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Stepping back to see the connection: Movement during problem solving facilitates creative insight

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Abstract

People thinking creatively will shift their bodies, wander around, move. Why? Here we investigate one explanation: Movement is a canny strategy for changing the information that is available visually, in ways that facilitate insight. We first analyzed video footage of mathematicians engrossed in creative thought. We found that sudden 'aha' insights were reliably preceded by movements away far from the blackboard, as if mathematicians were stepping back to 'see the big picture.' To confirm the causal impact of changing proximity on creativity, we conducted an experiment that manipulated proximity to a whiteboard while participants worked on insight puzzles represented by diagrams. Participants had greater creative success when they could survey the entire whiteboard from a distance. Whether in real-world expert reasoning or a controlled experiment, movements away and toward visual representations facilitated insight. Wandering is sometimes a kind of epistemic action, facilitating the discovery of novel connections.

Keywords: creativity, embodiment, insight, distributed cognition, epistemic action

Introduction

Rodin's classic sculpture *The Thinker* epitomizes a widespread vision of pure thought. The model's body is curled up into itself, the head bent forward, the entire sculpture evoking a sense of stillness. And yet descriptions of how people actually think tell a different story. Darwin, for instance, is renowned for his devotion to twice-daily walks (Browne, 1996). Many research institutes build walking paths to encourage perambulatory meetings, perhaps inspired by Aristotle's 'peripatetic' approach to lecturing — that is, lecturing by walking. Why do we tend to find so much physical activity wherever people are hard at work *thinking*?

This puzzling link between movement and mind is exemplified by the canonical experience of the physicist or mathematician: scribbling at a blackboard (P. Ball, 2017). Despite mathematics' reputation for silent reflection, its practice is almost always a form of manual labor — scribbling, sketching, erasing, gesturing (Marghetis et al., 2014). Analyses of the behavior of expert mathematicians 'in the wild' has confirmed that, even when deep in thought, they often remain in constant motion, stepping toward the blackboard to write and then stepping back to inspect their handiwork (Tabatabaeian, Deluna O'bi, Landy, & Marghetis, 2023). Tabatabaeian et al. (2023), for instance, reported that as time passes, mathematicians working on a proof spend more and more time *away* from the blackboard, so that while they start by spending the majority of their time at the blackboard, close enough to write, by the end of a proof session they spend nearly three-quarters of the time at a remove from the blackboard, either within touching distance of the blackboard or even farther.

What is the point of all this wandering? Why, when generating proofs for challenging mathematical conjectures, do expert mathematicians so reliably and constantly shift their proximity to the blackboard (Tabatabaeian et al., 2023). Is it just to combat boredom? Or might these movements contribute in some way to creative problem-solving, at even the highest levels of expertise and abstraction?

There are reasons to suspect that movement may facilitate creativity. Taking a walk, for instance, can be good for creativity (Oppezzo & Schwartz, 2014; Zhou, Zhang, Hommel, & Zhang, 2017; Kuo & Yeh, 2016; Frith, Miller, & Loprinzi, 2020). Walking outdoors can enhance divergent thinking (Oppezzo & Schwartz, 2014), especially when the route is not imposed so people can wander freely (Zhou et al., 2017). Some have argued that walking helps because abstract concepts are 'embodied' in our sensorimotor systems and walking activates the motor system (Matheson & Kenett, 2020; Kuo & Yeh, 2016). Others draw on notions of cognitive depletion (Reverberi, Toraldo, D'Agostini, & Skrap, 2005), arguing that walking exhausts control resources, thus reducing top-down control and increasing the free-flow of ideas (Zhou et al., 2017). Or perhaps the benefits of walking outdoors just reflect the salutory effects of fresh air.

These accounts do not address the way artists, scientists, and mathematicians wander about when working indoors, in studios and seminar rooms. This kind of expert creativ-

In L. K. Samuelson, S. L. Frank, M. Toneva, A. Mackey, & E. Hazeltine (Eds.), *Proceedings of the 46th Annual Conference of the Cognitive Science Society.* ©2024 The Author(s). This work is licensed under a Creative Commons Attribution 4.0 International License (CC BY).

ity is often accompanied by the generation of inscriptions, diagrams, sketches (Marghetis, Samson, & Landy, 2019; Tabatabaeian et al., 2023; P. Ball, 2017; Menary, 2015). When abstract thought is embedded in a rich ecosystem of visual representations — diagrams, equations, sketches, etc. — we propose another explanation of the ubiquity of seemingly aimless wandering: Movement changes the information that is visually accessible, and this change in visual information can hint at unexpected connections. Indeed, people are more likely to discover an analogical connection when relevant diagrams are juxtaposed visually (Tabatabaeian, Deluna, Landy, & Marghetis, 2022); movements that generate visual juxtapositions may thus facilitate sudden insights.

We investigate this proposal in two studies. In Study 1, we analyzed a video corpus of expert mathematicians thinking creatively in their natural habitats, at blackboards in their own offices or seminar rooms (Marghetis et al., 2019). These mathematicians sometimes experience 'aha' insights that appear to occur suddenly and unbidden (Tabatabaeian et al., 2022). The mathematicians also move around constantly (Tabatabaeian et al., 2023). They sometime stood close to the blackboard, at a distance that made writing easy. But initial inspection of the corpus revealed that they spent a considerable amount of time standing farther back, at a distance where writing was difficult but visual inspection of the blackboard was easy. To investigate whether mathematicians' insights are related systematically to changes in proximity and associated changes in blackboard visibility, we coded the mathematicians' moment-to-moment changes in location. If movement facilitates insight by changing the information that is juxtaposed visually, then sudden 'aha' insights should be preceded systematically by movements away from the blackboard, to a distance from where mathematicians can survey a range of previously disconnected inscriptions.

Study 1 is correlational. We thus designed an in-person experiment to test the causal relationship between physical proximity to visual representations and creative insight. In Study 2, participants attempted to solve two classic insight puzzles, one after the other, each accompanied by a corresponding diagram on a whiteboard. The puzzles were related such that the solution to the first puzzle could be used analogically to solve the second, though past work has found that people seldom notice the analogy without extensive hints (Gick & Holyoak, 1980, 1983; Grant & Spivey, 2003). If movement can facilitate insight by juxtaposing previously disconnected visual representations, then participants should be more likely to solve the second puzzle when situated at a distance that allows them to survey the entire whiteboard, thus viewing both diagrams at once.

Study 1: Corpus study of creative insight in expert mathematicians

Methods

Corpus Generation and Selection The Math Experts corpus consists of video footage of PhD-level mathematicians ($N = 8$ mathematicians working for 4 hours and 40 minutes) trying to prove various mathematical conjectures (Marghetis et al., 2019). Mathematicians, working alone at a blackboard in their own office or a nearby seminar room, were asked to prove mathematical conjectures from the William Lowell Putnam Mathematics competition that dealt with topics such as set theory, geometry, and analysis. They were encouraged to share their reasoning by speaking aloud.

A subset of this corpus (12 proof sessions lasting a total of 4 hours and 5 minutes) was coded previously for the occurrence of sudden 'aha' insights. Tabatabaeian et al. (2022) selected $n = 6$ mathematicians (3 women and 3 men) who had worked on the same two conjectures and identified every explicit expression of discovery (e.g. saying, "ohhhh, I see" or "aha!"). One coder viewed the entire corpus and identified candidate moments of discovery. Two coders then examined each of these instances within the larger context of the proof session to determine whether the mathematician had experienced an insight. This process identified 24 sudden insights.

Movement To quantify the mathematicians' movement throughout each proof session, we identified every moment they changed their proximity to the blackboard. Based on visual inspection of the videos, we decided that proximity could be coded as close, medium, or far, based on the kind of interaction that was afforded by that proximity. *Close* was defined as the distance the participant could write comfortably and interact with the board, typically less than an arms length away; this captured the distance at which inscription, a core part of mathematical practice, was possible and natural. *Medium* was defined as an extended arm's length from the board — too far to write comfortably, but at a distance that allowed the mathematician to survey the rest of the board. *Far* was defined as anything more than an arm's length from the board; this captured the periods during which direct interactions with the blackboard were not possible, but the blackboard as a whole was more visible. Changes in proximity were coded at a resolution of 1 second. The entire corpus was double-coded and interrater reliability was substantial (Cohen's $\kappa = .70, p < .001$). Disagreements were resolved by discussion.

Results

Mathematicians moved around — a lot: Mathematicians seldom stayed at the same location for long (Fig. 1A). On average, the time spent at a particular distance from the blackboard before moving to another distance — their dwell time — was only ten seconds ($M = 10.4$ sec., $SD = 12.0$ sec.). Since mathematicians in our corpus worked on each conjecture for approximately 20 minutes ($M = 19.2$ min., $SD = 8.1$ min.), they thus typically moved to a qualitatively new location more than a hundred times while working on each conjecture (number of changes in proximity: $M = 111 \pm 20$ SEM).

Dwell times at each of the three distances were, on average, all quite brief ($M_{close} = 15$ sec., $M_{medium} = 9$ sec., $M_{far} =$

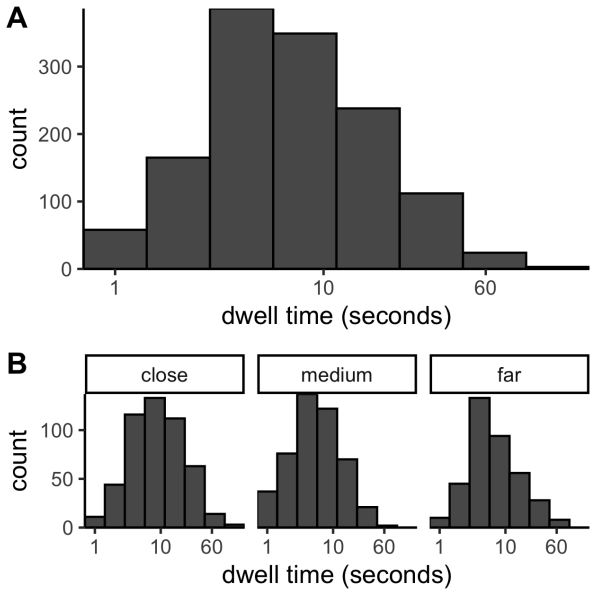

Figure 1: Mathematicians moved often. (A) Histogram of time spent at a particular distance from the blackboard before moving to another distance. Dwell times are plotted on a log-scale to visualize the heavy tail of dwell times. (B) Dwell times at each distance from the blackboard. Only rarely did mathematician stay in a particular location for long.

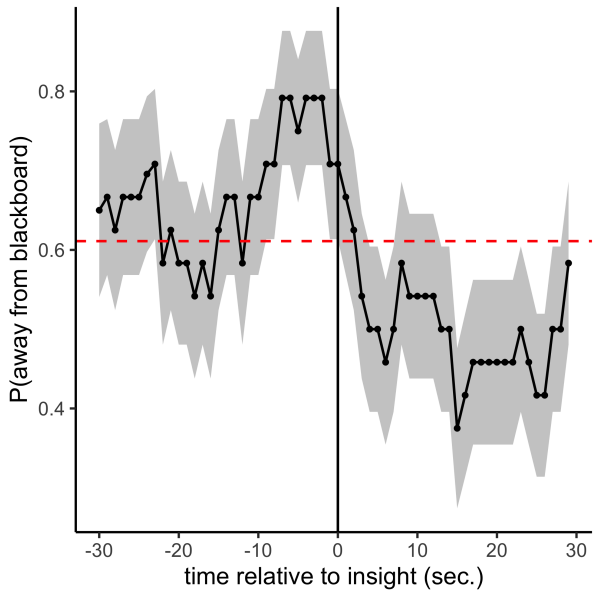

Figure 2: Time series of mathematicians' proximity to the blackboard relative to sudden 'aha' insights. Time (x-axis) was centered on the moment of insight. Mathematicians moved away from the blackboard before a sudden insight, and returned to the blackboard afterwards. (Error ribbon = standard error. Red dashed horizontal line = overall mean.

12 sec.; Fig. 1B). Bouts of time at the 'medium' distance (close enough to touch but not write on the blackboard) were significantly briefer than bouts at other distances (linear mixed model of dwell times as predicted by distance, with random intercepts and slopes by proof session, baselined on the 'medium' distance: $b_{close} = 6.2 \pm 1.5$ SEM, $p = .002$; $b_{far} = 3.1 \pm 1.2$ SEM, $p = .03$).

Mathematicians' situated activity, therefore, was marked by rapid and repeated wandering — movements toward and away from the blackboard. The critical question, then, is whether these movements were associated systematically with moments of sudden 'aha' insight.

Sudden insights were preceded by movement away from the blackboard: We investigated mathematicians' locations before and after experiencing a sudden 'aha' insight. We considered mathematicians to be 'away from the blackboard' if they were at either the 'medium' and 'far' location; otherwise, they were within writing distance of the blackboard. Inspection of the time periods surrounding the insights revealed a pattern where, around twenty seconds before the insight, mathematicians stepped away from the blackboard, far enough to survey the board, and then at the moment of insight returned rapidly to the blackboard (Fig. 2).

This pattern was confirmed by a logistic mixed model of the second-to-second time series of location. We modeled the probability of being away from the blackboard as predicted by three fixed effects: a continuous predictor for time (normalized within each proof session to range from 0 to 1); a dummy variable for whether the time was soon immediately an insight; and a dummy variable for whether it was immediately after an insight.[1] The model accounted for between-session variability with by-session random intercepts and random slopes for time. In the time period immediately before an insight, compared to periods away from an insight, mathematicians were significantly more likely to be away from the blackboard ($b = 0.89 \pm 0.16$ SE, $p < .001$). After the insight, however, they were significantly more likely than usual to be at the blackboard, within writing distance ($b = -0.33 \pm 0.08$ SE, $p < .001$). Verbal expressions of insight were thus associated with a systematic pattern of movement: before the insight, stepping back far enough to survey the blackboard; after, stepping forward to within writing distance of the blackboard.

Discussion

Using a video corpus of real-world mathematical activity, we found that moments of sudden insight were preceded by movements away from the board and followed by a return to within writing distance. We thus confirmed our hypothesis

[1] We operationalized 'immediately before [\after]' as the period of time 30 seconds before [\after] the insight occurred, but all results were remarkably robust to this parameter choice. To confirm that our results did not depend on this choice, we ran our analyses for values ranging from 20 to 60 seconds. This parameter sweep confirmed that all results remained qualitatively unchanged and statistically significant.

that mathematicians' wanderings are not random but related systematically to their reasoning. In particular, this pattern accords with our account of movement as a strategy for manipulating visual access to information — stepping away to literally 'see the big picture,' stepping forward to focus on a particular line of thought. On this account, movements away from the blackboard do not just prefigure the arrival of a sudden insight *but actively contribute to the insight*, a kind of epistemic action (Kirsh & Maglio, 1994). The results of Study 1, however, are correlational and cannot establish causality.

To test our causal claim, Study 2 systematically manipulated distance from the board — and thus access to visual information — while participants attempted to solve two classic insight puzzles. The second, critical puzzle could be solved by analogy with the first puzzle, if participants noticed that they were analogous. The insight puzzles were illustrated by diagrams, displayed on a whiteboard, that were designed to convey the structural similarities between the puzzles. Critically, we arranged standard office furniture in a way that forced people to stand either close to the whiteboard, far away, or at an optimal middle distance from where they could survey the entire whiteboard. We predicted that 'stepping back' to view the entire whiteboard would help participants solve the critical puzzle, by placing both diagrams within their field of vision.

Study 2: Experimental manipulation of distance and creative insight

Methods

Participants We were unsure of the effect size of manipulating distance. Since past work using the same materials (Gick & Holyoak, 1980) used between 20 and 30 participants per condition, we doubled the target sample size, out of an abundance of caution. We thus aimed to recruit 180 participants and stopped data collection the day this target was reached, with a total sample of $N = 181$ participants who completed the study. We removed participants who did not follow task instructions (e.g., argued with the experimenter and took a cellphone call; $n = 4$) and those who reported that they remembered the solution to the critical puzzle from a previous encounter ($n = 3$). This left a final sample of 174 participants (gender: 133 women, 38 men, 2 non-binary, 1 genderqueer; age: $M = 20$ years, $SD = 3$ years).

Materials Participants were tasked with solving two classic puzzles used to study insight problem solving: the Military puzzle and the Radiation puzzle (Gick & Holyoak, 1980).

In the Military puzzle, a general wants to attack an island fortress accessible by one of four bridges. To succeed, the general must attack simultaneously with their entire force, but none of the bridges is strong enough to hold all the soldiers at once. We illustrated the puzzle with a schematized diagram of the island fortress, the four bridges, and the surrounding water (Fig. 3D). The solution is to split the force into four smaller battalions so each of these smaller groups can attack simultaneously from a different bridge. A proposed solution is considered canonically correct if it mentions three features: splitting the full force into smaller groups, distributing them across the bridges, and attacking simultaneously.

The Radiation puzzle is superficially dissimilar to the Military puzzle but is analogous in its solution. A doctor wants to use radiation to treat a cancerous tumor that is surrounded by healthy tissue. To succeed, the doctor must attack simultaneously with a full dose of radiation, but a full dose of radiation would harm the surrounding tissue through which it would need to pass. We illustrated the puzzle with a schematized diagram of the tumor and the surrounding healthy tissue (Fig. 3D). The solution is to split the radiation into four smaller doses so each of these smaller doses can be applied to the tumor simultaneously from a different angle. A proposed solution is considered canonically correct if it mentions three features: splitting the full dose into smaller doses, distributing around the tumor, and attacking simultaneously.

People seldom transfer the Military puzzle's solution to the Radiation puzzle, even if the Military puzzle's solution was only recently explained to them (Gick & Holyoak, 1983, 1980). This pair of puzzles and their accompanying diagrams are thus a useful test-case for the hypothesis that stepping back can encourage people to notice connections among previously unconnected ideas.

Procedure Upon arrival, participants provided informed consent and were asked to solve the Military and Radiation puzzles. Each puzzle was accompanied by its illustrative diagram, printed on standard American-sized paper and attached to the whiteboard by a magnet. The diagrams were intentionally schematic to encourage analogical transfer. Each diagram was only revealed immediately before participants started to solve the associated puzzle. Participants could not write on the diagrams on the whiteboard. Participants had up to 5 minutes to solve each puzzle. The researcher judged whether proposed solutions included all three components of the canonical solution; if they did not, participants were instructed to continue searching for the solution.

In a between subjects design, we randomly assigned participants to stand at either Close, Far, or Flexible distances from the whiteboard (Fig. 3, A-C). In the Close condition, participants stood close to the board (2 ft). In the Far condition, participants stood far from the board (8 ft). In the Flexible condition, participants started at a middle distance from the board and could move forward or backward to change their proximity (between 2-8 ft) to the board as they desired. Proximity was imposed by cannily manipulating the placement of standard office furniture: three chairs that created a natural barrier (Fig. 3, A-C). This allowed us to manipulate participants' proximity to the board without explicitly instructing them to remain in a particular location.

Participants first attempted the Military puzzle (Fig. 3D) with only the Military diagram visible. Participants then shifted to the right, the Radiation diagram was revealed by

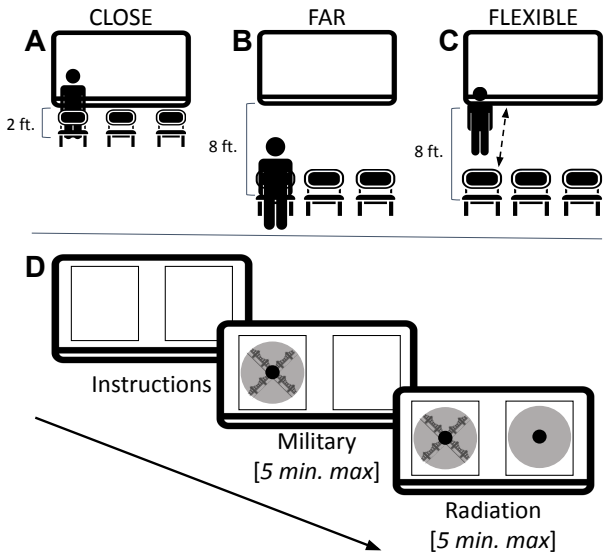

Figure 3: Study 2 Design. (A-C) Three conditions differed based on participants' proximity to the whiteboard: Close, Far, or Flexible. Participants in the Close condition were so close that they could only see the diagram for the puzzle they were currently working on; participants in Far and Flexible conditions had sufficient space to see both diagrams simultaneously. (D) Timeline of the experimental session. The diagrams, identical to those shown here, were printed on standard American papers and attached to the board by a magnet.

the researcher, and participants attempted the Radiation puzzle. During this second stage, the Military diagram remained on the board a few feet to the left of the Radiation diagram.

The distance manipulation was intended to change participants' access to visual information when attempting the Radiation puzzle. Participants in the Close condition could only see one diagram at a time; they had to turn their head to see the other. Participants in the Far condition could see both diagrams simultaneously but at a farther distance than was typical for mathematicians in the Math Experts corpus. Participants in the Flexible condition stood at a middle distance that allowed them to view both diagrams simultaneously; they could also adjust their distance.

After both puzzles, participants answered debriefing questions, including whether they had previously encountered the puzzles and recalled their solutions, and whether the Military diagram helped them solve the Radiation puzzle. The study ended with standard demographic questions.

Analysis Participants' solutions were categorized as *Correct* (1) if they mentioned all three components of the canonical solution, as *Partially Correct* (0.5) if they mentioned some but not all aspects of the canonical solution, and as *Incorrect* (0) otherwise. To analyze problem solving performance, solution quality was linearly regressed onto variables that indicated whether participants noticed a connection between the diagrams (yes = 1, no = 0) and participants' Distance condition (ordinal predictor, Close $<$ Far $<$ Flexible). To control for demographic variability, we also fit augmented models that included demographic predictors (age in years; gender, dummy coded to indicate if the participant was a woman; and an ordinal predictor for education, High School Graduate $<$ Associate's Degree $<$ Bachelor's Degree). To control for individual differences in problem solving, we fit an augmented model of success on the critical Radiation puzzle that added a predictor for solution quality on the Military puzzle.

Results

As predicted, manipulating proximity had no effect on the initial Military puzzle, since there were no other helpful diagrams on the whiteboard. Most participants produced solutions that were correct ($n = 118$), only a handful could not generate a solution that included any of the critical components of the correct solution ($n = 13$), and the remaining solutions were partially correct ($n = 43$). Critically, puzzle solving success was nearly identical across Distance conditions ($b = 0.00 \pm 0.03$ SE, $p = .97$) for the Military puzzle.

Performance on the critical Radiation puzzle, by contrast, was affected by the proximity manipulation. Solutions to the Radiation puzzle were significantly better when participants noticed the connection between the diagrams ($b = 0.78 \pm 0.05$ SE, $p < .001$). Critically, they were also better when standing at a location that allowed them to survey the whiteboard ($b = 0.05 \pm 0.03$ SE, $p = .035$). This effect of Distance was qualitatively unchanged after accounting for demographic factors ($b = 0.06 \pm 0.03$ SE, $p = .019$) and solution quality on the earlier Military puzzle ($b = 0.06 \pm 0.03$ SE, $p = .019$).

Post-hoc analyses suggested that standing at a distance from the whiteboard was especially effective at helping participants generate a solution that touched on *some* facets of the solution (logistic model of producing a solution that was at least partially correct: $b = 0.85 \pm 0.31$ SE, $p = .006$). Moreover, post-hoc pairwise comparisons suggested that the greatest benefits for solution quality came when standing at the middle Flexible distance (compared to Close: $b = 0.11 \pm 0.05$ SE, $p = .036$); participants in the Far condition were numerically but not significantly better than those in the Close condition ($b = 0.05 \pm 0.05$ SE, $p = .30$). Thus, standing at an optimal distance — with enough space to survey the board, but close enough to notice analogical connections — improved creative insight.

Discussion

Standing at a distance that allowed participants to see, simultaneously, both the Radiation diagram and the analogous Military diagram helped them solve the Radiation puzzle. Standing back to 'see the big picture' — or, more precisely, to see two related diagrams — helped people make an analogical connection and arrive at a creative insight.

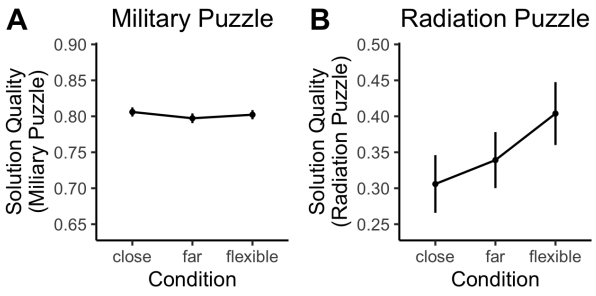

Figure 4: Solution quality for the insight puzzles in Study 2. (A) For the initial Military puzzle, solution quality was unaffected by viewing distance. (B) For the critical Radiation puzzle, solution quality was better when participants stood at an optimal viewing distance. (Circles indicate predictions from the model that accounts for demographic variability; error bars are standard errors. The panels have different ranges, but for comparison both use a vertical interval of 0.25.

General Discussion

We investigated the link between movement and creativity. In a corpus analysis, mathematicians were more likely to have a sudden insight after they had stepped back from the blackboard to a distance where they could survey the entire board. This suggested that movement may help by allowing the reasoner to see multiple visual representations at once. In a follow-up experiment, participants were more likely to solve an insight puzzle if they stood at a distance that allowed them to view both a diagrammatic representation of that insight puzzle and a diagrammatic representation of an analogous puzzle. When working creatively with visual representations, therefore, movement may help creativity by changing the information that is juxtaposed visually.

The benefits of movement may depend on the material contexts in which it occurs. The mathematicians in Study 1, for instance, stepped away from the blackboard before having insights — but, after the insight, they moved *forward*, returning to the blackboard. If the movement backwards helped them notice an unexpected connection, then the movement forward may have served to focus their attention on pursuing that connection. The cognitive benefits of movement in this context likely reflect both the spatial distribution of inscriptions and the type of reasoning (e.g., discovery, justification).

Participants in Study 2 performed best in the Flexible condition, where they started at an intermediate distance and had some flexibility in their proximity. This could reflect an optimal balance between distant visual juxtaposition and close visual inspection. It may also reflect the benefits of agency and interactivity (Kirsh, 2014), since participants in this condition could adjust their precise proximity from the blackboard, rocking forward to focus on certain elements, rocking backwards to notice connections. Anecdotally, participants in the Flexible condition did not move around much but did make small shifts in posture; future analyses will investigate the microdynamics of these postural shifts.

These results dovetail with studies that systematically manipulated visual access to the pair of puzzle diagrams used here (Tabatabaeian & Marghetis, 2023). In two computer-based studies, we manipulated whether the Military diagram was simultaneously visible while participants worked on the Radiation puzzle (Tabatabaeian & Marghetis, 2023). Even though all participants were shown both diagrams, those who could see both diagrams simultaneously were significantly more likely to notice the analogy and solve the Radiation puzzle. Together with the results of the current paper, the picture that emerges is one in which movement can strategically change the visual representations that are concurrently visible, and these canny changes in visual information can facilitate insight. Movement and interactivity may be mechanisms for self-generating hints during situated creativity (Kirsh, 2014; Ball & Litchfield, 2017).

On our account, however, movement is only one strategy for self-generating hints. Zooming in and out of a computer tablet, manipulating physical artifacts, or merely changing the orientation of one's head may all accomplish the same goal. The mathematicians in Study 1 may have relied on movement, but we suspect that creative thinkers may rely on a variety of strategies, including some not based on movement, to cannily modifying the available information.

These results raise serious concerns about the built environments in which children are taught mathematics. In the US, Canada, and many other countries, elementary schools use desks that limit the ability to move. If mathematical experts use movement strategically, and if the goal of mathematics education is to increase students' expertise, then we should engineer spaces so novices are able to deploy the same embodied practices as experts. Indeed, we suspect these strategies are not limited to mathematicians but are prevalent among architects, designers, and other experts. The spaces we use in education should, at minimum, allow pupils to engage in the interactivity that we observe in experts.

Conclusion

Apparently aimless movement may sometimes function as epistemic action, leveraging the body to change the available information in support of reasoning and insight (Kirsh & Maglio, 1994). These movements can be targeted, such as when mathematicians in Study 1 moved quickly to the blackboard after an insight. Or they may be exploratory, adding just enough noise to push a reasoner out of an impasse and toward an unexpected area of the solution space (Kirsh, 2014). This account is consistent with combinatorial theories of creativity (Koestler, 1964; Simonton, 2012; Thagard & Stewart, 2011; Hadamard, 1954; Mednick, 1962), which foreground the importance of unexpected connections. In fact, the mathematician Poincaré described creative insight as discovering an "unsuspected kinship between other facts, long known, but wrongly believed to be strangers to one another" (1913). Physical movement may help us discover such kinships.

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