengths are known for the transitions from the 0^+_3. The B(E2;$0^+_3 \rightarrow 2^+_1$)/B(E2;$2^+_1 \rightarrow 0^+_1$) value of 0.42(3) agrees well with the expected $0^+_\xi \rightarrow 2^+_1$ normalized transition strength of 0.49 predicted by the E(5) model.

The results of this work can now be summarized. On the basis of the known experimental information, possible E(5) candidates were identified in the A$\sim$100–110 and A$\sim$120–130 regions. It is interesting to note that nuclei in both these mass regions are expected to display transitional behavior from spherical vibration to γ–soft rotation [22,23]. There are no unambiguous examples of E(5) behavior in the A$\sim$100–110 region. The three candidates identified in that mass region all have significant discrepancies with the predicted E(5) behavior. Notably, the $2^+_2 \rightarrow 2^+_1$ strength is anomalously low, while in each of the three cases examined, the lowest two excited 0^+ cannot be mapped unambiguously to the 0^+_τ and 0^+_ξ states of the E(5) picture. Indeed, possible intruder configurations confuse the interpretation. In the A$\sim$120–130 region, ^{124}Te has a very low E(4^+_1)/E(2^+_1)$\simeq$2.07. However, interpreting it as an E(5) nucleus might provide a natural explanation for the higher than expected energy of the first excited 0^+ state. The best candidates for E(5) behavior are ^{128}Xe and the previously suggested example of ^{134}Ba. It is proposed that ^{128}Xe is a new example of an E(5) nucleus.

Several nuclei in the Xe, Ba, and Ce region have been described in terms of γ–soft rotational nuclei [23]. Note that ^{128}Xe lies exactly in a region where there should be a change from spherical vibration to γ–soft rotation. This is illustrated in Fig. 3, which shows plots of the $E(4_1^+)/E(2_1^+)$ ratios for the Z=54 (Xe) chain of isotopes and the N=74 isotones. The Xe isotope chain displays a long sequence of nuclei with $E(4_1^+)/E(2_1^+)\approx 2.50$ expected for γ–soft rotation. Indeed, the energies of states and the transition strengths in these nuclei (such as ^{124}Xe [24]) fit closely to the predictions of the γ–soft limit. The ratio gradually shifts to lower values upon approaching the N=82 spherical shell closure. The heavier N=74 isotones also have $E(4_1^+)/E(2_1^+)\approx 2.50$ and detailed studies have shown that cases such as ^{132}Ce [25] can be well described as γ–soft nuclei. The switch over to lower values $E(4_1^+)/E(2_1^+)$ occurs upon approaching the Z=50 spherical shell closure. Following either the isotopic or isotonic $E(4_1^+)/E(2_1^+)$ behavior, ^{128}Xe is found at a transitional point on the plot.

To conclude, the available data on even–even nuclei with $30\leq Z\leq 82$, $A\geq 60$ have been searched in an effort to find examples which display the predicted characteristics of E(5) critical point behavior. Of the $\approx$70 nuclei with $E(4_1^+)/E(2_1^+)$ values that might indicate such behavior, only six (namely, ^{102}Pd, ^{106,108}Cd, ^{124}Te, ^{128}Xe, and ^{134}Ba) have firmly assigned first and second excited 0^+ states in the range of excitation energy that might be expected for the 0_τ^+ and 0_ξ^+ excitations of the E(5) picture. The cases of ^{102}Pd and ^{134}Ba have already been discussed as possible examples of E(5) critical–point nuclei [14,12]. Of the remaining candidates the best agreement between experiment and the E(5) predictions is found to be with ^{128}Xe, which is proposed as a new example of an E(5) nucleus.

Future experimental investigations might focus on the properties of the excited 0^+ states in these candidate nuclei. As discussed, the pattern and strengths of the E2 decay from these states are characteristic of E(5) behavior. It would also be interesting to investigate their E0 decays which should also reflect E(5) symmetry properties. Since the 0_τ^+ is a member of a three–phonon multiplet its E0 decay to the 0_1^+ ground state should be forbidden, while the 0_ξ^+ level, which is the lowest member (zero phonon) of the ξ=2 family, should have an allowed E0 branch to the ground–state. A number of E0 transitions in our candidate nuclei are known but relevant E0 strengths have not been measured. These studies would be important for understanding the excitations in transitional nuclei regardless of the applicability of E(5).

ACKNOWLEDGMENTS

We would like to thank E. Browne and R.B.Firestone for their help and advice. This work has been supported by the U.S. DoE under Contract No. DE–AC03–76SF00098 (LBNL).

REFERENCES

[1] G. Scharff–Goldhaber and J. Weneser, Phys. Rev. 98 (1955) 212

[2] A. Bohr, Mat. Fys. Medd. K. Dan. Vidensk Selsk. 26 (1952)

[3] L. Wilets and M. Jean, Phys. Rev. 102 (1956) 788

[4] A.E.L. Dieperink, O. Scholten, and F. Iachello, Phys. Rev. Lett. 44 (1980) 1747

[5] F. Iachello, Phys. Rev. Lett. 85 (2000) 3580

[6] F. Iachello, Phys. Rev. Lett. 87 (2001) 052502

[7] R. Krücken et al., Phys. Rev. Lett. 88 (2002) 232501

[8] R.F. Casten and N.V. Zamfir, Phys. Rev. Lett. 87 (2001) 052503

[9] R. Bijker, R.F. Casten, N.V. Zamfir, and E.A. McCutchan, Phys. Rev. C 68 (2003) 064304

[10] R.M. Clark et al., Phys. Rev. C 67 (2003) 041302(R); Comment Phys. Rev. C 68 (2003) 059801

[11] R.M. Clark et al., Phys. Rev. C 68 (2003) 037301

[12] R.F. Casten and N.V. Zamfir, Phys. Rev. Lett. 85 (2000) 3584

[13] A. Frank, C.E. Alonso, and J.M.Arias, Phys. Rev. C 65 (2001) 014301

[14] N.V. Zamfir et al., Phys. Rev. C 65 (2002) 044325

[15] J.M. Arias et al., Phys. Rev. C 68 (2003) 041302(R)

[16] J.M. Arias, Phys. Rev. C 63 (2001) 034308

[17] J. Kern, P.E. Garrett, J. Jolie, H. Lehmann, Nucl. Phys. A 593 (1995) 21

[18] ENSDF, Evaluated Nuclear Structure Data File – a computer file of evaluated nuclear

structure data maintained by the National Nuclear Data Center, Brookhaven National Laboratory (file as of December 2002)

[19] C.A. Mallmann, Phys. Rev. Lett. 2 (1959) 507

[20] A. Gade, J. Jolie, and P. von Brentano, Phys. Rev. C 65 (2002) 041305(R)

[21] C. Doll, H. Lehmann, H.G. Börner, T. von Egidy, Nucl. Phys. A 672 (2000) 3

[22] Feng Pan and J.P. Draayer, Nucl. Phys. A 636 (1998) 156

[23] R.F. Casten and P. von Brentano, Phys. Lett. B 152 (1985) 22

[24] V. Werner, H. Meise, I. Wiedenhöver, A. Gade, P. von Brentano, Nucl. Phys. A 692 (2001) 451

[25] A. Gade et al., Nucl. Phys. A 643 (1998) 225

TABLES

TABLE I. Key observables for determining E(5) critical point behavior. The first column gives the quantity of interest. The second column gives the E(5) predictions from [5] using a linear quadrupole operator, $T_l(E2)$, in the quadrupole deformation. The third column gives the E(5) predictions using a quadratic quadrupole operator, $T_q(E2)$. The next four columns give the predictions from IBM calculations [15] at the critical point of the transition taking into account the finite boson number, N_B.

	E(5)		IBM			
	$T_l(E2)$	$T_q(E2)$	$N_B=4$	$N_B=5$	$N_B=6$	$N_B=7$
$\frac{E(4_1^+)}{E(2_1^+)}$	2.20	2.20	2.21	2.20	2.19	2.18
$\frac{E(0_\xi^+)}{E(2_1^+)}$	3.03	3.03	3.90	3.80	3.60	3.40
$\frac{E(0_\tau^+)}{E(2_1^+)}$	3.59	3.59	3.58	3.56	3.54	3.52
$\frac{B(E2;4_1^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$	1.68	1.56	1.30	1.34	1.40	1.46
$\frac{B(E2;0_\xi^+\rightarrow 2_1^+)}{B(E2;2_1^+\rightarrow 0_1^+)}$	0.86	0.49	0.42	0.47	0.53	0.60

TABLE II. The candidate nuclei with $30 \leq Z \leq 82$, $A \geq 60$, and $2.00 < E(4_1^+)/E(2_1^+) \leq 2.40$. The first column identifies the nucleus; the second gives the $E(4_1^+)/E(2_1^+)$ value; the third (fourth) column gives the energy of the first (second) excited 0^+ state, normalized to the energy of the 2_1^+ level, if it is known (parantheses indicate tentative assignments).

Nucleus	$E(4_1^+)/E(2_1^+)$	$E(0_2^+)/E(2_1^+)$	$E(0_3^+)/E(2_1^+)$
^{62}Zn	2.29	2.44	(4.19)
^{64}Zn	2.33	1.93	2.63
^{64}Ge	2.28	–	–
^{66}Zn	2.36	2.28	(2.92)
^{66}Ge	2.27	–	–
^{68}Ge	2.23	1.73	2.58
^{70}Zn	2.02	1.21	(2.42)
^{70}Ge	2.07	1.17	2.22
^{70}Se	2.16	(2.13)	–
^{72}Ge	2.07	0.83	2.43
^{74}Se	2.14	1.35	(2.61)
^{74}Kr	2.22	–	–
^{76}Se	2.38	2.01	(3.88)
^{80}Kr	2.33	(2.14)	–
^{82}Kr	2.35	1.92	(2.80)
^{82}Sr	2.32	2.29	4.65
^{84}Kr	2.38	2.08	–
^{84}Sr	2.23	1.89	2.62
^{84}Zr	2.34	–	–
^{86}Sr	2.07	1.96	2.05
^{86}Zr	2.22	–	–
^{88}Zr	2.02	1.44	2.10

^{90}Mo	2.11	2.09	2.58
^{96}Mo	2.09	1.48	1.71
^{98}Ru	2.14	2.03	–
^{100}Mo	2.12	1.30	2.81
^{100}Ru	2.27	2.10	3.23
^{102}Ru	2.32	1.99	3.87
^{102}Pd	2.29	2.86	2.98
^{104}Pd	2.38	2.40	(3.23)
^{106}Pd	2.20	2.22	3.33
^{106}Cd	2.36	2.84	3.39
^{108}Cd	2.38	2.72	3.02
^{110}Cd	2.34	2.23	(2.63)
^{112}Cd	2.29	1.98	2.32
^{114}Cd	2.30	2.03	2.34
^{114}Te	2.09	(2.62)	–
^{116}Cd	2.38	2.49	2.69
^{118}Cd	2.39	2.64	(2.99)
^{120}Cd	2.38	(2.75)	(3.45)
^{120}Te	2.08	1.97	2.88
^{122}Te	2.09	2.41	(3.10)
^{124}Te	2.07	2.75	3.12
^{126}Te	2.04	(2.81)	–
^{128}Xe	2.33	3.57	4.24
^{130}Xe	2.25	(3.35)	(3.76)
^{132}Xe	2.16	–	–
^{134}Xe	2.04	–	–
^{134}Ba	2.31	2.91	3.57

^{136}Ba	2.28	1.93	2.62
^{136}Ce	2.38	(1.95)	–
^{138}Ce	2.31	1.87	2.97
^{140}Xe	2.21	–	–
^{140}Nd	2.32	1.83	(2.77)
^{140}Sm	2.34	(1.87)	(3.07)
^{142}Ba	2.32	4.27	4.56
^{142}Sm	2.33	(1.89)	(2.83)
^{142}Gd	2.35	(2.66)	–
^{144}Ce	2.36	–	–
^{146}Nd	2.29	2.02	(3.46)
^{148}Sm	2.15	2.59	(3.69)
^{150}Sm	2.31	2.22	3.76
^{150}Gd	2.02	1.89	–
^{152}Gd	2.19	1.79	3.04
^{152}Dy	2.05	1.26	–
^{154}Dy	2.23	1.98	3.16
^{156}Er	2.31	2.70	–
^{176}Pt	2.14	(1.64)	–

FIGURES

FIG. 1. Level scheme calculated for the E(5) symmetry (top left – the 0^+_τ and 0^+_ξ levels are marked with a bold τ and ξ, respectively), and empirical schemes for ^{102}Pd (top right), ^{106}Cd (bottom left), and ^{108}Cd (bottom right). The excitation energies of states are normalized to the energy of the 2^+_1 level in each case. The numbers indicate E2 transition strengths, normalized to the B(E2;$2^+_1 \rightarrow 0^+_1$) value. Relevant branching ratios are indicated by two numbers separated by a slash.

FIG. 2. Level scheme calculated for the E(5) symmetry (top left – the 0^+_τ and 0^+_ξ levels are marked with a bold τ and ξ, respectively), and empirical schemes for ^{124}Te (top right), ^{128}Xe (bottom left), and ^{134}Ba (bottom right). The excitation energies of states are normalized to the energy of the 2^+_1 level in each case. The numbers indicate E2 transition strengths, normalized to the B(E2;$2^+_1 \rightarrow 0^+_1$) value. Relevant branching ratios are indicated by two numbers separated by a slash.

FIG. 3. Plots of the E(4^+_1)/E(2^+_1) ratios for the Z=54 (Xe) chain of isotopes (top panel) and the N=74 isotones (bottom panel). The expected values for an axially–symmetric rotor (3.33), a gamma–soft rotor (2.50), and an harmonic vibrator (2.00) are indicated by solid horizontal lines. The position of ^{128}Xe along the chains is indicated.