Optimizing Store-Brand Choices with Retail Competition and Sourcing Options

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A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Business Administration

in the

Graduate Division

of the

University of California, Berkeley

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Spring 2014

Abstract

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Retailers are introducing new store brands at a rapid pace, and annual sales of store brands in the U.S. now exceeds \$108 billion. In the literature on store brand decisions, it is commonly assumed that (1) the retailer is a downstream monopolist; (2) either the store brand quality level is fixed, or, the marginal cost of production is constant and independent of the quality level of the store brand; and (3) the retailer either produces the store brand in-house, or sources it from a non-strategic manufacturer. Although these assumptions significantly simplify the analysis, they do not capture what is commonly seen in practice. As a consequence, the insights from these studies may not apply more broadly. Assumption (1) needs to be relaxed in order to study retailers' product assortment decisions (in terms of store and national brands) and related pricing decisions at two competing retailers, along with pricing decisions of a leading national brand manufacturer. Assumption (1) and (2) need to be simultaneously relaxed in order to investigate a retailer's store brand quality-positioning decision when facing competition from another retailer that already carries a store brand. Assumptiona (2) and (3) need to be relaxed simultaneously in order to study how a retailer's optimal quality-positioning strategy changes across various sourcing arrangements and various pricing power relationships among retailers and manufacurers. This dissertation contributes to the store brand literature by analyzing models based on more realistic assumptions than those in the literature.

This dissertation consists of three stand-alone papers. The first paper (in Chapter 2) investigates a retailer's product assortment and pricing problem when she has the option to carry a store brand, a national brand, or both. I compare her decision when she is a downstream monopolist and when she faces competition from another retailer who may also offer the same national brand and a competing store brand. Specifically, I assume the quality levels of the products are exogenous and analyze a manufacturer-Stackelberg game involving a national brand manufacturer and two competing retailers. The national brand manufacturer sets a wholesale price for the national brand product (the same for both retailers; I assume they are similar in size and can therefore secure the same wholesale price). Then, observing the wholesale price, the retailers engage in a Nash game in which they set the retail prices for the product(s) they choose to carry. Finally, customers decide whether and what to purchase. Customers are heterogeneous in two dimensions: location, which can be interpreted as the degree of loyalty to one retailer or the other, and willingness to pay per unit of quality. Each customer visits the retailer where he can obtain the maximum surplus (willingness to pay less purchasing and transportation costs) among the offered products. After the customer

arrives at the selected retailer, the transportation cost is now sunk, so he buys the offered product with the larger difference between his willingness to pay for the product and its price, if it is non-negative.

The second paper (in Chapter 3) addresses store brand quality-positioning decisions for retailers facing retail competition. Specifically, I assume one of the two retailers (Retailer 2) already carries a store brand product whose quality level is fixed, and both retailers may offer the national brand product with a fixed quality level. The representation of customer preferences and the resultant demands are the same as in the first paper. I model the dynamics via a two-stage game. In the first stage, Retailer 1 decides whether to introduce a store brand product, and if so, its quality level. Then the three parties engage in a manufacturer-Stackelberg pricing game. Finally, customers decide whether and what to purchase. In the first stage, Retailer 1 anticipates the outcome of the second-stage game. I analyzed the second stage game in the first paper; it is a subproblem in the second paper. I also analyze a setting in which both retailers may choose the quality levels of their store brand products simultaneously.

The third paper (in Chapter 4) studies a retailer's equilibrium quality-positioning strategy under three sourcing structures, and for each sourcing structure, I consider three types of channel price leadership. Specifically, I study games between (among) a retailer, a national brand manufacturer and a strategic third-party manufacturer, where applicable. The retailer carries a product (with a fixed quality) offered by the national brand manufacturer, and is considering introducing a store brand whose quality can be decided. Customers are heterogeneous in their willingness-to-pay (WTP) per unit of quality. The utility a customer derives from either product equals her WTP per unit of quality times the product quality. Each customer chooses the product that gives her the greatest surplus (utility less price), provided that it is non-negative. The unit production cost of both products is strictly convex and increasing in the quality level of the product. I derive the retailer's equilibrium store-brand quality decision under three sourcing arrangements and three pricing power scenarios. The three sourcing arrangements are in-house (IH), a leading national brand manufacturer (NM) (whose product the retailer also carries), and a strategic third-party manufacturer (SM). The three power scenarios are the ones most commonly seen in the literature: Manufacturer-Stackelberg (MS), Retailer-Stackelberg (RS), and Vertical Nash (VN). In sum, I examine nine (i.e., three times three) combinations of sourcing and pricing power (or "game") scenarios, and compare the retailer's optimal quality positioning decision and other equilibrium results (including prices) across the nine scenarios. In all nine combinations of sourcing and pricing power scenarios, the retailer moves first in setting the quality of her store-brand (during the product development phase) before any pricing decisions are made. I derive subgame perfect equilibria for all scenarios. To the best of my knowledge, I am the first to present a comparison of equilibria for these nine realistic combinations of sourcing and pricing power in this context.

This dissertation makes several contributions to the literature on store brand strategies. First, the majority of papers on store brand strategies consider a monopolist retailer. The few papers that consider retail competition are based on restrictive assumptions concerning factors such as product quality (e.g., assuming store brand products have equal quality levels) or product offering (e.g., both retailers must offer the national brand product). My work in papers 1 and 2 takes a first step in presenting a model that is general enough to allow me to study retailers' strategies in a context with store and national brands, and with retail competition. Second, prior research utilizes demand models that are limited in their ability to capture customers' joint selection of a retailer and a product. My work in papers 1 and 2 is based on a model of customer preferences that allows me to incorporate both quality differentiation among the products and the degree of customer loyalty to retailers, both of which are important in my problem context. This model is flexible enough to support a fairly rich representation of demands. Third, in paper 3, I take a first step in studying the interaction between store-brand sourcing and positioning decisions, and the interplay of these decisions with the retailer's pricing power. From a comparison of the retailer's equilibrium store brand quality levels for the nine combinations of sourcing and game structure, I obtain a full characterization of the ordering of store-brand quality, retailer's profit, retail prices and consumer welfare across the nine combinations. To the best of my knowledge, I am the first to present a comparison of equilibria for these nine realistic combinations of sourcing and pricing power in the store brand context. I also show that sourcing of store brands plays a key role in the competitive interaction between a retailer and a national brand manufacturer. Whereas the marketing and economics literatures have emphasized the role of store brands in helping retailers elicit price concessions from national brand manufacturers, I find that having a preferable sourcing arrangement for a store brand product is more valuable than having pricing power.

CHAPTER 1

Introduction

Store brands account for a sizable percentage of sales at retailers. In 2012, sales of store brands in U.S. supermarkets alone totaled $59 billion, with a store brand unit share of 23.1% and a dollar share of 19.1% (PLMA, 2013). Furthermore, industry experts say that store brand sales could double in the next five to six years (Watson, 2012). Store brands help retailers in various ways. First, a store brand serves as a strategic weapon for the retailer by increasing the retailer's bargaining strength, thereby eliciting wholesale price reductions and non-price concessions from suppliers of competing products (Mills 1995 and 1999; Narasimhan and Wilcox 1998; Gabrielsen and Sorgard 2007). Second, they serve as differentiating tools that distinguish a retailer from its competitors (Corstjens and Lal 2000). Third, they help retailers to build store loyalty if customers repeatedly visit to purchase store-brand products, which are not available elsewhere (Bell et al. 1998). As a result, retailers actively engage in store brand development. As one example, Kroger is expanding its store brand selection, and this is contributing to its bottom line (Associated Press, 2013). As the trend of increasing store brand development continues, managing store-brand products has become more challenging for retailers. For this reason, this dissertation aims to provide managerial insights for retailers carrying store brands and to help them optimize their store brand strategies.

In the literature on store brand decisions, it is commonly assumed that (1) the retailer is a downstream monopolist; (2) either the store brand quality level is fixed, or, the marginal cost of production is constant and independent of the quality level of the store brand; and (3) the retailer either produces the store brand in-house, or sources it from a non-strategic manufacturer. Very few papers are based on substantially more general assumptions. Although the commonly-adopted assumptions significantly simplify the analysis, they do not capture what is typically seen in practice. As a consequence, the insights from these studies may not apply more broadly. We elaborate on this point below.

First, retail competition has become increasingly intense in recent years. For example, in June 2012, a few days after Kroger announced its plan to launch its store brand coffee pods for Keurig machines, Safeway launched Safeway brand Keurig coffee pods. In such a scenario, both retailers need to respond to their competitor's strategies, but the literature has little to say about how they should respond. To provide insights on this issue, one has to relax assumption (1) above. Second, each retailer needs to determine its quality level. As an example, Kroger has a "three-tier" store brand positioning strategy. Not only does Kroger need to decide the tier of a new store-brand product, but even for a given tier, it needs to decide the quality level at a more detailed level. This decision will not only affect the unit production cost Kroger incurs, but it will also affect the competitiveness of its store brand vis-a-vis competing national brand product(s) and the products sold at competing retailers. Assumptions (1) and (2) above need to be relaxed simultaneously in order to investigate optimal strategies in this commonly-occurring situation. Third, some store brands are sourced from large national brand manufacturers who also produce and offer competing products, and some are sourced from third-party manufacturers that have market power. For example, in the UK, the store-brand cola at a large supermarket chain is produced by Coca-Cola (Berges-Sennou 2006). And Overhill Farms, a third-party producer known for processing frozen foods, has been producing store-brand products for some major retailers (Mercury News, March 2013). Many such firms have gained considerable market presence and power as the demand for store brand products has grown. Given the variety of possible sourcing arrangements and the consequent variations in pricing power among the parties, one would think that the retailer's optimal choice of store brand quality should depend upon these factors. For one to investigate how the optimal store brand quality depends upon these factors, assumptions (2) and (3) above need to be relaxed simultaneously.

This dissertation contributes to the store brand literature by addressing some of the limitations of models in the literature that were mentioned above. Specifically, this dissertation consists of three stand-alone papers. The first paper investigates a retailer's product assortment and pricing problem when she has the option to carry a store brand, a national brand, or both. I compare her decision when she is a downstream monopolist and when she faces competition from another retailer who may also offer the same national brand and a competing store brand. The second paper addresses store brand quality-positioning decisions for retailers facing retail competition. The third paper studies a retailer's equilibrium quality-positioning strategy under three sourcing structures, and for each sourcing structure, I consider three types of channel price leadership. In the third paper, I assume the retailer is a downstream monopolist in order to isolate the interaction between the positioning decision and sourcing structure from the effect of retail competition.

Below, I present overviews of the models in the three papers. I primarily use analytic (game theoretic) approaches, and derive the equilibrium strategies for all parties participating in the game. In the first paper, I assume the quality levels of the products are exogenous and investigate a manufacturer-Stackelberg game between a national brand manufacturer and two retailers. The national brand manufacturer sets a wholesale price for the national brand product (the same for both retailers; I assume they are similar in size and can therefore secure the same wholesale price). Then, observing the wholesale price, the retailers engage in a Nash game in which they set the retail prices for the product(s) they choose to carry. Finally, customers decide whether and what to purchase. In the model, customers are heterogeneous in two dimensions: location, which can be interpreted as the degree of loyalty to one retailer or the other, and willingness to pay per unit of quality. In the first dimension, customers are distributed uniformly along a Hotelling line between the two retailers and they incur a transportation cost for visiting either retailer. Due to the transportation cost, customers are more loyal to the nearer retailer and the degree of loyalty increases as the transportation cost per unit distance increases. In the second dimension, customers' willingness to pay per unit of quality is uniformly distributed within an interval. Each customer visits the retailer where he can obtain the maximum surplus (willingness to pay less purchasing and transportation costs) among the offered products. Here, each customer's willingness to pay is obtained by multiplying his willingness to pay per unit of quality by the product's quality level. Likewise, the customer's transportation cost is equal to the transportation cost per unit distance multiplied by the customer's distance from the respective retailer. After the customer arrives at the selected retailer, the transportation cost is now sunk, so he buys the offered product with the larger difference between his willingness to pay for the product and its price, if it is non-negative.

In the second paper, I assume one of the two retailers (Retailer 2) already carries a store brand product whose quality level I assume to be fixed; and I investigate the other retailer's (Retailer 1's) store brand introduction decisions, specifically with respect to product quality and price. Both retailers can also offer the national brand and select its price. The quality of Retailer B's store brand is fixed, but he can adjust its price. The representation of customer preferences and the resultant demands are the same as in the first paper. I model the dynamics via a two-stage game. In the first stage, Retailer 1 decides whether to introduce a store brand product, and if so, its quality level. Then the three parties engage in a manufacturer-Stackelberg pricing game: the national brand manufacturer first sets a wholesale price for his product and the retailers then engage in a Nash game in which they set retail price(s). Finally, customers decide whether and what to purchase. In the first stage, Retailer 1 anticipates the outcome of the second-stage game. I analyzed the second stage game in the first paper; it is a subproblem in the second paper. In the second paper, I focus on Retailer 1's decision regarding her store brand quality and the implications of that choice for the various pricing decisions. I also explore characteristics of the equilibrium when the two competing retailers can simultaneously choose their store brand quality levels.

In the third paper, I study games between (among) a retailer, a national brand manufacturer and a strategic third-party manufacturer, where applicable. The retailer carries a product (with a fixed quality) offered by the national brand manufacturer, and is considering introducing a store brand whose quality can be decided. Customers are heterogeneous in their willingness-to-pay (WTP) per unit of quality. The utility a customer derives from either product equals her WTP per unit of quality times the product quality. Each customer chooses the product that gives her the greatest surplus (utility less price), provided that it is non-negative. The unit production cost of both products is strictly convex and increasing in the quality level of the product. I derive the retailer's equilibrium store-brand quality decision under three sourcing arrangements and three pricing power scenarios. The three sourcing arrangements are in-house (IH), a leading national brand manufacturer (NM) (whose product the retailer also carries), and a strategic third-party manufacturer (SM). The three power scenarios are the ones most commonly seen in the literature: Manufacturer-Stackelberg (MS), Retailer-Stackelberg (RS), and Vertical Nash (VN). Under MS, the national-brand manufacturer sets a wholesale price first, followed by the SM, where applicable. Under RS, the retailer sets her margin for the store- and the national brands before the manufacturer(s) set their wholesale prices. Under VN, the retailer and the manufacturer(s) engage in a Nash game, in which the retailer sets her margins while the manufacturer(s) set the wholesale prices. In sum, I examine nine (i.e., three times three) combinations of sourcing and pricing power (or "game") scenarios, and compare the retailer's optimal quality positioning decision and other equilibrium results (including prices) across the nine scenarios. In all nine combinations of sourcing and pricing power scenarios, the retailer moves first in setting the quality of her store-brand (during the product development phase) before any pricing decisions are made. I derive subgame perfect equilibria for all scenarios. To the best of my knowledge, I am the first to present a comparison of equilibria for these nine realistic combinations of sourcing and pricing power in this context.

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The remainder of the dissertation is organized as follows. Chapters 2, 3 and 4 contain papers 1, 2, and 3, respectively. In these three chapters, the equation, figure and Appendix referencing pertain to equations, figures and Appendices within the same chapter. Chapter 5 concludes the dissertation by summarizing the key results.

CHAPTER 2

Product Assortment and Price in the Presence of Retail Competition and Store Brands

1. Introduction

Sales of store brands have grown rapidly over the past decade, with an annual increase of 4.9% annually between 2009 and 2012 to over $108 billion annually. By comparison, sales of national brands have grown 2.1% annually during the same period (PLMA 2013a). What are the reasons underlying this trend? News articles and research reports suggest there is a positive reinforcement cycle of store brand growth leading to retailers' investments in store brands, which then leads to further growth. Specifically, during the recent economic recession, many consumers dropped national brands for store brands as they tightened their budgets. But once they switched, they tended not to switch back as they were "happy with" the new choices. Indeed, 46% of consumers think "it's foolish to spend more if comparable quality is available from a store brand" (Consumer Edge Insight 2011). And 97% of consumers favorably compared store brand products to their previous national brand choices (PLMA 2010).

Increased customer acceptance has benefited retailers, as their stores have become better differentiated because of store brands. Stronger store brands have helped them increase store traffic and build store loyalty (see, e.g., Corstjens and Lal 2000; Ailawadi et al. 2008). Recognizing the benefits, retailers continue to invest in quality, merchandising and space allocation for store brands. For example, Supervalu plans to increase its Essential Everyday store brand offerings to 2700 products by 2013 (Store Brand Decisions 2012). At Safeway, store brands can be found alongside competing national brands in categories "from dry cereal and frozen foods to paper towels and laundry detergent" (www.safeway.com). On its website, Safeway claims that the store brands are "the same as national brands but at a much lower price" (www.safeway.com).

An interesting case study concerning store brand assortment is the practice at Trader Joe's. As much as 80% of the products carried there in 2010 were store brands (Kowitt 2010) and this strategy has been successful. Significantly, 92.6% of shoppers there reported they were "not bothered" by the lack of national brands and many of them even objected to the addition of national brands there. On a news webpage about Trader Joe's product offerings, one customer commented, "Adding national brand products would be a huge mistake. Why offer a commodity you could purchase at any other retailer and therefore be at risk for 'price comparison'?" (Thayer 2009).

A key element missing from this picture of competition between store and national brands is retail competition. When retailers try to win over customers from their competitors by offering store brands, their competitors are likely to do the same. For example, in June 2012, a few days after Kroger announced its plan to launch its store brand coffee pods for Keurig machines, Safeway launched Safeway brand Keurig coffee pods. (Kroger owns the Ralph's supermarket chain that competes with Safeway in several major metropolitan areas.) Indeed, retail competition has become even more intense in recent years with the expansion of non-traditional advertising channels such as social media, which allow very rapid dissemination of prices, perceptions of quality, etc.

Retailers have begun to realize the need to consider retail competition when devising store brand strategies. Notably, 12% of retailers reported that the primary reason they accelerated store brand development was the need to respond to expanded efforts by their competitors (Canning and Chanil 2011). Retailers are expanding their store brands into new categories. Categories such as refrigerated and frozen foods were considered "unbrandable" by retailers years ago, but now they are among the fastest-growing store brand categories (PLMA 2012). Retailers are also eliminating national brands from some product categories: 60% of retailers reported that they either already or were planning to eliminate some national brands to make room for their store brands (Canning and Chanil 2011). But questions remain. How exactly should a retailer adjust her store brand strategies, or when should she drop a national brand, in response to related strategies at her competitors? Moreover, how does retail competition affect the pricing strategy and the profit of an upstream national brand manufacturer? Few analytical models exist to aid in answering these questions, and none considers the generality that our model offers.

We consider a setting with a manufacturer of a leading national-brand product and two retailers. Each retailer can offer the national-brand product and a competing store-brand product in the same product category. Each store brand is produced either in-house by the retailer or by a third-party manufacturer which is a non-strategic player in the game. Major grocery chains including Safeway and Kroger own manufacturing facilities that produce some of their store-brand products. Also, there are regional brand manufacturers that produce store brands for specific markets (PLMA 2013b) and do not have enough power to noticeably affect the outcome, except for a small markup over their variable costs. We leave scenarios with strategic store brand producers or production of store brands by the national brand manufacturer for future research.

We model the dynamics via a national brand manufacturer-Stackelberg game. First, the national brand manufacturer sets a single wholesale price offered to both retailers, taking retailers' reactions into consideration. Typically, it is large retailers who are able to offer their own store brands, so we assume that the retailers are of similar size and therefore the national brand manufacturer needs to offer them the same wholesale price. We assume the producer(s) of the store brand(s) is (are) non-strategic players. Given the wholesale price, the two retailers engage in a Nash pricing game for the products they choose to offer. If a retailer sets a sufficiently high price, this has the same effect as not carrying the product at all. In this way, retailers' pricing decisions endogenize their product assortment decisions. Finally, customers decide whether and what to purchase.

Customers are heterogeneous in two dimensions: location and willingness to pay per unit of quality. In the first dimension, customers are distributed uniformly on a Hotelling line between two retailers and they incur a transportation cost for visiting each retailer, which allows us to capture the degree of customer loyalty to one retailer or the other. In the second dimension, customers' willingness to pay is uniformly distributed within an interval. Each customer chooses a retailer to visit (if any) on the basis of which one offers the product offering the highest surplus (willingness to pay for the product less transportation cost less price). Upon arriving at the retailer, the transportation cost is sunk, so the customer then purchases the product that provides the higher difference between the willingness to pay for the product and its price, as long as it is nonnegative. Groznik and Heese (2010) use a similar model of demand; we explain the differences between their model and ours in more detail later.

From our analysis of the model, we offer insights into the following aspects of the equilibrium:

Product assortment at retailers

We study the product assortment and pricing problem facing competing retailers. We also compare how the retailer's decision differs under retail monopoly and duopoly settings. Not surprisingly, we find that, ignoring the fixed cost of store brand introduction, a retailer should always introduce her store brand unless the national brand manufacturer intentionally underprices the national brand to increase market share while ignoring profit considerations. There is also a threshold wholesale price for the national brand product above which the retailer does not offer the national brand product. Surprisingly, although these threshold wholesale prices differ for the two retailers, under mild conditions on the customers' transportation cost (which reflects the relative disutility of visiting the two retailers), for each retailer, the threshold is the same whether she is a monopolist or whether she faces competition from another retailer. If the national brand manufacturer offers a low wholesale price (below the smaller of the thresholds for two retailers), both retailers will offer the national brand. If the national brand manufacturer offers a high wholesale price (above the larger of the thresholds for the two retailers), neither retailer will offer the national brand. Between the two thresholds, only one retailer—the one with the store brand of lower quality—offers the national brand. This implies that, as the wholesale price of the national brand increases, the retailer with the higher-quality store brand will stop offering the national brand earlier. This partly explains the product assortment practice at Trader Joe's: because the quality of its own-label products is high, it does not carry national brands in as many categories as its competitors do.

Price gap between store and national brands

The price gap between the national and store brands conveys the "value for the money" of the store brand, but it simultaneously signals the quality gap to consumers. Because of this, practitioners have been very interested in determining a good "price gap" between the national and store brands.

Hoch and Lodish (1998) conducted a study of consumer's attitude toward prices of products in the analgesics category. Their results show that the price gap does not affect the consumers' choice of stores. Instead, only the price levels of the national and store brands at a retailer (compared to prices at the retailer's competitors) matter. Moreover, the national brand price matters more in customers' overall store choice probability. The authors thus recommended that retailers figure out whether their current prices are above or below the theoretical optimal values.

Our research can be used to aid in determining optimal prices for the national and store brands for a retailer under competition. From our equilibrium results, we also find that retailers should respond to competition by lowering the prices of both the store and national brands, and should lower the price of the national brand to a greater extent. In other words, in the face of competition, retailers experience greater pressure on their price for the national brand than on their price for store brands. This parallels the empirical findings of Hoch and Lodish.

National brand manufacturer's product distribution
The launch of the store brand coffee pods for Keurig machines described earlier caused the stock price of Green Mountain, a national brand, to plummet (Geller 2012). Thus, it is important for the national brand manufacturer to develop counterstrategies. A key question is: when both retailers offer store brands, should a national brand manufacturer sell through only one or both retailers?

We find that the answer to this question depends on the quality disparity between the two store brands. When the quality disparity is low, the national brand manufacturer prices in such a way that both retailers continue to sell the national brand. But when the quality disparity is greater than a threshold, the national brand manufacturer prices so that only the retailer with the lower store brand quality continues to offer the national brand product.

Sethuraman (2009) points out that few analytical papers derive results regarding national brand counterstrategies when facing competition from store brands; one notable exception is Mills (1999). Our work contributes to our understanding along these lines by characterizing, as part of our analysis of a three-party game, the national brand's optimal strategies and how they change as costs, product quality, and market parameters change.

Effect of customer loyalty
Many large retail chains invest heavily in customer loyalty programs; virtually every major grocery and drug store chain has one. What is the effect of greater customer loyalty in our environment? We find that when customers exhibit no loyalty, retailers end up in a prisoner's dilemma: both of them prefer a situation in which neither of them carries the national brand product. But at the equilibrium, both of them carry it, yet the perfectly competitive environment leads both of the retailers to price the national brand at cost, so all of their profit comes from their store brands.

The remainder of this paper is organized as follows. We review the related literature in Section 2. Our model is presented in Section 3. In Section 4, we present each retailer's problem of choosing her product assortment and prices, and properties of the equilibrium between the retailers for a given wholesale price of the national brand product. In Section 5, we analyze and discuss the national brand manufacturer's optimal pricing policy, which determines which retailer(s) choose to offer the national brand. A discussion of special cases and extensions appears in Section 6. We conclude the paper in Section 7.

2. Literature Review

The literature on competition between national and store brands is extensive but, as mentioned earlier, very few analytical models consider competition between retailers, nor do they consider differential quality of the store brand products or loyalty of customers to the retailers. In the interest of completeness, we first provide a comprehensive high-level overview of analytical (equilibrium) models of national- and store-brand competition. Near the end of this section, we provide more detail on the few papers that are closely-related to ours.

A major stream within the literature on store brands investigates the strategic benefits that retailers can gain from them. First, a store brand serves as a strategic weapon for the retailer by increasing the retailer's bargaining strength, thereby eliciting wholesale price reductions and non–price concessions (Du et al. 2005, Mills 1995, Narasimhan and Wilcox 1998, Mills 1999, Pauwels and Srinivasan. 2004, Steiner 2004, Tarziján 2004, Gabrielsen and Sørgard 2007). Second, store brands can be instruments for retailers to enhance store differentiation and store loyalty (Corstjens and Lal 2000, Sudhir and Talukdar 2004, Avenel and Caprice 2006, Geylani et al. 2009). Third, a store brand can help a retailer to better discriminate among consumers by serving as one additional product version in its category (Wolinsky 1987, Soberman and Parker 2004, Soberman and Parker 2006). Finally, retailers may carry a store brand because they have more control over its positioning and production (Bergès-Sennou and Rey 2008, Scott-Morton and Zettelmeyer 2004). We are most interested in the role of store brands as a retailer's strategic weapon in the vertical interaction with national brand manufacturers, particularly under retail competition.

We first discuss analytical models of the effect of store-brand introduction on national brand wholesale prices. These models are based on a manufacturer-Stackelberg game between a national brand manufacturer and one retailer whose store-brand product is available at a constant marginal cost. Under the assumption that the national brand manufacturer and the store brand producer share the same constant marginal cost, Mills (1995, 1999) finds that the wholesale price offered by the national brand manufacturer decreases as the quality of the store brand increases. When the quality of the store brand is not too low, the option of carrying a store brand imposes a threat, so the national brand manufacturer offers a lower wholesale price than when the retailer does not have a store-brand option and thereby successfully forecloses the store brand. If the quality of the store brand is very high, the store brand is sold, and as the quality of the store brand rises, the national brand manufacturer decreases its wholesale price to provide an incentive for the retailer to sell a fair amount of the national brand instead of the store brand alternative.

Unlike Mills, Bontems et al. (1999) find that the wholesale price is not necessarily monotonically decreasing in the quality of the store brand when the unit production cost is convex and increasing with its quality. This is because the store brand suffers from a cost disadvantage when its quality exceeds a threshold, so it no longer imposes a threat. Both Narasimhan and Wilcox (1998) and Gabrielsen and Sørgard (2007) consider a model with two customer segments: one is loyal to the national brand and the other (so-called "switchers") may choose the store brand if the price is attractive. Narasimhan and Wilcox (1998) assume that both segments have the same reservation price for the store and national brands, and obtain results that indicate the introduction of a store brand can only lead to a decrease in the national-brand wholesale price. Gabrielsen and Sørgard (2007) assume the loyal customers have a higher willingness to pay for quality, and their analysis indicates that the introduction of the store brand can lead to either an increase or a decrease in the wholesale price of the national brand, depending on the fraction of loyal customers. Soberman and Parker (2004, 2006) treat the level of advertising for the national brand as a decision variable and show that the direction of change in the average price for the product category after the store brand is introduced depends on whether advertising is expensive (with respect to its ability to increase the utility of the national brand among brand seekers). Fousekis (2010) considers a three-stage game in which the national brand manufacturer and retailer simultaneously set the quality level of their respective product (within a range) in the first stage of the game;

the author assumes the store brand quality is lower than that of the national brand. In the second stage, the national brand manufacturer chooses a wholesale price, and in the final stage the retailer sets prices. Consumers are heterogeneous in their willingness to pay per unit of quality and choose the product that gives them a higher surplus, if it is non-negative. He shows that, under his assumptions, it is optimal for both parties to choose the highest feasible quality level for their respective products.

Characteristics of vertical channel structures in the presence of store brands are also pertinent to our research. There are many analytical models of *traditional* vertical interactions between manufacturers and retailers (McGuire and Staelin 1983, Shugan 1985, Choi 1991, Lee and Staelin 1997), but we are not aware of any that address the specific characteristics of competition between store and national brands There is, however, substantial empirical research on the nature of competition between store and national brands, which suggests that the type of interaction and the degree of competition between national and store brands is idiosyncratic across categories: it depends on whether or not the national brand is a leading product as well as the quality of the store brand. See Putsis and Dhar (1998), Cotterill and Putsis (2000), Sayman et al. (2002), Meza and Sudhir (2010) and the references therein.

How do consumers choose among different stores? A rich stream of empirical research addresses this questions. Bell et al. (1998) develop and test an empirical model to investigate the store choice behavior of households visiting a set of stores over a certain time horizon, based on the assumption that each shopper is most likely to visit the store with the lowest total shopping cost. They find that, in order to provide a comprehensive theory of store choice, both fixed and variable costs are necessary. The fixed cost is the cost independent of the shopping list whereas the variable cost depends on the shopping list. The fixed cost depends upon the travel distance, the shopper's inherent preference for the store, and historic store loyalty, whereas the variable cost is the total expected cost of the items on the shopping list if purchased at the store.

A stream of research also examines how customers perceive store brands. Richardson et al. (1996) find that customers' reliance on extrinsic cues (such as price, packaging, and brand) adversely affects customers' propensity to purchase store brands. Baltas and Argouslidis (2007) find that quality plays a major role in the evaluation process from consumers develop their store brand preferences. Sayman et al. (2002) find that store brands are viewed as slightly more similar to secondary national brands than to the leading national brand. de Wulf et al. (2005) find that national brands enjoy brand equity while store brands do not.

Next, we review analytical models of demand when store and national brand(s) compete. In the literature, there are roughly five groups of demand models developed for such a context. Models in the first group derive demand from one representative consumer, or equivalently, by assuming consumers are homogenous (cf. Choi and Coughlan 2006 and Bergès-Sennou and Rey 2008). The second group of models uses an aggregate demand function which is linear with respect to price and is parameterized by differentiation and substitution factors (cf. McGuire and Staelin 1983, Choi 1991, Raju et al. 1995, Cotterill and Putsis 2000 and Sayman et al. 2002). Linear demand functions allow researchers to conduct sensitivity analysis on the cross-price sensitivity parameters and examine how they affect the equilibrium channel structure. However, linear demand functions are limited in their ability to accommodate the combination of complex customer choice behavior and interactions among multiple parties in a vertical channel.

The third group of models derives demand from individual utility functions of customers, which are assumed to be increasing with product quality and the customer's willingness to pay for quality, and decreasing with the product price (cf. Mills (1995 and 1999), Bontems et al. 1999, Tarziján 2004 and Avenel and Caprice 2006). When such a utility function is used, the store- and the national brands are assumed to differ in their quality levels, and the consumers' willingness to pay for quality varies among consumers (a uniform distribution is usually assumed). The fourth group of models segments customers into those who are loyal to the national brand and those who are more willing to switch (cf. Narasimhan and Wilcox 1998, Gabrielsen and Sørgard 2007, Soberman and Parker 2004, Soberman and Parker 2006 and Corstjens and Lal 2000). Articles in this group examine the role of store brand introduction and study how the equilibrium changes in response to a change in segment sizes. Finally, the remaining models do not fall into any of the above categories and are more context-specific (e.g., Scott-Morton and Zettelmeyer 2004, Du et al. 2005 and Bergès-Sennou 2006).

There has been little analytical research that incorporates retail competition when both national- and store-brand products are offered and the national brand manufacturer is a strategic player. Indeed, in a recent survey paper by Sethuraman (2009) on models of national- and store-brand competition, only one article with retail competition is mentioned (Corstjens and Lal (2000)), which we discuss later in this section. We offer a few comments on relevant articles, including those published after 2009, here. First, we discuss articles in which the retailer does not have an explicit product assortment decision. In the model of Choi and Fredj (2013), two retailers both offer the national brand and their respective store brand, so there is no product assortment decision. The authors assume there is competition between the store- and national-brand at each retailer, but there is very little price competition between the store brands (which is a salient feature of our model). They derive equilibrium prices under manufacturer-Stackelberg, Vertical Nash, Retailer Stackelberg and Retailer Double Stackelberg (in which one retailer moves first, then the other retailer, then finally the national brand manufacturer). They provide a comparison of various parties' profits under the four channel leadership arrangements. In general, the findings are similar to others in the literature: the retailers benefit from greater leadership but the national brand manufacturer is not necessarily better off being the leader.

Corstjens and Lal (2000) analyze a two-period setting with two retailers who offer the same national brand product and their own store-brand product; the store brand products are of similar quality, which is lower than that of the national brand. Customers are quality-sensitive and exhibit brand inertia. In each period, the retailer can choose which brand's price to advertise and how to set prices. Each customer's attraction to each retailer is influenced by the price information, but if he visits the same retailer in period 2 as he did in period 1, he will choose the same product unless the surplus differential exceeds the inertia threshold. Owing to this effect, store brands introduced in the first period play a role in store-differentiation in the second period. Moreover, retail competition in the first period is intensified because retailers can later extract profits from customers who tried and liked store brands in the first period and then continue to buy the same product in the second period due to inertia. We note that the retailers are essentially symmetric and the national brand manufacturer is not a strategic player in this model.

Colangelo (2008) studies a setting with asymmetric retailers and assumes that relevant parties can choose the level of advertising (or analogously, the quality level) of each product in the first stage of the game. Then the national brand manufacturer chooses wholesale price (or prices, when wholesale price discrimination is allowed) and fixed fees (if applicable), and finally the retailers choose quantities in a Cournot subgame. Because the model involves three products, the authors utilize a variant of the Dobson-Waterson (1996) utility model (Dobson and Waterson 1996) to derive retail demands, thereby enabling them to obtain closed-form solutions. Although the authors compare equilibria with and without private-label products, they do not explicitly incorporate the retailers' decisions regarding whether to offer a private label product.

We now turn to the few articles that address the assortment decision (including the option *not* to offer the national brand) in addition to pricing decisions. Fang et al. (2012) address this issue in a *single*-retailer setting. They derive conditions in which the retailer carries only the national brand product, only the store brand product, or both. They also propose a contract that coordinates the supply chain (i.e., achieves the first-best solution) when both products are offered.

Avenel and Caprice (2006) study a scenario in which two symmetric retailers can offer a (high quality) national brand product and/or an alternate low quality product (same for both retailers); the quality levels of the products are fixed and procurement costs are linear in the quantity. Customers are heterogeneous in their willingness to pay per unit of quality and choose the product that maximizes their surplus. The national brand manufacturer offers the retailers identical two-part tariffs and the retailer then compete in a Nash-Cournot game, choosing order quantities for the two products. (Prices are then implicitly defined as market-clearing prices.) In this framework, the national brand manufacturer implicitly chooses whether to sell to one or both retailers via his choice of the franchise fee (fixed portion of the two-part tariff).

Geylani et al. (2009) study store-brand introduction and pricing strategies for two competing retailers in the presence of "one-stop shopping" customers who visit only one retailer and view the national-brand and store-brand products as identical (except for price). The authors show that store brands enable a retailer to segment the market and thereby extract a higher price from national-brand loyal customers because the store brand can be sold to price-sensitive one-stop shoppers. They assume the national brand manufacturer may offer different wholesale prices to the two retailers, and do not focus on how the equilibrium is shaped by retail competition.

Groznik and Heese (2010) study the impact of retail competition on the retailers' decisions regarding store brand introduction. They show that under non-discriminatory pricing, store brand introductions (or the potential for them) increase the retailers' bargaining power vis–a–vis the national brand manufacturer, consistent with the result derived without retail competition. However, there are settings in which the retailers play a game of "chicken". Neither wants to introduce a store brand product but instead prfers that the competitor be the one to do so, thereby enabling both of them to secure a lower wholesale price.

To conclude, we emphasize that although competition between store- and national brands has been studied extensively, to the best of our knowledge, no research has considered the following factors simultaneously: (i) asymmetric store brand products; (ii) customers who are heterogeneous in terms of their willingness to pay per unit of quality and their loyalty to the retailers; (iii) national brand manufacturer is a strategic player (e.g., setting the wholesale price); (iv) retailers choose both product assortment (or store-brand introduction) and prices. All of these features are pervasive in practical settings.

3. The Model

We consider a scenario with a manufacturer of a leading national-brand product and two retailers, Retailer 1 (R1) and Retailer 2 (R2). Each retailer can offer the national-brand product and a competing store-brand product in the same product category. Each store brand is produced either in-house by the retailer or by a third-party manufacturer which is a non-strategic player in the game. We derive the equilibrium assuming that each retailer already has a store brand in place or that it is ready to be introduced. (If a retailer still needs to develop a store brand, then the firm can consider the results of the equilibrium analysis along with the fixed cost of store-brand introduction before making a decision.) Also, we assume that all parties have complete information.

The national brand manufacturer is the Stackelberg leader, and chooses the wholesale price, denoted by w_n, to offer to both retailers with the objective of maximizing his profit, taking into account both retailers' reactions. We assume that the national brand manufacturer offers them same wholesale price. (In general, it is large retailers that are able to offer their own store brands. We assume the two retailers are similar in size and can therefore secure the same wholesale price. Various U.S. laws require the same pricing under the same terms of trade.) For any wholesale price offered by the national brand manufacturer, the retailers engage in a Nash game, choosing which products to offer and at what price(s), with the objective of profit maximization. The retail prices of the national-brand product and that of the store-brand product at retailer i ($i = 1,\ 2$) are denoted by p_{ni} and p_{si}, respectively. In our model, choosing a very high price for either the store-brand or the national-brand product has the same effect as not offering the product at all. Therefore, when deriving the retail price equilibrium, we make the following assumption:

Assumption. *Whenever a retailer finds it optimal not to offer some product, she sets the price at the lowest level that drives the customer demand for that product to zero.*

In this way, given any wholesale price offered by the national brand manufacturer, we are implicitly modeling the product assortment decision via the Nash equilibrium in prices between the retailers.

The quality level of the national-brand product is denoted by q_n, and those of the store-brand products at R1 and R2 are q_{s1} and q_{s2}, respectively. Throughout our analysis, we assume these quality levels are exogenous, but we later explore how the quality levels and their differences affect the structure of the equilibrium. Without loss of generality, we assume $q_{s1} \leq q_{s2}$. We also assume that both q_{s1} and q_{s2} are less than q_n to concentrate our attention on store brand products that have quality levels below that of similar national brand products. This applies to most store brands except so–called "premium store brands." The marginal production cost of the national-brand product is denoted by c_n, and that of the store-brand product at retailer i ($i = 1,\ 2$) is denoted by c_{si}. We initially assume that the production cost of each product is proportional to its quality. That is, we assume $c_{si} = kq_{si}$ for $i = 1,\ 2$ and that $c_n = kq_n$ for some production parameter $k > 0$. In Section 6, we discuss results when this assumption is relaxed.

Customers are heterogeneous in two dimensions: location, which can be interpreted as the degree of loyalty to one retailer or the other, and willingness to pay per unit of quality. We discuss each dimension in turn. For ease of exposition, we assume that in the first dimension, customers are distributed uniformly along a Hotelling line between the two retailers. Customer loyalty is captured via a transportation cost for visiting either of the retailers. Each customer's transportation cost is the transportation cost per unit distance, t, multiplied by his distance from the respective retailer. (Heterogeneity in per-unit-distance transportation costs and a more general distribution of customers vis-a-vis loyalty to the two retailers can be captured by an appropriate adjustment of the customer's location. We discuss this further in Section 6. We assume $t > 0$ throughout the paper except in Section 6, where we consider the special case of of no customer loyalty.) Mathematically, a customer's location on a Hotelling line between the two retailers is denoted by $x_1 (x_1 \in [0,\ 1])$, with R1 located at $x_1 = 0$ and R2 at $x_1 = 1$. The customer's distance from R2 is denoted by $x_2 = 1 - x_1$.

In the second dimension, customers have a willingness to pay per unit of quality, θ, which is uniformly distributed in the interval $[0,\ \bar{\theta}]$. We assume $\bar{\theta} > k$ so that it is possible for the supply chain to profitably offer each product (in the absence of competition) to at least some customers. Mathematically, a customer with a willingness to pay per unit of quality θ derives utility (willingness to pay for the product) θq_n from a unit of the national brand, and utility θq_{si} from a unit of the store brand at retailer i $(i = 1, 2)$. This representation of the second dimension of customer heterogeneity is a standard modeling approach and has been used in Moorthy 1988 and many papers investigating store brand strategies. Let $\bar{v}_n \equiv \bar{\theta} q_n$ and $\bar{v}_{si} \equiv \bar{\theta} q_{si}$, i.e., $\bar{v}_n$ and $\bar{v}_{si}$ denote the highest utility derived from the national-brand and store-brand products at R1 and R2, respectively.

The total number of potential customers is normalized to 1. Each customer visits only one retailer and purchases at most one unit in the product category, either the national-brand or the store-brand product. When deciding which retailer to visit, each customer evaluates the maximum surplus he can derive from going to each of the retailers. At this stage, the customer calculates his willing to pay for the product under consideration as his willingness to pay per unit of quality multiplied by the product's quality level. The customer then subtracts the sum of the transportation cost for visiting the relevant retailer and the price of the product to determine his surplus from this product. Finally, he adds the expected surplus he can derive from purchasing products in other product categories, M, to determine the total surplus from going to each of the retailers. We assume each customer derives the same expected surplus from purchases in other product categories at either of the retailers, and that it is large enough that each customer visits one retailer or the other. Expressed mathematically, a customers' total surplus he can derive from going to retailer i $(i = 1,\ 2)$ is $\max\{\theta q_n - t x_i - p_{ni} + M,\ \theta q_{si} - t x_i - p_{si} + M\}$. If $\max\{\theta q_n - t x_1 - p_{n1} + M,\ \theta q_{s1} - t x_1 - p_{s1} + M\} \geq \max\{\theta q_n - t x_2 - p_{n2} + M,\ \theta q_{s2} - t x_2 - p_{s2} + M\}$, a customer located at x_1 chooses to visit R1. Otherwise, he/she visits R2.

After a customer located at a distance x_i from retailer i $(i = 1,\ 2)$ and a willingness to pay per unit of quality θ arrives at his "preferred" retailer i, the transportation cost is now sunk, so he buys the offered product that gives him the larger difference between his willingness to pay for the product and its price, if it is non-negative. That is, he buys the national-brand product if $q_n \theta - p_{ni} \geq q_{si} \theta - p_{si}$ and $q_n \theta - p_{ni} \geq 0$, or he buys the store-brand product if $q_n \theta - p_{ni} < q_{si} \theta - p_{si}$ and $q_{si} \theta - p_{si} \geq 0$. Otherwise, he does not buy a product in this category.

We note that Groznik and Heese (2010) use a demand model that is identical in characterizing customer heterogeneity, but they assume that customers will not purchase unless the single product under purchase consideration provides the customer a surplus large enough to compensate for the transportation cost. We assume, instead, that the product in question is only one product in a market basket (as would be the case for a typical grocery shopper) and that the surplus from the market basket will outweigh the transportation cost, so each customer will visit one retailer or the other. But the customer will buy one of the products only if her utility minus the price is non-negative. Our representation allows us to consider high transportation costs, representing strong customer loyalty, without simultaneously driving demands down to negligible quantities. As such, our representation provides more flexibility in exploring the effects of customer loyalty.

3.1. Customer Demand

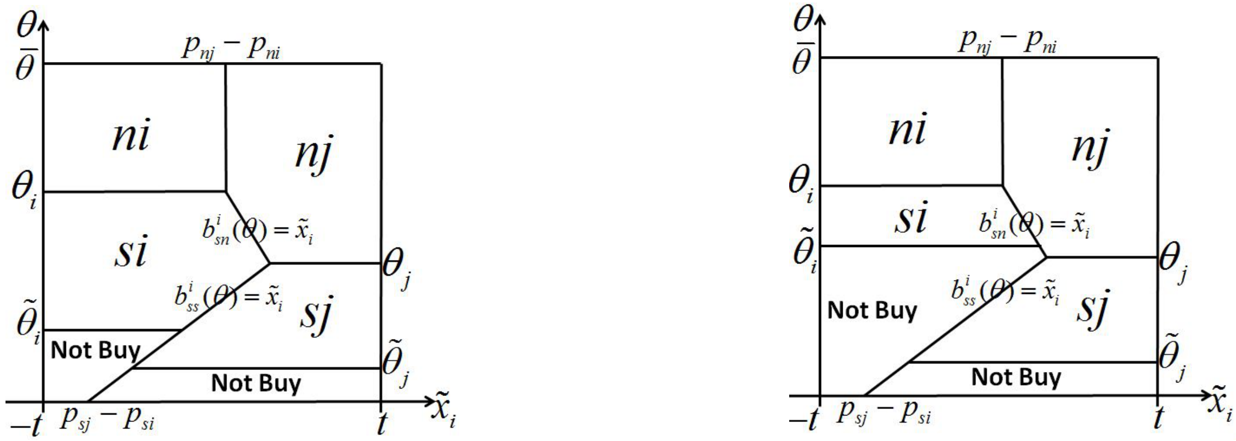
Now we are ready to derive the customers' demands given $(p_{ni},\ p_{si})$ at retailer $i = 1,\ 2$. In the remainder of the paper, we use ni and si $(i = 1,\ 2)$ to denote the national-brand and the store-brand product at retailer i respectively. Define $\theta_i \equiv \frac{p_{ni}-p_{si}}{q_n-q_{si}}$ and $\tilde{\theta}_i \equiv \frac{p_{si}}{q_{si}}$ for $i = 1, 2$. Then θ_i represents the threshold willingness to pay per unit of quality at which customers are indifferent between purchasing the national-brand and store-brand products at retailer i, and $\tilde{\theta}_i$ represents the threshold willingness to pay per unit of quality at which customers are indifferent between purchasing the store-brand product and purchasing nothing at retailer i. We then have:

Lemma 1. *For any positive wholesale price w_n, in any price equilibrium between the retailers, we have (i) $\theta_i \geq \tilde{\theta}_i$ for retailers $i = 1, 2$ and (ii) $\theta_1,\ \theta_2,\ \tilde{\theta}_1,\ \tilde{\theta}_2 \in [0,\ \bar{\theta}]$.*

All proofs are in the Appendices. Lemma 1 says that each retailer sets prices so that customers with high willingness to pay per unit of quality purchase the national brand, those with low willingness to pay per unit of quality purchase nothing, and those in between purchase the store brand.

Define $\tilde{x}_i \equiv t(x_i - x_j)$ for $i = 1, 2,\ j = 3 - i$, i.e., the difference between the travel cost a customer incurs from going to retailer i versus retailer j. Because customers' locations are distributed uniformly on the Hotelling line between the retailers, $\tilde{x}_i$ is uniformly distributed on $[-t,\ t]$. Also define $b^i_{sn}(\theta) \equiv \theta(q_{si} - q_n) - (p_{si} - p_{nj})$ for $i = 1, 2,\ j = 3 - i$. Then, for a customer with a willingness to pay per unit of quality θ who is located at $\tilde{x}_i$, $b^i_{sn}(\theta) - \tilde{x}_i$ is the difference between the customer's surplus from purchasing a unit of the store-brand at retailer i, and purchasing a unit of the national-brand product at retailer j. Clearly, $b^i_{sn}(\theta) - \tilde{x}_i = 0$ defines the customers who are indifferent between purchasing products si and nj. Define $b^i_{ss}(\theta) \equiv \theta(q_{si} - q_{sj}) - (p_{si} - p_{sj})$. Then, analogously, $b^i_{ss}(\theta) - \tilde{x}_i$ is the difference in the customer's surplus from purchasing a unit of the store-brand product from retailer i versus retailer j, and customers who are indifferent between si and sj are defined by $(\theta,\ \tilde{x}_i)$ that satisfy $b^i_{ss}(\theta) - \tilde{x}_i = 0$.

Figure 1 shows the partitioning of customer demand graphically on the $\theta - \tilde{x}_i$ plane. Here, without loss of generality, we assume $\theta_i > \theta_j$ $(i = 1$ or 2 and $j = 3 - i)$. Under this assumption, we need to divide our analysis into two cases: $\tilde{\theta}_i \leq \theta_j$ (shown in Figure 1(A)) and $\tilde{\theta}_i > \theta_j$ (shown in Figure 1(B)). Note that in Figure 1, although it is not explicitly stated, $\theta_i,\ \theta_j,\ \tilde{\theta}_i$ and $\tilde{\theta}_j$ are not parameters, but depend upon the retailers' pricing decisions. Expressions for the functions $b^i_{sn}(\cdot)$ and $b^i_{ss}(\cdot)$ also depend upon the pricing decisions. Also, for the diagrams in Figure 1, we are implicitly assuming that $t \geq \max\{|p_{n2} - p_{n1}|,\ |p_{s2} - p_{s1}|,\ |b^i_{sn}(\theta_j)|\}$, i.e., the degree of customer loyalty is not too low, which guarantees that none of the demand regions represented in the diagrams vanish. For now, we proceed with our analysis under this assumption about t, but address other cases later in this section.



(A) Customer demands if $\tilde{\theta}_i \le \theta_j$

(B) Customer demands if $\tilde{\theta}_i > \theta_j$

FIGURE 1. Graphical Representation of Customer Demands on the $\theta - \tilde{x}_i$ Plane

With Figure 1 at hand, we can easily write the expressions for the demands for the two products at each retailer given a price vector $\mathbf{p} = (p_{n1},\ p_{s1},\ p_{n2},\ p_{s2})$:

$$
\begin{aligned}
D_{ni}(\mathbf{p},\ \bar{\theta}) &= \begin{cases} D^H_{ni}, & \text{if } \theta_i \ge \theta_j \\ D^L_{ni}, & \text{if } \theta_i < \theta_j \end{cases} \\
D_{si}(\mathbf{p},\ \bar{\theta}) &= \begin{cases} D^H_{si}, & \text{if } \theta_i \ge \theta_j \\ D^L_{si}, & \text{if } \theta_i < \theta_j \end{cases}
\end{aligned} \tag{1}
$$

where

$$
\begin{aligned}
D^H_{ni}(\mathbf{p},\ \bar{\theta}) &= \frac{1}{2t\bar{\theta}}(p_{nj} - p_{ni} + t)(\bar{\theta} - \theta_i) \\
D^H_{si}(\mathbf{p},\ \bar{\theta}) &= \begin{cases} \frac{1}{2t\bar{\theta}}\{\frac{1}{2}[(p_{nj} - p_{ni} + t) + (b^i_{sn}(\theta_j) + t)](\theta_i - \theta_j)\} \\ +\frac{1}{2t\bar{\theta}}\{\frac{1}{2}[(b^i_{ss}(\tilde{\theta}_i) + t) + (b^i_{ss}(\theta_j) + t)](\theta_j - \tilde{\theta}_i)\}, & \text{if } \tilde{\theta}_i \le \theta_j \\ \frac{1}{2t\bar{\theta}}\{\frac{1}{2}[(p_{nj} - p_{ni} + t) + (b^i_{sn}(\tilde{\theta}_i) + t)](\theta_i - \tilde{\theta}_i)\}, & \text{if } \tilde{\theta}_i \ge \theta_j \end{cases} \\
D^L_{ni}(\mathbf{p},\ \bar{\theta}) &= \frac{1}{2t\bar{\theta}}\{(\bar{\theta} - \theta_j)(t - p_{ni} + p_{nj}) + \frac{1}{2}[(t - b^j_{ss}(\theta_i)) + (t - p_{ni} + p_{nj})](\theta_j - \theta_i)\} \\
D^L_{si}(\mathbf{p},\ \bar{\theta}) &= \frac{1}{2t\bar{\theta}}\{\frac{1}{2}[(t - b^j_{ss}(\theta_i)) + (t - b^j_{ss}(\tilde{\theta}_i))](\theta_i - \tilde{\theta}_i)\}
\end{aligned} \tag{2}
$$

and where, for example, D^H_{ni} is the demand for product ni if $\theta_i \ge \theta_j$. It should be understood that θ_i, θ_j and $\tilde{\theta}_i$ are functions of $\mathbf{p}$ as defined earlier. Similarly, $b^i_{ss}(\theta)$ and $b^i_{sn}(\theta)$ are functions of both $\mathbf{p}$ and θ. Although the full expressions for these variables are $\theta_i(\mathbf{p})$, $\theta_j(\mathbf{p})$, $\tilde{\theta}_i(\mathbf{p})$, $b^i_{ss}(\mathbf{p},\ \theta)$ and $b^i_{sn}(\mathbf{p},\ \theta)$, in both (6) and in many formulas in the remainder of the paper, we omit the variable $\mathbf{p}$ for brevity. It can be seen that we have four possible equilibrium scenarios, and from one scenario to another, the demand function $\mathbf{p} \longmapsto \mathbf{D} \equiv (D_{n1},\ D_{s1},\ D_{n2},\ D_{s2})$ is not continuous. Case 1 corresponds to $\theta_1 \ge \theta_2$ and $\tilde{\theta}_1 \ge \theta_2$, Case 2 corresponds to $\theta_1 \ge \theta_2$ and $\tilde{\theta}_1 < \theta_2$, Case 3 corresponds to $\theta_1 < \theta_2$ and $\tilde{\theta}_2 \ge \theta_1$, and Case 4 corresponds to $\theta_1 < \theta_2$ and $\tilde{\theta}_2 < \theta_1$.

The demands derived above enable us to formulate each retailer's pricing problem. Given a wholesale price $w_n > 0$, and given the retail prices $(p_{nj},\ p_{sj})$ set by the other retailer, retailer i ($i = 1, 2$ with $j = 3 - i$) seeks a pair of prices $(p_{ni},\ p_{si})$ that maximize her profit, $\pi_{ri}(w_n,\ \mathbf{p},\ \bar{\theta},\ c_{si})$. Retailer i's problem is:

$$\begin{aligned} &\max_{p_{ni} \geq w_n,\ p_{si} \geq c_{si}} && \pi_{ri}(w_n,\ \mathbf{p},\ \bar{\theta},\ c_{si}) \equiv (p_{ni} - w_n) D_{ni}(\mathbf{p},\ \bar{\theta}) + (p_{si} - c_{si}) D_{si}(\mathbf{p},\ \bar{\theta}) \\ &\text{subject to} && \theta_i \leq \bar{\theta} \\ & && \tilde{\theta}_i \leq \theta_i \end{aligned} \tag{3}$$

Although there are four prices, retailer i chooses only p_{ni} and p_{si}. Due to the form of the demand functions shown in (5), retailer i needs to solve two subproblems, one for $\theta_i \geq \theta_j$ and another for $\theta_i < \theta_j$, and then choose the better solution. Moreover, due to the form of $D^H_{si}(\mathbf{p},\ \bar{\theta})$, the subproblem for $\theta_i \geq \theta_j$ further divides into two subproblems, one for $\tilde{\theta}_i \geq \theta_j$ and one for $\tilde{\theta}_i < \theta_j$.

Let $p^*(w_n)$ denote the retailers' (joint) equilibrium reaction for any value of w_n.. Knowing $p^*(w_n)$, the national brand manufacturer seeks to optimizes his own profit by solving the following problem:

$$\max_{w_n \geq c_n} \pi_m(w_n,\ \bar{\theta},\ c_n) \equiv (w_n - c_n) \cdot \left(D_{n1}(\mathbf{p^*}(w_n),\ \bar{\theta}) + D_{n2}(\mathbf{p^*}(w_n),\ \bar{\theta}) \right) \tag{4}$$

Before analyzing the equilibrium for the case of two retailers, we first derive properties of the equilibrium for a benchmark scenario in which there is no retail competition. The single retailer has the option of selling her store-brand as well as the national-brand product. Later, we will compare these results with those that we derive under retail competition.

3.2. Single Retailer Case

All customers will visit the monopolist retailer, but each customer will purchase a unit in the product category (either the store-brand or the national-brand product) only if (i) it provides a higher surplus than the other product and (ii) that surplus is non-negative. The following lemma characterizes the retailer's optimal product assortment decision. Because there is only one retailer, we omit the subscript i for simplicity.

Lemma 2. *If $w_n \leq c_n$, a monopolist retailer carries only the national brand. She sets $p_n = \frac{1}{2}(\bar{v}_n + w_n)$ and $p_s = \frac{p_n}{q_n} q_s$ (at which price no customer would purchase the latter). If $w_n - c_s \geq \bar{v}_n - \bar{v}_s$, she carries only the store brand and sets $p_s = \frac{1}{2}(\bar{v}_s + c_s)$ and $p_n = p_s + (\bar{v}_n - \bar{v}_s)$ (at which price no customer would purchase the latter). If $c_n < w_n < c_s + \bar{v}_n - \bar{v}_s$, the retailer carries both products and sets $p_n = \frac{1}{2}(\bar{v}_n + w_n)$ and $p_s = \frac{1}{2}(\bar{v}_s + c_s)$.*

Lemma 2 states that a retailer's product assortment decision depends on the value of the national brand's wholesale price relative to two thresholds. This is consistent with the findings of Fang et al. (2012). More specifically, if $w_n \leq c_n$, then $\theta q_n - \theta q_{s \geq} w_n - c_s$ for all $\theta \geq k$. This implies that the utility premium the national brand provides to *all customers* to whom the retailer can sell the store brand without incurring a loss (i.e., those with willingness to pay per unit of quality greater than k) is greater than or equal to the cost premium the retailer pays for the national brand over the store brand. Therefore the retailer does not carry the store brand because she can earn a higher profit margin on the national brand while not losing any market share. If $\bar{v}_n - \bar{v}_s \leq w_n - c_s$, then we have $w_n - c_s \geq \theta q_n - \theta q_s$ for all $\theta \leq \bar{\theta}$. This implies that for every customer, the price premium he is willing to pay for the national brand over the store brand is less than the cost premium the retailer incurs by selling a unit of the national brand instead of the store brand. Therefore the retailer completely forgoes the national-brand product.

If the wholesale price falls between c_n and $c_s + \bar{v}_n - \bar{v}_s$, it is more profitable for the retailer to sell the national brand to the subset of customers who have a willingness to pay per unit of quality above a threshold and the store brand to the remaining customers, so the retailer carries both products. In this case, the retailer sets the retail prices in the same way as if each product were the sole product she carries. That is, she sets the price of the national (store) brand as half the sum of its procurement cost and the highest valuation for the national (store) brand among customers. As the wholesale price increases, $\tilde{\theta} = p_s/q_s$ (set by the retailer implicitly via her choice of p_s) stays the same whereas $\theta = (p_n - p_s)/(q_n - q_s)$ increases, leading to more store brand sales and fewer national brand sales while the total sales stays the same.

We characterize the manufacturer's equilibrium strategy and other equilibrium outcomes under this benchmark scenario in the following lemma.

LEMMA 3. *In a market with a monopolist retailer who has the option to offer a store brand, the equilibrium wholesale price is* $w_n = \frac{1}{2}(\bar{v}_n + c_n) - \frac{1}{2}(\bar{v}_s - c_s)$. *In response, the retailer carries both products and sets prices according to Lemma 2. The resulting customer demands are* $D_n = \frac{\bar{\theta}'}{4\bar{\theta}}$ *for the national brand and* $D_s = \frac{\bar{\theta}'}{2\bar{\theta}}$ *for the store brand, and the profits of the manufacturer and the retailer are, respectively,* $\pi_m = \frac{\bar{\theta}'}{8\bar{\theta}}\left[(\bar{v}_n - c_n) - (\bar{v}_s - c_s)\right]$ *and* $\pi_r = \frac{\bar{\theta}'}{16\bar{\theta}}\left[(\bar{v}_n - c_n) + 5(\bar{v}_s - c_s)\right]$.

4. Assortment and Pricing under Retail Competition

The focus of this section is on the retailers' optimal assortment and pricing strategies for a given national brand wholesale price. In Section 4.1, we derive the equilibrium and in Section 4.2, we study how the equilibrium prices and profits change with the level of customer loyalty.

4.1. Retailers' Product Assortment and Pricing

To simplify the analysis and for ease of exposition, we assume that for both retailers, their respective profit functions are quasi-concave in their prices, subject to certain conditions that we will explain later in this section. This is a common assumption and is supported by our observations from numerical examples. Given this, from results in game theory (see, for example, Osborne and Rubinstein (1994)) we know that a pure-strategy Nash equilibrium exists for the pricing game between the retailers. We now proceed to study its characteristics. The traditional approach to find a price equilibrium between two retailers is to (1) solve for the explicit expressions characterizing each retailer's best responses and (2) solve for the crossing points of the retailers' best responses and express them as explicit functions of w_n. However, in our model, the demand functions are discontinuous and nonlinear in the price variables (see the discussion in Section 3.2). Indeed, it is impossible to obtain explicit expressions for retailers' price responses and the equilibrium prices. We thus take an alternate approach: we derive and analyze the first-order-conditions for each retailer's profit-maximization problem to characterize the retailers' optimal strategies *without* explicitly solving the first-order-conditions. Details can be found in Appendix D. Here, we summarize the main results. We start by presenting each retailer's optimal product assortment strategy given any pricing and assortment strategy taken by the other retailer. This result facilitates our proof (presented later) of the uniqueness of the equilibrium.

PROPOSITION 1. *Each retailer's best response in terms of product assortment, given any strategy taken by the other retailer, is to carry only the national brand if $w_n \leq c_n$, to carry only the store brand if $w_n \geq c_{si} + (\bar{v}_n - \bar{v}_{si})$, and to carry both products otherwise.*

Proposition 1 implies that, under retail competition, for any given wholesale price, each retailer makes the product assortment decision in the same way as if she were a downstream monopolist. This result is certainly not intuitive. It implies that that a retailer's optimal product assortment decision is not affected by the existence of the other retailer, nor is it affected by the assortment or pricing decisions of the other retailer. Intuitively, we would imagine in a Nash game between the two retailers, one retailer's best response depends upon the strategy of the other retailer. But Proposition 1 says this is not so, which is quite surprising.

So what is the intuition behind Proposition 1? If it is less profitable for a retailer to sell one product (versus the other) to *all* customers, the retailer will not carry the less profitable product at all. For example, if $w_n \geq c_s + (\bar{v}_n - \bar{v}_{si})$, the per-unit cost premium retailer i incurs from selling the national brand (versus the store brand) is greater then the maximum per-unit price premium consumers are willing to pay. Thus, it is not worthwhile for the retailer to carry the national brand at all. This comparison between the two products remains the same whether a retailer is a monopolist or is facing competition. We will see later, however, that the national brand manufacturer takes into account the retailers' reactions (including the assortment decisions) when choosing the wholesale price, so the introduction of retail competition may ultimately affect retailers' assortment decisions. We study this issue in the next section.

Next, we establish the uniqueness of the equilibrium.

PROPOSITION 2. *There exists a t_0 with $0 < t_0 < +\infty$ such that, whenever $t > t_0$, the price equilibrium between retailer is unique for all w_n.*

In Appendix E, we prove the result for values of t for which the demand segmentation diagram has the form of Figure 1(A) or (B). As the value of t declines, the right and left boundaries in Figure 1(B) move inward, as shown by the dashed lines in Figure 2(A). The resulting extracted diagram is shown in Figure 2(B). Then finally, as t becomes very small, the demand segmentation approaches the form shown in Figure 2(C): the diagonal lines flatten out and become horizontal at $t = 0$. When this happens, each retailer is able to seize a large share of the national brand demand from the other retailer by reducing the price of the national brand slightly. As a result, at equilibrium each retailer sets the national brand price at the wholesale price and gains profit exclusively from her store brand. The equilibrium is also unique for $0 \leq t \leq t_0$ (i.e., for the demand segmentation diagrams shown in Figures 2(B) and 2(C)) but we have omitted the proofs for the sake of brevity.

How do the prices a retailer sets when facing competition compare to those she would set as a monopolist, and how do the respective demands compare? We answer this question by first introducing the following proposition, in which θ_i^m and $\tilde{\theta}_i^m$ denote the optimal values of θ and $\tilde{\theta}$, respectively, that retailer i would set when she is a downstream monopolist (cf. Lemma 3).

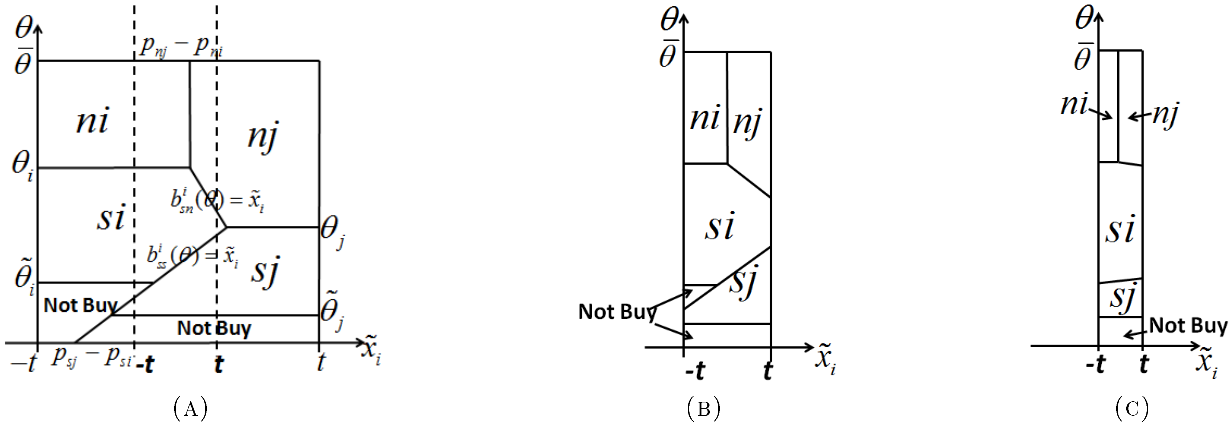


FIGURE 2. Graphical Representation of Customer Demands when t is small

PROPOSITION 3. *If retailer i decides to carry both the store and national brands, then she sets $\theta_i < \theta_i^m$ and $\tilde{\theta}_i < \tilde{\theta}_i^m$* for any pricing and assortment strategy of the other retailer $(i = 1, 2)$.

Proposition 3 states that, under retail competition, each retailer will set prices in such a way that the willingness to pay per unit of quality of the customer who is indifferent between the store brand and the national brand, as well as the willingness to pay per unit of quality of the customer who is indifferent between the store brand and the no-purchase option, are smaller than when the retailer is a downstream monopolist. From this, we can immediately obtain Corollary 1.

COROLLARY 1. *For any given value of w_n that induces both retailers to carry both the store and national brands, we have the following results under retail competition:*
(i) At each retailer, the retail price for each product as well as the price gap between the products are smaller than when the retailer is a downstream monopolist;
(ii) Total demand for the national brand and the aggregate demand for all products (s1, s2 and the national brand) are greater than the respective demands when R2 is a downstream monopolist.
(iii) The national brand manufacturer's profit is greater under retail competition than when R2 is a downstream monopolist. Moreover, there exists an $\epsilon > 0$ such that when $|q_{s2} - q_{s1}| < \epsilon$, the national brand manufacturer's profit is greater than that when R1 is a downstream monopolist.

Corollary 1 has several implications. First, although a retailer can ignore the other retailer without loss of optimality when deciding the assortment, it is suboptimal to do so when setting prices, as this would lead to prices that are too high and with too large a gap between them. Some reports in the business press have suggested that retailers should shrink this price gap to maximize profit (Nielsen 2011, SymphonyIRI 2011). Our results suggest one possible contributing reason for the larger-than-optimal price gaps is incomplete consideration of competition.

Second, when retail competition arises because the retailer with the lower store brand quality enters the market (i.e., R1 in our model), demand for the national brand increases at any given wholesale price. The reason is that the lower-quality store brand competes less directly with the national brand, so the retailers set prices in a way that is less unfavorable to the national brand compared to the scenario in which R2 is a downstream monopolist. Therefore, for any wholesale price, the demand for the national brand, and hence also the profit of the national brand manufacturer, are higher than in the scenario in which R2 is a downstream monopolist.

On the other hand, when retail competition arises because the retailer with the higher store brand quality (i.e., R2 in our model) enters the market, the national brand manufacturer's profit increases only if the two store brands have very similar quality (i.e., when $|q_{s2} - q_{s1}| < \epsilon$). Otherwise his profit decreases with the entry of the retailer with the higher store-brand quality, as the level of competition between the national brand and the store brands as a whole becomes fiercer.

Next, we study the effect of customer loyalty (or transportation cost), t, on the price equilibrium between the retailers. As before, we restrict our attention to positive t, but we examine how how the equilibrium prices, demands and profits change as t increases from zero.

4.2. Effect of t on Retailers' Product Assortments

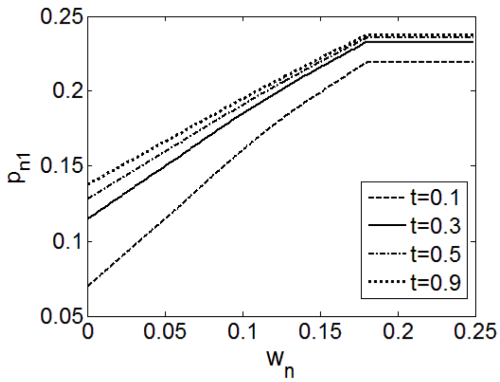
To study how equilibrium retail prices change with t, we resort to numerical approaches as it is impossible to derive closed form expressions for the equilibrium decisions. For any given input parameters $(\bar{\theta},\ k,\ q_n,\ q_{s1},\ q_{s2},\ t,\ w_n)$, we solve for the equilibrium by starting with an arbitrary pair of initial prices for R1 and alternately solving one retailer's problem given the other retailer's prices from the most recent iteration.

If t is sufficiently large, i.e., $t \geq \max\{|p_{n2} - p_{n1}|,\ |p_{s2} - p_{s1}|,\ |b^i_{sn}(\theta_j)|\}$ for all prices selected during the iterations, we can use the demand system (6) and this process converges. But we cannot guarantee that t will always satisfy this condition for all iterations before the equilibrium is identified. To address this problem, we utilize a more general representation of demand which is valid even if t is very small. This system of demand functions is discontinuous at more points than is the demand system (6). (We will illustrate this below.) We present these alternate expressions for demands in Appendix G. Just as in (6), the demand functions in the new system are discontinuous because θ_2 may be either greater or less than θ_1. However, if $\theta_2 \geq \theta_1$, we now may have either $\tilde{\theta}_2 \gtreqless \theta_1$, $p_{n1} - p_{n2} \in (-\infty,\ -t]$, $(-t,\ t]$ or $(t, +\infty)$, $b^2_{sn}(\theta_1) \in (-\infty,\ -t]$, $(-t,\ t]$ or $(t, +\infty)$, $b^2_{ss}(\tilde{\theta}_1) \in (-\infty,\ -t]$, $(-t,\ t]$ or $(t, +\infty)$, and $b^2_{ss}(\tilde{\theta}_2) \in (-\infty,\ -t]$, $(-t,\ t]$ or $(t, +\infty)$. Altogether, there are $2 \times 3 \times 3 \times 3 \times 3 = 162$ different cases if $\theta_2 \geq \theta_1$. (In contrast, in the demand system (6), there are only two subcases if $\theta_2 \geq \theta_1$, i.e., $\tilde{\theta}_2 \gtreqless \theta_1$.) Similarly, if $\theta_1 > \theta_2$, we may have either $\tilde{\theta}_1 \gtreqless \theta_2$, $p_{n2} - p_{n1} \in (-\infty,\ -t]$, $(-t,\ t]$ or $(t, +\infty)$, $b^1_{sn}(\theta_2) \in (-\infty,\ -t]$, $(-t,\ t]$ or $(t, +\infty)$, and $b^1_{ss}(\tilde{\theta}_2) \in (-\infty,\ -t]$, $(-t,\ t]$ or $(t, +\infty)$, again yielding 162 cases. In total, there are $162 \times 2 = 324$ different cases. Although it is possible to state the demands using common expressions for all cases (see Appendix G), it should be understood that the demand functions are discontinuous in the retail prices and therefore each retailer's profit function is, as well.

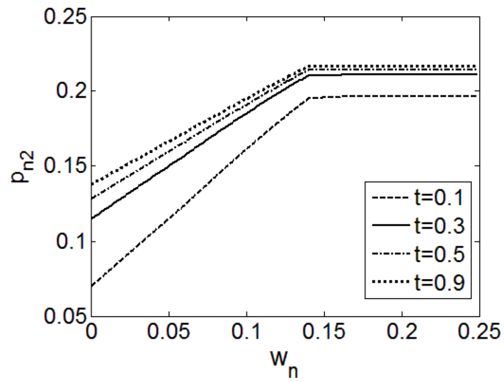
To solve each retailer's problem at a given iteration (given the other retailer's prices from the previous iteration), we first solve the problem for each of the aforementioned 324 cases using Sequential Quadratic Programming (SQP). We then compare the best feasible solutions for each of the 324 cases and choose the best among them. We repeat this process until the changes in both retailers' price vectors from one iteration to the next (measured as the Euclidean distance between them) is below a threshold. We use a threshold of 10^{-6}.

We next present some numerical examples to illustrate the impact of t on equilibrium prices and profits. We use parameter values $q_{s1} = 0.3$, $q_{s2} = 0.4$, $q_n = 0.6$, $\bar{\theta} = 0.5$ and $k = 0.1$. In Figures 3 through 5, we plot equilibrium retail prices, retailers' profit and customer demands, respectively, as a function of w_n for $t = 0.1, 0.3, 0.5$ and 0.9. (Other examples exhibit similar patterns.)

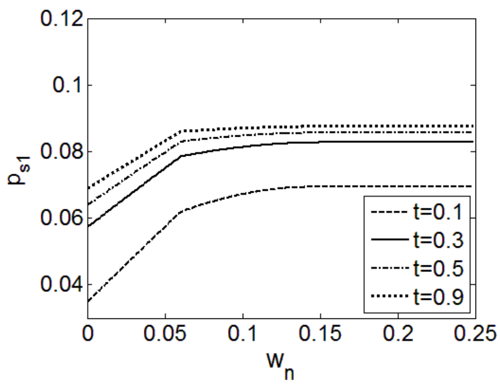
Figure 3 illustrates some characteristics of the price equilibrium. First, the retail price for



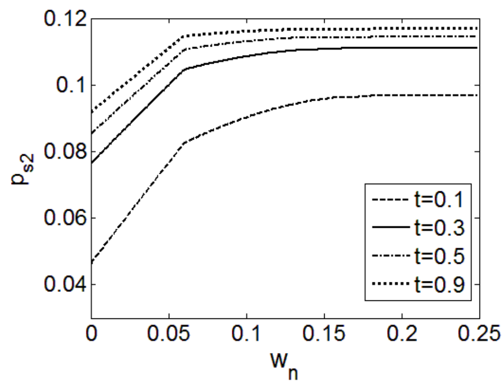
(A) Equilibrium p_{n1} for different levels of t



(B) Equilibrium p_{n2} for different levels of t



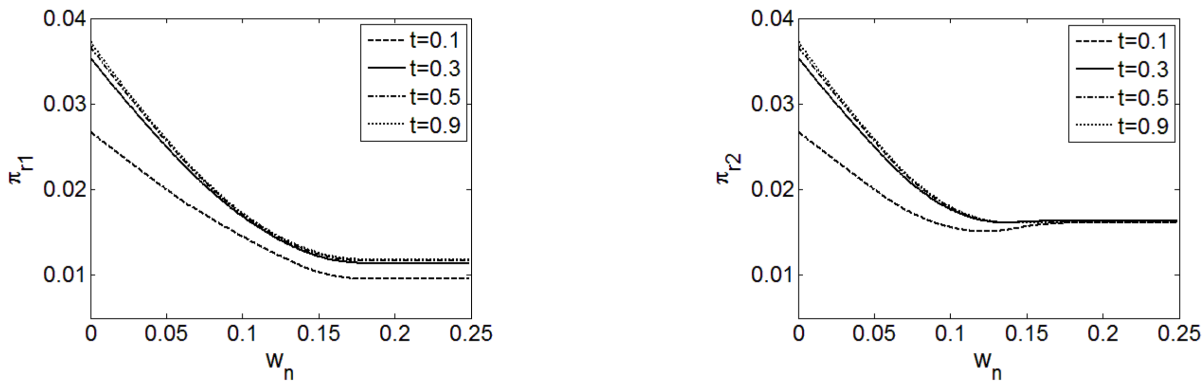
(C) Equilibrium p_{s1} for different levels of t



(D) Equilibrium p_{s2} for different levels of t

FIGURE 3. Equilibrium retail prices for different levels of t

the store brand at R2 is higher than that at R1, but the retail prices of the national brand exhibit the opposite relationship. Also, at each retailer, the retail prices for both the store and national brands strictly increase with w_n until w_n reaches the threshold at which the retailer stops carrying the national brand. Moreover, as predicted by Proposition 1, the retailers' product assortment decisions are not affected by a change in t. For any value of t, both retailers choose to offer the store brand if w_n exceeds $kq_n = 0.06$, as can be seen from the kink in each curve in Figures 3(C) and 3(D) at $w_n = 0.06$. The analogous kinks can also be identified in Figures 5(C) and 5(D). R1 drops the national brand if w_n exceeds $kq_{s1}+\bar{\theta}(q_n-q_{s1}) = 0.18$, which is evident from the kink in each curve in Figures 3(A) and 5(A) at $w_n = 0.18$. R2 stops carrying the national brand if w_n exceeds $kq_{s2} + \bar{\theta}(q_n - q_{s2}) = 0.14$, as can be identified from the kinks in the curves in Figures 3(B) and 5(B) at $w_n = 0.14$. These thresholds do not vary with the value of t. However, all of the retail prices increase with t, as expected, due to reduced competition.



(A) Equilibrium π_{r1} for different levels of t

(B) Equilibrium π_{r2} for different levels of t

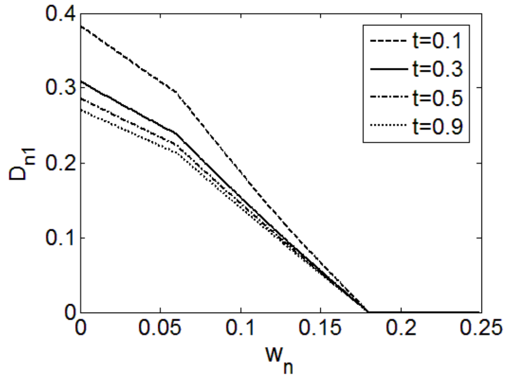
FIGURE 4. Retailers' profit for different levels of t

The retailers' profits for different values of t are shown in Figure 4. Comparing Figures 4(A) and 4(B), it can be seen that the profits of the two retailers are equal and both are decreasing with w_n for $w_n \leq 0.06$ (that is, when both of them carry only the national brand). For $w_n > 0.06$, R1's profit is smaller than that of R2. R1's profit keeps decreasing with w_n until the retailer stops carrying the national brand (at about $w_n = 0.18$). In comparison, R2's profit first decreases and then increases with w_n until both retailers stop carrying the national brand. The fact that R2's profit increases with w_n for a range of w_n is quite counterintuitive, as one would conjecture that both retailers should become worse off as w_n increases. What is happening here is the following: as w_n increases in this range, R2 prices so as to sell greater quantities of her store brand. This not only shields her from the declining margin on the national brand as w_n increases, but also lessens the degree of competition she faces from R1. As a result, R2 may be better off as w_n increases in this range. Note that this would never happen if R2 were a downstream monopolist. Finally, as t increases, both retailers' profits increase for all values of w_n as the level of retail competition declines.

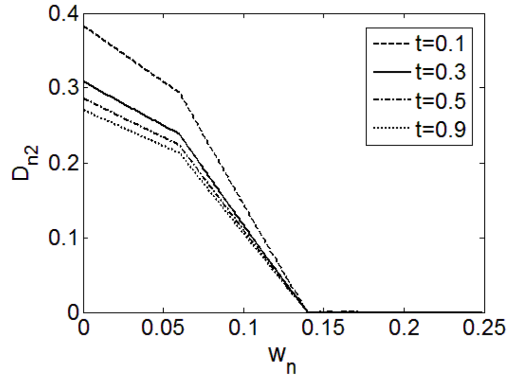
Figure 5 shows the customer demands at the equilibrium prices for different values of t. The national brand demand at each retailer strictly decreases with w_n before it reaches zero, as expected. The store brand demand at each retailer increases with w_n for the interval of w_n in which the retailer carries both products. Also, as t increases, the demands for all products decline due to lessened competition which drives up retail prices, which in turn drives down customer demands.

5. Manufacturer's Strategy under Retail Competition

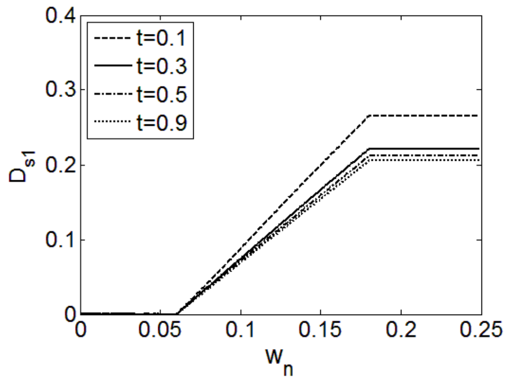
In this section, we investigate the manufacturer's strategy regarding the breadth of product distribution, i.e., whether to distribute through both retailers or only one. Due to the structure of each retailer's product assortment strategy (see Proposition 1), the national brand manufacturer can either sell through both retailers by setting the wholesale price in



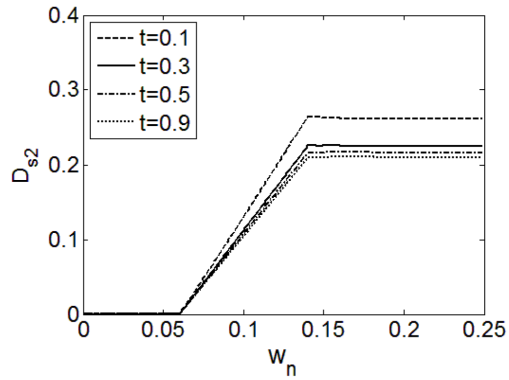
(A) Equilibrium D_{n1} for different levels of t



(B) Equilibrium D_{n2} for different levels of t



(C) Equilibrium D_{s1} for different levels of t



(D) Equilibrium D_{s2} for different levels of t

FIGURE 5. Equilibrium demands for different levels of t

the range $[c_n,\ c_{s2} + (\bar{v}_n - \bar{v}_{s2})]$, or sell it only through R1, whose store brand competes less directly with the national brand. The national brand manufacturer implements the latter decision by setting the wholesale price in the range $[c_{s2} + (\bar{v}_n - \bar{v}_{s2}),\ c_{s1} + (\bar{v}_n - \bar{v}_{s1})]$.

To understand when the manufacturer chooses each option, we start with the special case in which each customer is extremely loyalty to the closer retailer. We examine the general case subsequently.

PROPOSITION 4. *Define $H(q_{s2}) = 2(q_n - q_{s2})$. Then when $t \to +\infty$, the manufacturer distributes through both retailers if $q_{s2} - q_{s1} < H(q_{s2})$ and distributes through only R1 if $q_{s2} - q_{s1} > H(q_{s2})$. He is indifferent between these two options if $q_{s2} - q_{s1} = H(q_{s2})$.*

Proposition 4 states that, when the two retailers serve separate markets, the national brand manufacturer will distribute his product only through the retailer that competes less directly with him if the quality gap between the two store brands is greater than a threshold, $H(q_{s2})$. If the quality disparity is above this threshold, if the manufacturer sells through both retailers, he needs to compromise to a large degree when setting the wholesale price. (Ideally, he would like to offer different prices to the two retailers but he is not allowed to do so). In such a scenario, the manufacturer prefers to sell through only one retailer. The value of $H(q_{s2})$ decreases with q_{s2} because, as q_{s2} increases, the store brand at R2 competes more directly with the national brand. This makes selling through R2 less attractive to the national brand manufacturer.

Figure 6 shows the manufacturer's distribution breadth for different pairs of $(q_{s1},\ q_{s2})$ at different levels of t. (Although our discussion focuses on the half plane in which $q_{s2} \geq q_{s1}$, for completeness, we show the other half plane in Figure 6 as well.) We can see that the threshold $H(q_{s2})$ increases as t decreases, which implies that as the competition between retailers becomes fiercer, the manufacturer is willing to sell through both retailers when the quality levels of their store-brand products are even more disparate. Intuitively, when t is small, the national brand manufacturer can effectively create strong competition between retailers by selling his product through both of them, and the national brand manufacturer benefits from this competition. On the other hand, when t is relatively large, the level of competition between retailers remains low even if the national brand manufacturer sells through both of them. In this case the manufacturer is even more inclined to forgo the opportunity of distributing through R2.

Other than a change in $H(q_{s2})$, the qualitative insights from Proposition 4 carry over to the case of finite t. The national brand manufacturer sells through both retailers only if the quality disparity between the store brands is below a threshold. Proposition 5 formally establishes this result for the case where customer loyalty is large enough that the national brand manufacturer's profit function is piecewise concave.

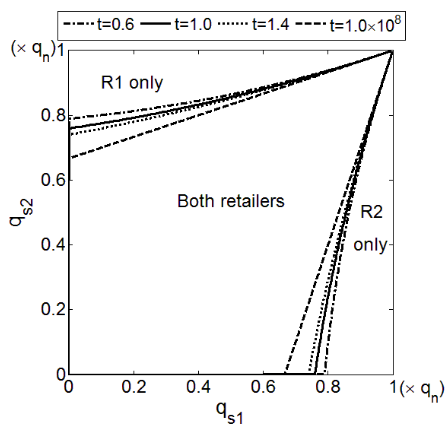


FIGURE 6. Manufacturer's distribution breadth for different $(q_{s1},\ q_{s2})$ pairs for different levels of t. Other parameters are $(k,\ \bar{\theta}) = (0.4,\ 2.0)$.

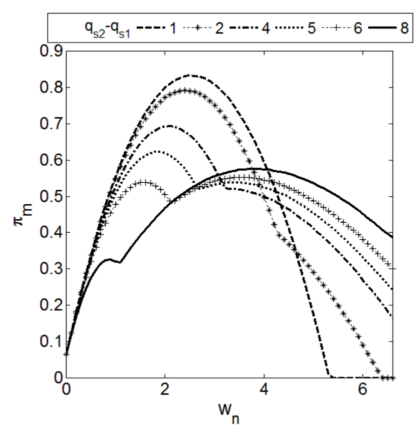


FIGURE 7. $\pi_m(w_n)$ for different $q_{s2} - q_{s1}$. Other parameters are $(q_n,\ t,\ \bar{\theta},\ k) = (9.64,\ 9.09,\ 1.06,\ 0.00)$.

PROPOSITION 5. *For any $\epsilon > 0$, there exists $t_0 = t_0(\epsilon) < +\infty$ such that whenever $t > t_0$, the manufacturer distributes through both retailers whenever $q_{s2} - q_{s1} \leq 2(q_n - q_{s2}) - \epsilon$ and distributes only through R1 whenever $q_{s2} - q_{s1} \geq 2(q_n - q_{s2}) + \epsilon$.*

Moreover, the threshold of $q_{s2} - q_{s1}$ at which the manufacturer changes his distribution breadth decreases with the absolute quality level of the higher-quality store brand (s2). In particular, when the quality of $s2$ is as high as that of the national brand, the threshold falls to zero, meaning that the manufacturer will sell through only R1 for all levels of the quality disparity between $s2$ and $s1$. Formally, we have the following result.

PROPOSITION 6. *For all positive values of t, if $q_{s2} = q_n$, the national brand manufacturer always distributes the national brand only through R1.*

Figure 7 shows the manufacturer's profit as a function of w_n for different levels of the quality disparity between the store brands while keeping the average store brand quality constant (at 4.6). We can see that the profit of the national brand manufacturer is piecewise concave with two modes, one for each distribution strategy. The national brand manufacturer chooses between the candidate wholesale prices corresponding to the two modes. For this example, he will choose the smaller candidate wholesale price and thus distribute the national brand through both retailers when $q_{s2}-q_{s1} = 1, 2, 4$, or 5. He will choose the higher candidate wholesale price, distributing through only R1 when $q_{s2} - q_{s1} = 6$ or 8. We note here that an interesting implication of this is that, keeping the other retailer's store brand quality fixed, as the retailer continues to improve its store brand quality, the wholesale price she needs to pay for the national brand will first decrease and then, at some point, jump up. Past research (Scott-Morton and Zettelmeyer 2004 and Mills 1995) suggests that a retailer is able to extract a lower wholesale price from a national brand manufacturer by introducing a store brand of similar quality. We find that this may not be the case when the national brand manufacturer has the option of selling solely through the competing retailer.

We close this section by noting that the manufacturer's wholesale price at equilibrium under retail competition is between the respective wholesale prices he would set with each retailer as a downstream monopolist. To understand the underlying intuition, notice that the higher is the average store-brand quality, the lower is the national brand's wholesale price. If R2 is introduced into the market in which R1 is a monopolist, the overall competitiveness of the store brands increases, and therefore the manufacturer lowers the wholesale price. Similarly, he increases the wholesale price if R1 enters the market in which R2 is initially a monopolist.

Because the equilibrium wholesale price differs under retail competition versus the two monopolist scenarios, the retailers' equilibrium assortments may change even though, for any given wholesale price, each makes the decision in the same way as if she were a monopolist retailer.

6. Special Cases, Extensions and Discussion

In this section, we briefly describe several additional results. Details, including proofs, are omitted here but are available from the authors.

6.1. No Customer Loyalty

When the retailers do not enjoy any store loyalty, i.e., when $t = 0$, if customers purchase the national brand product, they will do so at the retailer with the lower price. This has the effect of driving the retail prices of the national brand at both retailers down to the wholesale price. As a result, neither retailer can secure any profit from the national brand. (This result was also obtained by Moorthy (1988) in the absence of store brands.) On the other hand, if either retailer drops the national brand, her competitor can earn a profit by offering it, so at equilibrium, both retailers offer it, although both would prefer a scenario in which neither carries it.

With the national brand providing no profit, the store brands are the sole source of profit and the sole means by which the retailers are able to differentiate themselves. In practice, major retailers almost always enjoy some level of customer loyalty. Therefore we rarely see retailers getting no margins from national brands. However, for retailers facing strong head-to-head competition, this result implies that the markup on national brands can be fairly low and that retailers are essentially deriving most of their profit from store brands. Indeed, there is empirical evidence showing that some major retailers set high markup ratios on store brands, while at the same time their markup ratios for national brands are close to 1 (see Barsky et al. 2003). Although there may be other explanations for this phenomenon (for example, differences in marginal costs for the products), it is consistent with our findings. Thus, retail competition could be one factor that leads to low markup ratios on national brands compared to those on store brands.

We conclude that when retailers compete head-to-head, the structure of retailers' product assortment and pricing differs from when each enjoys some degree of customer loyalty. Therefore, for the sake of answering the questions raised in Section 1, it is important to include a positive parameter t in our model to capture reality.

6.2. Production Cost

Thus far, we have assumed that both retailers and the manufacturer share the same production cost function, i.e., $c = kq$. If one retailer has a cost advantage over the other, we can let $k_1 \neq k_2$. Without loss of generality, we assume $k_i < k_j$ ($i \in \{1, 2\}$ and $j = 3 - i$). In this scenario, the retailers' product assortment strategy can be characterized as a generalization of Proposition 2:

COROLLARY 2. *If $c_{si} = k_i q_{si}$ for $i = 1, 2$ and $k_1 \neq k_2$, each retailer's best response in terms of product assortment is to carry only the national brand if $w_n \leq k_i q_n$, to carry only the store brand if $w_n \geq c_{si} + (\bar{v}_n - \bar{v}_{si})$, and to carry both products otherwise.*

Recall that when the retailers have the same production cost parameters, the lower national brand wholesale price threshold (above which the retailers choose to carry a store brand) coincide with each other. This no longer holds if their production cost parameters differ. Retailer i (i.e., the retailer with the smaller cost parameter) starts to carry the store brand at a lower wholesale price than the other retailer. When $w_n \in (k_i q_n,\ k_j q_n]$, retailer i carries both the national and the store brand and retailer j carries the national brand only. This phenomenon has additional implications for the national brand manufacturer's strategy, as we delineate below.

Case (1): $k \geq \max\{k_1, k_2\}$. In this case, the national brand manufacturer never sets the wholesale price such that one or both retailer(s) only carry the national brand product. Therefore, just as in our basic model, he chooses between two candidate wholesale prices, which is equivalent to choosing between distributing the national-brand product through one or both retailers. At either candidate wholesale price, both retailers will carry their respective store brand.

Case (2): $k_i < k < k_j$. In this case, the national brand manufacturer chooses among three candidate wholesale prices, each corresponding to a different distribution strategy: selling through both retailers while foreclosing the store brand at retailer j , selling through both retailers but foreclosing neither of the store brands, and selling through only the retailer with the lower-quality store brand while not foreclosing her store brand.

Case (3): $k < k_i < k_j$. In this case, the national brand manufacturer chooses among four candidate wholesale prices, each corresponding to a different distribution strategy. One strategy is selling through both retailers while foreclosing the store brand at both of them. The other three are the same as those mentioned above for the case of $k_i < k < k_j$.

Mills (1995, 1999) finds that it is optimal for the national brand manufacturer to foreclose the store brand if its quality falls into a particular range. We note here that this result is contingent on his assumption that the national brand has an advantage in terms of cost-per-unit-of-quality (as in Case (2) or (3) above). If the national brand does not enjoy a cost advantage over either of the store brands (as in Case (1)), the national brand manufacturer would never find it optimal to foreclose a store brand because he would need to set the wholesale price below his production cost to accomplish this. This effect has been found by Fang et al. (2012) in the case of a single retailer. We have shown that it also applies when when retail competition is introduced.

6.3. Heterogeneity in Transportation Costs

In our basic model, we have assumed that customers are uniformly distributed on a Hotelling line between the retailers. Our model can be extended to capture heterogeneity in transportation costs by an appropriate placement of each customer on the Hotelling line between the two retailers and using a general distribution of customers along the Hotelling line. To implement this, one can use the demand expressions in Appendix G and then substitute a general distribution of customers' locations into the demand expressions. The analysis could then be conducted in a similar fashion. From numerical examples, we have found that retailers' product assortment strategy (i.e., Proposition 1) and the manufacturer's strategy regarding distribution breath (i.e., distributing through both retailers only if the quality disparity between the two store brands is low) remain valid for common symmetric bell-shaped distributions of customers along the Hotelling line.

6.4. Market Share versus Profitability

In Section 5, we establish that it is sometimes profitable for the manufacturer to sell through only one of retailers. In reality, however, national brand manufacturers may still sell through both retailers to maintain market share even though they realize that doing so may have an adverse effect on profit. Indeed, national brand manufacturers are facing a tradeoff between market share and profitability when making the decision of whether to sell through a retailer with a very high quality store brand. For example, a senior manager of a large national brand revealed to us that he chooses to continue to sell through all the major retailers although profit may suffer in the short run. Maintaining a large market share has benefits if it reduces competition in the long-run, but whether this actually occurs is an ongoing topic of discussion in the literature. Many researchers have suggested that increasing market share at a cost may not prove to be profitable in the long-term (Fruhan 1972, Hamermesh et al. 1978, Woo and Cooper 1981). For example, Wernerfelt (1986) showed that firms do well by attacking in the early stages of the product life cycle but are better off not overplaying their cards in the stable periods of the life cycle.

7. Conclusions

In this paper, we study a retailer's product assortment and pricing problem when she has the option to carry a store brand, a national brand, or both. We first derive the structure of the retailer's optimal assortment and prices in two settings: (1) when she is a downstream monopolist and (2) when she faces competition from another retailer who may also offer the same national brand and a competing store brand. Past research has established that store brands help generate store traffic and help a store better differentiate itself. As a result, one would expect that a retailer would be more likely to introduce a store brand as a competitive strategy when she faces retail competition. In contrast, we find that in the presence of retail competition, for any given wholesale price of the national brand product, a retailer makes the product assortment decision in the same way as if she were a downstream monopolist. The underlying reason for this result is that a retailer's assortment decision is determined by a comparison between the profitability of her store and national brands, which does not involve the other retailer. However, the presence of multiple retailers affects the national brand manufacturer's choice of a wholesale price, which then affects the ultimate assortments and prices at the retail level.

We also characterize how each retailer's optimal assortment decision depends on the national brand's wholesale price. For wholesale prices below a lower threshold, the retailer carries only the national brand and for wholesale prices above an upper threshold, the retailer carries only the store brand. For wholesale prices between the lower and upper thresholds, the retailer carries both brands. The thresholds may, in general, differ by retailer, but if the production cost per unit of quality is the same for both retailers, then the lower thresholds are the same for both retailers. Although a retailer in a duopoly is able to make the optimal assortment decision in the same way as if she were a monopolist, she needs to take into account retail prices set by her competitor to set the optimal retail prices. Failing to do so may have a sizable effect on profits.

Not surprisingly, the equilibrium retail prices of all offered products decline with the introduction of retail competition, but interestingly, the optimal gap between the prices of the store and national brands also decline. Thus, the introduction of retail competition puts greater pressure on the retail price of the national brand than it does on the retail prices of the store brands. Several news reports and market research suggest that the retailers are essentially "leaving money on the table" by setting store-brand prices too low. (See, e.g., Kumar and Steenkamp 2007).

We also study the national brand manufacturer's optimal pricing decision which affects the retailers' product assortments. From our characterization of how each retailer's assortment changes as a function of the wholesale price, we can infer that the national brand manufacturer needs to choose between two regimes: (1) selling through both retailers and optimizing the wholesale price within the interval in which both retailers choose to offer the national brand (along with their respective store brand), and (2) selling through only the retailer with the lower-quality store brand and optimizing the wholesale price within the relevant price interval.

Our results suggest that the manufacturer should sell his national brand through only one retailer if the quality disparity between the two store brands exceeds a threshold. With a large quality disparity, the manufacturer would charge the retailers different wholesale prices if he were allowed to do so. He would need to compromise a lot when setting a single wholesale price, so he may be better off distributing through one retailer.

We also study the effect of customer loyalty. When there is no customer loyalty, both retailers stop carrying the national brand when the wholesale price exceeds the same threshold. Thus, the national brand manufacturer always distributes through both retailers. The retailers end up in a prisoner's dilemma at the equilibrium: both of them could have earned a higher profit if neither of them had carried the national brand, but they both end up carrying it. As the degree of customer loyalty increases, the national brand manufacturer may or may not distribute through both retailers depending upon the quality levels of the two store-brand products, as discussed above.

As discussed, our model can be extended to handle different production cost functions (as a function of quality) for the various products as well as heterogeneity in customer loyalty to the retailers.

In this paper, we assume that store-brand quality levels are exogenous and that the retailers already offer or are ready to offer their respective store brands. Further research is needed to determine how a retailer should choose the quality of a new store brand when facing competition from a retailer that has, or can offer, her own store brand, along with the national brand. We are pursuing research along these lines. Further research is also needed to study retail competition in settings in which the manufacturers of the store-brand products—both national brand manufacturers and third-party producers—are strategic players. For these settings, equilibria are much more difficult to derive because there will be four or more parties in the competitive game.

APPENDICES

Appendix A: Proof of Lemma 1

If any of the relations in (i) or (ii) fails to hold for retailer i at the equilibrium, she *could* decrease one of the retail prices without affecting either her own profit or the pricing decisions of the other retailer. Then given Assumption 1, she will decrease the relevant price.

Appendix B: Transformed Problems

In this Appendix, we establish the equivalence of a market with parameters $(\bar{\theta},\ k)$ and a transformed market with parameters $(\bar{\theta}',\ 0)$. Define $\mathbf{p}' \equiv (p'_{n1},\ p'_{s1},\ p'_{n2},\ p'_{s2}) \equiv \mathbf{p} - \mathbf{c}$ where $\mathbf{c} \equiv (c_n,\ c_{s1},\ c_n,\ c_{s2})$, $w'_n \equiv w_n - c_n$, $\bar{\theta}' \equiv \bar{\theta} - k$, $\theta'_i \equiv \theta_i - k$ $(= \frac{p'_{ni}-p'_{si}}{q_n-q_{si}})$ and $\tilde{\theta}'_i = \tilde{\theta}_i - k$ $(= \frac{p'_{si}}{q_{ni}})$ for $i = 1,\ 2$. Then we have the following lemma, which can be obtained from straightforward algebraic manipulations.

Lemma 4. *$D_{ni}(\boldsymbol{p}, \bar{\theta}) = \frac{\bar{\theta}'}{\bar{\theta}} D_{ni}(\boldsymbol{p}', \bar{\theta}')$. Moreover, $\pi_{ri}(w_n, \boldsymbol{p}, \bar{\theta}, c_{si}) = \frac{\bar{\theta}'}{\bar{\theta}} \pi_{ri}(w'_n, \boldsymbol{p}', \bar{\theta}', 0)$ and $\pi_m(w_n, \bar{\theta}, c_n) = \frac{\bar{\theta}'}{\bar{\theta}} \pi_m(w'_n, \bar{\theta}', 0)$.*

As such, if the equilibrium prices for the transformed market are $(w'_n,\ \mathbf{p}'(w'_n))$, we can immediately write the equilibrium prices of the original market as $(w'_n + c_n,\ \mathbf{p}'(w'_n) + \mathbf{c})$. As such, here and throughout the remaining Appendices, we will only present proofs of the lemmas and propositions for a market with parameters $(\bar{\theta},\ 0)$ and omit the proofs for the original markets (i.e., the markets with parameters $(\bar{\theta},\ k)$). Nevertheless, the results immediately generalize with appropriate transformations of the prices.

Appendix C: Proofs of Lemmas 2 and 3

The customer demands under this scenario are $D_n(\mathbf{p},\bar{\theta}) = \frac{1}{\bar{\theta}}(\bar{\theta} - \frac{p_n - p_s}{q_n - q_s})$ and $D_s(\mathbf{p},\bar{\theta}) = \frac{1}{\bar{\theta}}(\frac{p_n - p_s}{q_n - q_s} - \frac{p_s}{q_s})$. It is easy to confirm that $\pi_r(w_n,\ \mathbf{p},\ \bar{\theta},\ 0)$ is concave in the retail prices. Setting the first derivatives of $\max_{\mathbf{p}=(p_n,p_s)\geq \mathbf{0}} \pi_r(w_n,\ \mathbf{p},\ \bar{\theta},\ 0)$ with respect to the prices equal to zero, we obtain $p_n(w_n) = \frac{1}{2}(\bar{\theta}q_n + w_n)$ and $p_s(w_n) = \frac{1}{2}\bar{\theta}q_s$. It remains to verify whether the solution is an *interior* point in the region of $(p_n,\ p_s)$ satisfying

$$\bar{\theta} > \frac{p_n - p_s}{q_n - q_s} \tag{5}$$

and

$$\frac{p_n}{q_n} > \frac{p_s}{q_s} \tag{6}$$

Otherwise, we have a boundary solution which corresponds to a case in which either the store brand is not sold at equilibrium (which occurs if (6) is violated), or the national brand is not carried by the retailer (which occurs if (5) is violated). It is easy to confirm that (6) is satisfied if and if only $w_n > 0$, and (5) is satisfied if and only if $w_n \leq (q_n - q_s)\bar{\theta}$. Therefore the retailer sells both products if $0 < w_n < (q_n - q_s)\bar{\theta}$ and sets $p_n(w_n) = \frac{1}{2}(\bar{\theta}q_n + w_n)$ and $p_s(w_n) = \frac{1}{2}\bar{\theta}q_s$. She sells only the national brand if $w_n \leq 0$ and sets $p_n(w_n) = \frac{1}{2}(\bar{\theta}q_n + w_n)$ and $p_n(w_n) = q_s \cdot \frac{p_n(w_n)}{q_n}$. She sells only the store brand if $w_n \geq (q_n - q_s)\bar{\theta}$ and sets $p_s(w_n) = \frac{1}{2}\bar{\theta}q_s$ and $p_n(w_n) = \bar{\theta}(q_n - q_s) + p_s(w_n)$. Taking the reaction of the retailer into account, the objective of the national brand manufacturer is $\max_{(q_n - q_s)\bar{\theta} \geq w_n \geq 0} \frac{w_n}{\bar{\theta}}(\bar{\theta} - \frac{\frac{1}{2}(\bar{\theta}q_n + w_n) - \frac{1}{2}\bar{\theta}q_s}{q_n - q_s})$. Solving this optimization problem, we obtain the equilibrium wholesale price. Equilibrium demands and profits follow directly.

Appendix D: Proof of Proposition 1

We utilize the following change of variables: $b_i \equiv \frac{q_{si}}{q_n}$, $d_i \equiv \frac{q_n - q_{si}}{q_n}$ $(= 1 - b_i)$, $\alpha \equiv \frac{t/\bar{\theta}}{q_n}$, $\beta \equiv \frac{w_n/\bar{\theta}}{q_n}$, $\gamma_i \equiv \frac{\beta}{d_i}$, $\hat{D}_{ni} = 2D_{ni}$ and $\hat{D}_{si} = 2D_{si}$ for $i = 1, 2$. Also define $\hat{D}_{nsi} \equiv \hat{D}_{ni} + \hat{D}_{si}$ and $\hat{\pi}_{ri} = \frac{2}{\bar{\theta}q_n}\pi_{ri}$. Then the problem facing retailer i $(i = 1, 2)$ can be written as

$$\max_{0\leq\tilde{\phi}_i\leq\phi_i\leq 1} \hat{\pi}_{ri}(\phi_i, \tilde{\phi}_i) = d_i\hat{D}_{ni}(\phi_i, \tilde{\phi}_i)\cdot(\phi_i - \gamma_i) + b_i\hat{D}_{nsi}(\phi_i, \tilde{\phi}_i)\cdot(\tilde{\phi}_i) \tag{7}$$

where $\hat{D}_{ni}(\phi_i, \tilde{\phi}_i)$ and $\hat{D}_{nsi}(\phi_i, \tilde{\phi}_i)$ can be derived directly from (5) and (6). Without loss of generality, assume the indices $l, h \in \{1, 2\}$ are such that $\theta_l \leq \theta_h$. Then we have:

$$\begin{aligned}
\hat{D}_{nl} &= (1 - \phi_l) + \frac{1}{\alpha}[(1 - \phi_l)A_l + \frac{1}{2}d_h(1 - \phi_l)^2 - \frac{1}{2}d_h(1 - \phi_h)^2] \\
\hat{D}_{nsl} &= (1 - \tilde{\phi}_l) + \frac{1}{\alpha}[(1 - \tilde{\phi}_l)A_l + \frac{1}{2}d_h(1 - \phi_l)^2 - \frac{1}{2}d_h(1 - \phi_h)^2 - \frac{1}{2}(b_l - b_h)(\phi_l - \tilde{\phi}_l)^2] \\
\hat{D}_{nh} &= (1 - \phi_h) + \frac{1}{\alpha}\left[(1 - \phi_h)A_h\right] \\
\hat{D}_{nsh} &= \begin{cases} (1 - \tilde{\phi}_h) + \frac{1}{\alpha}[(1 - \tilde{\phi}_h)A_h + (\phi_l - \tilde{\phi}_h)(d_h\phi_h - d_l\phi_l) & \\ +\frac{1}{2}(\phi_l^2 - \tilde{\phi}_h^2)(d_l - d_h) + \frac{1}{2}d_h(\phi_h - \phi_l)^2] & , \tilde{\phi}_h \leq \phi_l \\ (1 - \tilde{\phi}_h) + \frac{1}{\alpha}\left[(1 - \tilde{\phi}_h)A_h + \frac{1}{2}d_h(\phi_h - \tilde{\phi}_h)^2\right] & , \tilde{\phi}_h > \phi_l \end{cases}
\end{aligned} \tag{8}$$

where $A_l \equiv -(b_l\tilde{\phi}_l - b_h\tilde{\phi}_h) + (b_l - b_h)\phi_l$ and $A_h \equiv (b_l\tilde{\phi}_l - b_h\tilde{\phi}_h) - (d_h\phi_h - d_l\phi_l)$.

To establish Proposition 1, we only need to show that the solution to (7) satisfies (i) $\phi_i = 1$ when $\gamma_i \geq 1$ and $\phi_i < 1$ when $\gamma_i < 1$; and (ii) $\tilde{\phi}_i = \phi_i$ when $\gamma_i \leq 0$ and $\tilde{\phi}_i < \phi_i$ when $\gamma_i > 0$.

We first prove (i). Notice that for any $\tilde{\phi}_i \in [0,\ \phi_i]$ we have

$$\frac{\partial\hat{\pi}_{ri}}{\partial\phi_i}|_{\phi_i\to1^-} = d_i[\hat{D}_{ni}(\phi_i,\tilde{\phi}_1)|_{\phi_i\to1^-} + \frac{\partial\hat{D}_{ni}}{\partial\phi_i}|_{\phi_i\to1^-} \times (\phi_i - \gamma_i)|_{\phi_i\to1^-}] + b_i[\tilde{\phi}_i\frac{\partial\hat{D}_{nsi}}{\partial\phi_i}|_{\phi_i\to1^-}]$$

It is easy to confirm that $\hat{D}_{ni}(\phi_i,\tilde{\phi}_1)|_{\phi_i\to1^-} = 0$, $\frac{\partial\hat{D}_{nsi}}{\partial\phi_i}|_{\phi_i\to1^-} = 0$, and $\frac{\partial\hat{D}_{ni}}{\partial\phi_i}|_{\phi_i\to1^-} = -\frac{1}{t}(t - b_{ss}^2(\theta_i)) < 0$. Therefore

$$\frac{\partial\hat{\pi}_{ri}}{\partial\phi_i}\begin{cases} > 0 & \text{if } \gamma_i > 1 \text{ and as } \phi_i \to 1^-; \\ < 0 & \text{if } \gamma_i < 1,\ \phi_i > \gamma_i \text{ and as } \phi_i \to 1^-; \\ = 0 & \text{if } \gamma_i = 1 \text{ and as } \phi_i \to 1^-. \end{cases}$$

Thus, when $\gamma_i > 1$, a retailer i who chooses ϕ_i slightly less than 1 can locally improve her profit by increasing ϕ_i to 1. Along with the quasi-concavity of the profit function, this implies the solution to (7) satisfies $\phi_i = 1$. Similarly, when $\gamma_i < 1$, retailer i can obtain a local improvement by slightly decreasing ϕ_i Therefore the solution to (7) satisfies $\phi_i < 1$. Finally, when $\gamma_i = 1$, we have $\frac{\partial\hat{\pi}_{ri}}{\partial\phi_i}|_{\phi_i\to1^-} = 0$, and therefore the solution to (7) satisfies $\phi_i = 1$.

We next show (ii). Define $x_i \equiv d_i\phi_i + b_i\tilde{\phi}_i$ and $y_i = d_i\phi_i - b_i\tilde{\phi}_i$. Then problem (7) is equivalent to one of maximizing $\tilde{\pi}_{ri}(x_i(\phi_i,\tilde{\phi}_i),\ y_i(\phi_i,\tilde{\phi}_i)) \equiv \hat{\pi}_{ri}(\phi_i,\tilde{\phi}_i)$ with respect to x_i and y_i subject to $0 \leq x_i \leq b_i + d_i$ and $-(b_i - d_i)x_i \leq y_i \leq \frac{2b_id_i}{b_i+d_i} - (b_i - d_i)x_i$. So we have

$$\begin{aligned}
&\frac{\partial\tilde{\pi}_{ri}}{\partial y_i}(x_i(\phi_i,\tilde{\phi}_i),\ y_i(\phi_i,\tilde{\phi}_i)) \\
=&\frac{1}{2}\left[\frac{1}{d_i}\cdot\frac{\partial\hat{\pi}_{ri}}{\partial\phi_i} - \frac{1}{b_i}\cdot\frac{\partial\hat{\pi}_{ri}}{\partial\tilde{\phi}_i}\right] \\
=&\frac{1}{2}[\hat{D}_{si}(\phi_i,\tilde{\phi}_i) + (\frac{\partial\hat{D}_{si}}{\partial\tilde{\phi}_i}\tilde{\phi}_i + \frac{\partial\hat{D}_{si}}{\partial\phi_i}\phi_i) + (\frac{\partial\hat{D}_{ni}}{\partial\tilde{\phi}_i} - \frac{b_i}{d_i}\frac{\partial\hat{D}_{nsi}}{\partial\phi_i})\tilde{\phi}_i \\
&-(\frac{\partial\hat{D}_{nsi}}{\partial\phi_i} - \frac{d_i}{b_i}\frac{\partial\hat{D}_{ni}}{\partial\tilde{\phi}_i})\phi_i + (\frac{\partial\hat{D}_{ni}}{\partial\phi_i} - \frac{d_i}{b_i}\frac{\partial\hat{D}_{ni}}{\partial\tilde{\phi}_i})\gamma_i]
\end{aligned}$$

For any $x_i \in [0,\ b_i + d_i]$, as $y_i \to [-(b_i - d_i)x_i]^+$(equivalently, as $\tilde{\phi}_i \to \phi_i^-$), it is easy to confirm that we have $\hat{D}_{si}(\phi_i,\tilde{\phi}_i) \to 0$, $\frac{\partial\hat{D}_{ni}}{\partial\tilde{\phi}_i} - \frac{b_i}{d_i}\frac{\partial\hat{D}_{nsi}}{\partial\phi_i} \to 0$, $\frac{\partial\hat{D}_{nsi}}{\partial\phi_i} - \frac{d_i}{b_i}\frac{\partial\hat{D}_{ni}}{\partial\tilde{\phi}_i} \to 0$, $\frac{\partial\hat{D}_{ni}}{\partial\phi_i} - \frac{d_i}{b_i}\frac{\partial\hat{D}_{ni}}{\partial\tilde{\phi}_i} > 0$, and $\frac{\partial\hat{D}_{si}}{\partial\tilde{\phi}_i}\tilde{\phi}_i + \frac{\partial\hat{D}_{si}}{\partial\phi_i}\phi_i \to 0$. Therefore,

$$\frac{\partial\tilde{\pi}_{ri}}{\partial y_i}\begin{cases} > 0 & \text{if } \gamma_i < 0 \text{ and as } y_i \to [-(b_i - d_i)x_i]^+ \text{ (equivalently, as } \tilde{\phi}_i \to \phi_i^-); \\ < 0 & \text{if } \gamma_i > 0 \text{ and as } y_i \to [-(b_i - d_i)x_i]^+ \text{ (equivalently, as } \tilde{\phi}_i \to \phi_i^-); \\ = 0 & \text{if } \gamma_i = 0 \text{ and as } y_i \to [-(b_i - d_i)x_i]^+ \text{ (equivalently, as } \tilde{\phi}_i \to \phi_i^-). \end{cases}$$

In words, if $\gamma_i < 0$, a retailer i who sets her $\tilde{\phi}_i$ slightly less than ϕ_i can locally improve her profit by increasing $\tilde{\phi}_i$. Along with the quasi-concavity of the profit function, this implies that if $\gamma_i < 0$, the solution to (7) satisfies $\tilde{\phi}_i = \phi_i$. Similarly, if $\gamma_i > 0$, retailer i can strictly increase her profit by decreasing $\tilde{\phi}_i$ from a value very close to ϕ_i. Therefore the solution to (7) satisfies $\tilde{\phi}_i < \phi_i$. Finally, when $\gamma_i = 0$, the solution to (7) satisfies $\tilde{\phi}_i = \phi_i$.

Appendix E: Proof of Proposition 2

We present a proof for the assortment scenario in which both retailers carry both the store and the national brands (i.e. when $0 \leq \gamma_1,\ \gamma_2 \leq 1$). The uniqueness of the price equilibrium under the other assortment scenarios can be shown in a similar way. For this case, the first order conditions of the two retailers can be written as $\boldsymbol{\phi} = T(\boldsymbol{\phi})$ where $\boldsymbol{\phi}$ represents vector $(\phi_1, \tilde{\phi}_1, \phi_2, \tilde{\phi}_2)^T \in [0,\ 1]^4$. We can confirm that $\lim\limits_{\alpha \to +\infty} T(\boldsymbol{\phi}) = \left[\ \frac{1}{2}(1+\gamma_1) \quad \frac{1}{2} \quad \frac{1}{2}(1+\gamma_2) \quad \frac{1}{2}\ \right]^T \in [0,\ 1]^4$. Therefore there exists α_1 with $0 < \alpha_1 < +\infty$ such that for all α satisfying $\alpha > \alpha_1$, $T(\boldsymbol{\phi}) \in [0,\ 1]^4$. Therefore, for all α satisfying $\alpha > \alpha_1$, $T(\boldsymbol{\phi})$ is a mapping from $[0,\ 1]^4$ to $[0,\ 1]^4$. Define $q(\alpha) \equiv \sup\limits_{\boldsymbol{\phi}_x, \boldsymbol{\phi}_y \in [0\ ,1]^4} \Gamma(\boldsymbol{\phi}_x, \boldsymbol{\phi}_y)$ where $\Gamma(\boldsymbol{\phi}_x, \boldsymbol{\phi}_y) \equiv \begin{cases} \frac{||T(\boldsymbol{\phi}_x) - T(\boldsymbol{\phi}_y)||}{||\boldsymbol{\phi}_x - \boldsymbol{\phi}_y||} & \text{if } \boldsymbol{\phi}_x \neq \boldsymbol{\phi}_y \\ \frac{dT(\boldsymbol{\phi})}{d\boldsymbol{\phi}}|_{\boldsymbol{\phi}=\boldsymbol{\phi}_x} & \text{if } \boldsymbol{\phi}_x = \boldsymbol{\phi}_y \end{cases}$.

Then, because $\lim\limits_{\alpha \to +\infty} T(\boldsymbol{\phi}) = \left[\ \frac{1}{2}(1+\gamma_1) \quad \frac{1}{2} \quad \frac{1}{2}(1+\gamma_2) \quad \frac{1}{2}\ \right]^T$ for any $\boldsymbol{\phi} \in [0,\ 1]^4$, we have $\lim\limits_{\alpha \to +\infty} q(\alpha) = 0$. Then we can immediately establish that there exists α_2 with $0 < \alpha_2 < +\infty$ such that, for all $\alpha > \alpha_2$, there exists a $q \equiv q(\alpha)$ with $0 < q < 1$ such that $||T(\boldsymbol{\phi}_x) - T(\boldsymbol{\phi}_y)|| \leq q||\boldsymbol{\phi}_x - \boldsymbol{\phi}_y||$ for any $\boldsymbol{\phi}_x, \boldsymbol{\phi}_y \in [0\ ,1]^4$. Define $\alpha_0 \equiv \max\{\alpha_1,\ \alpha_2\}$. Then for all $\alpha > \alpha_0$, $T(\bullet)$ is a contraction mapping on $[0\ ,1]^4$. By the Banach Fixed Point Theorem, there exists a unique solution to the system of the retailers' first order conditions. This implies that for all $t > t_0 \equiv \alpha_0 q_n \bar{\theta}$, the price equilibrium between the retailers is unique.

Appendix F: Proof of Proposition 3

First, notice that $\hat{D}_{ni}$ and $\hat{D}_{nsi}$ are both strictly decreasing and convex in ϕ_i, and that given ϕ_i, $\hat{D}_{ni}$ and $\hat{D}_{nsi}$ are both strictly decreasing and convex in $\tilde{\phi}_i$ $(i = 1, 2)$. Next, from a Taylor series expansion of $\hat{D}_{ni}(1, \tilde{\phi}_i)$, we have $\hat{D}_{ni}(1,\ \tilde{\phi}_i) = \hat{D}_{ni}(\phi_i, \tilde{\phi}_i) + \frac{\partial \hat{D}_{ni}(\phi_i, \tilde{\phi}_i)}{\partial \phi_i} \cdot (1 - \phi_i) + \frac{\partial^2 \hat{D}_{ni}(\eta_i,\ \tilde{\phi}_i)}{\partial \phi_i^2} \cdot \frac{(1-\phi_i)^2}{2}$ where $\eta_i \in (\phi_i, 1)$. Because $\hat{D}_{ni}(1,\ \tilde{\phi}_i) = 0$ and $\frac{\partial^2 \hat{D}_{ni}(\eta_i,\ \tilde{\phi}_i)}{\partial \phi_i^2} > 0$, we have $\hat{D}_{ni}(\phi_i, \tilde{\phi}_i) + \frac{\partial \hat{D}_{ni}(\phi_i, \tilde{\phi}_i)}{\partial \phi_i} \cdot (1 - \phi_i) = -\hat{D}_{ni}(1,\ \tilde{\phi}_i) - \frac{\partial^2 \hat{D}_{ni}(\eta_i,\ \tilde{\phi}_i)}{\partial \phi_i^2} \cdot \frac{(1-\phi_i)^2}{2} < 0$. This proves that for any $\tilde{\phi}_i$, we have $\hat{D}_{ni}(\phi_i, \tilde{\phi}_i) + \frac{\partial \hat{D}_{ni}}{\partial \phi_i} \cdot (1 - \phi_i) < 0$. Similarly, we can show for any ϕ_i, we have $\hat{D}_{nsi} + \frac{\partial \hat{D}_{nsi}}{\partial \tilde{\phi}_i}(1 - \tilde{\phi}_i) < 0$. Now, the first order condition of problem (7) with respect to ϕ_i can be written as $0 = d_i \left[\frac{\partial \hat{D}_{ni}}{\partial \phi_i}\left((\phi_i - \gamma_i) - (1 - \phi_i)\right)\right] + d_i \left[\hat{D}_{ni} + \frac{\partial \hat{D}_{ni}}{\partial \phi_i}(1 - \phi_i)\right] + b_i \frac{\partial \hat{D}_{nsi}}{\partial \phi_i} \tilde{\phi}_i$ after mathematical manipulation. Because $\phi_i^m (\equiv \theta_i^m / \bar{\theta}) = \frac{1}{2}(1 + \gamma_i)$ as derived in Lemma 3, we have $2d_i \frac{\partial \hat{D}_{ni}}{\partial \phi_i}(\phi_i^m - \phi_i) = d_i \left[\hat{D}_{ni} + \frac{\partial \hat{D}_{ni}}{\partial \phi_i}(1 - \phi_i)\right] + b_i \frac{\partial \hat{D}_{nsi}}{\partial \phi_i} \tilde{\phi}_i$. As we have just shown above that $\hat{D}_{ni} + \frac{\partial \hat{D}_{ni}}{\partial \phi_i}(1 - \phi_i) < 0$, $\frac{\partial \hat{D}_{nsi}}{\partial \phi_i} < 0$ and $\frac{\partial \hat{D}_{ni}}{\partial \phi_i} < 0$, we can see immediately that $\phi_i^m - \phi_i > 0$, which proves $\theta_i^m > \theta_i$. The proof of $\tilde{\theta}_i^m > \tilde{\theta}_i$ follows similarly.

Appendix G: Another Way to Express Customer Demand

Denote the p.d.f. and c.d.f. of $\tilde{x}_i$ as $f(\cdot)$ and $F(\cdot)$, respectively, and suppose $\theta_i \geq \theta_j$ $(i = 1, 2,\ j = 3 - i)$. Then for all finite and positive values of t, the demand functions are

$$D_{ni} = \int_{\theta_i}^{\bar{\theta}} \frac{1}{\bar{\theta}} d\theta \int_{-\infty}^{p_{nj}-p_{ni}} dx f(x)$$

$$D_{nj} = \int_{\theta_i}^{\bar{\theta}} \frac{1}{\bar{\theta}} d\theta \int_{p_{nj}-p_{ni}}^{+\infty} dx f(x) + \int_{\theta_j}^{\theta_i} \frac{1}{\bar{\theta}} d\theta \int_{b_{sn}^i(\theta)}^{+\infty} dx f(x)$$

$$D_{si} = \begin{cases} \int_{\tilde{\theta}_i}^{\theta_i} \frac{1}{\bar{\theta}} d\theta \int_{-\infty}^{b_{sn}^i(\theta)} dx f(x) & \text{if } \theta_j < \tilde{\theta}_i \\ \int_{\theta_j}^{\theta_i} \frac{1}{\bar{\theta}} d\theta \int_{-\infty}^{b_{sn}^i(\theta)} dx f(x) & \text{if } \theta_j \geq \tilde{\theta}_i \end{cases}$$

$$D_{sj} = \int_{\tilde{\theta}_j}^{\theta_j} \frac{1}{\bar{\theta}} d\theta \int_{b_{ss}^i(\theta)}^{+\infty} dx f(x)$$

These expressions can be integrated explicitly as θ is uniformly distributed on $(0,\ \bar{\theta})$. We obtain the following demand functions:

$$D_{ni}^H = \frac{1}{\bar{\theta}}(\bar{\theta} - \theta_i) F(p_{nn}^j)$$

$$\begin{aligned} D_{ni}^L = &\frac{1}{\bar{\theta}}\{(\theta_j \bar{F}(b_{sn}^j(\theta_j)) - \theta_i \bar{F}(b_{sn}^j(\theta_i))) + \frac{1}{h_{sn}^j}\bar{X}(b_{sn}^j(\theta_i), b_{sn}^j(\theta_j)) + \frac{p_{sn}^j}{h_{sn}^j}(F(b_{sn}^j(\theta_j)) \\ &- F(b_{sn}^j(\theta_i))) + \frac{1}{\bar{\theta}}(\bar{\theta} - \theta_j)\bar{F}(p_{nn}^i)\} \end{aligned}$$

$$\begin{aligned} D_{si}^L = &\frac{1}{\bar{\theta}}\{(\theta_i \bar{F}(b_{ss}^j(\theta_i)) - \tilde{\theta}_i \bar{F}(b_{ss}^j(\tilde{\theta}_i))) + \frac{1}{h_{ss}^j}\bar{X}(b_{ss}^j(\tilde{\theta}_i), b_{ss}^j(\theta_i)) + \frac{p_{ss}^j}{h_{ss}^j}(F(b_{ss}^j(\theta_i)) \\ &- F(b_{ss}^j(\tilde{\theta}_i)))\} \end{aligned}$$

$$D_{si}^H = \begin{cases} \frac{1}{\bar{\theta}}\{\theta_i F(b_{sn}^i(\theta_i)) - \theta_j F(b_{sn}^i(\theta_j)) - \frac{1}{h_{sn}^i}\bar{X}(b_{sn}^i(\theta_j), b_{sn}^i(\theta_i)) & \\ -\frac{p_{sn}^i}{h_{sn}^i}(F(b_{sn}^i(\theta_i)) - F(b_{sn}^i(\theta_j))) + \theta_j F(b_{ss}^i(\theta_j)) - \tilde{\theta}_i F(b_{ss}^i(\tilde{\theta}_i)) & \\ -\frac{1}{h_{ss}^i}\bar{X}(b_{ss}^i(\tilde{\theta}_i),\ b(\theta_j)) - \frac{p_{ss}^i}{h_{ss}^i}(F(b_{ss}^i(\theta_j)) - F(b_{ss}^i(\tilde{\theta}_i)))\}, & \text{if } \tilde{\theta}_i \leq \theta_j \\ & \\ \frac{1}{\bar{\theta}}\{\theta_i F(b_{sn}^i(\theta_i)) - \tilde{\theta}_i F(b_{sn}^i(\tilde{\theta}_i)) - \frac{1}{h_{sn}^i}\bar{X}(b_{sn}^i(\tilde{\theta}_i),\ b_{sn}^i(\theta_i)) & \\ -\frac{p_{sn}^i}{h_{sn}^i}(F(b_{sn}^i(\theta_i)) - F(b_{sn}^i(\tilde{\theta}_i)))\} & \text{if } \tilde{\theta}_i \geq \theta_j \end{cases}$$

D_{ni}^H, D_{si}^H, D_{ni}^L and D_{si}^L were defined in (5) and $\bar{X}(a, b) \equiv \int_a^b f(x)dx$. The advantages of the above demand functions in comparison to those in (6) are that (i) the above demand functions hold for any distribution of $\tilde{x}_i$ whereas (6) holds only when $\tilde{x}_i$ has a uniform distribution; and (ii) (6) represents demands for the case where $t \geq \max\{|p_{n2} - p_{n1}|,\ |p_{s2} - p_{s1}|,\ |b_{sn}^i(\theta_j)|\}$, whereas the above demand functions are more general and are valid for all positive, finite values of t. We thus use the above demand functions when performing our numerical analysis. In our numerical analysis, given all the exogenous parameters and a set of retail prices, we substitute expressions for $F(\tilde{x})$, $f(\tilde{x})$ and $\bar{X}(\tilde{x}_1, \tilde{x}_2)$ into the above demand functions to calculate the ultimate demand for each product. Specifically, for the case of uniform distributions, $F(\tilde{x}) = \frac{\tilde{x}+t}{2t} \cdot 1\{-t < \tilde{x} < t\}$, $f(\tilde{x}) = \frac{1}{2t} \cdot 1\{-t < \tilde{x} < t\}$, and $\bar{X}(\tilde{x}_1, \tilde{x}_2) = \bar{X}(-t,\ \tilde{x}_2) \cdot 1\{\tilde{x}_1 \leq -t\} + \frac{\tilde{x}_2^2 - \tilde{x}_1^2}{4t} \cdot 1\{-t < \tilde{x}_1 < \tilde{x}_2 < t\} + \bar{X}(\tilde{x}_1,\ t) \cdot \{\tilde{x}_2 \geq t\}$.

Appendix H: Proof of Proposition 4

When $t \to +\infty$, we have $\hat{D}_{ni} = 1 - \phi_i$, $\hat{D}_{nsi} = 1 - \tilde{\phi}_i$, $\hat{\pi}_{ri} = d_i(1-\phi_i)(\phi_i - \gamma_i) + b_i(1-\tilde{\phi}_i)\tilde{\phi}_i$ for $i = 1, 2$ and $\hat{\pi}_m \equiv \frac{2}{\bar{\theta} q_n}\pi_m = [(1-\phi_1) + (1-\phi_2)]\,\beta$. Given β , retailer i's problem is $\max\limits_{\phi_i,\ \tilde{\phi}_i} \hat{\pi}_{ri}$. The solution is $(\phi_i,\ \tilde{\phi}_i) = (1,\ \frac{1}{2})$ if $\gamma_i \geq 1$, $(\phi_i,\ \tilde{\phi}_i) = (\frac{1+\beta/d_i}{2},\ \frac{1}{2})$ if $\gamma_i \in (0,\ 1)$ and $(\frac{1+\beta}{2},\ \frac{1+\beta}{2})$ if $\gamma_i \leq 0$. Knowing retailers' responses given β, the manufacturer's problem becomes $\max\limits_{\beta \in [0,\ d_2]} \hat{\pi}_m = \left[(1 - \frac{1+\beta/d_1}{2}) + (1 - \frac{1+\beta/d_2}{2})\right]\beta$ and $\max\limits_{\beta \in [d_2,\ d_1]} \hat{\pi}_m = \left[0 + (1 - \frac{\beta/d_1}{2})\right]\beta$. The solutions are $\beta_1^* \equiv \underset{\beta \in [0,\ d_2]}{\operatorname{argmax}} \hat{\pi}_m = \frac{d_1 d_2}{d_1 + d_2}$, $\beta_2^* \equiv \underset{\beta \in [d_h,\ d_l]}{\operatorname{argmax}} \hat{\pi}_m = d_1/2$ if $2d_2 \leq d_1$ and $\beta_2^* = d_2$ otherwise. Therefore $\hat{\pi}_m(\beta_1^*) = \frac{1}{2} \cdot \frac{d_1 d_2}{d_1 + d_2}$. $\hat{\pi}_m(\beta_2^*) = \frac{1}{8} d_1$ if $2d_2 \leq d_1$ and $\hat{\pi}_m(\beta_2^*) = \frac{1}{2} d_2 (1 - \frac{d_2}{d_1})$ otherwise. It can be easily verified that $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) > 0$ whenever $2d_2 > d_1$ or when $2d_2 \leq d_1 \leq 3d_2$. Therefore, the manufacturer sets his price at β^* if $d_1 \leq 3d_2$ (which is equivalent to $q_{s2} - q_{s1} < 2(q_n - q_{s2})$) or at β_2^* otherwise.

Appendix I: Proof of Proposition 5

When both retailers carry both the store and the national brands, the first order conditions of two retailers can be simplified to $\phi_l = \frac{1}{2}\left[(1+\gamma_l) + \frac{B_l}{\alpha + C_l}\right]$, $\tilde{\phi}_l = \frac{1}{2}\left[1 + \frac{\tilde{B}_l}{\alpha + \tilde{C}_l}\right]$, $\phi_h = \frac{1}{2}\left[(1+\gamma_h) + \frac{B_h}{\alpha + C_h}\right]$, and $\tilde{\phi}_h = \frac{1}{2}\left[1 + \frac{\tilde{B}_h}{\alpha + \tilde{C}_h}\right]$ where B_l, $\tilde{B}_l$, B_h and $\tilde{B}_h$ are quadratic functions of $\boldsymbol{\phi}$. C_l, $\tilde{C}_l$, C_h and $\tilde{C}_h$ are linear functions of $\boldsymbol{\phi}$. Observe that none is a function of α. With the above expressions for the first order conditions, we next establish Proposition 5 via a series of lemmas.

Lemma 5. *There exists $t_1 < +\infty$ such that whenever $t > t_1$, $\hat{\pi}_m(\beta)$ is concave on $[0,\ d_2]$.*

Proof. First, notice that $\forall \beta \in [0,\ d_2]$,

$$\begin{aligned} \hat{\pi}_m(\beta) &= [\hat{D}_{n1} + \hat{D}_{n2}]\beta \\ &= (1 - \phi_1(\beta)) + (1 - \phi_2(\beta)) + \frac{1}{\alpha} P(\boldsymbol{\phi}(\beta)) \\ &= 2 - \frac{1 + \frac{\beta}{d_1}}{2} - \frac{1 + \frac{\beta}{d_2}}{2} - \frac{1}{2}\left(\frac{B_1(\boldsymbol{\phi}(\beta))}{\alpha + C_1(\boldsymbol{\phi}(\beta))} + \frac{B_2(\boldsymbol{\phi}(\beta))}{\alpha + C_2(\boldsymbol{\phi}(\beta))}\right) + \frac{1}{\alpha} P(\boldsymbol{\phi}(\beta)) \end{aligned} \tag{9}$$

where $P(\boldsymbol{\phi})$ is a quadratic polynomial of ϕ_1, ϕ_2, $\tilde{\phi}_1$ and $\tilde{\phi}_2$. Define

$$p(\beta) \equiv -\frac{1}{2} \frac{B_1(\boldsymbol{\phi}(\beta))}{\alpha + C_1(\boldsymbol{\phi}(\beta))} \tag{10}$$

Because $P(\boldsymbol{\phi}(\beta))$, $C_1(\boldsymbol{\phi}(\beta))$, $C_2(\boldsymbol{\phi}(\beta))$, $B_1(\boldsymbol{\phi}(\beta))$ and $C_2(\boldsymbol{\phi}(\beta))$ are bounded and continuously differentiable for $\forall \beta \in [0,\ d_2]$, we know that $\lim\limits_{\alpha \to +\infty} p(\beta) = 0$, $\forall \beta \in [0,\ d_2]$. Therefore, for any $\epsilon > 0$, there exists $\alpha_1 = \alpha_1(\epsilon) < +\infty$ such that whenever $\alpha > \alpha_1$, $|p'(\beta)| = |p''(\beta)| < \epsilon$ for $\forall \beta \in [0,\ d_2]$. If we take $\epsilon = 2$, then whenever $\alpha > \alpha_1(2)$, $\hat{\pi}''_m(\beta) = -(\frac{1}{d_1} + \frac{1}{d_2}) + p''(\beta) < 0$ for $\forall \beta \in [0,\ d_2]$. This implies that there exists $t_1 \equiv (\bar{\theta} q_n)\alpha_1(2) < +\infty$ such that whenever $t > t_1$, $\hat{\pi}_m(\beta)$ is concave on $[0,\ d_2]$.

Lemma 6. *There exists $t_2 < +\infty$ such that whenever $t > t_2$, $\hat{\pi}_m(\beta)$ is concave on $[d_2,\ d_1]$.*

Proof. The proof is similar to that of Lemma 5 and is omitted here.

Define $\beta_1^* = \max_{\beta\in[0,\ d_2]} \hat{\pi}_m(\beta)$ and $\beta_1^* = \max_{\beta\in[d_2,\ d_1]} \hat{\pi}_m(\beta)$, we then have the following:

LEMMA 7. *There exists $t_3 < +\infty$ such that whenever $t > t_3$, β_1^* is an interior point in $[0,\ d_2]$.*

Proof. Notice that $\hat{\pi}_m'(\beta) = 1 - \beta(\frac{1}{d_1} + \frac{1}{d_2}) + p'(\beta)$ for $\beta \in (0,\ d_2)$. Then for any $d_1,\ d_2 > 0$, when $\beta \to 0^+$, $\hat{\pi}_m'(\beta) = 1 + p'(\beta) > 0$ whenever $\alpha > \alpha_1(1)$. When $\beta \to d_2^-$, $\hat{\pi}_m'(\beta) = -\frac{d_2}{d_1} + p'(\beta)$. Then whenever $\alpha > \alpha_1(d_2)$, $\hat{\pi}_m'(\beta) < 0$. Moreover, whenever $\alpha > \alpha_1(d_1)$, we also have $\alpha > \alpha_1(2)$, and hence from the proof of Lemma 5, $\hat{\pi}_m(\beta)$ is concave on $[0,\ d_2]$. Therefore $\exists t_3 \equiv (\bar{\theta}q_n)\alpha_1(d_2) < +\infty$ such that $\forall t > t_3$, β_1^* is an interior point in $[0,\ d_2]$.

LEMMA 8. *For any $\epsilon > 0$, there exists $t_4(\epsilon) < +\infty$ such that whenever $t > t_4$, β_2^* is an interior point in $(d_2,\ d_1)$ if $d_2 < (\frac{1}{2} - \epsilon)d_1$ and equals d_2 if $d_2 > (\frac{1}{2} + \epsilon)d_1$.*

Proof. First, when $\beta \in [d_2,\ d_1]$, $\hat{\pi}_m(\beta) = [(1 - \frac{1+\frac{\beta}{d_1}}{2}) + p_1(\beta)]$ in which $p_1(\beta) \equiv -\frac{1}{2}\frac{B_1(\boldsymbol{\phi}(\beta))}{\alpha + C_1(\boldsymbol{\phi}(\beta))} + \frac{1}{\alpha}[(1-\phi_1(\beta))A_1 + \frac{1}{2}d_2(1-\phi_1(\beta))^2 - \frac{1}{2}d_2(1-\phi_2(\beta))^2]$. Because $C_1(\boldsymbol{\phi}(\beta))$, $B_1(\boldsymbol{\phi}(\beta))$ and $\phi(\beta)$ are bounded and continuously differentiable for $\forall\beta \in [d_2,\ d_1]$, we know that $\lim_{\alpha\to+\infty} p_1(\beta) = 0$, $\forall\beta \in [d_2,\ d_1]$. Therefore, for any $\epsilon > 0$, there exists $\alpha_2 = \alpha_2(\epsilon) < +\infty$ such that whenever $\alpha > \alpha_2$, $|p_1'(\beta)| < \epsilon$ for $\forall\beta \in [d_2,\ d_1]$. If we take $\epsilon = \frac{1}{2}$, then $\hat{\pi}_m'(d_1) \left(\equiv \frac{\partial\hat{\pi}_m}{\partial\beta}|_{\beta=d_1}\right) = -\frac{1}{2} + p_1'(\beta) < 0$ whenever $\alpha > \alpha_2(\frac{1}{2})$.

Now, because $\hat{\pi}_m'(d_2) = \frac{1}{2} - \frac{d_2}{d_1} + p_1'(\beta)$. For $\forall\epsilon > 0$, using the fact that $|p_1'(\beta)| < \epsilon$ whenever $\alpha > \alpha_2(\epsilon)$, we have $\frac{1}{2} + |p_1'(\beta)| \in [\frac{1}{2} - \epsilon,\ \frac{1}{2} + \epsilon]$ whenever $\alpha > \alpha_2(\epsilon)$. Therefore, if $\frac{d_2}{d_1} > \frac{1}{2} + \epsilon$, we must have $\frac{d_2}{d_1} > \frac{1}{2} + p_1'(\beta)$ which gives $\hat{\pi}_m'(d_2) < 0$, and if $\frac{d_2}{d_1} < \frac{1}{2} - \epsilon$, we must have $\frac{d_2}{d_1} < \frac{1}{2} + p_1'(\beta)$ which gives $\hat{\pi}_m'(d_2) > 0$. If $\hat{\pi}_m'(d_2) < 0$, β_2^* is set at d_2. If $\hat{\pi}_m'(d_2) > 0$, β_2^* is an interior point in $(d_2,\ d_1)$.

Finally, $\forall\epsilon > 0$, if we take $t_4(\epsilon) \equiv (\bar{\theta}q_n)\max\{\alpha_2(\frac{1}{2}),\ \alpha_2(\epsilon)\}$, then we have established Lemma 8.

LEMMA 9. *For any $\epsilon > 0$, there exists $t_0 = t_0(\epsilon) < +\infty$ such that whenever $t > t_0$, the manufacturer sets β_1^* whenever $3d_2 > d_1 + \epsilon$ and sets β_2^* whenever $3d_2 < d_1 - \epsilon$.*

Proof. $\forall\epsilon > 0$, define $\epsilon_0 = \frac{\epsilon}{16}$. From the lemmas above, we know that $\beta_1^* = \frac{d_1 d_2}{d_1 + d_2} + O(\frac{1}{t})$, $\beta_2^* = \frac{d_1}{2} + O(\frac{1}{t})$ if $\frac{d_2}{d_1} < \frac{1}{2} - \epsilon_0$ and $\beta_2^* = d_2$ if $\frac{d_2}{d_1} > \frac{1}{2} + \epsilon_0$. We can thus conclude that $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) = \frac{1}{2}\frac{d_1 d_2}{d_1 + d_2} - \frac{1}{8}d_1 + O(\frac{1}{t})$ if $\frac{d_2}{d_1} < \frac{1}{2} - \epsilon_0$. If $\frac{d_2}{d_1} < \frac{1}{2} - \epsilon_0$ fails to hold, we know that $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq \frac{1}{2}\frac{d_1 d_2}{d_1 + d_2} - \frac{1}{8}d_1 + O(\frac{1}{t})$ because β_2^* sometimes cannot assume the value yielding the interior local optimum. When this happens, from mathematical manipulation we get $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq \frac{1}{2}\frac{d_1 d_2}{d_1 + d_2} - \frac{1}{8}d_1 + O(\frac{1}{t}) = \frac{1}{2}d_1(\frac{3}{4} - \frac{1}{1 + d_2/d_1}) + O(\frac{1}{t}) \geq \frac{1}{2}d_1(\frac{3}{4} - \frac{2}{1 + \frac{1}{2} - \epsilon_0}) + O(\frac{1}{t}) > 0$ for small ϵ_0 values. Therefore the manufacturer always sets the wholesale price at β_1^* when $\frac{d_2}{d_1} < \frac{1}{2} - \epsilon_0$ fails to hold.

If $\frac{d_2}{d_1} < \frac{1}{2} - \epsilon_0$, there exists $t_5 = t_5(\epsilon_0) < +\infty$ such that whenever $t > t_5 \equiv t_0(\epsilon)$, $\frac{1}{2}\frac{d_1 d_2}{d_1 + d_2} - \frac{1}{8}d_1 - \epsilon_0 \leq \hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq \frac{1}{2}\frac{d_1 d_2}{d_1 + d_2} - \frac{1}{8}d_1 + \epsilon_0$. With this, if $3d_2 \geq d_1 + 8\epsilon_0\frac{d_1 + d_2}{d_1}$, then $\frac{1}{2}\frac{d_1 d_2}{d_1 + d_2} - \frac{1}{8}d_1 - \epsilon_0 \geq 0$, which leads to $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq 0$. If $3d_2 \leq d_1 - 8\epsilon_0\frac{d_1 + d_2}{d_1}$, then $\frac{1}{2}\frac{d_1 d_2}{d_1 + d_2} - \frac{1}{8}d_1 + \epsilon_0 \leq 0$, which leads to $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq 0$. Define $\epsilon_1 = 8\epsilon_0\left|\sup_{\{d_1, d_2\}}\frac{d_1 + d_2}{d_1}\right| =$

$16\epsilon_0$ and $\epsilon_2 = 8\epsilon_0 \left| \inf\limits_{\{d_1,d_2\}} \frac{d_1+d_2}{d_1} \right| = 8\epsilon_0$; then $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq 0$ if $3d_2 \geq d_1 + \epsilon_1$ and $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq 0$ if $3d_2 \leq d_1 - \epsilon_2$. Recall that $\epsilon = 16\epsilon_0$, so $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \geq 0$ if $3d_2 \geq d_1 + \epsilon$ and $\hat{\pi}_m(\beta_1^*) - \hat{\pi}_m(\beta_2^*) \leq 0$ if $3d_2 \leq d_1 - \epsilon$.

Recall that $d_i = \frac{q_n - q_{si}}{q_n}$ for $i = 1, 2$. With this, we can immediately obtain Proposition 5 from Lemma 9.

Appendix J: Proof of Proposition 6

If $q_{s1} < q_{s2} = q_n$, the store brand and the national brand are treated as exactly the same product by R2. She will carry only the one with the lower variable cost. Therefore the national brand manufacturer needs to set a wholesale price less than c_n in order to distribute the national brand through R2, but then he will get a nonpositive profit. Hence the national brand manufacturer distributes the national brand through R1 only.

CHAPTER 3

Positioning a Store Brand against National and Store Brands under Retail Competition

1. Introduction

Store brands account for a sizable percentage of sales at retailers. In 2012, sales of store brands in the U.S. totaled $108 billion, with a 23.1 percent unit share (PLMA, 2014). Furthermore, industry experts say that store brand sales could double in the next five to six years (Watson, 2012). Store brands help retailers in various ways. First, a store brand serves as a strategic weapon for the retailer by increasing the retailer's bargaining strength, thereby eliciting wholesale price reductions and non-price concessions from suppliers of competing products. Second, they serve as differentiating tools that distinguish a retailer from its competitors. Third, they help retailers to build store loyalty if customers repeatedly visit to purchase store-brand products, which are not available elsewhere. As a result, retailers actively engage in store brand development. As one example, Kroger is expanding its store brand selection, and this is contributing to its bottom line (Associated Press, 2013). See Kumar and Steenkamp (2007) for a discussion of these and other related benefits of offering store brands.

Before launching a store-brand product, the retailer needs to design and position it. This involves choosing a quality level, which entails both the physical design of the product and the associated product tier or "image." In parallel with these decisions, retailers also seeks to choose a profit maximizing price, considering customers' willingness to pay for a product of the chosen quality level and the associated production cost. In this paper, we study how a retailer should position and price its store brand product when that retailer may offer the leading national brand product, and faces competition against a retailer that offers its own store brand product and the leading national brand product in the same product category.

Recently, many retailers have introduced high quality store brands. For example, Target's Archer Farms has successfully established itself as an affordable luxury convenience brand, offering gourmet goat cheese pizza, Key Lime Cookie Straws and organic milk. Walgreen is also trying to turn its Nice! into a "high quality everyday product at a way-better price" (Davis, 2013). High-quality store brands build a good store image and also increase the retailer's negotiating power versus the national brand manufacturer. However, a lower-quality store brand might be a better choice if the retailer can gain market share by differentiating its store brand from both the national brand and the store brand at the other retailer. In short, the optimal brand positioning of a store brand may change with the quality positioning of the retail competitor's store brand.

Many researchers have studied the question of how a retailer should position its store brand versus a competing national brand product (or in a few articles, two national brand products), but to the best of our knowledge, there is very little research on settings in which a retailer wants to introduce a store brand and needs to position the product not only against national brands but also against existing store brand products at other retailers. Indeed, prior literature does not provide complete equilibrium analysis of the retailer's quality positioning decisions, in concert with relevant pricing equilibria, when there are two competing retailers, both of which can offer their own store band. This paper aims to answer this question. Specifically, we focus on vertical product positioning in which product quality is the key decision variable, along with related pricing decisions, but we also capture customer heterogeneity in terms of their loyalty to competing retailers.

The problem of positioning a store brand against both national and competing store brands differs from the problem of positioning it against only national brands because a store brand product at a competing retailer enhances customers' tendencies to visit that retailer, as store-brand products increase store differentiation and build store loyalty, whereas a leading national brand product does not have such features, as it is available almost everywhere. By positioning a store brand effectively against incumbent national brands, a retailer addresses only vertical competition. But when positioning its store brand effectively against both national and store brands, a retailer needs to address competition both in the vertical (quality) and the horizontal (competing retail channels) dimensions. We capture both of these dimensions in our model.

For retailers, setting the "right" quality level for store brand products is more challenging when competing retailers may offer both their respective store brand as well as the national brand. One reason is that the national brand manufacturer is now both a supplier and competitor to both retailers. Moreover, the retailers become not only competitors, but also "collaborators," because the combined power of the two retailers contributes to eliciting a good wholesale price from the national brand manufacturer. The power of the two retailers depends upon the quality levels of the two store-brand products and the prices that the retailers set for the products that they choose to offer.

To analyze this problem, we develop a game-theoretic, two-retailer model in which one of the retailers chooses the quality level of its store brand given the quality levels of the national brand product (which either retailer may offer) and a store brand at a competing retailer. In the first stage of the game, the retailer of interest decides whether to introduce a store brand product, and if so, what its quality level should be. With the quality levels of all three products now fixed, in the second stage of the game, the national brand manufacturer and the retailers engage in a manufacturer-Stackelberg game in which the national brand manufacturer chooses a single wholesale price for its product to offer to both retailers. Most retailers that are able to offer store brands are relatively large, and we consequently assume that they can secure the same wholesale price for the the national brand product. Then, observing the wholesale price, the retailers engage in a Nash game in which they simultaneously set retail prices for they products they carry. Finally customers make purchasing decisions. The second stage game has been studied in Chapter 2. In this paper, we focus on the retailer's decision of setting the store brand quality in the first stage of the game. We assume complete and perfect information and seek subgame perfect equilibria. We derive a full characterization of the equilibria which then helps us to identify how the store brand positioning decision should be made.

It is not intuitively obvious how such a decision should be made. Past literature suggests that a monopolist retailer often has an incentive to position its store brand as close to the leading national brand as possible, as this endows the retailer with bargaining power which enables her to get a lower wholesale price from the national brand manufacturer. However, when there are two competing retailers, carrying a store brand with too high a quality level could result in the national brand manufacturer increasing its wholesale price to a level that is unattractive to the retailer in question but still acceptable to the competing retailer. The national brand manufacturer would choose such a high wholesale price if it prefers to distribute its product only through the retailer whose store brand competes less directly with the national brand product. If this occurs, the retailer's introduction of his high-quality store brand could backfire, leading to lower, rather than higher, profits. Via our analysis in Sections 4 and 5, we explore what types of equilibria can arise and the conditions that lead to those outcomes.

The remainder of this paper is organized as follows. In Section 2, we present a review of closely-related literature. In Section 3, we describe our basic model and present a formulation of each party's decision problem. Section 4 presents a full equilibrium analysis for the case of $t \to \infty$, i.e., when each retailer has an ample number of loyal customers and the retailers cannot compete on price alone. In Section 5, we discuss the case of finite t. We show that the qualitative structure of the equilibrium remains the same and explain how the retailer should optimally adjust the quality level of her store brand product for smaller t (greater competition). In Section 6, we study the retailer's preference regarding the quality of her competitor's store brand product. In Section 7, we discuss how our results can be generalized to handle scenarios in which the national brand manufacturer and the two retailers have different production cost parameters. We also discuss how our results can be used to obtain the equilibrium for a simultaneous quality-setting game between two retailers. We conclude the paper in Section 8.

2. Related Literature

Although a substantial literature exists on store brand strategies, including introduction, positioning, pricing, and the competitive response of national brand manufacturers, little research has been done to analyze settings with competing retailers, each of which can offer its own store brand in addition to the leading national brand. Before discussing this literature, we first briefly discuss research on store brand positioning at a monopolist retailer.

Models with One or Two National Brands and without Competing Retailers

A few papers present equilibrium positioning (quality) and/or pricing results for settings with one retailer who is introducing a store brand that will compete against a single product offered by a one national brand manufacturer. We refer the reader to Bontems et al. (1999)), Sethuraman (2009), Fousekis (2010) and Chen et al. (2011). In these settings, the optimal positioning of the store brand depends heavily upon the relationship between the unit cost of the product and its quality. Generally speaking, when the unit cost is linear in the quality of the product, the retailer sets a high quality level to gain negotiating leverage, but if the unit cost is convex increasing in the quality, then the retailer sets a lower quality level, which also provides some benefits of gaining greater market share via offering a differentiated product.

Scott-Morton and Zettelmeyer (2004) study a setting in which a retailer currently sells two competing national brands targeted at distinct customer segments of different sizes and must decide which one to eliminate when introducing a new store brand. Customers' utility for the store brand product is less than that for the national brand, and all products are assumed to have zero variable cost. From their equilibrium analysis, the authors conclude that the retailer should eliminate the weaker national brand and position its store brand as close to the leading national brand as possible. The authors also present results from an empirical study which suggests that retailers are trying to imitate the national brand: they often choose packaging features such as color, size and shape to mimic the national brand. Furthermore, retailers often place the store brand adjacent to the national brand it is mimicking. Du et al. (2005) extend Scott Morton and Zettelmeyer's model to incorporate a more general representation of customer heterogeneity and describe scenarios in which it is optimal to position the store brand below the quality level of the leading national brand.

Other researchers have addressed the problem of positioning a store brand against two national brand products that will continue to be offered. Within this stream, the retailer first chooses the positioning of the store brand, the national brand manufacturers next set wholesale prices, and finally the retailer sets retail prices. Sayman et al. (2002) examine a setting in which the effect of product characteristics on demands is modeled (indirectly) via cross-price sensitivity parameters. His analysis is based on the assumption that all variable costs are zero and therefore are not affected by product quality levels. He derives conditions in which the retailer has an incentive to position the store brand as close to the stronger national brand as possible. Sayman also reports results from an empirical study. Two findings are pertinent to our research. First, Sayman's results regarding the targeting and/or imitation of the leading national brand—also focusing on extrinsic features and not explicitly on product quality—parallel those of Scott Morton and Zettelmeyer. Second, his results also indicate that consumers perceive the quality of the store brand to be lower than that of the leading national brand, even when the retailer has targeted the store brand at the leading national brand. Choi and Coughlan (2006) present a model in which products are differentiated both vertically and horizontally, with the horizontal differentiation capturing product features. Consumers' utility functions are parameterized by the quality levels of the products and the degree of substitutability between pairs of products. Although the authors offer a general model in terms of differentiation, nearly all of their analysis is based on the assumption of zero, and therefore equal, marginal costs for the products. Under this simplifying assumption, they show that (i) when the national brands are differentiated in terms of quality, the retailer should maximize the quality of the store brand; (ii) when the national brands are differentiated in terms of features, the retailer should aim for minimum feature differentiation from one of the national brands; and (iii) when the national brand products have undifferentiated features, the retailer should feature-differentiate its store brand.

None of the papers discussed above considers competition between retailers. Indeed, Sethuraman (2009) points out that "almost all analytical models that recommend close store brand positioning to the national brand: (i) assumed demand is linear in price; (ii) were based on aggregate demand function that did not evolve from individual consumer behavior; (iii) did not explicitly incorporate consumer heterogeneity; (iv) assumed marginal cost of national brand and store brand to be equal and set it to 0; (v) did not explicitly consider category expansion; (vi) did not incorporate non-price variables such as advertising, and (vii) did not consider store competition." The author also says of his own model that "our analytical model attempts to investigate this (the store brand positioning) question by relaxing assumptions (i)-(vi), but we do not consider store competition." Our model relaxes all of the assumptions above except that we do not consider advertising and category expansion.

Models with a National Brand and a Retailer with a Competing Store Brand

The stream of research on settings involving two competing retailers and both national and store brands is limited and much of it is fairly recent. As we will see, although several articles present equilibrium pricing results that account for product quality via parameters in the demand functions, none of the articles explicitly addresses the retailers' *decisions* regarding store brand quality.

Corstjens and Lal (2000) study a two-period game between two retailers, both of whom carry the same national brand product and a store brand of their own. In each period, each retailer decides the retail price for both products and chooses to advertise the price of either the national or the store brand. The authors do not address the retailers' quality decisions directly. They do, however, show that even if the store brand is of similar quality to the national brand, has no cost advantage over the national brand and does not affect the wholesale price of the national brand, if customers exhibit heterogenous levels of brand inertia, the competing retailers can still be better off if both introduce a store brand. We explain why this may occur. In the first period, retailers compete intensively with respect to the price of the store brand to induce customers try the store brand. Once customers have tried the store brand and a portion of them like it, the store brand serves as a tool for store differentiation for the retailer in the second period and thus increases retailers' profit over the two periods. Indeed, brand inertia keeps some customers from purchasing from the other store even if the quality and price are more favorable.

Colangelo (2008) investigates the welfare implications of retailers carrying store brands, which may be differentiated due to quality and/or advertising. He analyzes a manufacturer-Stackelberg game between a national brand manufacturer and one or two retailers. In his model, product differentiation is captured via two parameters, one representing the differentiation between the national brand and one of the store brands, and the other representing the differentiation between the two store brands. He shows that when the retailer(s) carry a store brand(s), the channel profit under a downstream duopoly is sometimes lower than the channel profit with a downstream monopoly. Moreover, when the national brand manufacturer is able to engage in perfect price discrimination, it sometimes sets highly discriminatory wholesale prices, even when the downstream retailers are symmetric, to induce only one retailer to carry the national brand. The author also considers a game in which the national brand can choose a level of "advertising" at the outset that affects the extent of product differentiation; this is accomplished indirectly by optimizing the substitution factors in the demand functions. Explicit optimization of product quality and any associated increase in the variable cost are not incorporated into this game.

Geylani et al. (2009) study store-brand introduction and pricing strategies for competing retailers in the presence of "one-stop" customers who visit only one retailer and view the store and the national brand as exactly the same except for price. The authors show that both retailers have an incentive to introduce a store brand because they can then segment the market: they can sell the national brand product with a high price and margin to customers with high willingness to pay while selling the store-brand product to the price-sensitive "one-stop" shoppers. They assume the wholesale prices offered to the two retailers may differ. They do not explore in depth how the equilibrium is shaped by retail competition, nor do they study the retailers' quality decisions for their store brands.

Groznik and Heese (2010) show that introducing a store brand increases a retailer's bargaining power in the supply chain vis-a-vis the national brand manufacturer just as in the single-retailer case. However, under retail competition, each retailer prefers that the other retailer introduce a store brand because the other retailer then would incur the fixed cost of store-brand introduction and yet both retailers benefit from a reduced wholesale price for the national brand. The authors assume exogenous quality levels for the store brand products.

Choi and Fredj (2013) study a setting with two retailers, both of which offer a national brand product and their own (potentially) differentiated store brand product. They develop equilibrium results for pricing problems, implicitly assuming that the quality levels of the products are exogenous. More specifically, they assume a linear demand function in which two parameters capture the substitutability between (i) the national brands at the two stores, and (ii) the national brand and the store brand at a given store. There is no price competition between the two retailers' store brands in their model. The authors compare the equilibrium outcome for four games with different forms of price leadership: Manufacturer-Stackelberg, Retailer-Stackelberg, Vertical Nash and Retailer Double Stackelberg (in which one of the retailers has price leadership over the other retailer). The authors show that each party earns more profit with greater price leadership. Via sensitivity analysis on the substitutability factors in the demand functions, they show that each retailer should seek minimum differentiation between their own store brand and the national brand.

Other work on models involving two retailers who offer the same national brand product as well as their own store brand has been done by Avenel and Caprice (2006)) whose model does not incorporate direct competition between the retailers, and Moner-Colonques et al. (2011) who investigate conditions in which duopolist retailers will replace a national brand product with a store brand product in their product lines.

When a retailer has the flexibility to choose (or adjust) his store brand quality, he needs to consider the product assortment at the competing retailer as well as the quality levels of the products offered there, and the degree of loyalty of customers to him and/or his competitor. We consider all of these factors in our model.

3. Model

3.1. Basic Setup

We consider a scenario with one manufacturer of a leading national-brand product and two retailers, retailer 1 (R1) and retailer 2 (R2). Retailer 2 offers the national-brand product whose quality is q_n and a competing store-brand product in the same product category whose quality is q_{s2}. We initially assume that R2 cannot change q_{s2} in the short term but we later explore a situation in which both retailers can set their quality levels simultaneously. Retailer 1 currently carries only the national brand and is considering the possibility of introducing a store brand whose quality level can be chosen, and all parties (the national brand manufacturer and retailers) can set prices. Therefore, we treat q_n and q_{s2} as exogenous parameters whereas q_{s1} is to be decided by R1. We assume that R1 seeks to maximize equilibrium profits in two ways, with and without the introduction of the store brand, in order to assess whether the difference covers the fixed cost of introduction. In the concluding section, we explain how our results have immediate implications for the structure of the equilibrium in a setting in which both retailers can choose the quality levels of their store brands simultaneously. Such a situation can occur when store brand products are first introduced, or when retailers are in a phase of repositioning their store brand products. We assume that each store brand is produced either in-house by the retailer or by a third-party manufacturer which is a non-strategic player in the game. Also, we assume that all parties have complete and perfect information.

The sequence of events in the game is as follows. First, R1 decides whether to introduce a store-brand product, and if so, what its quality level (q_{s1}) should be. Second, the national brand manufacturer chooses the wholesale price, denoted by w_n, to offer to both retailers. We assume the wholesale price offered to both retailers is the same. (Typically, it is only large retailers that are able to offer their own store brands, so it is reasonable to assume that the national brand manufacturer needs to offer the same price to both retailers.) Third, the retailers observe the wholesale price and engage in a Nash game, choosing which product(s) to offer and at what price(s). Finally customers make purchase decisions. Both retailers and the national brand manufacturer make decisions with the objective of profit maximization. In all of our analysis, we seek subgame perfect equilibria.

The retail prices of the national-brand and store-brand products at retailer i ($i = 1,\ 2$) are p_{ni} and p_{si}, respectively. Notice that choosing a very high price for any product has the same effect as not offering the product at all. Therefore, when deriving the retail price equilibrium, we assume that whenever a retailer finds it optimal not to offer some product, it sets the price at the lowest value that drives the demand for that product to zero. In this way, we are implicitly modeling the product assortment decision via the prices chosen by the retailers.

We focus our attention on store brand products whose quality levels are at or below that of leading national brand products. We therefore assume $q_{s2} < q_n$ and that R1 sets q_{s1} constrained to $q_{s1} \leq q_n$. This assumption applies to most store brands except for those called "premium store brands." The marginal production cost of the national-brand product is denoted by c_n, and that of the store-brand product at retailer i ($i = 1,\ 2$) is denoted by c_{si}. We assume that the unit production cost of each product is an increasing and convex function of its quality. That is, we assume $c_{si} = kq_x^2$ for $x \in \{s1,\ s2,\ n\}$ for some production parameter $k > 0$. However, we will compare our results with those for the case where the unit cost is linear in quality, i.e., $c_{si} = kq_x$ for $x \in \{s1,\ s2,\ n\}$.

We use the same demand model as in Chapter 2. For completeness, in the following subsection, we summarize the structure of customers' preferences and how they make buying decisions. For a more detailed treatment of this material, see their paper.

3.2. Customer-Demand

We begin this section with a description of our customer choice model. We assume that each consumer derives enough utility from other items to be purchased on his/her shopping trip that he will visit one retailer, but he/she does not necessarily make a purchase in the category under study. If the customer chooses to buy a product in the category, he purchases at most one unit of the product, either the national-brand or the store-brand product. We capture the effect of customer loyalty via a transportation cost, which represents the disutility of visiting a more distant retailer. When deciding which retailer to visit, each customer evaluates the maximum surplus that he/she can derive from going to each of the retailers. At this stage, the customer calculates his willingness to pay for the product being studied as the utility derived from the product's quality less the sum of the transportation cost for visiting the relevant retailer and the price of the product to determine his surplus from this product category. The customer then visits the retailer offering the product that provides the highest surplus. After the customer arrives at the store, the transportation cost is now sunk, so he/she purchases the offered product with the higher surplus (if there is more than one product available), provided that the surplus (not considering transportation cost) is positive. We now provide a more detailed description of our customer choice model.

Customers are heterogeneous in two dimensions: location, which also can be interpreted as the degree of loyalty to one retailer or the other, and taste (or willingness to pay) for quality. In the first dimension, customers are distributed uniformly along a Hotelling line connecting the two retailers. A customer's location on the Hotelling line is denoted by x_1 ($x_1 \in [0,\ 1]$), with R1 located at $x_1 = 0$ and R2 at $x_1 = 1$. We use $x_2 = 1 - x_1$ to denote the customer's distance from R2. On the second dimension, customers have a willingness to pay (WTP) per unit of quality, θ, which is uniformly distributed within an interval $[0,\ \bar{\theta}]$. The total number of potential customers is normalized to 1. We assume $\bar{\theta} > 2kq_n$ so that the "efficiency" of producing a product (highest utility provided to consumers less production cost) increases with the product quality within $[0,\ q_n]$. If this efficiency of producing a product is not increasing on the entire interval $[0,\ q_n]$, then the retailer would never consider setting the store brand quality equal to that of the national brand, and there would be a different upper bound on the quality of the store brand.

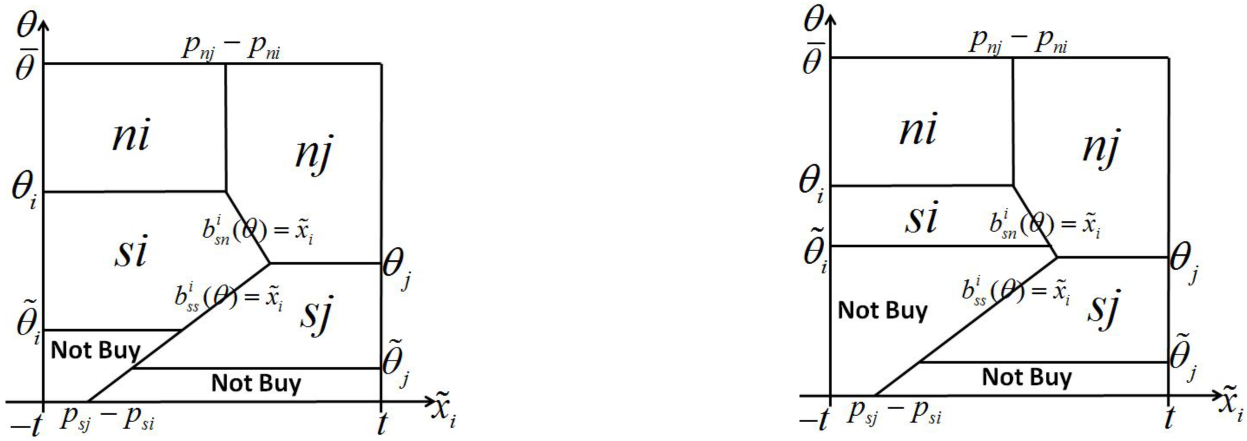
Customers' utility derived from each product is the product of the quality level and his/her WTP per unit of quality. That is, a customer with a WTP for quality θ derives utility θq_n from a unit of the national brand, and utility θq_{si} from a unit of the store brand at retailer i ($i = 1, 2$). Let $\bar{v}_n \equiv \bar{\theta} q_n$ and $\bar{v}_{si} \equiv \bar{\theta} q_{si}$, i.e., $\bar{v}_n$ and $\bar{v}_{si}$ denote the highest utility derived from the national-brand product and the store-brand product at retailer i ($i = 1,\ 2$) respectively.

The customer's transportation cost is equal to the transportation cost per unit distance multiplied by the customer's distance from the respective retailer. We denote the transportation cost per unit distance by t, which is the same for all customers.

The surplus a customer derives from purchasing products in other product categories in the shopping basket, denoted by M, is assumed to be the same from going to either of the retailers for all customers. We assume M is large enough so that it guarantees all customers benefit from a shopping trip. Summing up, a customer's total surplus he/she can derive from going to retailer i ($i = 1,\ 2$) is $\max\{\theta q_n - tx_i - p_{ni} + M,\ \theta q_{si} - tx_i - p_{si} + M\}$. Because M is large enough, a customer always derives positive total surplus from going to at least one of the retailers. Each customer chooses to visit the retailer who offers the product that provides the customer the highest net surplus, taking the transportation cost into consideration. More specifically, if $\max\{\theta q_n - tx_1 - p_{n1} + M,\ \theta q_{s1} - tx_1 - p_{s1} + M\} \geq \max\{\theta q_n - tx_2 - p_{n2} + M,\ \theta q_{s2} - tx_2 - p_{s2} + M\}$, a customer located at x_1 chooses to visit R1. Otherwise, he/she visits R2.

After a customer with distance x_i from retailer i ($i = 1,\ 2$) and a WTP for quality of θ arrives at his "preferred" retailer i, the transportation cost is now sunk, so he buys the offered product that gives him the larger difference between his willingness to pay for the product and its price, if it is non-negative. That is, he buys the national-brand product if $q_n\theta - p_{ni} \geq q_{si}\theta - p_{si}$ and $q_n\theta - p_{ni} \geq 0$, or he buys the store-brand product if $q_n\theta - p_{ni} < q_{si}\theta - p_{si}$ and $q_{si}\theta - p_{si} \geq 0$. Otherwise, he walks away from the product category being studied (but keeps shopping from other product categories before ending the shopping trip).

Now we are ready to present the customers' demands given a set of prices (p_{ni}, p_{si}) at retailer $i = 1,\ 2$. In the remainder of the paper, we use ni and si ($i = 1,\ 2$) to denote the national-brand and the store-brand product at retailer i, respectively, and use D_{ni} and D_{si} to represent the customer demand for the corresponding product at retailer i. A complete



(A) Customer demands if $\tilde{\theta}_i \leq \theta_j$

(B) Customer demands if $\tilde{\theta}_i > \theta_j$

FIGURE 1. Graphical Representation of Customer Demands on the $\theta - \tilde{x}_i$ Plane

derivation of the demand function is included in Appendix A. Here, we only define the key parameters in the demand function and present the customer demand graphically. Define $\theta_i \equiv \frac{p_{ni}-p_{si}}{q_n-q_{si}}$ and $\tilde{\theta}_i \equiv \frac{p_{si}}{q_{si}}$ for $i = 1, 2$. Then θ_i represents the threshold WTP for quality at which customers are indifferent between purchasing a national-brand and a store-brand product at retailer i, and $\tilde{\theta}_i$ represents the threshold WTP for quality at which customers are indifferent between purchasing a unit of the store-brand product and purchasing nothing at retailer i.

We also define $\tilde{x}_i \equiv t(x_i - x_j)$, $b^i_{sn}(\theta) \equiv \theta(q_{si} - q_n) - (p_{si} - p_{nj})$ and $b^i_{ss}(\theta) \equiv \theta(q_{si} - q_{sj}) - (p_{si} - p_{sj})$ for $i = 1, 2,\ j = 3 - i$. Then $\tilde{x}_i$ denotes the difference between the travel cost a customer incurs from going to retailer i and retailer j. $b^i_{sn}(\theta) - \tilde{x}_i = 0$ defines the customers who are indifferent between purchasing product si and product nj. Similarly, $b^i_{ss}(\theta) - \tilde{x}_i$ is the difference in the total surplus between purchasing a unit of the store-brand product from retailer i and retailer j, and customers who are indifferent between si and sj are defined by $(\theta,\ \tilde{x}_i)$ that satisfy $b^i_{ss}(\theta) - \tilde{x}_i = 0$.

We show in Figure 1 the customer demand partitioning graphically on the $\theta - \tilde{x}_i$ plane. Without loss of generality, we assume $\theta_i > \theta_j$ ($i = 1$ or $i = 2$, $j = 3 - i$) in Figure 1. Under $\theta_i > \theta_j$, we need to further separate the analysis into two cases to accurately represent the demand functions on the $\theta - \tilde{x}_i$ plane: $\tilde{\theta}_i \leq \theta_j$ (shown in Figure 1(A)) and $\tilde{\theta}_i > \theta_j$ (shown in Figure 1(B)). Notice that in Figure 1, although it is not explicitly shown, θ_i, θ_j, $\tilde{\theta}_i$ and $\tilde{\theta}_j$ are not parameters, but are affected by the retailers' pricing decisions. Expressions for functions $b^i_{sn}(\cdot)$ and $b^i_{ss}(\cdot)$ are also affected by the pricing decisions. Also, notice that we are implicitly assuming $t \geq \max\{|p_{n2} - p_{n1}|,\ |p_{s2} - p_{s1}|,\ |b^i_{sn}(\theta_j)|\}$ in Figure 1. This assumption guarantees that none of the product demand regions represented in Figure 1 vanishes. Intuitively, this implies that the degree of customer loyalty is not too low, so that retailers have positive demand for both the store and the national brands if they decided to carry them, and all of the demand regions shown in Figure 1 are non-empty. We perform our equilibrium analysis under this assumption because the presence or absence of the regions is affected not only by the value of t but also by the prices, which are decisions in our model. In Section 5, we show numerically that characteristics of the equilibrium retain the same qualitative features for finite t.

3.3. Problem Formulation

We are now ready to formulate each party's problem at each stage of the game using backward induction. Recall that the stages in the game are:
1. Retailer 1 decides whether to introduce a store brand and if so, chooses its quality level.
2. The national brand manufacturer chooses its wholesale price.
3. The retailers engage in a Nash game, choosing retail prices (and implicitly, which products to offer).
4. Customers choose which retailer to visit and which product to purchase, if any.

The customers' aggregate response to the retailer's price decisions was presented in Section 3.2. We now discuss the third stage of the game.

Given any wholesale price $w_n > 0$ and the quality level of R1's store brand product, q_{s1}, each retailer sets its retail prices while observing the retail prices selected by the other retailer. That is, retailer i ($i = 1, 2$, $j = 3 - i$) seeks a pair of prices $(p_{ni},\ p_{si})$ that maximize its profit $\pi_{ri}(q_{s1},\ w_n,\ \mathbf{p})$. That is, retailer i solves the following problem:

$$\begin{aligned} \max_{p_{ni}\geq w_n,\ p_{si}\geq c_{si}} \quad & \pi_{ri}(q_{s1},\ w_n,\ \mathbf{p}) \equiv (p_{ni} - w_n)D_{ni}(\mathbf{p}) + (p_{si} - c_{si})D_{si}(\mathbf{p}) \\ \text{subject to} \quad & \frac{p_{ni} - p_{si}}{q_n - q_{si}} \leq \bar{\theta} \\ & \frac{p_{si}}{q_{si}} \leq \frac{p_{ni} - p_{si}}{q_n - q_{si}} \end{aligned} \tag{1}$$

Although there are four prices, retailer i chooses only p_{ni} and p_{si}. Due to the form of demand functions, retailer i needs to solve two subproblems, one for $\theta_i \geq \theta_j$ and the other for $\theta_i < \theta_j$, and then choose the better solution. Moreover, due to the form of $D_{si}^H(\mathbf{p},\ \bar{\theta})$, the subproblem for $\theta_i \geq \theta_j$ further divides into two subproblems, one for $\tilde{\theta}_i \geq \theta_j$ and one for $\tilde{\theta}_i < \theta_j$. For any q_{s1} and w_n, equilibrium retail prices $\mathbf{p^*}(q_{s1},\ w_n) \equiv (p_{n1}^*,\ p_{s1}^*,\ p_{n2}^*,\ p_{s2}^*)$ satisfy $(p_{ni}^*,\ p_{si}^*) \in \underset{p_{ni},p_{si}\geq 0}{argmax}\ \pi_{ri}(q_{s1},\ w_n,\ (p_{ni},\ p_{si},\ p_{nj}^*,\ p_{sj}^*))$ for $i = 1, 2$ and $j = 3 - i$.

We now describe the national brand manufacturer's problem in the second stage of the game. Given any q_{s1} set by R1, the national brand manufacturer selects the wholesale price that optimizes his own profit, $\pi_m(q_{s1},\ w_n)$, taking into consideration $\mathbf{p^*}(q_{s1},\ w_n)$. That is, the national brand manufacturer solves the following problem:

$$\max_{w_n\geq c_n} \pi_m(q_{s1},\ w_n) \equiv (w_n - c_n) \cdot (D_{n1}(\mathbf{p^*}(q_{s1},\ w_n)) + D_{n2}(\mathbf{p^*}(q_{s1},\ w_n))) \tag{2}$$

Given any q_{s1}, the equilibrium wholesale price $w_n^*(q_{s1})$ satisfies $w_n^*(q_{s1}) \in \underset{w_n\geq 0}{argmax}\ \pi_m(q_{s1}, w_n)$.

Finally, we formulate R1's problem in the first stage of the game, i.e., choosing the optimal store brand quality, taking into consideration $w_n^*(q_{s1})$:

$$\begin{aligned} \max_{0\leq q_{s1}\leq q_n} & \pi_{r1}(q_{s1},\ w_n^*(q_{s1}),\ \mathbf{p^*}(q_{s1},\ w_n^*(q_{s1}))) \\ & \equiv (p_{n1}(q_{s1},\ w_n^*(q_{s1})) - w_n^*(q_{s1}))D_{n1}(\mathbf{p^*}(q_{s1},\ w_n^*(q_{s1}))) \\ & \quad + (p_{s1}(q_{s1},\ w_n^*(q_{s1})) - c_{s1})D_{s1}(\mathbf{p^*}(q_{s1},\ w_n^*(q_{s1}))) \end{aligned} \tag{3}$$

In the next section, we present a full equilibrium analysis for the case of $t \to +\infty$.

4. Model Analysis When $t \to +\infty$

In this section, we assume $t \to +\infty$ and investigate R1's optimal quality-setting strategy. We first present results regarding the manufacturer's strategy, and then we present R1's optimal quality-setting strategy under a benchmark scenario in which the unit production cost is linear in the product quality. Finally, we characterize her optimal strategy under the assumption that the unit production cost is strictly convex and increasing in the quality level.

4.1. Manufacturer's Strategy

In Chapter 2, we show how the retailers' product assortment decisions depend on the wholesale price when the unit cost is linear in the quality level. We can show in a similar way that, under any non-decreasing relationship between the unit production cost and the quality level, retailer i ($i = 1, 2$) carries only the national brand if $w_n \leq \frac{c_{si}}{q_{si}} q_n$, both the national and store brands if $\frac{c_{si}}{q_{si}} q_n \leq w_n \leq c_{si} + (\bar{v}_n - \bar{v}_{si})$ and only the store brand if $w_n > c_{si} + (\bar{v}_n - \bar{v}_{si})$. Define $l,\ h \in \{1, 2\}$ such that $q_{sh} > q_{sl}$. With this specification of indices, both retailers carry the national brand if the wholesale price lies in $[c_n,\ c_{sh} + (\bar{v}_n - \bar{v}_{sh})]$ (which we refer to as "below the wholesale price threshold" for short), whereas only the retailer with the lower-quality store brand carries the national brand if the wholesale price is set within $[c_{sh} + (\bar{v}_n - \bar{v}_{sh}),\ c_{sl} + (\bar{v}_n - \bar{v}_{sl})]$ (which we refer to as "above the wholesale price threshold" for short). Therefore the national brand manufacturer faces a tradeoff between market share and per-unit profit margin.

To see specifically how the manufacturer's profit margin differs under the two scenarios, let $m_x \equiv \bar{v}_x - c_x$ represent the highest possible utility a unit of product x ($x \in \{s1,\ s2,\ n\}$) provides to customers less its cost of production. m_x ($x \in \{s1,\ s2,\ n\}$) then represents the competitiveness of product x compared to other products in the market. Then, after solving the problem in (2) with the wholesale price constrained tobe above the wholesale price threshold, we can see that the manufacturer's profit margin derived from choosing an optimal wholesale price above the threshold is $m_n - m_{sl}$, the difference between the competitiveness of the national brand and the store brand at this retailer. This relationship arises because the national brand competes directly with the low-quality store brand in this case. Therefore the optimal profit margin for the national brand manufacturer (based on the optimal wholesale price) decreases as the competitiveness of the low-quality store brand increases. In comparison, the manufacturer's profit margin derived from choosing an optimal wholesale price below the wholesale price threshold is the difference between the competitiveness of the national brand and a weighted average of the competitiveness of the two store brand products, i.e., $m_n - m_{sm}$ where $m_{sm} \equiv \frac{\frac{1}{\bar{v}_n - \bar{v}_{s1}}}{\frac{1}{\bar{v}_n - \bar{v}_{s1}} + \frac{1}{\bar{v}_n - \bar{v}_{s2}}} m_{s1} + \frac{\frac{1}{\bar{v}_n - \bar{v}_{s2}}}{\frac{1}{\bar{v}_n - \bar{v}_{s1}} + \frac{1}{\bar{v}_n - \bar{v}_{s2}}} m_{s2}$.

In the formula for the weighted average of the competitiveness of the two store brand products (i.e., m_{sm}), the weight placed on the competitiveness of store brand i increases with the quality level of store brand i, because stronger store brands pose more competition for the national brand product. Because $m_{sh} > m_{sl}$, it is obvious that the national brand manufacturer's profit margin is higher when he sets the wholesale price above the wholesale price threshold. Moreover, the larger is the difference between m_{sh} and m_{sl}, the larger is the difference between m_{sm} and m_{sl}, implying that, the reduction in the national brand manufacturer's margin in the selling-through-both-retailers scenario from his margin in the selling-through-one-retailer scenario is more significant. Lemma 1 summarizes how the national brand manufacturer's choice depends upon the aforementioned margins. We note that although we have discussed the national brand manufacturer's decisions in terms of the competitiveness of the two store-brand products, the competitiveness of each of these products is directly related to its quality: because $\bar{\theta} > 2kq_n$, them efficiency of producing each product, or the competitiveness of each product, is strictly increasing with its quality level. All proofs appear in the Appendix.

LEMMA 1. *The national brand manufacturer sells through both retailers if and only if*

$$\frac{m_n - m_{sm}}{m_n - m_{sl}} \geq \left(\frac{\bar{v}_n - \bar{v}_{sm}}{2(\bar{v}_n - \bar{v}_{sl})}\right)^{1/2} \tag{4}$$

and sells exclusively through the retailer with the lower-quality store brand otherwise. His per-unit profit margin in the former case is $m_n - m_{sm}$ and in the latter case is $m_n - m_{sl}$.

Observe that the ratio on the left hand side of the inequality (4) in Lemma 1 decreases as m_{s1} and m_{s2} become more disparate, which occurs as q_{s1} and q_{s2} become more disparate.

PROPOSITION 1. *For any q_{s2} and q_n, there exist $q_{sl}^0 \in [0,\ q_{s2})$ and $q_{sh}^0 \in (q_{s2},\ q_n)$ (both are functions of q_{s2} and q_n) such that the national brand manufacturer sells through both retailers if and only if $q_{s1} \in [q_{sl}^0,\ q_{sh}^0]$. If q_{s1} is equal to q_{sl}^0 or q_{sh}^0, the manufacturer is indifferent between selling through both or only one retailer.*

Proposition 1 has important implications for R1. Because of the structure of the manufacturer's pricing policy which determines whether he sells through both retailers or only one of them, R1's quality-setting problem consists of three subproblems, namely, maximizing her profit constrained to $q_{s1} \in [0,\ q_{sl}^0)$, $q_{s1} \in [q_{sl}^0,\ q_{sh}^0]$ and $q_{s1} \in (q_{sh}^0,\ q_n)$, respectively, because her profit function is discontinuous at the boundaries between these intervals. Before we look at the subproblems in detail, we introduce Proposition 2 which shows how the two thresholds that define the three subproblems change with the quality level of the other retailer's store brand (q_{s2}), the production cost parameter (k), and customers' mean WTP for per unit of quality ($\frac{\bar{\theta}}{2}$). We observe that the cost parameter and customers' mean WTP for quality always affect the equilibrium variables in opposite directions, and therefore in the remainder of the paper, we express the results in terms of the ratio $r \equiv \frac{k}{\bar{\theta}}$.

PROPOSITION 2. *q_{sh}^0 is increasing in q_{s2} and decreasing in r. q_{sl}^0 is non-decreasing in both q_{s2} and r. Moreover, q_{sh}^0 and q_{sl}^0 both approach q_n as q_{s2} approaches q_n.*

The value of q_{sh}^0 is increasing in q_{s2} because the higher is the quality of the lower-quality store brand, the less the national brand manufacturer changes his wholesale price in response to an increase in the quality of the higher-quality store brand. q_{sl}^0 is non-decreasing in q_{s2} because the higher is the quality level of the higher-quality store brand, the more likely the manufacturer is to sell exclusively through the retailer with the lower-quality store brand. The value of q_{sh}^0 is decreasing in r, and q_{sl}^0 is non-decreasing in r, because the higher is the unit production cost of the national brand, more likely the national brand manufacturer is to decrease his production quantity by concentrating on selling through the retailer with the lower-quality store brand.

When the unit production cost is linear in quality, both q_{sh}^0 and q_{sl}^0 can be expressed as closed-form functions of q_{s2} and q_n). When the unit production cost is strictly convex and increasing in quality, expressions for q_{sh}^0 and q_{sl}^0 are too complex to convey meaningful insights. However, we are able to show that they are bounded by some simple closed-form expressions. We present all of the relevant expressions in Proposition 3.

PROPOSITION 3. *When the unit production cost is linear in quality,* $q_{sh}^0 = \frac{2}{3}q_n + \frac{1}{3}q_{s2}$ *and* $q_{sl}^0 = 3q_{s2} - 2q_n$. *When the unit production cost is strictly increasing and convex in quality,* $\frac{3-\sqrt{5}}{2}q_n + \frac{\sqrt{5}-1}{2}q_{s2} < q_{sh}^0 < \frac{2}{3}q_n + \frac{1}{3}q_{s2}$ *and* $3q_{s2} - 2q_n < q_{sl}^0 < \frac{\sqrt{5}+1}{2}q_{s2} - \frac{\sqrt{5}-1}{2}q_n$.

When the production cost is linear in quality, we can reexpress q_{sh}^0 implicitly via the equality $q_{sh}^0 - q_{s2} = 2(q_n - q_{s2})$ and similarly, q_{sl}^0 via the equality $q_{s2} - q_{sl}^0 = 2(q_n - q_{s2})$. We can then conclude that the manufacturer sells through both retailers if and only if $q_{sh} - q_{sl} < 2(q_n - q_{sh})$, i.e., only if the quality disparity between the two store brands is below a threshold. Furthermore, it is clear that the threshold decreases with quality of the higher-quality store brand. When the unit production cost is a quadratic function of the quality level, expressions for q_{sh}^0 and q_{sl}^0 are much more complex. But Proposition 3 says that q_{sh}^0 is bounded above by $\frac{2}{3}q_n + \frac{1}{3}q_{s2}$, where $\frac{2}{3}q_n + \frac{1}{3}q_{s2}$ is exactly q_{sh}^0 derived under the scenario in which the unit production cost is linear. Also, q_{sh}^0 is bounded below by $\frac{3-\sqrt{5}}{2}q_n + \frac{\sqrt{5}-1}{2}q_{s2}$ ($\approx 0.38q_n + 0.62q_{s2}$). One can easily confirm that $\frac{3-\sqrt{5}}{2}q_n + \frac{\sqrt{5}-1}{2}q_{s2}$ is exactly q_{sh}^0 derived under the scenario in which $k = \frac{\bar{\theta}}{2q_n}$. (Recall that we assume $k < \frac{\bar{\theta}}{2q_n}$ so that the "efficiency" of producing a product increases with the product quality.) Similarly, q_{sl}^0 is bounded below by q_{sl}^0 derived under the scenario in which unit production cost is linear, and bounded above by q_{sl}^0 derived under the scenario in which $k = \frac{\bar{\theta}}{2q_n}$.

We next explain why the linear-unit-production-cost scenario and the $k = \frac{\bar{\theta}}{2q_n}$ scenarios provide upper (lower) and lower (upper) bounds on q_{sh}^0 and q_{sl}^0, respectively. When the unit production cost is linear in quality, producing one unit of the national brand is less costly than when the unit production cost is strictly convex and increasing, therefore the region in which the national brand manufacturer prefers to sell through both retailers (and hence obtains a higher demand) expands. On the other hand, the solution for $k = \frac{\bar{\theta}}{2q_n}$ provides a bound because we are interested in the region in which $k < \frac{\bar{\theta}}{2q_n}$, and by using $k = \frac{\bar{\theta}}{2q_n}$, we obtain a smaller region in which the national brand manufacturer prefers to sell through both retailers. In Figure 2, we present a numerical example showing q_{sh}^0 and q_{sl}^0 as functions of q_{s2} when the unit production cost is strictly convex and increasing in quality. We use parameter values $r = 0.4$ and $q_n = 1$ in this example. Figure 2 also shows the upper and lower bounds on q_{sh}^0 and q_{sl}^0.

4.2. Retailer 1's Quality-Setting Strategy

We now examine the three subproblems that R1 needs to solve; she needs to choose the best solution among them. When $q_{s1} \in [0,\ q_{sl}^0)$ (i.e., in the low range), the national brand manufacturer sets his price above the wholesale price threshold so as to sell through R1 only and R1's profit is derived from her sales of both the store and national brands. When $q_{s1} \in [q_{sl}^0,\ q_{sh}^0]$ (i.e., in the medium range), R1's profit still comes from both products, but the national brand manufacturer sets his price below the wholesale price threshold to cater to both retailers. When $q_{s1} \in (q_{sh}^0,\ q_n]$ (i.e., in the high range), the national brand manufacturer sets the wholesale price above the threshold, so R1 chooses not to carry the national brand and only carries its store brand.

Let $\pi_{r1}^l(q_{s1})$ denote R1's profit function when q_{s1} is in the low range and its unconstrained maximizer as q_{s1}^l. Similarly, let $\pi_{r1}^m(q_{s1})$ ($\pi_{r1}^h(q_{s1})$) denote R1's profit function when q_{s1} is in

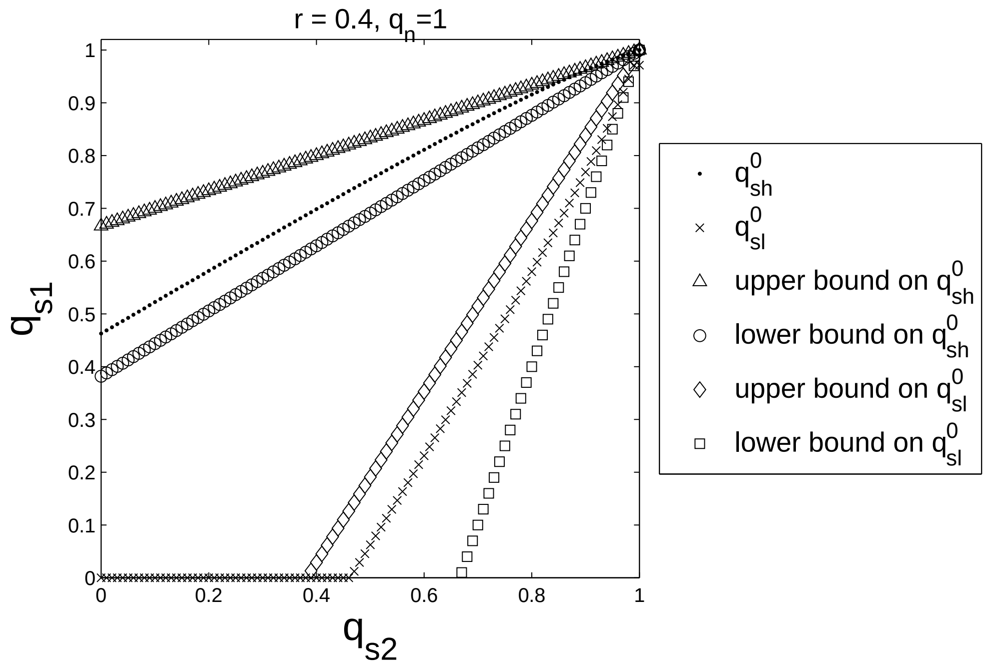


FIGURE 2. q^0_{sh}, q^0_{sl} and upper and lower bounds on them

the medium (high) range and its maximizer is q^m_{s1} (q^h_{s1}). We characterize the global optimal quality level q^*_{s1} by deriving the three unconstrained solutions, one for each region, and their relationship to the two quality thresholds, q^0_{sh} and q^0_{sl}. Before doing so, we first analyze R1's optimal store-brand quality under a benchmark scenario in which the unit cost of production is linear in the product quality.

4.2.1. Optimal Quality When the Unit Production Cost is Linear in Quality

Unit production costs that are linearly increasing in product quality arise in product categories in which the production cost depends heavily on the quantity or density of desirable raw materials. For example, the production cost of bed sheets rises approximately linearly with the threadcount, and the production cost of orange-juice-based breakfast beverages rises approximately linearly with the percentage of real orange juice.

We first present results on the optimal quality level.

PROPOSITION 4. *When the unit production cost is linear in the product quality (i.e., $c_x = kq_x$ for $x \in \{s1,\ s2,\ n\}$, for all $k \in (0,\ \bar{\theta})$), π^l_{r1}, π^m_{r1} and π^h_{r1} are strictly increasing in q_{s1} on $[0,\ q_n)$ and hence q^*_{s1} should be set as close to q_n as possible.*

Here, we are considering the case in which both the unit production cost and customers' utility derived from the store brand at R1 increase linearly with its quality level. Therefore, any increase in the unit cost resulting from an increase in quality level can be more than offset by an increase in the price. Moreover, the retailer has no incentive to differentiate its store brand from the national-brand product under linear production costs. To see why, observe that with an arbitrary unit production cost c_{si} and arbitrary wholesale price, R1 will set $\tilde{\theta}$ at $\frac{1}{2}(\bar{\theta} + \frac{c_{s1}}{q_{s1}})$ at the equilibrium. Therefore, R1's total demand (combining the national and store brands) is $\frac{1}{2}(1 - \tilde{\theta}) = \frac{1}{4}(\bar{\theta} - \frac{c_{s1}}{q_{s1}})$. When the unit production cost is linear in the product quality, $c_{s1} = kq_{s1}$ and thus the total retail demand is $\frac{1}{4}(\bar{\theta} - k)$, which does not change with the quality of her store brand. Thus, because quality differentiation has no effect on total unit sales and R1 earns a higher profit margin from both the store-brand product (as explained above) and the national-brand product (because the national brand manufacturer lowers his wholesale price in response to increased competition), R1 increases the quality of her store brand product to match that of the national brand product.

Although the retailer does not derive any value from differentiating her product from the national-brand product when the unit production cost is linear in the quality, she does derive value from doing so when the unit production cost is strictly convex in the quality, as we show in the next subsection.

4.2.2. Optimal Quality Level When the Unit Production Cost is Strictly Increasing and Convex in Quality

In this section, we discuss the optimal quality-setting strategy for R1 when the unit production cost is strictly convex and increasing in product quality. This cost relationship is applicable in settings where increasingly higher quality requires increasingly rarer input materials, skill, or technical sophistication to produce the product, such as cosmetics, computer "chips" or leather products. We assume costs have the form $c_{s1} = kq_{s1}^2$. Under this assumption, the total demand at R1 is $\frac{1}{4}(\bar{\theta} - \frac{c_{s1}}{q_{s1}}) = \frac{1}{4}(\bar{\theta} - kq_{s1})$ which increases as q_{s1} decreases. Therefore, under these conditions, by differentiating her store brand from the national brand, the retailer is able to achieve price discrimination between customers with high and low WTP per unit of quality and enjoy a larger demand base. However, the more differentiated the store brand is from the national brand, the less competition the store brand poses for the national brand manufacturer. Thus, R1 has an incentive to introduce a store brand that is a close substitute for the national brand in order to gain the greatest price concession on the national brand product. Thus, R1 faces a tradeoff; we elaborate further below.

When r is very low, the retailer incurs very low unit cost of producing a high-quality store brand. She therefore always sets the quality level of her store brand at q_n and sells the store brand product exclusively. On the other hand, when r is greater than a threshold, the retailer prefers a quality level less than q_n, which keeps the unit production cost down and allows the retailer to benefit from some product differentiation. When making this decision, the retailer is aware that, depending on whether her store-brand quality level is much lower, much higher, or not much different from the other store brand, the national brand manufacturer prices differently to induce either only her, only the other retailer, or both of them, to offer the national brand. The retailer's optimal choice of her store-brand quality therefore depends heavily on the quality level of the store brand at the other retailer. Proposition 5 states conditions under which the optimal quality level of the retailer in question falls into the low, medium and high intervals under the condition that r exceeds a threshold.

PROPOSITION 5. *If $r > \frac{1}{3q_n}$, there exist $\underline{q_{s2}}(r)$ and $\overline{q_{s2}}(r)$ such that q_{s1}^* takes a value in $[q_{sh}^0,\ q_n]$ if $q_{s2} \in [0,\ \underline{q_{s2}}(r)]$, a value in $(q_{sl}^0,\ q_{sh}^0)$ if $q_{s2} \in (\underline{q_{s2}}(r),\ \overline{q_{s2}}(r))$ and a value in $(0,\ q_{sl}^0]$ if $q_{s2} \in [\overline{q_{s2}}(r),\ q_n)$.*

This proposition states that for any r above the stated threshold, there is an interval of high (medium, low) quality levels for R2's store brand product that maps to an interval of low (medium, high) quality levels for R1's store brand product. It is also straightforward to show that when R1's store brand quality takes a value in the medium range, the optimal value is strictly decreasing in q_{s2}.

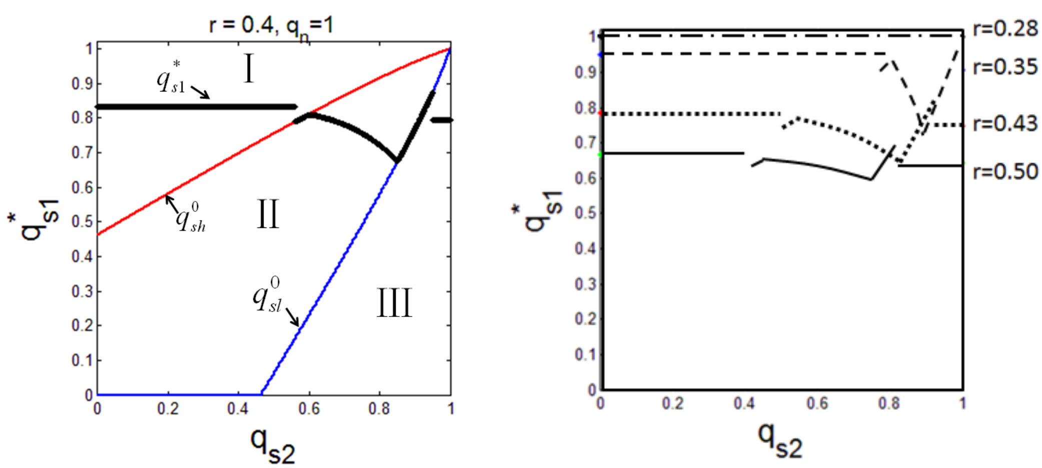
Intuitively, when the quality level of the other store brand is low (i.e., $q_{s2} \in [0,\ \underline{q_{s2}}(r)))$, selling the national brand is not a good choice for R1. Recall that the national brand manufacturer's optimal wholesale price increases as the weighted average of the competitiveness of the two store brands decreases. When q_{s2} is very low, the competitiveness of the store brand at R2 is very low, and hence the wholesale price charged by the national brand manufacturer is very high because he faces little competition. R1 would prefer to carry only her store brand in this scenario. If the quality level of the other store brand (s2) is moderate ($q_{s2} \in (\underline{q_{s2}}(r),\ \overline{q_{s2}}(r))$), on the other hand, not only is its competitiveness higher, but the national brand manufacturer also puts more weight on it when calculating the weighted average of the competitiveness of the two store brand products (because $m_{sm} = \frac{\frac{1}{\bar{v}_n - \bar{v}_{s1}}}{\frac{1}{\bar{v}_n - \bar{v}_{s1}} + \frac{1}{\bar{v}_n - \bar{v}_{s2}}} m_{s1} + \frac{\frac{1}{\bar{v}_n - \bar{v}_{s2}}}{\frac{1}{\bar{v}_n - \bar{v}_{s1}} + \frac{1}{\bar{v}_n - \bar{v}_{s2}}} m_{s2}$). As a result, R1 is able to obtain a fairly low wholesale price from the national brand manufacturer. In this case she will carry both the national brand and her store brand, while at the same time R2 also carries both products. Finally, if the quality level of the store brand at R2 is very high ($q_{s2} \in (\overline{q_{s2}}(r),\ q_n)$), the national brand manufacturer finds it optimal to distribute through R1 exclusively and he accomplishes this by setting a high wholesale price that only R1 finds acceptable. Therefore, in this case, R1's optimal quality level is the one that maximizes π^l_{r1}, and she is the only retailer to carry both products.

Now that we have established the conditions under which R1's optimal quality falls into each of the three intervals, we turn to the question of what value the optimal quality level should take in each region and how it changes with q_{s2} and r. Proposition 6 provides answers to these questions.

PROPOSITION 6. *The optimal quality levels have the following properties: (i) Magnitudes of q^h_{s1} and q^l_{s1}: $q^h_{s1} = \frac{1}{3r}$ and $q^{h*}_{s1} = \min\{q^h_{s1},\ q_n\}$, $q^l_{s1} = \frac{1}{9r}[6 + q_n r - (9 + 12 q_n r - 8 q_n^2 r^2)^{\frac{1}{2}}]$ and $q^{l*}_{s1} = \min\{q^l_{s1},\ q_n\}$. (ii) Relationship among q^h_{s1}, q^l_{s1} and q^m_{s1}: $q^m_{s1}, q^l_{s1} < q^h_{s1}$. Moreover, if $q_{s2} > q^l_{s1}$, then $q^m_{s1} < q^l_{s1} < q^h_{s1}$. (iii) How q^h_{s1}, q^l_{s1} and q^m_{s1} change with q_{s2}: q^h_{s1} and q^l_{s1} are invariant in q_{s2}, whereas q^m_{s1} is strictly decreasing in q_{s2}; (iv) How q^h_{s1}, q^l_{s1} and q^m_{s1} change with r: if $q^{h*}_{s1} < q_n$, $q^{h^*}_{s1}$ and q^{l*}_{s1} are strictly decreasing in r. q^m_{s1} is decreasing in r if and only if $(q_n - q^m_{s1})(q_n - q^m_{s1} + 2(q_n - q_{s2})) > q_n^2 - q_{s2}^2$.*

Proposition 6 has several implications. First, from (iii), we see that q_{sh} and q_{sl} are invariant in q_{s2}. This can be explained as follows. First, when $t \to +\infty$ (so each retailer has a large number of loyal customers), the level of customer demand at R1 is not affected by the value of q_{s2}. So a change in q_{s2} can only affect R1's profit through a change in the national brand wholesale price. But, if R1 sets q_{s1} in the high range, R1 does not carry the national brand at all. Therefore, R1's profit (when she sets q_{s1} in the high range) is invariant in q_{s2}. On the other hand, if R1 sets q_s1 in the low range, the wholesale price itself is invariant in q_{s2} (as the national brand manufacturer does not sell through R2). As a result, R1's profit is also invariant in q_{s2} when q_{s1} is set in the low range. Second, from (ii), we see that $q^l_{s1} < q^h_{s1}$. The reason is that when R1 carries only the store brand, she sets the quality at a level that caters to customers with relatively high WTP per unit of quality, but she is deterred from setting the quality level extremely high because of the convex and increasing structure of the unit production cost. On the other hand, when R1 sets her quality level in the low range, this corresponds to a setting in which she offers both products and faces a tradeoff between maintaining bargaining power with respect to the national brand manufacturer and differentiating her store-brand product from the national brand. Because of her incentive to differentiate her product (and therefore achieve price discrimination across consumers with different WTP per unit of quality), she sets the quality level of her store brand lower than when she carries the store brand exclusively. However, the quality level is just slightly smaller because as q_{s1} declines, the bargaining power of R1 vis-a-vis the national brand manufacturer becomes weaker and the wholesale price for the national brand increases. That is, $w_n = \frac{1}{2}[\bar{v}_n + c_n - m_{s1}]$ increases as q_{s1} declines).

Finally, from the properties of q_{s1}^m in (ii) and (iii), we observe that when R1 sets the quality of her store brand in the mid-range, which corresponds to a setting in which both R1 and R2 offer the national brand and their respective store brand products, she differentiates the quality level of her store brand to a greater degree (i.e., $q_{s1}^m < q_{s1}^l$) and her optimal quality level depends on q_{s2}. In particular, as q_{s2} increases, R1 decreases the quality of her own brand. This is because, as q_{s2} increases, the national brand manufacturer is forced to decrease her wholesale price. That is, $w_n = \frac{1}{2}[\bar{v}_n + c_n - m_{sm}]$ decreases as q_{s2} increases. Recall that m_{sm} is the weighted average of m_{s1} and m_{s2}, and the weight on m_{s2} increases as q_{s2} increases, and m_{s2} itself also increases in q_{s2}). Thus R1 maintains her bargaining power even if the quality level of her own brand is low. Consequently, in this case, she is able to focus on increasing market share via product differentiation.



(A) Optimal q_{s1} as a function of q_{s2}

(B) Optimal q_{s1} as a function of r

FIGURE 3. Relationship of Optimal q_{s1} to q_{s2} and r

We close this subsection with numerical examples. Figure 3(A) shows how q_{s1}^* changes with q_{s2} when $q_n = 1$ and $r = 0.4$. When $(q_{s2},\ q_{s1})$ falls in Region I (III), the national brand manufacturer chooses a wholesale price which leads to only R2 (R1) carrying the national brand. When $(q_{s2},\, q_{s1})$ falls in Region II, the national brand manufacturer chooses her wholesale price such that both retailers carry the national brand. Knowing this, when q_{s2} is low, R1 sets the quality level of her store brand in Region I (the high range).

When q_{s2} is moderate, R1 sets q_{s1}^* in Region II (the medium range). In this case, setting the store brand quality high, so that it falls into Region I, would backfire for R1, as it would induce the national brand manufacturer to change his pricing strategy from setting a price below the wholesale price threshold which appeals to both retailers to setting a high wholesale price which leads to only R2 carrying the national brand. Effectively, this forecloses R1 from selling the national brand, although it would be in R1's best interest to carry both the national and store brands. Notice that R1 would not need to worry that setting the store brand quality high will backfire in this way if she were a downstream monopolist, as the national brand manufacturer would only lower the wholesale price as the quality level of the store brand increases in that scenario.

Finally, when q_{s2} is high, R1 sets q_{s1}^* in Region III (the low range). Interestingly, R1 sometimes sets the quality level along the boundary between regions. In particular, in the region of the upward sloping response curve along the boundary between Regions II and III in Figure 3(A), R1 prefers a constrained solution within Region II, where both retailers offer both the national and their respective store brands to the optimal unconstrained solution in Region III, where R1 sells both the national and store brands but R2 sells only its store brand. The main reason for this phenomenon is that when both retailers offer both the national brand and their respective store brands, the combined competition against the national brand generated by the two store brands enables R1 to secure a much lower wholesale price from the national brand manufacturer than would be offered when R2 does not sell the national brand.

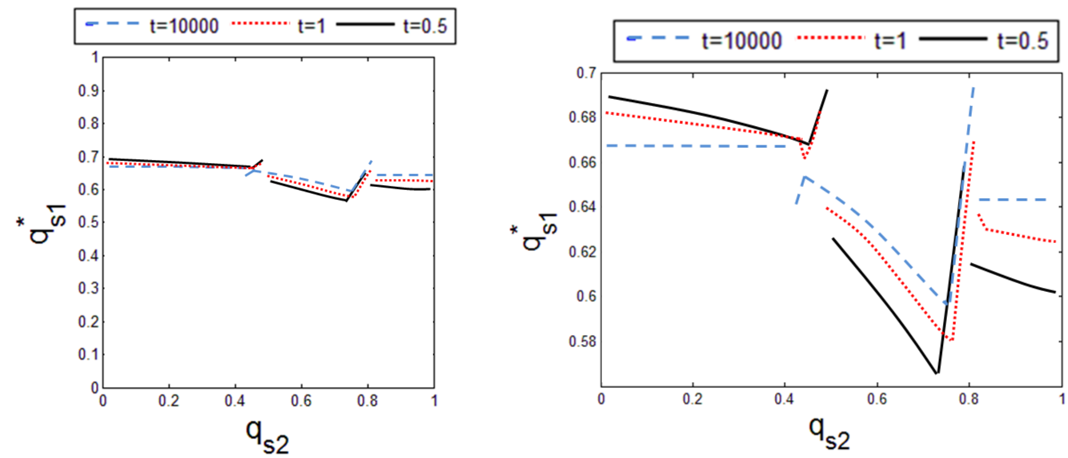
Figure 3(B) shows how q_{s1}^* changes with q_{s2} for different values of r, with q_n fixed at 1. As r increases, the overall level of q_{s1}^* decreases. In our example, $r = \frac{1}{3}$, so in Figure 3(B) we see that for r above this threshold, R1's optimal response has the shape of the black curve shown in Figure 3(A). For $r = 0.28 < \frac{1}{3}$, however, we see that the optimal response does not take on this characteristic three-region form; instead, R1 always sets the quality of her store brand as close to that of the national brand as possible. This is not surprising: when r is less than the stated threshold (which happens when the production cost parameter, k, is very small or when the mean WTP per unit of quality across consumers is very high), R1 has such a strong incentive to choose a very high store-brand quality that the competitor's store-brand quality level becomes irrelevant.

We now turn to a discussion of the case of finite t.

5. Analysis for finite t

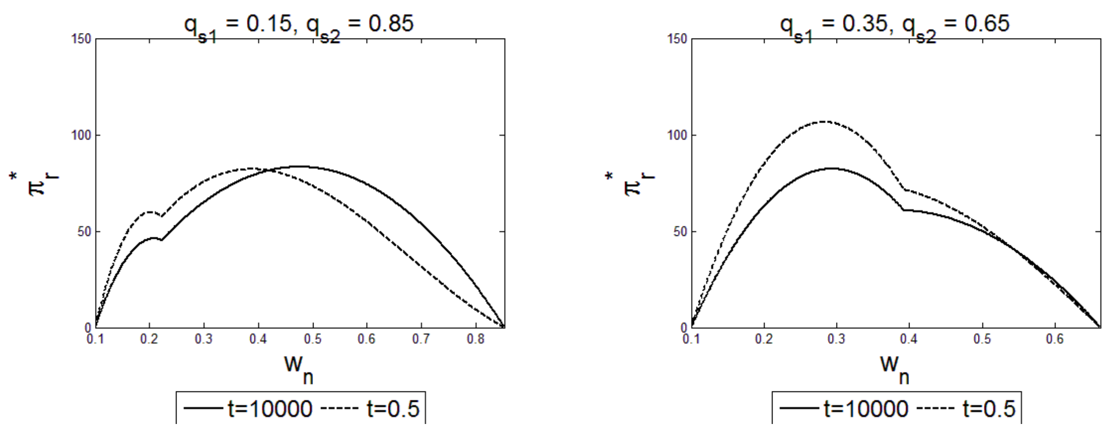
In this section, we first explore whether and how a finite t changes the structure of the retailer's quality-setting problem. We then investigate how the value of t affects the national brand manufacturer's wholesale price, and finally, its effect on the optimal retail prices and demands. Recall that t denotes customers' transportation cost per unit distance and it affects customer demands. Specifically, a customer with WTP per unit of quality, θ, prefers purchasing a unit of the store-brand product from retailer i over purchasing a unit of the store-brand product from retailer j if and only if $\theta(q_{si} - q_{sj}) - (p_{si} - p_{sj})$ is greater than the difference between the transportation cost the customer incurs from going to retailer i versus retailer j. As t increases, the customer is more inclined to purchase from the retailer located closer to him. Therefore t represents the level of customer loyalty to one retailer or another.

We conducted numerical studies to investigate how Retailer 1's optimal quality positioning changes when t is no longer infinite. Figure 4 shows Retailer 1's optimal quality level for its store brand as a function of q_{s2} for three different values of t (t= 0.5, 1 and 10,000). Other parameter values for this example are $r = 0.4$ and $q_n = 1$. Figure 4(B) is an enlargement of a portion of Figure 4(A) where the most interesting changes occur. As can be seen, when t is finite, the qualitative structure of the equilibrium described in Proposition 5 still holds. That is, the plot of Retailer 1's optimal quality choice as a function of q_{s2} has three sections with jump discontinuities between sections, and the general shapes of the curves remain the same for all t.



(A) (B)

FIGURE 4. How optimal q_{s1} changes with q_{s2} for different values of t



(A) (B)

FIGURE 5. Comparison of manufacturer's profit functions for $t = 10000$ and $t = 0.5$

The retailer does, however, make adjustments in the quality level to account for t being finite rather than infinite. These adjustments differ markedly across the three regions, as shown in Figure 4. R1's choice of quality level for q_{s2} in the low range (corresponding to Region I) increases whereas her quality choices in Regions II and III decrease. In Region I, R1 sells only her high-quality store brand. As t decreases, retail competition becomes more intense, so she is forced to lower the retail price of her store brand if she does not have the flexibility to adjust its quality. When R1 has the flexibility to adjust the quality of her store

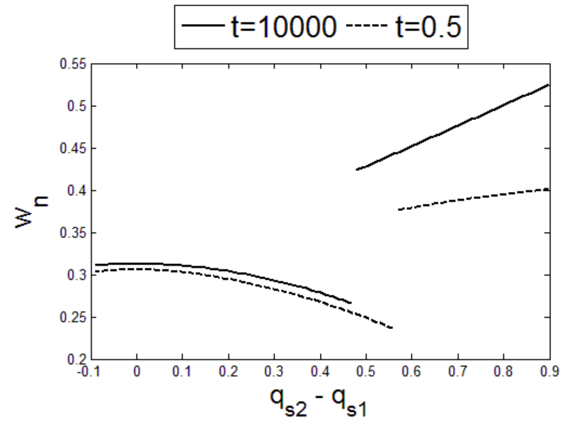
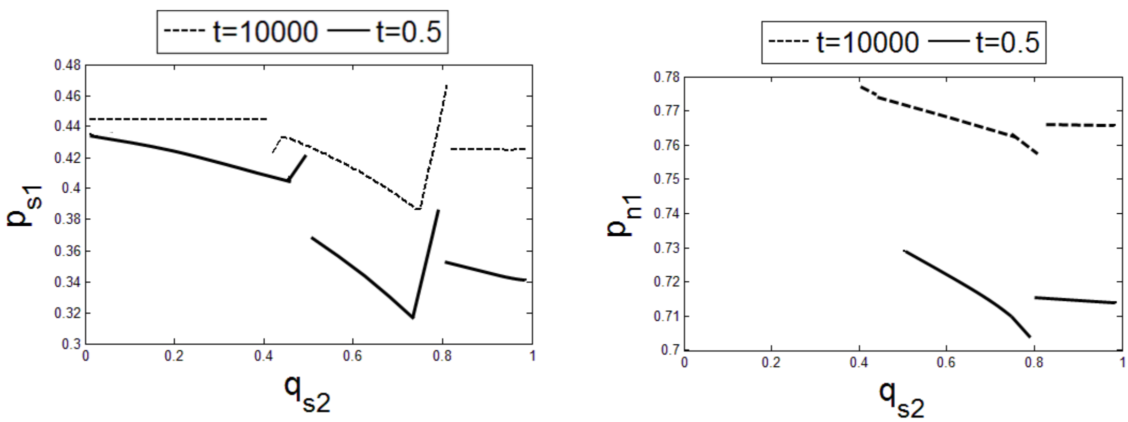


FIGURE 6. Manufacturer's optimal wholesale prices for $t = 10000$ and $t = 0.5$ holding $q_{s1} + q_{s2}$ constant)

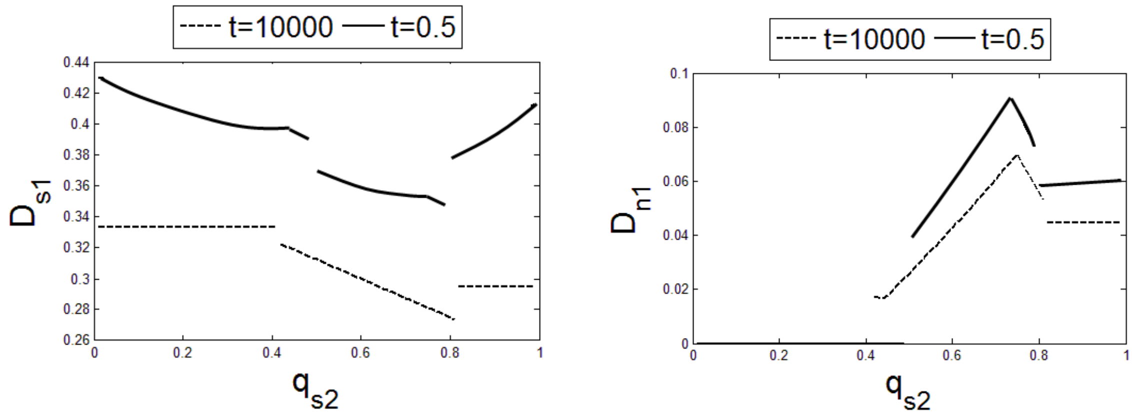


(A)

(B)

FIGURE 7. Comparison of retail prices for $t = 0.5$ and $t = 10000$

brand, instead of lowering the price a lot, she uses a different strategy. She simultaneously increases the quality level, making her product more competitive and reduces its price, but the price reduction is less than she would have implemented in the absence of a quality adjustment. In Regions II and III (i.e., the regions in which R1 sells both the store and national brands), R1 chooses to decrease the quality level of her store brand as t declines. The rationale is as follows. As customers become less loyal to their nearer retailer, the national brand manufacturer is forced to lower the wholesale price (as we will explain in more detail shortly). This reduction in the wholesale price allows R1 to be less concerned about maintaining bargaining power versus the national brand manufacturer. R1 can then focus more on differentiating the products sold at her store by decreasing quality level of her store brand. Not surprisingly, the retailer's adjustment in the store brand quality to account for finite t is complicated because it is intertwined with her pricing decisions, which in turn, are affected by the national brand manufacturer's wholesale price strategy. For this reason, we now discuss how the national brand's equilibrium wholesale price is affected by t before



(A) (B)

FIGURE 8. Comparison of customer demands for $t = 0.5$ and $t = 10000$

presenting a more comprehensive view of how the retailer's response is affected by the value of t.

Figures 5(A) and 5(B) show the national brand manufacturer's profit for a range of wholesale prices when $(q_{s1},\ q_{s2}) = (0.15q_n,\ 0.85q_n)$, and $(q_{s1},\ q_{s2}) = (0.35q_n,\ 0.65q_n)$, respectively. Other parameter values for this example are $r = 0.1$ and $q_n = 1$). In Figure 5(A), because the quality disparity between store brands is high, the national brand manufacturer chooses a price above the wholesale price threshold, and thus sells through only the retailer with the lower-quality store brand. In Figure 5(B), the quality disparity between store brands is low, so the national brand manufacturer chooses a price below the wholesale price below threshold and thus sells the national brand through both retailers. As can be seen in the Figures, whether the national brand manufacturer distributes though one retailer or both, the optimal wholesale price is smaller when t is finite than when it is infinite, but the price difference is smaller in the latter case. The reason for the national brand's wholesale price being smaller when t is finite is clear: as t declines, competition increases and the national brand manufacturer has no choice but to reduce his wholesale price. The reason why the price difference between $t \to \infty$ and a finite t is larger when the national brand manufacturer distributes through only one retailer can be explained as follows. When t is finite, by reducing his wholesale price, which leads to a reduction in the retail price, the national brand manufacturer can be more competitive against the store brand at the retailer where his product is being offered, but also against the store brand at the other retailer. The latter effect does not exist when $t \to \infty$. These patterns are illustrated in Figure 6. (The other parameter values for this example are $r = 0.1$ and $q_n = 1$.) When the quality disparity between the store brands is small, the national brand manufacturer sells through both retailers at a price below the wholesale price threshold and has little incentive to offer an even lower price even as competition increases (i.e., as t declines). On the other hand, when the quality disparity between the store brands is large, the national brand manufacturer finds it profitable to reduce his wholesale price, often significantly, as t declines because he can compensate via increased sales volume. In the figure, there is an intermediate interval of the quality disparity between the store brands for which the national brand manufacturer sells though both retailers when $t \to \infty$ but only one retailer when $t = 0.5$. Here, the wholesale price differential is significant and is mostly due to a change in the national brand manufacturer's retail distribution strategy and but also partly due to a change in the degree of competition.

Now that we have a better understanding of how the national brand manufacturer's wholesale price is affected by t, we can examine how equilibrium retail prices and demand demands are affected by t. With this as a backdrop, we next provide a more detailed explanation of the reasons underlying the retailer's equilibrium quality choices, using Figures 7 and 8 to illustrate our results. These two figures show (1) R1's equilibrium retail prices (respectively, demands) at the optimal quality level for $t \to +\infty$ and (2) R1's optimal retail prices (demands) at the optimal quality level for $t = 0.5$. Other parameter values for this example are $r = 0.5$ and $q_n = 1$.

Earlier in this section, we explained why R1 chooses a higher quality level in Region I when t is finite than when t is infinite: she increases the quality to avoid a large price decrease in the face of increased competition. We now explain why the optimal quality level in Region I is decreasing in q_{s2} for a fixed, finite t. The logic is as follows. As the retailer increases her quality level in response to t being small, she suffers from a reduction in customer demand (compared to that in the absence of a quality adjustment) As q_{s2} increases, R2's store brand becomes more competitive, so each increment in q_{s1} (and the corresponding price increase for product $s1$) causes a greater reduction in demand for R1. Therefore, when t is finite, as q_{s2} increases in Region I, R1 decreases her optimal q_{s1}. This, together with the phenomenon mentioned earlier, that is, R1's optimal q_{s1} in Region I when t is finite is greater than that when t is $+\infty$ (for all value of q_{s2}), explains the change in q_{s1} in Region I as t declines from $+\infty$ to a finite value.

In Regions II and III, R1's optimal reaction to a smaller t is dramatically different than in Region I: her store brand quality level is smaller for finite t. In both of these regions, the retailer carries both the national and store brands. In Region I, R1 does not carry the national brand. Therefore the change in the wholesale price does not affect her quality-positioning decision as directly as it did in Regions II and III. Due to the change in the national brand manufacturer's wholesale price in Regions II and III, the retailer decreases her quality. To complement the lower quality level, she sets a lower store brand retail price and a higher national brand price than in the case of $t \to +\infty$, as illustrated in Figure 7(B)). In essence, when there is stiff competition (small t), the retailer adopts a strategy of differentiating the two products so as to increase market share, and is aided by the national brand manufacturer's choice of a relatively low wholesale price in response to the competition. We note that R1's demands in Region II differ from those in Region III (see Figure 8) because R2 offers a lower price on her store brand in Region II (where both retailer offer both the national brand and their respective store brand products) than in Region III (where R2 carries only the store brand).

We close this section by exploring whether R1 can safely choose her store brand quality level assuming $t \to +\infty$ if all of the subsequent pricing decisions are made optimally conditional on the suboptimal quality level and the true t. Using the same numerical example mentioned above (with $\bar{\theta} = 1, q_n = 1$), we calculate, for all values of q_{s2}, the retailer's percentage profit reduction when she ignores the finiteness of t when deciding the store-brand quality (but other decisions are made based on the true value of t) from the optimal profit level when all decisions are made optimally for $t = 0.5$. For our example, we find that when the retailer's suboptimal quality level does not trigger the national brand manufacturer to choose a different distribution strategy, the retailer's profit reduction is less than 0.3%. When the value of q_{s2} is such that, if the retailer makes quality positioning decision sub-optimally, the national brand manufacturer uses a different distribution strategy than he would when under the optimal quality decision, the percentage reduction in R1's profit is less than 0.96%. For this example, in both cases, the retailer's profit loss is small. This suggests that the optimal quality in the $t \to +\infty$ case can serve as a good starting point for the retailer to search for optimal store brand quality when the true t is finite.

6. Retailer 1's Preference about q_{s2}

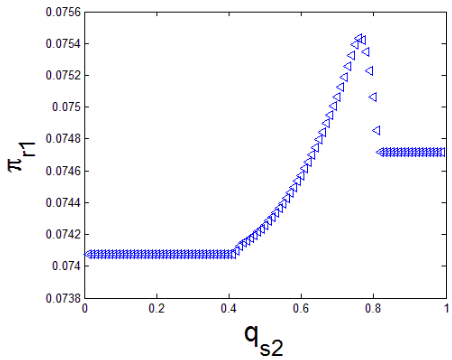
In the previous section, we derived R1's optimal quality decision when $t \to +\infty$ and showed through numerical studies that the structure of her optimal quality decision does not change significantly when t is finite. From a numerical example, we also observed that R1's percentage profit reduction if she sets the quality level ignoring the finiteness of t but makes the subsequent pricing decisions optimally, is relatively small.

One interesting question to ask is: what is R1's preference regarding the value of q_{s2}? We present R1's equilibrium profit as a function of q_{s2} in Figures 9(A) and 9(B) for the cases of $t \to \infty$ and $t = 0.5$, respectively. Other parameters for this example are $r = 0.5$ and $q_n = 1$. In both figures, the retailer's profit is calculated assuming that she sets quality and prices optimally. In Figure 9(C), we display the profit functions from Figures 9(A) and 9(B) together using the same scale to facilitate comparison.

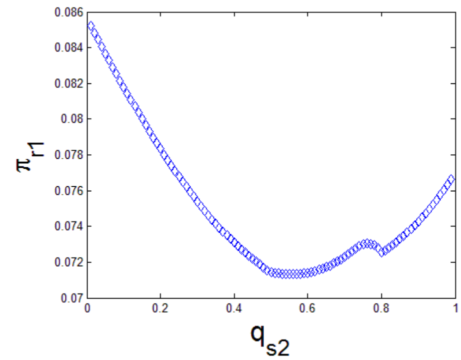
When $t \to +\infty$, the retailer prefers that q_{s2} take the largest value within Region II. The reason is that in Region II, any increase in q_{s2} forces the national brand manufacturer to lower the national brand wholesale price. As q_{s2} increases within Region II, R1 enjoys the benefit of a lowered national brand wholesale price without having to increase her own store brand quality level, which in turn, allows her to enjoy a larger market share. This benefit does not exist in the other two regions. If the value of q_{s2} is high enough that it falls in Region III, the national brand manufacturer sets a high wholesale price and sells through R1 only. If the value of q_{s2} is low enough that it falls in Region I, R2's store brand product does not pose enough competition for the national brand product to induce the national brand manufacturer to offer a wholesale price that is sufficiently attractive to R1. Thus, although R2 offers a store brand product, R1 does not get the benefit of a "free ride" from price concessions made by the national brand manufacturer in response to R2's store brand product. R1 thus chooses to sell her store brand only.

In contrast, when $t = 0.5$, R1 prefers q_{s2} to take the lowest possible value. There are two reasons. First, if q_{s2} is very low, the store brand at R2 is much less competitive than the store brand at R1. Therefore R1 is able to gain more market share. This effect does not exist when t is infinite. Second, if q_{s2} is very low, the national brand manufacturer will set a very high wholesale price. This leads R2 to set a high price on the national brand, which, in turn, reduces the competitiveness of the national brand at R2. Many customers located near R2 will then choose to buy products at R1. This effect also does not exist when t is infinite.

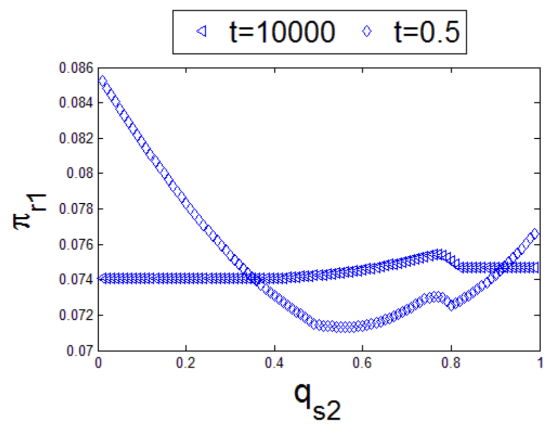
Comparing R1's profit when t is infinity and when t is 0.5 in Figure 9(C), we find some interesting phenomena. If q_{s2} is either very low or very high, R1's optimal profit when customers are not very loyal (represented by the $t = 0.5$ scenario) is greater than her optimal profit when customers are strictly loyal to the retailer located nearer them (represented by



(A) when t is infinity



(B) when $t = 0.5$



(C) Comparison

FIGURE 9. Retailer 1's optimal profit for different levels of q_{s2}

the $t \to +\infty$ scenario). This is quite counterintuitive, as one might think that both retailers would earn a lower profit when retail competition is more intense. Indeed, if q_{s2} is too low to be competitive, or if q_{s2} is too high to be differentiated enough from the national brand, more customers located near R2 will turn to purchase products at R1, and R1 can thus earn a higher profit than in the case in which customers are all loyal to the retailer located closer to them. As we will see in the next section, if R2 also sets his quality level optimally, he will not set q_{s2} either very low or very high.

We conclude this section by noting that, when customers are very loyal to the retailer located closer to them (i.e. t is large), R1 prefers the other retailer to introduce a medium-high level store brand so that R1 can enjoy the benefit of a lowered wholesale price. When customers are less loyal to the retailer located close to them, R1 prefers the other retailer to introduce a store brand with low quality (or, not introduce a store brand at all). When the other retailer does not have a store brand or has a store brand with very low quality level, the national brand manufacturer will set very high wholesale price for the national brand so that it will be sold by R2 only. R2 therefore has no choice but to set a high price for the national brand. R1, on the other hand, sells its low-cost store brand only. As a result, R1 gains much more market share, which leads to a large profit.

7. Asymmetric k and Competing Retailers' Quality-Setting

In this section, we first discuss how our results can be generalized to handle scenarios in which the national brand manufacturer and the two retailers have different production cost parameters. Then we discuss how our results can be used to obtain the equilibrium for a simultaneous quality-setting game between two retailers, under both symmetric and asymmetric cost parameters.

Thus far, we have assumed that both retailers and the national brand manufacturer share the same "k" in the unit production cost function, $C(q) = kq^2$. However, in reality, the retailer making the quality decision may have either a cost advantage or disadvantage versus the other retailer or/and the national brand manufacturer. When this happens, the quality-setting problem for the retailer is more complicated, mostly due to a change in the national brand manufacturer's pricing strategy. More specifically, retailer i ($i = 1, 2$) carries only the national brand if $\underline{w_{ni}} \leq w_n \leq \overline{w_{ni}}$, in which $\underline{w_{ni}} = \frac{c_{si}}{q_{si}} q_n$ and $\overline{w_n} = c_{si} + (\bar{v}_n - \bar{v}_{si})$. When both retailers and the national brand manufacturer share the same cost parameter, we have $\underline{w_{nl}} < \underline{w_{nh}} < c_n < \overline{w_{nh}} < \overline{w_{nl}}$ (recall that $\{h, l\} \in \{1, 2\}$ such that $q_{sh} > q_{sl}$). The national brand manufacturer will never set its wholesale price below c_n, nor will he ever set the wholesale price above $\overline{w_{nl}}$; therefore, the manufacturer chooses between only two wholesale prices, each corresponding to the optimal wholesale price constrained to $(c_n, \overline{w_{nh}})$ and $[\overline{w_{nh}}, \overline{w_{nl}})$, respectively.

However, when the national brand manufacturer's cost parameter (henceforth denoted by k_0) differs from the retailer's cost parameters (denoted by k_1 and k_2), there are multiple orderings among $\underline{w_{nl}}$, $\underline{w_{nh}}$, c_n, $\overline{w_{nh}}$ and $\overline{w_{nl}}$. Depending on the ordering, the national brand manufacturer may need to choose among three or even four candidate prices. For example, if $k_0 < \frac{q_{sl}}{q_n} k_l$ and $k_1 = k_2$, the ordering of c_n, $\underline{w_{ni}}$ and $\overline{w_{ni}}$ ($i = 1, 2$) becomes $c_n < \underline{w_{nl}} < \underline{w_{nh}} < \overline{w_{nh}} < \overline{w_{nl}}$. In this case, the national brand manufacturer will choose among four candidate wholesale prices, each corresponding to a different distribution strategy: selling through both retailers while foreclosing the store brand at R2, selling through both retailers but foreclosing neither of the store brands, selling through both retailers but foreclosing the store brand at both of them, and selling through only the retailer with the lower-quality store brand while not foreclosing that retailer's store brand.

This change in the structure of the national brand manufacturer's pricing strategy will affect R1's quality setting problem. The retailer needs to be more careful when choosing quality levels, especially when a small change in her quality level causes the national brand manufacturer's to alter his wholesale price enough that the set of retailers offering the national brand product changes. However, for small cost parameter distinctions among the manufacturer and two retailers, the qualitative insights on how the retailer should set her quality level stay the same as in our base model. Figure 10 shows results for a numerical example with $q_n = 1$, $\bar{\theta} = 1$, $t = 10000$ and with k_0 held constant at 0.4. The figure displays R1's optimal quality as a function of q_{s2} for different degrees of dispersion between the cost parameters of R1 and R2. Not surprisingly, R1's best response curve (i.e., $q^*_{s1}(q_{s2})$) is higher when she enjoys an advantage in production efficiency, and is lower when she has a disadvantage.

We now discuss how our results can be utilized to derive equilibria for quality-setting games between two retailers. Consider a situation in which both retailers engage in a Nash game in which they choose quality levels of their respective store brands, anticipating all

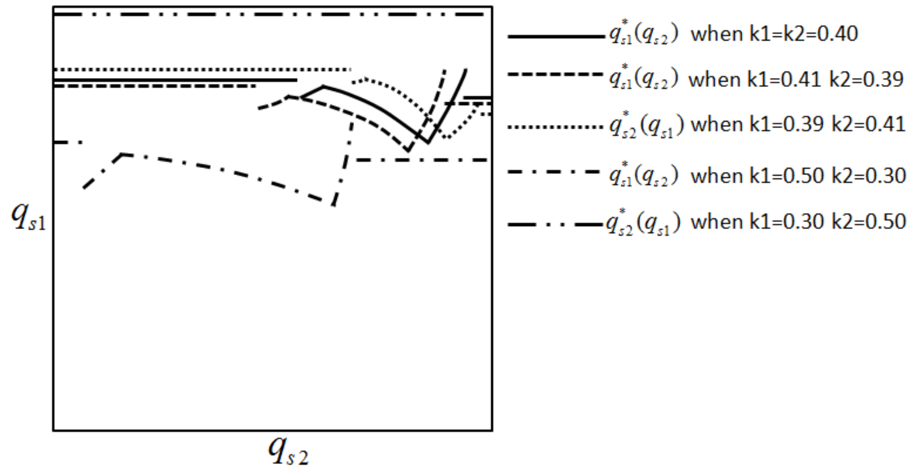


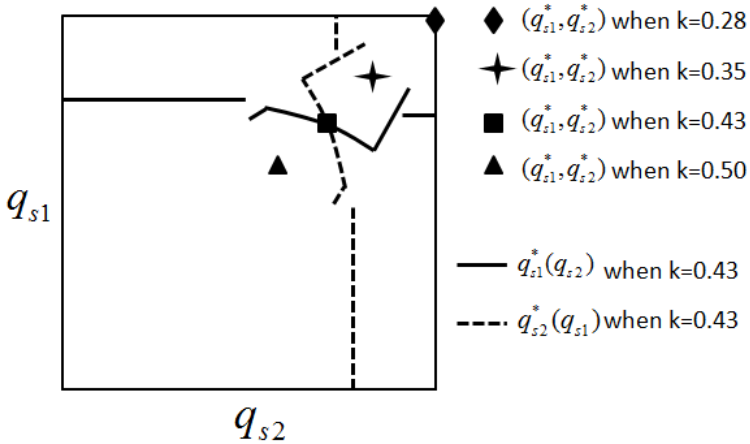
FIGURE 10. Retailer 1's optimal quality as a function of q_{s2} for different degrees of dispersion between k_1 and k_2

subsequent stages of the game (i.e., the manufacturer's wholesale-price-setting, and the retailers' retail-price-setting). Note that we have already derived R1's best response in this quality-setting game, that is, her optimal quality for any quality level of R2's store brand. If the retailers' cost parameters are symmetric, R2's best response is exactly the same as R1's. In most cases, there exists a unique equilibrium. To enable the reader to better visualize the equilibria, in Figure 11(A), we present the equilibrium for several different values of k shared by the national brand manufacturer and the two retailers. For any given k, both retailers set the same quality level for their respective store brands at the equilibrium. Other parameters for this example are $q_n = 1$, $\bar{\theta} = 1$, and $t = 10000$.

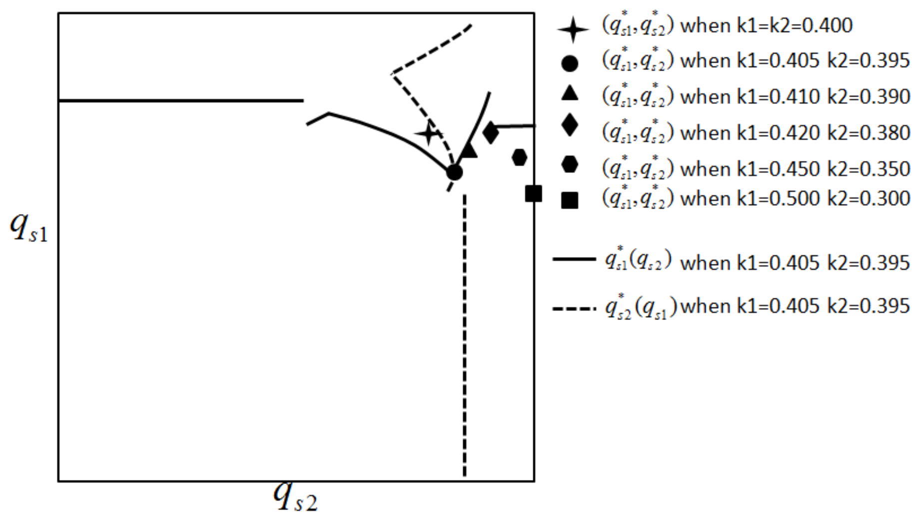
Our model can be also modified to accommodate asymmetric cost parameters at two retailers. In Figure 11(B), we hold the national brand manufacturer's cost parameter constant at 0.4, and plot several quality-setting equilibria for different degrees of disparity in cost parameters of the two retailers . Here, we assume that $k_1 > k_0 > k_2$. Other parameters for this example are $q_n = 1$, $\bar{\theta} = 1$, and $t = 10000$. Not surprisingly, the retailer who enjoys a cost advantage always sets a higher quality level than the other at the equilibrium. But identification of the equilibrium is not easy: due to the discontinuous and non-monotonic nature of each retailer's best response curve, the equilibrium changes non-monotonically as the two retailers' production efficiency levels become more disparate. Thus, the discontinuous and non-monotonic responses that we observe in the symmetric case become more complex when the parties have asymmetric cost parameters, and consequently, the equilibria become more difficult to predict. This is where our characterization of the structure of the optimal response function can provide value to a decision-maker.

8. Conclusions

This paper addresses store brand quality-positioning decisions for retailers facing retail competition. We analyze a situation in which a retailer is considering introducing a store brand product in a product category in which her retail competitor already offers a store brand, and both retailers can offer the same national brand product. We initially assume that the quality levels of the national brand and the competing retailer's store brand are fixed. The



(A)



(B)

FIGURE 11. How optimal q_{s1} changes with q_{s2} for different levels of t

retailer in question first makes a decision regarding the store-brand quality level, anticipating the manufacturer-Stackelberg pricing game that follows. In that game, the national brand manufacturer first chooses a wholesale price that will be offered to both retailers, and the retailers then engage in a Nash game to choose retail prices for the product(s) they choose to offer. Their product offering decisions are implicit in the prices: if a retailer chooses not to offer a product, he/she can do so by setting a price that drives demand for that product to zero. We assume customers are heterogeneous in two dimensions—location (which represents a customer's degree of loyalty to one retailer or another) and willingness to pay (WTP) per unit of quality. in our equilibrium analysis, we derive the optimal product assortment and pricing strategy for the retailer of interest as well as the national brand manufacturer's wholesale pricing strategy, for all levels of the store brand quality chosen by the retailer of interest. With an understanding of these optimal responses, we derive the retailer's optimal store-brand quality level.

We find that, given a quality level of the store brand set by the retailer of interest, the manufacturer has two candidate wholesale prices that may be optimal. At the lower candidate wholesale price, the national brand manufacturer sells through both retailers. At the higher candidate wholesale price, the national brand manufacturer gets a higher margin, but only sells through the retailer whose store brand competes less directly with the national brand. We find that, the national brand manufacturer chooses the higher wholesale price (and thus distributes only through the retailer with the lower-quality store-brand which competes less directly with the national brand product) if the quality gap between the two store brands is greater than a threshold. The underlying reasons are as follows. The higher is the quality level of the store brand at a retailer, the lower the national brand manufacturer needs to set its wholesale price in order for this retailer to carry the national brand. If the quality levels at the two retailers differ, ideally, the manufacturer would like to offer different prices to the two retailers, but he is not allowed to do so. The higher is the quality disparity between the two store brands, the greater is the compromise the manufacturer needs to make if he wants to sell through both retailers. The nature of the compromise can be explained as follows. When the quality disparity is above the threshold, offering a price that is sufficiently low to induce both retailers to offer the national brand leads to a situation in which the retailer with the higher-quality store brand is selling a product that is quite competitive with the national brand, and yet the national brand manufacturer is charging a low wholesale price for it. At the same time, the other retailer is offering a low-quality store brand and benefits from the product differentiation that the low-quality store brand provides in capturing the portion of customers with a low WTP per unit of quality.

The threshold on the quality disparity decreases with the absolute quality level of the higher-quality store brand. If the quality of the better store brand is high, it poses strong competition for the national brand product. This makes it unattractive for the national brand manufacturer to sell through this retailer. Even if the quality disparity between the two store brands is low, the national brand manufacturer will choose to sell through the retailer with the lower-quality store brand because he otherwise will not sell anything at all, and if he sells through both retailers, the competition posed by their two strong store brands will force him to drop his wholesale price significantly. In the special case where the quality level of the higher-quality store brand is equal to that of the national brand, the threshold on the quality disparity falls to zero, meaning that the manufacturer will sell only through the retailer with the lower-quality store brand.

The national brand manufacturer's wholesale pricing strategy has important implications for how the retailer of interest should make quality-positioning decisions. The retailer knows that if the quality level of her store brand is much higher than that of the other store brand, the manufacturer will choose a (high) wholesale price which will make it no longer optimal for her to carry the national brand. If the quality level of her store brand is much lower than that of the other store brand, the manufacturer will choose a wholesale price which will cause the other retailer drop the national brand. If the store brand quality level at the other retailer and herself are not too disparate, the manufacturer will choose a wholesale price such that both retailers carry the national brand. It turns out that both retailers tend to be better off when they both carry the respective store brands as well as the national brand because together, each with a competing store-brand product available to sell, they are able to elicit greater price concessions from the national brand manufacturer while simultaneously reaching customers with low WTP for quality via their store brands.

The finer details of the retailer's quality decision for her store-brand depend on how the unit production cost changes with the quality level. If the unit production cost of store brand is linearly increasing in the product quality, the retailer always sets the quality level of her store brand the same as that of the national brand. The underlying reason is that with a linear cost relationship, the retailer does not derive any value from differentiating her product from the national-brand product. Stated another way, the total number of units that she sells does not change with the quality level of the store brand. The retailer therefore increases the quality level of the store brand and thus earns a higher profit margin on each unit of the product sold.

On the other hand, if the unit production cost of is strictly convex and increasing in the product quality, the retailer's quality decision becomes more complex, and it depends heavily on the quality level of the store brand at the other retailer. Indeed, as the quality level of the competing store brand increases, the optimal quality level set by the retailer of interest is non-monotonic and discontinuous. A small increase in the quality level of the competing store brand could lead to a "jump" in her optimal quality level. Specifically, if the quality level at the other retailer is low (high), the retailer will set the quality level of her store brand in a high (low) range. If the quality level at the other retailer is moderate, the retailer will set the quality level of her store brand in the moderate range. When the retailer's quality level is within the either the high or the low range, her optimal quality level is invariant with the quality level of the other store brand; whereas when her optimal optimal quality level falls into the moderate range, the quality level is strictly decreasing with the quality level of the store brand at the other retailer. Intuitively, as the quality level of the other store brand increases, the national brand manufacturer is forced to reduce his wholesale price. The retailer of interest thus has less incentive to increase quality, as her incentive to increase quality arises from her quest to obtain a lower wholesale price from the national brand manufacturer.

Our results contribute to the literature by showing how the presence of retail competition leads to optimal store brand quality-positioning strategies that can be quite different from the common wisdom but also much more complex than what has been reported in past research.. The past literature on store brand strategies has suggested that a monopolist retailer should set the quality level of her store brand as high as possible, as it can increase her bargaining power versus the national brand manufacturer. We find this is not the case if the retailer is facing competition. Under retail competition, setting a high quality level for the store brand can backfire. In particular, increasing the quality level of the store brand does not necessarily increase a retailer's bargaining power. If the retailer sets too high a quality level, the national brand manufacturer could charge a high wholesale price, effectively foreclosing the retailer of interest from selling the national brand (because it is uneconomical for the retailer to offer the national brand product), although it would have been in the retailer's best interest to sell a moderate- or low-quality store brand product alongside the national brand product.

We also study how the intensity of competition affects the equilibrium. As retail competition becomes more heated, the retailer adjusts the quality level of her store brand in order to alleviate price competition between the product(s) sold at her store and the product(s) sold at the other retailer. More specifically, if it is optimal for the retailer to carry only the store brand at her store (which occurs if the other store brand is low in quality), she would increase the quality level of her store brand to become more differentiated from the store brand at the other retailer. If it is optimal for the retailer to carry both the store and the national brand at her store (which occurs if the other store brand is high or moderate in quality), she would decrease the quality level of her store brand.

We generalize our model to allow the retailers and the national brand manufacturer to have different production cost parameters and show how this asymmetry affects the equilibria. We also explain how our results can be utilized to identify equilibria–for both symmetric and asymmetric production cost parameters— for a game in which the two retailers simultaneously set their store-brand quality levels.

As we mentioned early in this paper, little work has been done to analyze how retailers should set store-brand quality levels in the face of retail competition, and the results in this paper represent a first step in studying this issue. It would be interesting to study how the equilibria are affected by the structure of competitive interactions among the retailers and the national brand manufacturer as well as by different sourcing arrangements for the store brand(s).

APPENDICES

Appendix A: Derivation of the demand function for finite t

With Figure 1 (see Section 3.2) at hand, we can easily write expressions for demands for the two products at each retailer given a price vector $\mathbf{p} = (p_{n1},\ p_{s1},\ p_{n2},\ p_{s2})$:

$$
\begin{aligned}
D_{ni}(\mathbf{p},\ \bar{\theta}) &= \begin{cases} D_{ni}^H, & \text{if } \theta_i \geq \theta_j \\ D_{ni}^L, & \text{if } \theta_i < \theta_j \end{cases} \\
D_{si}(\mathbf{p},\ \bar{\theta}) &= \begin{cases} D_{si}^H, & \text{if } \theta_i \geq \theta_j \\ D_{si}^L, & \text{if } \theta_i < \theta_j \end{cases}
\end{aligned} \tag{5}
$$

where

$$
\begin{aligned}
D_{ni}^H(\mathbf{p},\ \bar{\theta}) &= \frac{1}{2t\bar{\theta}}(p_{nj} - p_{ni} + t)(\bar{\theta} - \theta_i) \\
D_{si}^H(\mathbf{p},\ \bar{\theta}) &= \begin{cases} \frac{1}{2t\bar{\theta}}\left\{\frac{1}{2}\left[(p_{nj} - p_{ni} + t) + (b_{sn}^i(\theta_j) + t)\right](\theta_i - \theta_j)\right\} \\ +\frac{1}{2t\bar{\theta}}\left\{\frac{1}{2}\left[(b_{ss}^i(\tilde{\theta}_i) + t) + (b_{ss}^i(\theta_j) + t)\right](\theta_j - \tilde{\theta}_i)\right\}, & \text{if } \tilde{\theta}_i \leq \theta_j \\ \frac{1}{2t\bar{\theta}}\left\{\frac{1}{2}\left[(p_{nj} - p_{ni} + t) + (b_{sn}^i(\tilde{\theta}_i) + t)\right](\theta_i - \tilde{\theta}_i)\right\}, & \text{if } \tilde{\theta}_i \geq \theta_j \end{cases} \\
D_{ni}^L(\mathbf{p},\ \bar{\theta}) &= \frac{1}{2t\bar{\theta}}\left\{(\bar{\theta} - \theta_j)(t - p_{ni} + p_{nj}) + \frac{1}{2}\left[(t - b_{ss}^j(\theta_i)) + (t - p_{ni} + p_{nj})\right](\theta_j - \theta_i)\right\} \\
D_{si}^L(\mathbf{p},\ \bar{\theta}) &= \frac{1}{2t\bar{\theta}}\left\{\frac{1}{2}\left[(t - b_{ss}^j(\theta_i)) + (t - b_{ss}^j(\tilde{\theta}_i))\right](\theta_i - \tilde{\theta}_i)\right\}
\end{aligned} \tag{6}
$$

and where, for example, D_{ni}^H is the demand for product ni if $\theta_i \geq \theta_j$. It should be understood that θ_i, θ_j and $\tilde{\theta}_i$ are functions of $\mathbf{p}$ as defined earlier. Similarly, $b_{ss}^i(\theta)$ and $b_{sn}^i(\theta)$ are functions of both $\mathbf{p}$ and θ. Although the full expressions for these variables are $\theta_i(\mathbf{p})$, $\theta_j(\mathbf{p})$, $\tilde{\theta}_i(\mathbf{p})$, $b_{ss}^i(\mathbf{p},\ \theta)$ and $b_{sn}^i(\mathbf{p},\ \theta)$, we omit the variable $\mathbf{p}$ in (6) for brevity. It can be seen that we have four possible equilibrium scenarios, and from one scenario to another, the demand function $\mathbf{p} \longmapsto \mathbf{D} \equiv (D_{n1},\ D_{s1},\ D_{n2},\ D_{s2})$ is not continuous. Case 1 corresponds to $\theta_1 \geq \theta_2$ and $\tilde{\theta}_1 \geq \theta_2$, Case 2 corresponds to $\theta_1 \geq \theta_2$ and $\tilde{\theta}_1 < \theta_2$, Case 3 corresponds to $\theta_1 < \theta_2$ and $\tilde{\theta}_2 \geq \theta_1$, and Case 4 corresponds to $\theta_1 < \theta_2$ and $\tilde{\theta}_2 < \theta_1$.

Appendix B: Proofs of Lemma 1 and Propositions 1, 2 and 3

Appendix B.1: Proofs of Lemma 1 and Propositions 1, 2 and 3 when the unit production cost is linear in quality

When the unit production cost is linear in quality, the condition stated in Lemma 1 can be easily reinterpreted in terms of the original parameters. We have the following identities: $m_{sm} = (\bar{\theta}-k)\frac{2}{\frac{1}{q_n-q_{s1}}+\frac{1}{q_n-q_{s2}}}$, $m_n - m_{sl} = (\bar{\theta}-k)(q_n - q_{sl})$, $\bar{v_n} - \bar{v_{sm}} = \bar{\theta}\frac{2}{\frac{1}{q_n-q_{s1}}+\frac{1}{q_n-q_{s2}}}$ and $\bar{v_n} - \bar{v_l} = \bar{\theta}(q_n - q_{sl})$. The condition $\frac{m_n-m_{sm}}{m_n-m_{sl}} \geq \left(\frac{\bar{v}_n-\bar{v}_{sm}}{2(\bar{v}_n-\bar{v}_{sl})}\right)^{1/2}$ therefore simplifies to $\frac{\frac{2}{q_n-q_{sl}}}{\frac{1}{q_n-q_{s1}}+\frac{1}{q_n-q_{s2}}} \geq (\frac{\frac{1}{q_n-q_{sl}}}{\frac{1}{q_n-q_{s1}}+\frac{1}{q_n-q_{s2}}})^{\frac{1}{2}}$. After mathematical manipulation, this further simplifies to $q_{sh} - q_{sl} \leq 2(q_n - q_{sh})$. This condition has been derived in Proposition 4 in Section 5 of Chapter 2. From this condition, it immediately follows that $q^0_{sh} = \frac{2}{3}q_n + \frac{1}{3}q_{s2}$ and $q^0_{sl} = 3q_{s2} - 2q_n$. From these expressions, we can immediately infer that the statements in Lemma 1 and in Propositions 1, 2 and 3 hold when the unit production cost is linear in quality. Therefore, in the remaining proofs in this Appendix, we focus on the scenario in which the unit production cost is strictly increasing and convex in quality.

Appendix B.2: Proof of Lemma 1 when the unit production cost is strictly increasing and convex in quality

The proof proceeds as follows. We first present some variable transformations to simplify the problem. We then derive the retailer's optimal response to any set of wholesale prices. We find that he manufacturer needs to solve two subproblems, corresponding to selling or not selling the national brand product, compare their respective profits, and choose the better alternative. We present analyses of both subproblems.

Because of the structure of the retailer's potential responses, the national brand manufacturer has to anticipate the possibility of selling through both or only one of the retailers, optimize his wholesale price under each of the scenarios, and choose the better option. We present a full analysis of each of the national brand manufacturer's subproblems and then go on to derive conditions in which he prefers one or the other assuming that the solutions to the subproblems are interior points. Finally, we show that these conditions remain valid when one or both of the solutions to the subproblems is (are) boundary point(s).

Transformation of variables to simplify the problem

Define $\hat{D}_{ni} \equiv 2D_{ni} = 1-\phi_i$ and $\hat{D}_{nsi} \equiv 2(D_{ni}+D_{si}) = 1-\tilde{\phi}_i$. Define $\theta_i = \frac{p_{ni}-p_{si}}{q_n-q_{si}}$,$\tilde{\theta}_i = \frac{p_{si}}{q_{si}}$, $\phi_i = \frac{\theta_i}{\bar{\theta}}$, $\tilde{\phi}_i = \frac{\tilde{\theta}_i}{\bar{\theta}}$, $b_i = \frac{q_{si}}{q_n}$, $d_i = \frac{q_n-q_{si}}{q_n}$, $\beta_i = \frac{w_n-c_{si}}{\bar{\theta}q_n}$, $\tilde{\beta}_i = \frac{c_{si}}{\bar{\theta}q_n}$, $\gamma_i = \frac{\beta_i}{d_i}$ and $\tilde{\gamma}_i = \frac{\tilde{\beta}_i}{b_i}$. ($\tilde{\gamma}_i \leq 1$ because we have assumed that the production of store brands is efficient for the market.)

The retailer i's profit function can then be written as:

$$\begin{aligned}\pi_{R_i} &= 0.5\hat{D}_{ni}(p_{ni}-w_n)+\hat{D}_{si}(p_{si}-c_{si})\\ &= 0.5\bar{\theta}q_n\left[d_i\hat{D}_{ni}(\phi_i-\gamma_i)+b_i\hat{D}_{nsi}(\tilde{\phi}_i-\tilde{\gamma}_i)\right]\end{aligned} \tag{7}$$

Therefore $\hat{\pi}_{Ri} \equiv \frac{2\pi_{R_i}}{\bar{\theta}q_n} = d_i\hat{D}_{ni}(\phi_i-\gamma_i)+b_i\hat{D}_{nsi}(\tilde{\phi}_i-\tilde{\gamma}_i) = d_i(1-\phi_i)(\phi_i-\gamma_i)+b_i(1-\tilde{\phi}_i)(\tilde{\phi}_i-\tilde{\gamma}_i)$.

The retailer's optimal response

Retailer i's optimization problem is $\max_{0\le\tilde{\phi}_i\le\phi_i\le 1}\hat{\pi}_{Ri}$. The optimal solution depends upon the relationship among $\tilde{\gamma}_i$, γ_i and 1. There are three possibilities: (i) $\phi_i^* = \frac{1}{2}(1+\gamma_i)$ and $\tilde{\phi}_i^* = \frac{1}{2}(1+\tilde{\gamma}_i)$ if $\tilde{\gamma}_i \le \gamma_i \le 1$; (ii) $\phi_i^* = 1$ and $\tilde{\phi}_i^* = \frac{1}{2}(1+\tilde{\gamma}_i)$ if $\tilde{\gamma}_i \le 1 \le \gamma_i$; or (iii) $\phi_i^* = \tilde{\phi}^* = \frac{1}{2}(1+\frac{d_i\gamma_i+b_i\tilde{\gamma}_i}{d_i+b_i})$ if $\gamma_i \le \tilde{\gamma}_i \le 1$. Notice that $\tilde{\gamma}_i \le \gamma_i$ is equivalent to $w_n \ge \frac{c_{si}}{q_{si}}q_n$ and $\gamma_i \le 1$ is equivalent to $w_n \le c_{si}+\bar{\theta}(q_n-q_{si})$. Define the indices $\{l,\ h\}\in\{1,2\}$ such that $q_{sl} < q_{sh}$. Then we can see immediately that $\frac{c_{sl}}{q_{sl}}q_n \le \frac{c_{sh}}{q_{sh}}q_n$. Define $\underline{w_{ni}} \equiv \frac{c_{si}}{q_{si}}q_n$ and $\overline{w_{ni}} \equiv c_{si}+\bar{\theta}(q_n-q_{si})$. Then retailer i sells only the store brand if $w_n \ge \overline{w_{ni}}$, sells only the national brand if $w_n \le \underline{w_{ni}}$ and sells both the store and the national brand otherwise. Therefore, the national brand manufacturer has two subproblems to solve.

Analysis of the national brand manufacturer's two subproblems

Because $k < \frac{\bar{\theta}}{2q_n}$, we have $q_{s1}+q_{s2} < \frac{\bar{\theta}}{k}$, which is equivalent to $\overline{w_{nh}} \le \overline{w_{nl}}$. Similarly, we have $q_n+q_{sh} \le \frac{\bar{\theta}}{k}$, which is equivalent to $c_n < \overline{w_{nh}}$. Moreover, because $\underline{w_{nh}} = \frac{c_{sh}}{q_{sh}}q_n = kq_{sh}q_n$, we have $\underline{w_{nh}} < c_n$. Therefore, we have $\underline{w_{nl}} < \underline{w_{nh}} < c_n < \overline{w_{nh}} < \overline{w_{nl}}$. The national brand manufacturer will never set its wholesale price below c_n. Also, the national brand manufacturer will never set its wholesale price above $\overline{w_{nl}}$ because neither retailer will offer the national brand facing such a high wholesale price. Therefore the manufacturer needs to solve the following two subproblems and compare the resulting profit.

$$\max_{c_n\le w_n\le\overline{w_{nh}}}\pi_{M1} = \frac{1}{2}[\left(1-\frac{1}{2}(1+\frac{w_n-c_{s1}}{\bar{\theta}(q_n-q_{s1})})\right)+\left(1-\frac{1}{2}(1+\frac{w_n-c_{s2}}{\bar{\theta}(q_n-q_{s2})})\right)](w_n-c_n) \tag{8}$$

$$\max_{\overline{w_{nh}}\le w_n\le\overline{w_{nl}}}\pi_{M2} = \frac{1}{2}[\left(1-\frac{1}{2}(1+\frac{w_n-c_{sl}}{\bar{\theta}(q_n-q_{sl})})\right)](w_n-c_n) \tag{9}$$

Solving the first-order necessary condition to maximize (8), we get $w_{n1} = \frac{1}{2}(\hat{v}+\hat{c}_{ns})$, where $\hat{v} \equiv \frac{2}{\frac{1}{\bar{v}_n-\bar{v}_{s1}}+\frac{1}{\bar{v}_n-\bar{v}_{s2}}}$ is the Harmonic mean of $(\hat{v}_n-\hat{v}_{s1})$ and $(\hat{v}_n-\hat{v}_{s2})$ and $\hat{c}_{ns} \equiv \frac{\frac{c_n+c_{s1}}{\bar{v}_n-\bar{v}_{s1}}+\frac{c_n+c_{s2}}{\bar{v}_n-\bar{v}_{s2}}}{\frac{1}{\bar{v}_n-\bar{v}_{s1}}+\frac{1}{\bar{v}_n-\bar{v}_{s2}}}$ is a weighted average of (c_n+c_{s1}) and (c_n+c_{s2}). It is straightforward to confirm that $\pi_{M1}(w_n)$ is concave. Therefore, w_{n1} *is* the optimal solution for problem (8) as long as it is an interior point. It is easy to verify that w_{n1} is always greater than c_n . Now w_{n1} is an interior point if $w_{n1} < \overline{w_{nh}}$, which is equivalent to $d_{t1} < \frac{\bar{\theta}}{kq_n}$ where $d_{t1} \equiv (d_h+d_l)(1+b_h)+b_l^2-b_h^2$. Therefore,

$$w_{n1}^* \equiv \arg\max_{c_n\le w_n\le\overline{w_{nh}}}\pi_{M1} = \begin{cases} w_{n1} & \text{if } d_{t1} \le \frac{\bar{\theta}}{kq_n}(2d_h) \\ \overline{w_{nh}} & \text{if } d_{t1} > \frac{\bar{\theta}}{kq_n}(2d_h)\end{cases} \tag{10}$$

Similarly, after solving the first-order condition to maximize $\pi_{M2}(w_n)$, verifying concavity of the objective function, and checking boundary conditions, we get

$$w_{n2}^* \equiv \underset{c_n \le \overline{w_{nh}} \le \overline{w_{nl}}}{\arg\max}\ \pi_{M2} = \begin{cases} w_{n2} & \text{if } d_{t2} \ge \frac{\bar{\theta}}{kq_n}(2d_h - d_l) \\ \overline{w_{nh}} & \text{if } d_{t2} < \frac{\bar{\theta}}{kq_n}(2d_h - d_l) \end{cases} \tag{11}$$

where $w_{n2} = \frac{1}{2}\left[(c_n + c_{sl}) + (\bar{v}_n - \bar{v}_{sl})\right]$ and $d_{t2} \equiv d_h(1 + b_h) + (b_l^2 - b_h^2)$.

It is straightforward to show that if the condition for $w_{n1}^* = \overline{w_{nh}}$ is satisfied, then we must have $w_{n2}^* = w_{n2}$. Also, if the condition for $w_{n2}^* = \overline{w_{nh}}$ is satisfied, then we must have $w_{n1}^* = w_{n1}$. Therefore, we have

$$(w_{n1}^*,\ w_{n2}^*) = \begin{cases} (w_{n1},\ w_{n2}) & \text{if } d_{t1} \le \frac{\bar{\theta}}{kq_n}(2d_h) \text{ and } d_{t2} \ge \frac{\bar{\theta}}{kq_n}(2d_h - d_l) \\ (\overline{w_{nh}},\ w_{n2}) & \text{if } d_{t1} > \frac{\bar{\theta}}{kq_n}(2d_h) \\ (w_{n1},\ \overline{w_{nh}}) & \text{if } d_{t2} < \frac{\bar{\theta}}{kq_n}(2d_h - d_l) \end{cases} \tag{12}$$

Conditions in which the manufacturer prefers one of the subproblem solutions when both are interior

Both w_{n1}^* and w_{n2}^* take the corresponding interior solution when $d_{t1} \le \frac{\bar{\theta}}{kq_n}(2d_h)$ and $d_{t2} \ge \frac{\bar{\theta}}{kq_n}(2d_h - d_l)$,. The manufacturer compares the profits from the two solutions and chooses the better one. The manufacturer's profit is $\pi_{M1}^* = \frac{1}{8\hat{v}}(\hat{v} + c_{sm} - c_n)^2$ at w_{n1} and $\pi_{M2}^* = \frac{1}{16(\bar{v}_n - \bar{v}_{sl})}(\bar{v}_n - \bar{v}_{sl} + c_{sl} - c_n)^2$ at w_{n2}. Define $\bar{v}_{sm} \equiv \bar{v}_n - \hat{v} = \frac{\frac{\bar{v}_{s1}}{\bar{v}_n - \bar{v}_{s1}} + \frac{\bar{v}_{s2}}{\bar{v}_n - \bar{v}_{s2}}}{\frac{1}{\bar{v}_n - \bar{v}_{s1}} + \frac{1}{\bar{v}_n - \bar{v}_{s2}}}$, the weighted average of $\bar{v}_{s1}$ and $\bar{v}_{s2}$, and define c_{sm} as $\frac{\frac{c_{s1}}{\bar{v}_n - \bar{v}_{s1}} + \frac{c_{s2}}{\bar{v}_n - \bar{v}_{s2}}}{\frac{1}{\bar{v}_n - \bar{v}_{s1}} + \frac{1}{\bar{v}_n - \bar{v}_{s2}}}$, that is, the weighted average of c_{s1} and c_{s2}. Then we can express π_{M1}^* as $\frac{1}{8(\bar{v}_n - \bar{v}_{sm})}((\bar{v}_n - c_n) - (\bar{v}_m - c_{sm}))^2$. Also define $m_x \equiv \bar{v}_x - c_x$ for $x \in \{n,\ s1,\ s2,\ sm\}$. That is, m_x is the utility that product x provides to a customer less its production cost. Then

$$\begin{cases} \pi_{M1}^* & = \frac{1}{8(\bar{v}_n - \bar{v}_{sm})}(m_n - m_{sm})^2 \\ \pi_{M2}^* & = \frac{1}{16(\bar{v}_n - \bar{v}_{sl})}(m_n - m_{sl})^2 \end{cases} \tag{13}$$

In this way, we have constructed an "aggregate" product, sm, whose competitiveness, m_{sm}, is the weighted average of the competitiveness of $s1$ and $s2$. From (13), we conclude that when the solution to both of the manufacturer's subproblems, (8) and (9), are interior points, the manufacturer sets the wholesale price at w_{n1} when $\frac{m_n - m_{sm}}{m_n - m_{sl}} \ge \left(\frac{\bar{v}_n - \bar{v}_{sm}}{2(\bar{v}_n - \bar{v}_{sl})}\right)^{1/2}$ and the wholesale price at w_{n2} otherwise.

Proof that the results remain the same when the solution to either subproblem is on a boundary

As stated earlier, the manufacturer's profit functions with the wholesale price constrained to either below or above the threshold, $\pi_{M1}(w_n)$ and $\pi_{M2}(w_n)$ (as defined in (8) and (9)), are both concave. Hence his profit function is piecewise concave. Also, his profit function is continuous on $[c_n, \overline{w_{nl}}]$. Therefore, when the solution of one subproblem is an interior point and the solution of the other subproblem is a boundary solution, the interior solution is optimal. We need to show that, when this is the case, the condition the "optimal solution w_{n1}^* is chosen over w_{n2}^* if $\frac{m_n - m_{sm}}{m_n - m_{sl}} \ge \left(\frac{\bar{v}_n - \bar{v}_{sm}}{2(\bar{v}_n - \bar{v}_{sl})}\right)^{1/2}$" still holds. That is, what remains to be proved are two relationships: (1) $w_{n1}^* = \overline{w_{nh}} \Rightarrow (\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 \geq \frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}}$ and (2) $w_{n2}^* = \overline{w_{nh}} \Rightarrow (\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 \leq \frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}}$.

To facilitate the proof, we define $f_i \equiv \frac{1}{\bar{v}_n - \bar{v}_{si}}$ for $i = 1, 2$; $y \equiv \frac{q_{sh} - q_{sl}}{q_n - q_{sl}}$; $\alpha \equiv \frac{\frac{1}{\bar{v}_n - \bar{v}_{sh}}}{\frac{1}{\bar{v}_n - \bar{v}_{sh}} + \frac{1}{\bar{v}_n - \bar{v}_{sl}}}$ and $x \equiv \frac{m_{sh} - m_{sl}}{m_n - m_{sl}}$. Clearly, $y \in (0, 1)$, $\alpha \in (\frac{1}{2},\ 1)$ and $x \in (0, 1)$. Moreover, we can express relationships among f_i, x, y, α and the m_x's (for $x \in \{s1, s2, n\}$) as $\alpha \equiv \frac{1}{2-y}$, $x \equiv y \cdot \frac{(1 - r \cdot 2q_{sl}) - r(q_n - q_{sl})y}{1 - r(q_n + q_{sl})}$, $\frac{m_n - m_{sh}}{m_{sh} - m_{sl}} \equiv \frac{1}{x} - 1$, $\frac{(\bar{v}_n - \bar{v}_{sm})}{2(\bar{v}_n - \bar{v}_{sl})} = \frac{f_l}{f_1 + f_2} = 1 - \alpha$, and $(\frac{m_n - m_{sm}}{m_n - m_{sl}})^2 = (1 - \alpha x)^2$.

To show (1), note that $w_{n1}^* = \overline{w_{nh}}$ if and only if $\frac{f_1 + f_2}{f_l} < \frac{m_{sh} - m_{sl}}{m_n - m_{sh}}$. Moreover, $\frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}} = \frac{f_1 + f_2}{f_l}$. Therefore we only need to show that $(\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 \geq \frac{m_{sh} - m_{sl}}{m_n - m_{sh}}$, which is equivalent to $\frac{m_n - m_{sh}}{m_{sh} - m_{sl}} \geq (\frac{m_n - m_{sm}}{m_n - m_{sl}})^2$. Rewriting the last inequality in terms of x and α, the condition becomes $\frac{1}{x} - 1 \geq (1 - \alpha x)^2$. Now, because $x \in (0, 1)$, a sufficient condition for the inequality to hold is $1 - x = x(\frac{1}{x} - 1) \geq (1 - \alpha x)^2$, which is equivalent to $x \leq \frac{2\alpha - 1}{\alpha^2} = y(2 - y)$. Now, substituting for x and dividing both sides of the inequality by y, the condition becomes $\frac{(1 - r \cdot 2q_{sl}) - r(q_n - q_{sl})y}{1 - r(q_n + q_{sl})} \leq 2 - y$, i.e., $2 \geq \frac{(1 - 2rq_{sl}) + y(1 - 2rq_n)}{1 - r(q_n + q_{sl})}$. This is always true because the right-hand-side in increasing in y and equals 2 when $y = 1$.

To show (2), we first note that $w_{n2}^* = \overline{w_{nh}}$ if and only if $m_n - 2m_{sh} + m_{sl} > 0$, or equivalently $m_n - m_{sl} > 2m_{sh} - 2m_{sl}$, which can then be rewritten as $\frac{m_{sh} - m_{sl}}{m_n - m_{sl}} < \frac{1}{2}$. That is, $x < \frac{1}{2}$. Also, using the definition of α and x, we can express $\frac{(\bar{v}_n - \bar{v}_{sm})}{2(\bar{v}_n - \bar{v}_{sl})}$ as $1 - \alpha$ and express $(\frac{m_n - m_{sm}}{m_n - m_{sl}})^2$ as $(1 - \alpha x)^2$. Hence, $(\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 \leq \frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}}$ is equivalent to $(1 - \alpha x)^2 \geq 1 - \alpha$. Therefore, to establish our claim, we only need to show that $x < \frac{1}{2} \Rightarrow (1 - \alpha x)^2 \geq 1 - \alpha$. This is obviously true because $\alpha \in (\frac{1}{2},\ 1)$ and $x \in (0, 1)$.

Appendix B.3: Proof of Proposition 1 when the unit production cost is strictly increasing and convex in quality

We need to show that for any fixed q_{sl} and q_n, there exists a unique $q_{sh}^0 \equiv q_{sh}^0(q_{sl},\ q_n) \in (q_{sl},\ q_n)$ at which $(\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 = \frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}}$ is satisfied, and that for any fixed q_{sh} and q_n, there exists a unique $q_{sl}^0 \equiv q_{sl}^0 < q_{sh}$ at which $(\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 = \frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}}$ is satisfied. We define $y \equiv \frac{q_{sh} - q_{sl}}{q_n - q_{sl}}$ as in the proof of Lemma 1. Also define $\tilde{y} \equiv 1 - y = \frac{q_n - q_{sh}}{q_n - q_{sl}}$ and $z_1 \equiv \frac{1 - 2rq_{sl}}{1 - r(q_n + q_{sl})} (\in (1, 2))$. After tedious mathematical manipulation, one can show that (i) at any $q_{sh} \in (q_{sl},\ q_n)$ that constitutes a solution to $(\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 = \frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}}$, the corresponding $y \equiv \frac{q_{sh} - q_{sl}}{q_n - q_{sl}} \in (0,\ 1)$ is the solution to

$$z_1 y - (z_1 - 1)y^2 = \sqrt{2 - y}(\sqrt{2 - y} - \sqrt{1 - y}); \tag{14}$$

and (ii) at any $q_{sl} < q_{sh}$ that constitutes a solution to $(\frac{m_n - m_{sl}}{m_n - m_{sm}})^2 = \frac{2(\bar{v}_n - \bar{v}_{sl})}{\bar{v}_n - \bar{v}_{sm}}$, the corresponding $\tilde{y} \equiv \frac{q_n - q_{sh}}{q_n - q_{sl}} \in (0,\ 1)$ is the solution to

$$1 - \tilde{y} \cdot \frac{1 - r(q_n + q_{sh})}{1 - r(2q_n - \frac{q_n - q_{sh}}{\tilde{y}})} = \sqrt{1 + \tilde{y}}(\sqrt{1 + \tilde{y}} - \sqrt{\tilde{y}}) \tag{15}$$

Then we only need to show that both (14) and (15) have a unique solution (y and $\tilde{y}$, respectively) within $(0,\ 1)$. The left hand side of (14) is increasing and concave in y and passes though $(0,\ 0)$ and $(1,\ 1)$, whereas the right hand side is increasing and convex in y

and passes through $(0,\ 2-\sqrt{2})$ and $(1,\ 1)$. Therefore, for all $z_1 \in (1,2)$, there exists a unique y_0 within $(0,1)$ that satisfies the above equation.

Both the left and right hand sides of (15) are decreasing in $\tilde{y}$ and both cross (0,1). However, at $\tilde{y}=1$, the left hand side is smaller than the right hand side. This implies that there exists a solution to (15) with $\tilde{y} \in (0,1)$ that value is the solution. It is straightforward to verify that the left-hand-side is concave and the right-hand-side is convex. Therefore the solution within (0,1) is unique. However, if the solution to (15) falls below $1-\frac{q_{sh}}{q_n}$, the q_{sl} corresponding to the solution $\tilde{y}$ is negative. In this case, q_{sl}^0 should be equal to zero. A strictly positive q_{sl}^0 exists if and only if the solution to (15) is greater than $1-\frac{q_{sh}}{q_n}$.

Appendix B.4: Proof of Proposition 2 when the unit production cost is strictly increasing and convex in quality

In this proof, we show that $q_{sh}^0(q_{sl},\ q_n)$ is increasing in q_{sl} and decreasing in r. The proof for $q_{sl}^0(q_{sl},\ q_n)$ being an increasing function of both q_{sh} and r is similar.

We first show that $q_{sh}^0(q_{sl},\ q_n)$ increases in q_{sl}. The proof consists of three steps. First, we show z_1 is decreasing in q_{sl}. Second, we show that the solution to (14), y_0, is decreasing in z_1. These two statements together imply that y_0 is increasing in q_{sl}. Then, in the third step, we show that y_0 being increasing in q_{sl} implies that q_{sh} is increasing in q_{sl}. To establish the statement in the first step, notice that $\frac{d\ z_1}{d\ q_{sl}} = -\frac{r(1-2rq_n)}{[1-r(q_n+q_{sl})]^2} < 0$. Therefore z_1 is decreasing in q_{sl}. To prove the claim in the second step, we can rewrite (14) as $z_1 = 1+\frac{1}{y}(2-\sqrt{1+\frac{1}{1-y}})$. As y increases, both $1+\frac{1}{1-y}$ and $\frac{1}{y}$ decreases, therefore z_1 decreases. For the statement in the third step, we can rewrite the expression defining y, i.e., $y=\frac{q_{sh}-q_{sl}}{q_n-q_{sl}}$, as $q_n - q_{sh} = (q_n-q_{sl})(1-y)$. As we just showed, as q_{sl} increases, y increases. Therefore, as q_{sl} increases, $(q_n-q_{sl})(1-y)$ increases (i.e., the right-hand side of the last equation increases), which implies that the left-hand side of the last equation (i.e., $q_n - q_{sh}$) increases. This proves that q_{sh} decreases as q_{sl} increases, which is the statement in the third step.

Next, we show that $q_{sh}^0(q_{sl},\ q_n)$ is decreasing in r. As r increases, z_1 increases, and therefore the solution to (14), y_0, decreases (because y_0 is decreasing in z_1, as noted earlier). This implies that q_{sh}^0 is decreasing in r.

Appendix B.5: Proof of Proposition 3 when the unit production cost is strictly increasing and convex in quality

In this proof, we derive upper and lower bounds on q_{sh}^0 and q_{sl}^0 when the unit production cost is strictly increasing and convex in quality. Expressions for q_{sh}^0 and q_{sl}^0 have been derived in Chapter 2 when the unit production cost is linear and convex in quality, it is obvious that $q_{s1} = q_{sl}^0(q_{s2})$ and $q_{s1} = q_{sl}^0(q_{s2})$ are symmetric on the $q_{s1}-q_{s2}$ plane with respect to the line $q_{s1}=q_{s2}$. In other words, $q_{s1} = q_{sl}^0(q_{s2})$ and $q_{s1} = q_{sl}^0(q_{s2})$ are inverse functions of each other. Also, notice that the lower (upper) bound provided for $q_{s1} = q_{sl}^0(q_{s2})$ is symmetric with the upper (lower) bound provided for $q_{s1} = q_{sl}^0(q_{s2})$ with respect to the line $q_{s1}=q_{s2}$ on the $q_{s1}-q_{s2}$ plane. In other words, the lower (upper) bound provided for $q_{s1} = q_{sl}^0(q_{s2})$ and the upper (lower) bound provided for $q_{s1} = q_{sl}^0(q_{s2})$ are inverse functions of each other. Therefore, we only need to show that the upper and lower bound provided for q_{sh}^0 are valid.

From the proof of Proposition 2, we know that q_{sh}^0 can be found by solving $z_1 = 1+\frac{1}{y}[2-(1+\frac{1}{1-y})^{\frac{1}{2}}]$. From the definition of z_1 in the proof of Proposition 1, we know $z_1 \in$

$(1,2)$. (We have $z_1 > 1$ because $q_{sl} < q_n$ and $z_1 < 2$ because $k < \frac{\bar{\theta}}{2q_n}$.) This implies that $\frac{1}{y}[2-(1+\frac{1}{1-y})^{\frac{1}{2}}] \in (0,1)$. Via tedious mathematical manipulation, it can be shown that this condition is equivalent to $y \in (\frac{3-\sqrt{5}}{2}, \frac{2}{3})$. Using the fact that $y = \frac{q_{sh}-q_{sl}}{q_n-q_{sl}}$ (as defined earlier), $y < \frac{2}{3}$ and $y > \frac{3-\sqrt{5}}{2}$ can be transformed to the respective upper and lower bounds on q^0_{sh} provided in the statement of Proposition 3.

Appendix C: Proof of Proposition 4

The retailer solves three subproblems and chooses the one that provides the highest profit. In this proof, we first present the three subproblems R1 needs to solve, assuming an arbitrary increasing relationship between the unit production cost and the quality level. Then, we show that when the unit production cost is linearly increasing in product quality, R1's profit function is strictly increasing in the quality of her store brand.

Under unit production costs that are general increasing functions of the product quality, R1 solves the following three subproblems.

$$\begin{aligned}
&\max_{q_{s1}} \pi^h_{r1} \equiv \frac{q_{s1}}{4q_n}(1-\frac{kq_{s1}}{\bar{\theta}})^2 \;\; \text{s.t. } q_{s1} > q^0_{sh}(q_{s2},\ q_n) \\
&\max_{q_{s1}} \pi^l_{r1} \equiv \frac{d_1}{4}(1-\frac{\frac{1}{2}(\bar{v}_n-\bar{v}_{s1}+c_n+c_{s1})-c_{s1}}{\bar{v}_n-\bar{v}_{s1}})^2+\frac{b_1}{4}(1-\frac{c_{s1}}{\bar{v}_{s1}})^2 \\
&\qquad\qquad \text{s.t. } q_{s1} < \max\left(q^0_{sl}(q_{s2},\ q_n),\ 0\right) \\
&\max_{q_{s1}} \pi^m_{r1} \equiv \frac{d_1}{4}(1-\frac{\frac{1}{2}(\bar{v}_n-\bar{v}_{sm}+c_n+c_{sm})-c_{s1}}{\bar{v}_n-\bar{v}_{s1}})^2+\frac{b_1}{4}(1-\frac{c_{s1}}{\bar{v}_{s1}})^2 \\
&\qquad\qquad \text{s.t. } \max\left(q^0_{sl}(q_{s2},\ q_n),\ 0\right) \le q_{s1} \le q^0_{sh}(q_{s2})
\end{aligned}$$

where π^h_{r1} is R1's profit if q_{s1} is set at a (high) level such that the national brand manufacturer responds with a wholesale price that is unattractive to R1, so R1 does not offer the national brand; π^l_{r1} is R1's profit if q_{s1} is in an intermediate range and the national brand manufacturer prices at w_{n1}, leading both retailers to offer the national brand product; π^m_{r1} is R1's profit if q_{s1} is set low enough that the national brand manufacturer prices at w_{n2}, which is attractive to R1 but not to R2, so only R1 sells the national brand product.

Now, suppose that $c_x = kq_x$ for $x = s1, s2, n$. Then, with some algebra, we obtain $\pi^l_{r1} = \frac{q_{s1}}{4q_n}(1-\frac{kq_{s1}}{\bar{\theta}})^2 = \frac{q_{s1}}{4q_n}(1-r)^2$, $\pi^h_{r1} = \frac{d_1}{4}(1-\frac{\frac{1}{2}(\bar{v}_n-\bar{v}_{s1}+c_n+c_{s1})-c_{s1}}{\bar{v}_n-\bar{v}_{s1}})^2+\frac{b_1}{4}(1-\frac{c_{s1}}{\bar{v}_{s1}})^2 = \frac{q_n-q_{s1}}{4q_n}(\frac{1-r}{2})^2+\frac{q_{s1}}{4}(1-r)^2 = \frac{(1-r)^2(q_n+3q_{s1})}{16q_n}$ and $\pi^m_{r1} = \frac{d_1}{4}(1-\frac{\frac{1}{2}(\bar{v}_n-\bar{v}_{sm}+c_n+c_{sm})-c_{s1}}{\bar{v}_n-\bar{v}_{s1}})^2+\frac{b_1}{4}(1-\frac{c_{s1}}{\bar{v}_{s1}})^2 = \frac{q_n-q_{s1}}{4q_n}(\frac{1-r}{2}-\frac{1}{2}\frac{q_{s1}-q_{s2}}{q_n-q_{s1}+q_n-q_{s2}}(1-r))^2+\frac{q_{s1}}{4}(1-r)^2$. Obviously the optimal solutions to both π^l_{r1} and π^h_{r1} is q_n because these profit functions are strictly increasing in q_{si}. Now π^m_{r1} can be written as $\pi^m_{r1} = \frac{(1-r)^2}{16q_n}[4q_n-2(q_n-q_{s1})-\frac{2(q_n-q_{s2})}{1+\frac{q_n-q_{s2}}{q_n-q_{s1}}}]^2$, which is also increasing in q_{s1}. So we have established that when the unit production cost is linearly increasing in the product quality, R1 should set q_{s1} to the maximum possible quality level.

Appendix D: Proof of Proposition 5

We sketch a proof below. First, the retailer's profit when the quality level in the high, medium and low ranges can be expressed as:

$$
\begin{aligned}
q_{s1} \in (q_{sh}^0,\ q_n] &: \pi_{r1}^h(q_{s1}) \equiv \frac{q_{s1}}{4q_n}(1-\frac{c_{s1}}{\bar{v}_{s1}})^2 \\
q_{s1} \in [q_{sl}^0,\ q_{sh}^0] &: \pi_{r1}^m(q_{s1}) \equiv \frac{q_n - q_{s1}}{4q_n}(1-\frac{\frac{1}{2}(\bar{v}_n-\bar{v}_{sm}+c_n+c_{sm})-c_{s1}}{\bar{v}_n-\bar{v}_{s1}})^2 + \frac{q_{s1}}{4q_n}(1-\frac{c_{s1}}{\bar{v}_{s1}})^2 \\
q_{s1} \in [0,\ q_{sl}^0) &: \pi_{r1}^l(q_{s1}) \equiv \frac{q_n - q_{s1}}{4q_n}(1-\frac{\frac{1}{2}(\bar{v}_n-\bar{v}_{s1}+c_n+c_{s1})-c_{s1}}{\bar{v}_n-\bar{v}_{s1}})^2 + \frac{q_{s1}}{4q_n}(1-\frac{c_{s1}}{\bar{v}_{s1}})^2
\end{aligned}
\tag{16}
$$

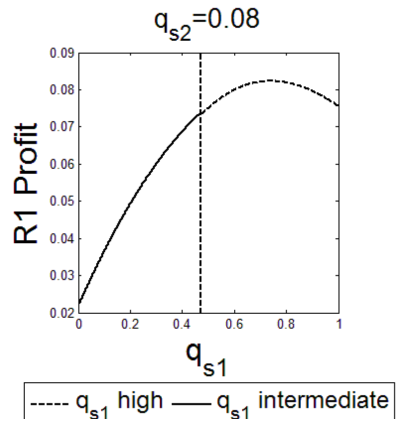
We first prove that whenever q_{s1}^m is an interior point in $(q_{sl}^0,\ q_{sh}^0)$, q_{s1}^m is, indeed, the global optimum of the retailer's quality-positioning problem. (Recall that q_{s1}^m denotes the *unconstrained* optimal solution for maximizing $\pi_{r1}^m(q_{s1})$.) Then we show that q_{s1}^m falls into $(q_{sl}^0,\ q_{sh}^0)$ if and only if q_{s2} falls into the intermediate range (i.e., $q_{s2} \in (\underline{q_{s2}}(r),\ \overline{q_{s2}}(r))$ as in the statement of Proposition 5). Finally, we establish that the optimal value of q_{s1} constrained to $[q_{sl}^0,\ q_{sh}^0]$ is q_{sh}^0 if $q_{s2} \in [0,\ \underline{q_{s2}}(r)]$, and is q_{sl}^0 if $q_{s2} \in [\overline{q_{s2}}(r),\ q_n)$.

To see why q_{s1}^m being an interior point on $(q_{sl}^0,\ q_{sh}^0)$ implies that q_{s1}^m is the global optimum for R1's quality-positioning problem, observe from (16) that (a) $\pi_{r1}^m(q_{s1}) > \pi_{r1}^h(q_{s1})$ for all q_{s1}, and (b) $\pi_{r1}^m(q_{s1}) > \pi_{r1}^l(q_{s1})$ whenever $q_{s1} < q_{s2}$. Because $q_{sl}^0 < q_{s2}$, (b) implies that $\pi_{r1}^m(q_{s1}) > \pi_{r1}^l(q_{s1})$ whenever $\pi_{r1}^l(q_{s1})$ is relevant. Therefore, we know that (i) $\pi_{r1}^m(q_{s1}) > \pi_{r1}^l(q_{s1})$ whenever $\pi_{r1}^l(q_{s1})$ is relevant; (ii) $\pi_{r1}^m(q_{s1}) > \pi_{r1}^h(q_{s1})$ whenever $\pi_{r1}^h(q_{s1})$ is relevant and (iii) $\pi_{r1}^m(q_{s1}) = \pi_{r1}^m(q_{s1})$ whenever $\pi_{r1}^m(q_{s1})$ is relevant. The combination of (i), (ii) and (iii) implies that, whenever the *unconstrained* solution for maximizing $\pi_{r1}^m(q_{s1})$ falls in $[q_{sl}^0,\ q_{sh}^0]$, it *is* the global optimum of R1's quality-positioning problem.

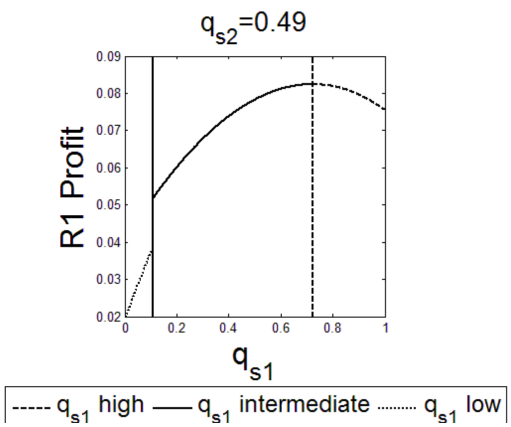
We next show that $q_{s1}^m \in (q_{sl}^0,\ q_{sh}^0)$ occurs if and only if q_{s2} is in the intermediate range (i.e., $q_{s2} \in (\underline{q_{s2}}(r),\ \overline{q_{s2}}(r))$). It is easy to verify that when $r > \frac{1}{3q_n}$, q_{s1}^m is strictly greater than q_{sh}^0 at $q_{s2} = 0$, and it is strictly smaller than q_{sl}^0 when $q_{s2} = q_n$. So we only need to show that both $q_{s1}^m - q_{sl}^0$ and $q_{s1}^m - q_{sh}^0$ are monotonic in q_{s2}. But this is true because q_{sh}^0 and q_{sl}^0 increase with q_{s2} (as shown in Proposition 2) and q_{s1}^m decreases with q_{s2} (as will be shown in part (iv) of Proposition 6).

On the other hand, if $q_{s2} \in [0,\ \underline{q_{s2}}(r)]$, the optimal q_{s1} constrained to $[q_{sl}^0,\ q_{sh}^0]$ is q_{sh}^0, therefore the global optimum, q_{s1}^*, must take a value within $[q_{sh}^0,\ q_n]$. Similarly, if $q_{s2} \in [\overline{q_{s2}}(r),\ q_n)$, the optimal q_{s1} constrained to $[q_{sl}^0,\ q_{sh}^0]$ is q_{sl}^0. Therefore the global optimal q_{s1}^* must take a value in $(0,\ q_{sl}^0]$. This complete the proof of Proposition 5. $\square$

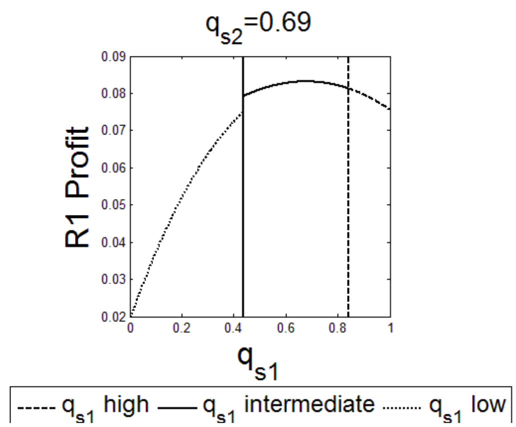
To help the reader visualize the statement and the proof of Proposition 5, we present a numerical example in Figure 12. In this example, $\bar{\theta} = 1$, $k = 0.35$ and $q_n = 1$. We show results for five different values of q_{s2} in Figures 12(A) to (E). The dashed vertical line represents $q_{s1} = q_{sh}^0$ and the solid vertical line represents $q_{s1} = q_{sl}^0$. As can be seen from the figures, when the quality level of the other store brand is low, the global optimal quality level of the store brand at R1 corresponds to the optimizer of π_{r1}^h (as in Figure 12(A)). When the quality level of the other store brand is high, R1's global optimum is the optimizer of π_{r1}^l (as in Figure 12(E)). When the quality level of the other store brand is intermediate, q_{s1}^* is the optimizer of π_{r1}^m (as in Figure 12(C)). In Figures 12(B) and (d), q_{s1}^* should be set at the boundary between Regions I and II, and between Regions II and II, respectively. That is, in Figure 12(B), $q_{s1}^* = q_{sh}^0$ and in Figure 12 (D), $q_{s1}^* = q_{sl}^0$.



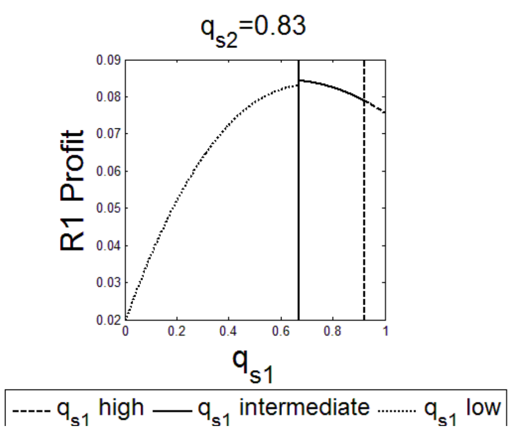
(A) $q_{s1}^{*} = q_{s1}^{h}$



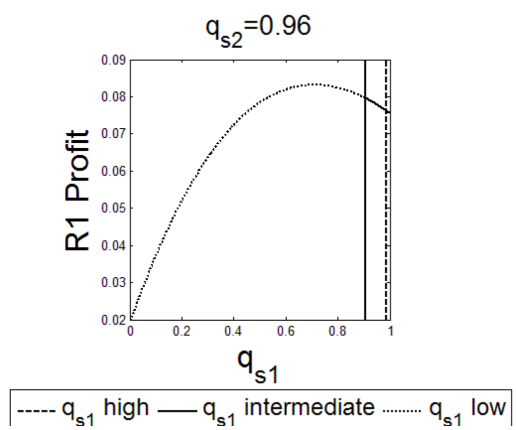
(B) $q_{s1}^{*} = q_{sh}^{0}$



(C) $q_{s1}^{*} = q_{s1}^{m}$



(D) $q_{s1}^{*} = q_{sl}^{0}$



(E) $q_{s1}^{*} = q_{s1}^{l}$

FIGURE 12. Retailer's profit as a function of q_{s1} for different values of q_{s2}

Appendix E: Proof of Proposition 6

First, (i) can be derived by finding the solutions to the subproblems $\max\limits_{q_{s1}} \pi_{r1}^{l}$ and $\max\limits_{q_{s2}} \pi_{r1}^{h}$ and confirming that π_{r1}^{l} and π_{r1}^{h} are both concave in q_{s1}.

We now prove (iv) because the result is also required in the proof of (ii). Notice that at q_{s1}^m, the sign of $\frac{dq_{s1}}{dq_{s2}}$ is the same as the sign of $\frac{\partial^2\tilde{\pi}_m}{\partial q_{s1}\partial q_{s2}}$ because $\frac{dq_{s1}}{dq_{s2}} = -\frac{\frac{\partial}{\partial q_{s2}}(\frac{\partial\tilde{\pi}_m}{\partial q_{s1}})}{\frac{\partial}{\partial q_{s1}}(\frac{\partial\tilde{\pi}_m}{\partial q_{s1}})} = -\frac{\frac{\partial^2\tilde{\pi}_m}{\partial q_{s1}\partial q_{s2}}}{\frac{\partial^2\tilde{\pi}_m}{\partial q_{s1}^2}}$. Therefore, we only need to show that at the solution to $\frac{\partial\tilde{\pi}_m(q_{s1},\ q_{s2})}{\partial q_{s1}} = 0$, $\frac{\partial^2\tilde{\pi}_m}{\partial q_{s1}\partial q_{s2}}$ is negative. To facilitate the proof, we define $f \equiv (1-r(q_n+q_{s1})) - \frac{q_{s1}-q_{s2}}{(q_n-q_{s1})+(q_n-q_{s2})}(1-r(q_{s1}+q_{s2}))$ and $g \equiv (q_n-q_{s1})+(q_n-q_{s2})$, then after mathematical manipulations, we get

$$\frac{\partial^2\pi_m}{\partial q_{s1}\partial q_{s2}} = 2q_n\left\{\frac{\partial f}{\partial q_{s2}}\cdot(q_n-q_{s1})\cdot\frac{\partial f}{\partial q_{s1}} + f\left[(q_n-q_{s1})\frac{\partial^2 f}{\partial q_{s1}\partial q_{s2}} - \frac{\partial f}{\partial q_{s2}}\right]\right\}$$

If $q_{s1}^m > q_{s2}$, $\frac{\partial^2 f}{\partial q_{s1}\partial q_{s2}} < 0$, therefore we immediately get $\frac{\partial^2\tilde{\pi}_m}{\partial q_{s1}\partial q_{s2}} < 0$. If $q_{s1}^m < q_{s2}$, because $(q_n-q_{s1})\frac{\partial^2 f}{\partial q_{s1}\partial q_{s2}} = \frac{-2(q_{s1}-q_{s2})(1-2rq_n)}{g^2\cdot(1+\frac{q_n-q_{s2}}{q_n-q_{s1}})} < -\frac{2}{g^2}(q_{s1}-q_{s2})(1-2rq_n)$ (when $q_{s1}^m < q_{s2}$) , we have $(q_n-q_{s1})\frac{\partial^2 f}{\partial q_{s1}\partial q_{s2}} - \frac{\partial f}{\partial q_{s2}} < \frac{1}{g^2}[2(q_{s2}-q_{s1})(1-2rq_n) - 2(q_n-q_{s1})(1-2rq_{s2}) - r(q_{s1}-q_{s2})^2] = \frac{1}{g^2}[-2(q_n-q_{s2})(1-2rq_{s1}) - r(q_{s1}-q_{s2})^2] < 0$; therefore, we still get $\frac{\partial^2\tilde{\pi}_m}{\partial q_{s1}\partial q_{s2}} < 0$.

To show (ii), we only need to establish that $q_{s1}^m < q_{s1}^h$when $q_{s2} = 0$ because q_{s1}^m is decreasing in q_{s2} and q_{s1}^h does not change with q_{s2}. To demonstrate this, we only need to show that $\frac{\partial\tilde{\pi}_{r1}^m}{\partial q_{s1}} < 0$ at $q_{s1}^h = \frac{1}{3r}$. At $q_{s1}^h = \frac{1}{3r}$, $\frac{\partial\tilde{\pi}_{r1}^m}{\partial q_{s1}} = \frac{q_n-q_{s1}}{2}f\frac{\partial f}{\partial q_{s1}} - \frac{1}{4}f^2 + (1-rq_{s1})(1-3rq_{s1}) = \frac{q_n-q_{s1}}{2}f\frac{\partial f}{\partial q_{s1}} - \frac{1}{4}f^2$. Therefore we only need to confirm that $\frac{\partial f}{\partial q_{s1}} < 0$. But when $q_{s2} = 0$, we have $\frac{\partial f}{\partial q_{s1}} = -r - \frac{(1-2rq_{s1})(2q_n-q_{s1})+q_{s1}(1-rq_{s1})}{(2q_n-q_{s1})^2} < 0$. This shows that $q_{s1}^m < q_{s1}^h$ for all q_{s2}.

We show (iii) by proving that at q_l (i.e., at the unconstrained solution to $\max\limits_{q_{s1}} \pi_{r1}^l$), $\partial\pi_{r1}^m/\partial q_{s1} < 0$. To facilitate the analysis, we further define $\tilde{f} = 1 - r(q_n+q_{s1})$. Then

$$\pi_{r1}^l = 4q_n\left[\frac{q_n-q_{s1}}{4}\tilde{f}^2 + q_{s1}(1-rq_{s1})^2\right]$$
$$\pi_{r1}^m = 4q_n\left[\frac{q_n-q_{s1}}{4}f^2 + q_{s1}(1-rq_{s1})^2\right]$$

and

$$\frac{\partial\pi_{r1}^l}{\partial q_{s1}} = 4q_n\left[\frac{q_n-q_{s1}}{2}\tilde{f}\frac{\partial\tilde{f}}{\partial q_{s1}} - \frac{1}{4}\tilde{f}^2 + (1-rq_{s1})(1-3rq_{s1})\right]$$
$$\frac{\partial\pi_{r1}^m}{\partial q_{s1}} = 4q_n\left[\frac{q_n-q_{s1}}{2}f\frac{\partial f}{\partial q_{s1}} - \frac{1}{4}f^2 + (1-rq_{s1})(1-3rq_{s1})\right]$$

It is straightforward to confirm that $\frac{\partial\tilde{f}}{\partial q_{s1}} - \frac{\partial f}{\partial q_{s1}} = \frac{1}{g^2}\left[r\cdot g^2 + 2(q_n-q_{s2})(1-2rq_{s1})\right] > 0$. Therefore, if $q_{s1} < q_{s2}$, we have $f > \tilde{f}$ and $f\frac{\partial f}{\partial q_{s1}} < \tilde{f}\frac{\partial\tilde{f}}{\partial q_{s1}} < 0$, which then gives us $\frac{\partial\pi_{r1}^m}{\partial q_{s1}} < \frac{\partial\pi_{r1}^l}{\partial q_{s1}}$. This implies that at q_{s1}^l, $\frac{\partial\pi_{r1}^m}{\partial q_{s1}} < 0$, which then implies $q_{s1}^m < q_{s1}^l$.

Next, we show (v). This is obvious for q_{s1}^l because $q_{s1}^l = \frac{1}{3r}$. Define $\alpha \equiv \frac{1}{r}$. Then for q_{s1}^h, to determine the sign of $\frac{dq_{s1}^l}{d\alpha}$, we only need to determine the sign of $\frac{\partial^2\pi_{r1}^l}{\partial q_{s1}\partial\alpha}$ at q_{s1}^l because $\frac{dq_{s1}}{d\alpha} = -\frac{\partial^2\pi_{r1}^l}{\partial q_{s1}\partial\alpha}/\frac{\partial^2\pi_{r1}^l}{\partial q_{s1}\partial\alpha}$ at $q_{s1} = q_{s1}^l$. After mathematical manipulation, we get $\frac{\partial^2\pi_{r1}^l}{\partial q_{s1}\partial\alpha} = -\frac{8q_n}{\alpha^3}[\frac{3}{4}(\alpha - q_{s1})(\alpha-3q_{s1}) - 2q_nq_{s1} + q_n^2] + \frac{4q_n}{\alpha^2}[\frac{3}{4}((\alpha-q_{s1})+(\alpha-3q_{s1}))] = \frac{4q_n}{\alpha^2}[\frac{3}{4}((\alpha-q_{s1})+(\alpha-3q_{s1}))]$

at q_{s1}^l. Because $q_{s1}^l < \frac{1}{3r} = \frac{\alpha}{3}$, we have $\frac{\partial^2 \tilde{\pi}_{r1}^l}{\partial q_{s1} \partial \alpha} > 0$ at q_{s1}^l. Therefore q_{s1}^l increases in α, which means it decreases in r.

To prove (vi), we note that $\frac{\partial q_{s1}^m}{\partial\ r}$ has the same sign as $\frac{\partial^2 \pi_{r1}^m}{\partial q_{s1} \partial r}$. We have

$$\begin{aligned}
\frac{\partial^2 \pi_{r1}^m}{\partial q_{s1} \partial r} &= -\frac{8q_n}{g^2}\left[2q_n(q_n - q_{s1}) - (q_{s1} + q_{s2}) \cdot g\right] \\
&= -\frac{8q_n}{g^2}\left[(q_n - q_{s1})\left((q_n - q_{s1}) + 2(q_n - q_{s2})\right) - (q_n^2 - q_{s2}^2)\right]
\end{aligned}$$

which is negative if and only if $(q_n - q_{s1})\left((q_n - q_{s1}) + 2(q_n - q_{s2})\right) - (q_n^2 - q_{s2}^2) > 0$. Now $q_n^2 - q_{s2}^2$ is positive and $q_{s1} \leq q_n$ so it is sufficient to show that at $q_n - q_{s1} = 0$, $(q_n - q_{s1})((q_n - q_{s1}) + 2(q_n - q_{s2})) - (q_n^2 - q_{s2}^2) < 0$ and when $q_n - q_{s1} = q_n$, $(q_n - q_{s1})\left((q_n - q_{s1}) + 2(q_n - q_{s2})\right) - (q_n^2 - q_{s2}^2) > 0$, both of which are easy to confirm. Therefore $\frac{\partial^2 \pi_{r1}^m}{\partial q_{s1} \partial r}$ must change signs for at some value of q_{s1} between zero and q_n. This implies that, as q_{s1} increases from zero to q_n, q_{s1}^m first decreases in r and then later increases in r.

CHAPTER 4

Store Brand Positioning under Various Sourcing Structures

1. Introduction

Managing store brands is very important for retailers. In 2012, store brand sales in U.S. supermarkets alone totaled \$59 billion, with a store brand unit share of 23.1% and dollar share of 19.1% (PLMA, 2013b). Retailers have spent heavily to develop their own brands in recent years, building test kitchens, hiring culinary experts, improving packaging and testing and retesting their products with consumers (Strom, 2013). New lines of store-brand products are introduced every year. For example, in 2010, Kroger more than doubled its offerings of store-brand cosmetics and shampoos (Sewell, 2010).

Before introducing a store-brand product, the retailer needs to determine its quality level. As an example, Kroger has a "three-tier" store brand positioning strategy. From the economy tier (i.e., the Value brand), to mid-tier store brands (such as Kroger, Ralphs and King Soopers), to the Private Selection brand introduced in 2000, the quality level increases. Not only does Kroger need to decide the tier of a new store-brand product, but even for a given tier, it needs to decide the quality level at a more detailed level, which would then determine the nature and quality of the ingredients, manufacturing processes, etc.

Some store brands are produced in-house in a manufacturing facility owned or controlled by the retailer; we call this arrangement "in-house" or IH. Some are sourced from large national brand manufacturers who also produce and offer competing products ("national (brand) manufacturer" or NM for short). As one example, in the UK, the large supermarket chain ASDA, has been having Coca-Cola produce its store-brand cola (Bergès-Sennou, 2006). Some are sourced from third-party manufacturers that have market power, either because of their overall size or because of their strength in a particular product category, and can therefore price strategically (called "strategic (third-party) manufacturer" or SM for short). One example of a specialized store-brand manufacturer is Overhill Farms, a company known for making frozen foods (Choi, 2013). We note that relatively weak third-party manufacturers that are not strategic players in their interactions with retailers can be represented in the same way as as IH. We elaborate on this point in more detail later.

Choosing the quality level of a new store-brand product is an important strategic decision that the retailer is unlikely to change quickly. All of the existing literature is based on the assumption that the product is obtainable at an exogenously-specified constant marginal cost. This representation is adequate if the retailer produces the product in-house or procures it from a weak third-party producer who offers a fixed wholesale price (e.g., cost-plus pricing) that does not depend upon any competitive factors. Given the variety of possible sourcing arrangements and the consequent variations in pricing power among the parties, one would think that the retailer's optimal choice of store brand quality should depend upon these factors. This is the focus of our study, and as our analysis proceeds, we show that the retailer's optimal decision differs markedly depending upon both the sourcing arrangement and pricing power situation.

More specifically, we study the retailer's equilibrium quality-positioning strategy under the three sourcing structures mentioned above, and for each sourcing structure, we consider the three types of channel price leadership most commonly seen in the literature: Manufacturer-Stackelberg (MS), Retailer-Stackelberg (RS), and Vertical Nash (VN). Thus, we will examine nine (i.e., three times three) combinations of sourcing and pricing power (or "game") scenarios, and compare the retailer's optimal quality positioning decision and other equilibrium results (including prices) across the nine scenarios. In all nine combinations, the retailer moves first in setting the quality of her store-brand (during the product development phase) before any pricing decisions are made. We derive subgame perfect equilibria for all scenarios.

Typically, when a retailer is introducing a new store brand product, she has a choice among sources, even if she does not hold much pricing power. We build upon our analysis of the nine combinations of sourcing and game structure to show how the availability of other sources as outside options changes the equilibrium.

Our paper makes several contributions to the literature on store brand strategies. First, we take a first step in studying the interaction between store-brand sourcing and positioning decisions, and the interplay of these decisions with the retailer's pricing power. From a comparison of the retailer's equilibrium store brand quality levels for the nine combinations of sourcing and game structure, we obtain a full characterization of the ordering of store-brand quality level, retailer's profit, retail prices and consumer welfare across the nine combinations. To the best of our knowledge, we are the first to present a comparison of equilibria for these nine realistic combinations of sourcing and pricing power in the store brand context. Second, we show that sourcing of store brands plays a key role in the interactions between a retailer and a national brand manufacturer. Whereas the marketing and economics literatures have emphasized the role of store brands in helping retailers elicit price concessions from national brand manufacturers, we find that having a preferable sourcing arrangement for a store brand product is more valuable than having pricing power.

Our results also show that having a store brand product provides a retailer no additional leverage in dealing with the national band manufacturer if the retailer sells the national brand product and sources the store brand from the national brand manufacturer. This is in contrast to the vast majority of the literature on store-brand introduction, which concludes that a retailer can use her store brand as a bargaining chip and thereby elicit price concession from the national brand manufacturer. Finally, for two common scenarios, we show how the equilibrium—both the retailer's optimal store brand quality level and his profit— are affected by the retailer's outside option of an alternate sourcing arrangement.

The remainder of this paper is organized as follows. In Section 2, we review the literature that is pertinent to our research. In Section 3, derive and compare the equilibria for the nine combinations of sourcing and game structures mentioned above. In Section 4, for two common scenarios, we investigate how the equilibrium is affected by the retailer's outside option vis-a-vis sourcing. In Section 5, we discuss the relationship between the results in our paper and those on outsourcing in similar contexts. We also discuss how supply chain profit differs across the nine scenario. Section 6 concludes the paper.

2. Related Literature

In this literature review, we first briefly discuss papers that investigate reasons why and conditions under which retailers introduce store brands. Then we review papers studying store-brand positioning. We then discuss the literature on issues surrounding store brand sourcing. Finally, we discuss the only model of which we are aware that investigates channel competition under different price leadership structures when store brands and national brands compete.

We concentrate on analytical papers, but also review a few empirical papers whose results are pertinent to the development of our models. For more extensive reviews on store brand strategies, we refer readers to the book by Kumar and Steenkamp (2007), the review by Sethuraman (2009) and the literature review in Chapter 2.

A major stream within the analytical literature on store brands investigates why a monopolist retailer would introduce store brands. In this stream, the most well-accepted result is that introducing a store brand (or having the option of introducing a store brand) helps a retailer elicit price concessions from upstream national brand manufacturers. In this stream, the models either do not incorporate store brand quality explicitly, or simply assume the store brand quality level to be exogenous. Also, the models are based on the assumption of a constant marginal production cost for the store brand which is unaffected by the quality. More specifically, Mills (1995, 1999) under the same assumptions about demand as in our model, finds that the wholesale price set by the national brand manufacturer is non-increasing in the store brand quality We find that Mills' result depends heavily on his assumption that the store brand in his model is not supplied by the national brand manufacturer. In our model, when the retailer sources the store brand from the national brand manufacturer, the national brand manufacturer does not change his wholesale price for the national brand as store brand quality increases.

Narasimhan and Wilcox (1998) and Gabrielsen and Sørgard (2007) segment customers into those who only consider purchasing the national brand and those who consider both brands ("switchers"). Narasimhan and Wilcox find that if the number of switchers is large, the national brand manufacturer will reduce his wholesale price upon introduction of a store brand. Gabrielsen and Sorgard allow the national brand manufacturer to condition his wholesale price on whether the store brand is introduced. They find that, if the number of switchers is large, the national brand manufacturer will reduce its wholesale price in order to prevent the retailer from introducing the store brand. We find that if the retailer carries the national brand product and outsources production of its store brand product to the national brand manufacturer, the retailer is unable to elicit any price reductions, irrespective of the quality of the store brand product.

Other papers studying reasons why retailers introduce store brands have different foci (and are less relevant to our research). Some focus on the role store brands play in coordinating a supply chain consisting of a national brand manufacturer and a retailer (Kurata et al. 2007, Colangelo 2008, Chen et al. 2011); some focus on retailers' store-brand assortment decisions, including the option not to offer the national brand product (Fang et al. 2012, Moner-Colonques et al. 2011, and Chapter 2 of this dissertation); some focus on the impact of retail competition on the retailer's store brand introduction decisions (Lal and Narasimhan 1996, Corstjens and Lal 2000, Geylani et al. 2009, Groznik and Heese 2010).

Analytical papers on store-brand positioning when the store brand competes against one or more national brands can be divided into two groups. Articles in the first (and larger) group generally offer the conclusion that store brands should seek minimal quality differentiation from the national brand(s), while articles in the other group offer different conclusions. We now discuss papers in the first group, followed by papers that falls into the second. All of the papers discussed here are based on the assumption that the retailer and the national brand manufacturer(s) engage in a manufacturer-Stackelberg (MS) game after the retailer makes the store-brand quality decision.

We first discuss papers whose results support minimal differentiation from the national brand product. Sayman et al. (2002), Scott-Morton and Zettelmeyer (2004), Choi and Coughlan (2006) and Fousekis (2010) conclude that store brands should seek minimal quality differentiation from the national brand. Fousekis assumes the unit production cost of store brand to be linearly increasing in the product quality, whereas all other authors assume the unit production cost to be zero or constant (and therefore not affected by the quality level). Sayman, Scott-Morton and Zettelmeyer, and Choi and Coughlan all study a retailer's problem of positioning a store brand against two national brands. Sayman assumes linear demand functions and poses the retailer's quality positioning problem as one of choosing two parameters corresponding to the price sensitivity between the store brand and each of the national brands, respectively. Scott-Morton and Zettelmeyer assume the two existing national brands are targeted at distinct customer segments. The retailer needs to eliminate one national brand if she decides to introduce the store brand. Both Sayman and Scott-Morton and Zettelmeyer show that the retailer has an incentive to position its store brand as close to the leading national brand as possible because doing so increases her bargaining power versus the national brand. Choi and Coughlan study a retailer's problem of positioning a store brand against two national brands in both the quality and the feature dimensions. They assume a linear demand function, parameterized by the quality levels of the three products and the degree of substitutability between each pair of them. The authors find that the retailer should maximize the quality level of its store brand. They also report results on store brand positioning in the feature dimension. Fousekis (2010) studies a game in which a national brand manufacturer and a retailer simultaneously choose the quality levels of the national and the store brand products, respectively, before a manufacturer-Stackelberg pricing game takes place. They find that the national brand manufacturer chooses the highest quality level for the national brand irrespective of the choice of quality level made by the retailer. The retailer also chooses the highest possible quality level for all levels of the national brand quality (but always smaller than the national brand quality, as assumed in their model). This implies that both the retailer and the national brand manufacturer choose the highest quality level for their products, implying that the retailer seeks minimal quality differentiation from the national brand and the national brand manufacturer seeks maximal quality differentiation from the store brand.

We now discuss the second group of articles on store-brand positioning. Both Du et al. (2005) and Sethuraman (2002) assume the marginal cost of production to be constant and independent of the quality level of the product. Du et al. (2005) study an extension of Scott Morton and Scott-Morton and Zettelmeyer (2004) model with a more general representation of customer heterogeneity. They describe scenarios in which the retailer should position the store brand close to the weaker national brand or in between the two national brands. Sethuraman (2002) studies a game between a national brand manufacturer and one retailer in which the retailer first decides the quality of the store brand, then the national brand manufacturer sets an investment level for advertising (which will increase customers' reservation prices for products in the category). Following this, the parties engage in a manufacturer-Stackelberg pricing game. They find that positioning a store brand close to the national brand may not be profitable if the manufacturer can significantly increase category demand via advertising, or if a significant portion of the market consists of consumers with low reservation prices who are unwilling to pay for the national brand. Bontems et al. (1999) also study the store brand positioning problem by modeling a manufacturer-Stackelberg game between a national brand manufacturer and a retailer. The authors assume the retailer incurs a unit cost which is convex and increasing in the quality of the store brand, and identify two types of effects of increasing store brand quality: the retailer is able to get a lower wholesale price from the national brand manufacturer but the store brand is less competitive due to its increased marginal cost. They conclude that it is not always optimal for a retailer to position its store brand close to the leading national brand. The authors do not report any analytical results on optimal store brand quality.

The academic literature on store-brand sourcing choices by retailers is sparse. The only paper of which we are aware on this topic is by Bergès-Sennou (2006). In this paper, two retailers carry a national-brand product produced by a national brand manufacturer. Only one of the retailers has the option of carrying a store brand, whose quality level is fixed. This retailer needs to choose between producing the store brand in-house and outsourcing production to the national brand manufacturer, which can produce the store brand at a lower unit production cost. There are four customer segments, and customers within a segment are homogenous. The first segment only considers buying the national brand at the retailer without a store brand. The second segment only considers buying the store brand from the retailer that carries a store brand. The third segment buys either the store brand or the national brand at the retailer with a store brand, whichever provides them a higher surplus, and the fourth segment considers buying all three products, choosing the product that provides them the highest surplus. The retailer with the option to offer a store brand first decides the store-brand sourcing. Then, she and the national brand manufacturer negotiate over the wholesale price for the national brand (and the store brand, if applicable), and a franchise fee, assuming an axiomatic Nash bargaining framework. If the parties fail to reach an agreement after negotiating, the retailer will not sell any product(s) produced by the national brand manufacturer. Not surprisingly, the authors find that the retailer will entrust store brand production to the national brand manufacturer if the bargaining power of the national brand manufacturer is below a threshold.

Finally, we briefly review models investigating channel competition under different price leadership structures, with an emphasis on models that include store-brand products. There are many analytical models of traditional vertical interactions between manufacturers and retailers (e.g., McGuire and Staelin 1983, Shugan 1985, Moorthy and Fader 2012, Choi 1991, Lee and Staelin 1997). There is substantial empirical research on the nature of competition between store and national brands in the presence of store brands, which suggests that the type of interaction and the degree of competition between national and store brands is idiosyncratic across categories. See Putsis and Dhar (1998), Cotterill and Putsis (2000), Sayman et al. (2002), Meza and Sudhir (2010), Dhar and Ray (2004) and the references therein.

We are aware of only one paper that specifically studies the effect of different price leadership structures in the context of store-national brand competition. Choi and Fredj (2013) study a game between a national brand manufacturer and two retailers. Each retailer offers the national brand and a store brand of their own. The store brands are assumed to have a fixed quality level, and each retailer incurs zero marginal cost for their own store brand. The national brand manufacturer is also assumed to incur zero marginal cost for the national brand. They assume linear demand functions, in which there are two parameters capturing the price substitutability between (1) the national brand at the two stores, and (2) the national brand and the store brand at each store. The authors compare the equilibrium outcomes among four types of games with different price leadership structures: Manufacturer-Stackelberg, Retailer-Stackelberg, Vertical Nash and Retailer Double Stackelberg. The authors show that each party earns more profit by having price leadership. Via sensitivity analysis on the substitubility factors in the demand functions, they show that each retailer should seek minimum differentiation between their own store brand and the national brand (in order to achieve a high demand level).

3. Quality Positioning under Various Sourcing Structures

We study games between (among) a retailer, a national brand manufacturer and a strategic third-party manufacturer, where applicable. The retailer carries a product (with a fixed, high quality) offered by the national brand manufacturer, and is considering introducing a store brand whose quality can be decided. The retailer also chooses retail prices. We derive and compare the retailer's equilibrium store-brand quality levels under three sourcing arrangements and three pricing power scenarios. As discussed in the Introduction, the three sources are in-house (IH), a leading national brand manufacturer (NM) whose product the retailer also carries, and a strategic third-party manufacturer (SM). The three power scenarios are the ones most commonly seen in the literature: Manufacturer-Stackelberg (MS), Retailer-Stackelberg (RS), and Vertical Nash (VN).

Under MS with store-brand production in-house, NM first sets the national-brand wholesale price, and then the retailer sets retail prices for both the national and the store brands. At this step, if the retailer decides to not introduce the store brand, she sets the retail price for the store brand high enough to drive demand to zero. In this way, we can implicitly model the store-brand introduction decision via the retailer's pricing decision. Under MS with store-brand production by NM, NM sets the wholesale prices for both the store and national brands, and the retailer then sets retail prices. Under MS with store-brand production by SM, NM and SM engage in a Nash game to set wholesale prices for the national and store brands, respectively, and the retailer then sets retail prices. Under RS, the retailer sets her margins on the two products before the manufacturer(s) set their wholesale prices. Under VN, the retailer and the manufacturer(s) engage in a Nash game in which the retailer sets her margins while the manufacturer(s) set the wholesale prices. In sum, we examine nine (i.e., 3 x 3) combinations of sourcing and pricing power (or "game") scenarios, and compare the retailer's optimal quality level and other equilibrium results (including prices) across the nine scenarios. In all nine scenarios, the retailer moves first in setting the quality of her store-brand (during the product development phase) before any pricing decisions are made, and, after all pricing decisions are made, customers make purchases. We seek subgame perfect equilibria.

Customers are heterogeneous in their willingness-to-pay (WTP) per unit of quality, which is denoted by the parameter θ. We assume θ has a uniform distribution between 0 and $\overline{\theta}$. The utility a customer derives from either product equals her WTP per unit of quality times the product quality. Each customer chooses the product that gives her the greatest surplus (utility less price), provided that it is non-negative. We denote the quality level of the national brand by q_n, the quality level of the store brand by q_s, the retail price of the national brand by p_n and the retail price of the store brand by p_s. Thus, a customer with WTP for quality θ derives surplus $\theta q_n - p_n$ from purchasing the national brand or $\theta q_s - p_s$ from purchasing the store brand. She purchases the national brand if and only if $\theta q_n - p_n \geq \max\{\theta q_s - p_s,\ 0\}$ and purchases the store brand if and only if $\theta q_s - p_s \geq \max\{\theta q_n - p_n, 0\}$. Therefore, given the retail prices and the quality levels of the two products, customer demand for the store brand is $D_s(p_n, p_s) = \max\{\frac{1}{\bar{\theta}}[\frac{p_n - p_s}{q_n - q_s} - \frac{p_s}{q_s}],\ 0\}$ and customer demand for the national brand is $D_n(p_n, p_s) = \max\{\frac{1}{\bar{\theta}}[\bar{\theta} - \frac{p_n - p_s}{q_n - q_s}],\ 0\}$. We assume the quality level of the store brand does not exceed q_n. This assumption applies to the majority of the store brand products except for those which are called "premium store brands." In our model, this assumption is captured implicitly: the utility provided to consumers less the unit production cost of the store brand is decreasing in the product quality for $q_s > q_n$, so the retailer has no incentive to choose q_s above this threshold.

We assume that each product has a unit production cost which is strictly convex and increasing in the quality level. This cost relationship is applicable in settings where increasingly higher quality require increasingly rarer input materials, skill or processes to produce the product. Many consumer-packaged goods fall into this category. We also assume the national and store brands share the same production cost function, and that the cost function is identical across the nine scenarios studied. In reality, the national brand and third-party manufacturers can sometimes produce the store brand at a lower unit production cost due to economies of scale or skill specialization. However, some major retailers have also invested heavily in their own production facilities and have strengthened their own production capabilities. Although they may still not be able to achieve the same level production efficiency as a NM or SM, the difference in the total variable cost (production plus transportation) per unit is likely to be small, especially when the cost of transportation to the retail stores is considered, as retailers tend to locate their manufacturing facilities close to the bulk of their store locations, whereas national brand manufacturers cannot locate their manufacturing facilities close to every retail chain.

We denote the production cost of product x ($x = s$ or n, denoting store and the national brand respectively) as $c_x = C(q_x)$, where $C(\cdot)$ is a convex and increasing function. For ease of analysis, we assume $C(q_x) = kq_x^2$, where k is the production cost parameter. We also assume that $k \leq \frac{\bar{\theta}}{2q_n}$, so that the "efficiency" of producing a product (maximum utility provided to a consumer less production cost) increases with the product quality within the interval $[0,\ q_n]$. This is a mild assumption that ensures sensible equilibria. If the efficiency of producing a product is decreasing with the product quality, the retailer will set its quality level at zero. If the efficiency of producing a product is first increasing with the product quality within $[0,\ q_n']$ and then decreasing within $(q_n',\ q_n]$ for some $q_n' < q_n$, the national brand manufacturer would have an incentive to lower the quality of the national brand below q_n. In our model, we assume the quality of the national brand product is fixed at q_n, and our assumption ensures that this exogenous choice would also be incentive compatible if the choice were endogenized.

The remainder of this section is organized as follows. In Sections 3.1.1 through 3.1.3, we we provide a detailed analysis of each party's decision problem and the equilibria under the MS game structure for sourcing under IH, SM, and NM, respectively. Next, in Section 3.1.4, we compare equilibrium quality and profit levels for these three scenarios. Then, in Section 3.2, we present equilibria under the three sourcing structures under RS. In Section 3.3, we present equilibrium outcomes under all three sourcing structures under VN. We present only summaries of the equilibria for RS and VN; details appear in Appendix F. In Section 3.4, we provide a comparison of equilibrium quality levels, profit levels, and consumer welfare across all nine scenarios. Throughout the analysis, we use q_n and q_s to denote quality levels, w_n and w_s to denote wholesale prices, p_n and p_s to denote retail prices, and, m_n and m_s to denote retail margins, for the national and the store brands. respectively, where the superscript s denotes the store brand and n denotes the national brand. Throughout most of the analysis we omit the sourcing and game structures in the notation when this information is clear from the context.

3.1. Quality-positioning for the Three Sourcing Arrangements under MS

3.1.1. Quality Positioning under IH+MS

Given a wholesale price w_n for the national brand product, for any store brand quality level, q_s, the retailer chooses retail prices to maximize profit:

$$\max_{p_s, p_n} (p_n - w_n) D_n(p_n,\ p_s;\ q_s) + (p_s - C(q_s))) D_s(p_n,\ p_s;\ q_s) \tag{1}$$

If $w_n < C(q_s) + \bar{\theta}(q_n - q_s)$ and $w_n > \frac{C(q_s)}{q_s} q_n$, i.e., if the wholesale price is between lower and upper threshold values, the retailer sells both products, setting $p_n(w_n) = \frac{1}{2}(\bar{\theta} q_n + w_n)$ and $p_s(q_s) = \frac{1}{2}(\bar{\theta} q_s + C(q_s))$. If $w_n \geq C(q_s) + \bar{\theta}(q_n - q_s)$, the retailer sells only her store brand. If $w_n > \frac{C(q_s)}{q_s} q_n$, the retailer sells only the national brand.

For any store brand quality level q_s, the national brand manufacturer solves the following problem to maximize his profit, taking into account the retailer's price reaction:

$$\max_{w_n} (w_n - c_n) \cdot D_n(p_n(w_n),\ p_s(q_s);\ q_s) \tag{2}$$

Setting the first derivative with respect to w_n equal to zero and confirming concavity of the objective function, we obtain $w_n(q_s) = \frac{1}{2}[\bar{\theta} q_n + c_n - (\bar{\theta} q_s - C(q_s))]$. Because $k \leq \frac{\bar{\theta}}{2q_n}$, we can verify that $w_n(q_s)$ satisfies the conditions under which the retailer sells both products.

Taking NM's wholesale pricing strategy into account, the retailer solves the following quality-setting problem.

$$\begin{aligned} \max_{q_s \in [0,\ q_n]} \pi_r(q_s) \equiv & \left[p_n\left(w_n(q_s)\right) - w_n(q_s)\right] D_n\left[p_n\left(w_n(q_s)\right),\ p_s\left(q_s\right);\ q_s\right] \\ & + \left[p_s\left(q_s\right) - C(q_s)\right] D_s\left[p_n\left(w_n(q_s)\right),\ p_s\left(q_s\right);\ q_s\right] \end{aligned} \tag{3}$$

Substituting for $w_n(q_s)$, $p_n(w_n)$, $p_s(q_s)$ and the demand functions in $\pi_r(q_s)$, we obtain $\pi_r(q_s) = \frac{1}{16\bar{\theta}}[k^2(q_n^3 + q_n^2 q_s - q_n q_s^2 + 3q_s^3) - 2k(q_n^2 + 3q_s^2)\bar{\theta} + (q_n + 3q_s)\bar{\theta}^2]$. Therefore $\frac{\partial \pi_r(q_s)}{\partial q_s} = \frac{1}{16\bar{\theta}}[9k^2 q_s^2 - (2kq_n + 12\bar{\theta})kq_s + (3\bar{\theta}^2 + k^2 q_n^2)]$. Solving $\frac{\partial \pi_r(q_s)}{\partial q_s} = 0$ and verifying that the solution is a global maximum, we obtain the optimal quality level under IH+MS as $\min\{q_n,\ \frac{1}{9k}[kq_n + 6\bar{\theta} - \sqrt{-8k^2 q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2}]\}$. The retailer never finds it optimal to set the quality level above q_n, because the only way this could happen is if the national brand manufacturer decreases the national brand wholesale price below $C(q_s) + \bar{\theta}(q_n - q_s)$ which is less than its production cost when $q_s > q_n$. The retailer therefore sells only the store brand, in which case she also has no incentive to set q_s above q_n because the utility provided to consumers less the unit production cost of the store brand is decreasing in the product quality for $q_s > q_n$.

3.1.2. Quality Positioning under SM+MS

In the retail pricing stage of the game, the retailer solves the same problem as (1), but with with w_s in place of $C(q_s)$. Performing the same type of analysis as in the previous subsection, we find that if $w_n \le w_s + \bar{\theta}(q_n - q_s)$ and $w_n > \frac{C(q_s)}{q_s} q_n$, the retailer sells both products, setting $p_n(w_n) = \frac{1}{2}(\bar{\theta} q_n + w_n)$ and $p_s(w_s,\ q_s) = \frac{1}{2}(\bar{\theta} q_s + w_s)$. Otherwise, if $w_n > w_s + \bar{\theta}(q_n - q_s)$, the retailer sells only her store brand; and if $w_n < \frac{C(q_s)}{q_s} q_n$, the retailer sells only her national brand.

Knowing the retailer's price response, both NM and SM engage in a Nash game, setting the wholesale price for the product they supply. NM's problem is:

$$\max_{w_n}\ (w_n - c_n) \cdot D_n(p_n(w_n),\ p_s(w_s,\ q_s);\ q_s) \tag{4}$$

for which the optimal response, $w_n(w_s$, is $q_s) = \frac{1}{2}(\bar{\theta} q_n + c_n - (\bar{\theta} q_s - w_s))$. SM's problem is:

$$\max_{w_s}(w_s - C(q_s))D_s(p_n(w_n),\ p_s(w_s,\ q_s);\ q_s) \tag{5}$$

We obtain the optimal response, $w_s(w_n)$, from the first-order necessary condition as $q_s = \frac{1}{2}(\frac{w_n}{q_n} q_s + C(q_s)$. Solving two equations in two unknowns, we obtain the equilibrium wholesale prices as:

$$\begin{cases} w_n(q_s) = & \frac{2q_n}{4q_n - q_s}\left(\bar{\theta} q_n - \bar{\theta} q_s + c_n + \frac{C(q_s)}{2}\right) \\ w_s(q_s) = & \frac{q_s}{4q_n - q_s}\left(\bar{\theta} q_n - \bar{\theta} q_s + c_n + \frac{C(q_s)}{2}\right) + \frac{C(q_s)}{2} \end{cases} \tag{6}$$

Knowing the pricing strategy of NM and SM, the retailer's quality-setting problem is:

$$\begin{aligned} \max_{q_s \in [0,\ q_n]}\ \pi_r(q_s) \equiv & \left[p_n\left(w_n(q_s)\right) - w_n(q_s)\right] D_n\left[p_n\left(w_n(q_s)\right),\ p_s\left(w_s(q_s),\ q_s\right);\ q_s\right] \\ & + \left[p_s\left(w_s(q_s),\ q_s\right) - C(q_s)\right] D_s\left[p_n\left(w_n(q_s)\right),\ p_s\left(w_s(q_s),\ q_s\right);\ q_s\right] \end{aligned} \tag{7}$$

The first derivative of the objective function is $\frac{\partial \pi_r(q_s)}{\partial q_s} = \frac{q_n^2}{4(4q_n - q_s)^3 \bar{\theta}}[3k^2(4q_n^3 + 3q_n^2 q_s + 12 q_n q_s^2 - q_s^3) - 6kq_n(4q_n + 11q_s)\bar{\theta} + (28q_n + 5q_s)\bar{\theta}^2]$. It can be shown that $\frac{\partial \pi_r(q_s)}{\partial q_s}$ is positive on $q_s \in [0,\ q_n]$, so the retailer's profit is strictly increasing in q_s on $[0,\ q_n]$. Therefore, when the retailer sources the store brand from SM under MS, its quality should be set to the maximum quality level, q_n, i.e., equal to that of the national brand.

3.1.3. Quality Positioning under NM+MS

Under MS, when the retailer sources the store brand from NM, she makes pricing decisions using the same approach as she does when sourcing from SM, except that the wholesale prices differ. If $w_n \le w_s + \bar{\theta}(q_n - q_s)$ and $w_n \ge \frac{w_s}{q_s} q_n$, the retailer's optimal prices are $p_n(w_n) = \frac{1}{2}(\bar{\theta} q_n + w_n)$ and $p_s(w_s,\ q_s) = \frac{1}{2}(\bar{\theta} q_s + w_s)$. Otherwise, if $w_n > w_s + \bar{\theta}(q_n - q_s)$, the retailer sells only her store brand; and if $w_n < \frac{C(q_s)}{q_s} q_n$, the retailer sells only her national brand. Knowing the retailer's price response, NM's problem is:

$$\max_{w_n, w_s}\ (w_n - c_n) \cdot D_n(p_n(w_n), p_s(w_s, q_s);\ q_s) + (w_s - C(q_s)) \cdot D_s(p_n(w_n), p_s(w_s, q_s);\ q_s) \tag{8}$$

Jointly optimizing the prices using the first order necessary conditions, we obtain $w_n = \frac{1}{2}(\bar{\theta} q_n + c_n)$ and $w_s(q_s) = \frac{1}{2}(\bar{\theta} q_s + C(q_s))$. (It is easy to verify that the objective function is jointly concave.) We can easily confirm that w_n and $w_s(q_s)$ satisfy the conditions under which the retailer sells both products. Therefore w_n and $w_s(q_s)$ indeed constitute NM's optimal solution. Given the NM's price response, the retailer's quality-setting problem is:

$$\begin{aligned}\max_{q_s \in [0,\ q_n]} \pi_r(q_s) \equiv & \left[p_n(w_n) - w_n\right] D_n\left[p_n(w_n),\ p_s(w_s(q_s),\ q_s);\ q_s\right] \\ & + \left[p_s(w_s(q_s),\ q_s) - C(q_s)\right] D_s\left[p_n(w_n),\ p_s(w_s(q_s),\ q_s);\ q_s\right]\end{aligned} \tag{9}$$

The objective function can be rewritten as $\pi_r(q_s) = \frac{q_n}{16\bar{\theta}}[k^2(q_n^2 + q_n q_s - q_s^2) - 2kq_n\bar{\theta} + \bar{\theta}^2]$, from which we obtain $\frac{\partial \pi_r(q_s)}{\partial q_s} = \frac{k^2 q_n}{16\bar{\theta}}(q_n - 2q_s)$. We can confirm that $\pi_r(q_s)$ is concave on $[0,\ q_n]$. Therefore, setting the first derivative of the objective function to zero, we obtain $q_s = \frac{1}{2}q_n$.

3.1.4. Comparison of Equilibria for the Three Sourcing Arrangements under MS

In this section, we compare the retailer's optimal quality level across the three sourcing structures. We use superscripts "IH", "SM" and "NM" to distinguish the equilibria. Our earlier analysis established that the retailer's optimal quality levels are $q_s^{IH} = \min\{q_n,\ \frac{1}{9k}[kq_n + 6\bar{\theta} - \sqrt{-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2}]\}$, $q_s^{SM} = q_n$ and $q_s^{NM} = \frac{1}{2}q_n$. The next proposition states how they compare.

PROPOSITION 1. *Under MS, $q_s^{NM} < q_s^{IH} \leq q_s^{SM}$. Moreover, the last inequality holds strictly for all $k \geq \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$.*

All proofs appear in the Appendix. Before discussing the intuition behind Proposition 1, we introduce the following Lemma.

LEMMA 1. *The optimal quality level for the store brand in a centralized supply chain consisting of the retailer and the supplier(s) of the national and store brands (as applicable), is $q_n/2$.*

Lemma 1 states that the optimal store brand quality level in a centralized supply chain is $\frac{q_n}{2}$. To understand this result, notice that the quality level of the store brand has two types of effects on the profit of the supply chain. The first is product differentiation: as q_s decreases, the store brand becomes more differentiated from the national brand, and thus total demand increases. The second effect is to increase the efficiency of the product. (Because $k \leq \frac{\bar{\theta}}{2q_n}$, the maximum utility provided a customer less production cost increases with the product quality on $[0,\ q_n]$). As a result, the maximum obtainable margin on store brand increases as q_s increases. The centralized decision-maker must trade off these two effects; the resulting optimal quality level is $\frac{q_n}{2}$.

In addition to the two effects just discussed, there is a third effect of q_s when the retailer either produces the store brand IH or sources it from SM: a larger q_s increases the retailer's negotiating power vis-a-vis the national brand manufacturer. Due to the increased competition as the retailer raises q_s, the national brand manufacturer is forced to decrease the wholesale price as if the retailer had either produced the store brand IH or sourced it from SM. As a result, the retailer is able to get a bigger slice of the pie when she increases q_s above the optimal quality level for the centralized supply chain. Therefore, both q_s^{IH} and q_s^{SM} are greater than $\frac{q_n}{2}$. In contrast, when the retailer sources the store brand from NM, third effect does not exist because NM sets a same wholesale price for the national brand irrespective of q_s. As such, the retailer sets q_s^{NM} in exactly the same way as in the centralized supply chain.

It is surprising that the retailer's optimal quality level is higher when she sources from SM than when she produces IH under MS. One might think that the introduction of double marginalization under production by SM would cause the retailer to compensate by lowering quality (as she does when sourcing from NM), thereby reducing the unit production cost incurred by SM. Instead, increasing q_s heats up the competition between NM and SM which forces them to decrease their markups, and, as a result, the retailer earns a greater profit. Indeed, when store brand quality is set at the same level as that of the national brand, both SM and NM set their wholesale prices equal to the unit production cost, thereby completely eliminating double marginalization. The next proposition states these results formally.

PROPOSITION 2. *When sourcing from SM (under MS), both NM's incremental markup on the national brand above that under IH as well as SM's markup on the store brand decrease in q_s, $q_s \in [\frac{7}{13}q_n,\ q_n]$ (and therefore, decrease in q_s, $q_s \in [q_s^{IH},\ q_n]$, where q_s^{IH} is retailer's optimal quality level under IH). If $q_s = q_n$, both NM and SM get zero margin on their products.*

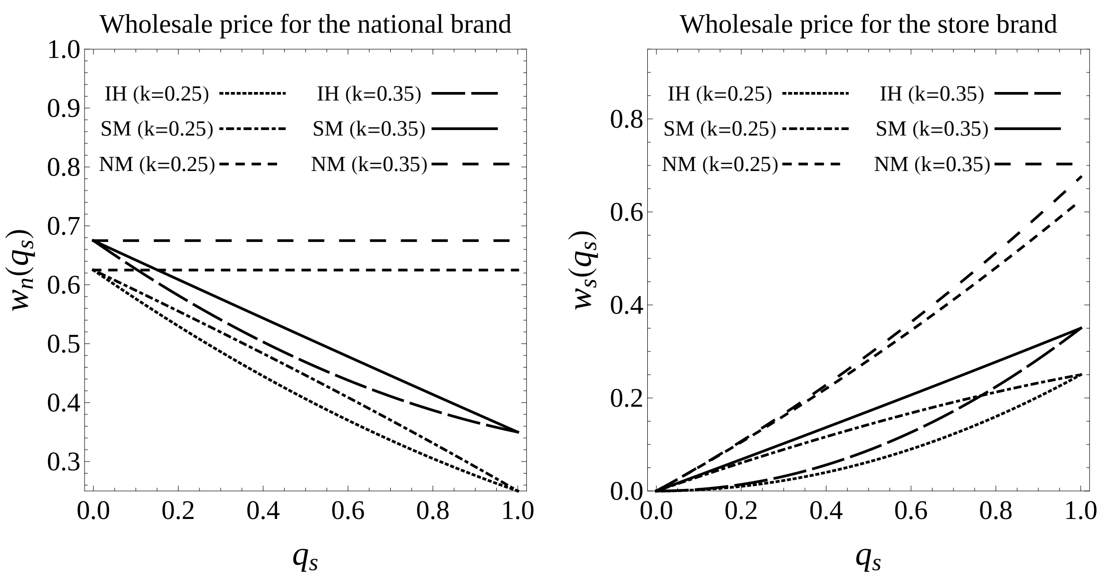
To help the reader visualize this effect, we show in Figure 1 a numerical comparison of wholesale prices for a given quality level under the three sourcing structures. The parameters used for this numerical study are $\bar{\theta} = 1$ and $q_n = 1$. One can see that as q_s increases above $\frac{7}{13}q_n$, the difference between w_s under SM and that under IH (which represents SM's markup), as well as the difference between w_n under SM and that under IH (which represents NM's markup under SM above and beyond his markup under IH), shrink. Moreover, when $q_s = q_n$, both wholesale prices for the national brand and the store brand equal the unit production cost of the product, and this holds under both SM and IH. In short, we conclude that, when sourcing from SM, the retailer increases the store brand quality level above q_s^{IH} due to the benefit she gains from generating increased competition between NM and SM. The heated competition forces both NM and SM to decrease their markups, which then enables the retailer achieve a greater profit.

To provide a more complete picture, we also compare wholesale prices and retail margins for the store and national brands for a given quality of the store brand under the three sourcing structures. Let $m_n^x(q_s)$ and $m_s^x(q_s)$ ($x \in \{IH,\ SM,\ NM\}$) denote the retailer's equilibrium profit margin on the national and store brands, respectively, when the store brand quality is q_s. Let $m_n^x(q_s^x)$ and $m_s^x(q_s^x)$ ($x \in \{IH,\ SM,\ NM\}$) denote the retailer's equilibrium profit margins when the store brand quality is chosen optimally under sourcing arrangement x. Similarly, let $w_n^x(q_s)$ and $w_s^x(q_s)$ denote wholesale prices when the store brand quality level is q_s, and let $w_n^x(q_s^x)$ and $w_s^x(q_s^x)$ denote the wholesale prices when the store brand quality is chosen optimally. We have the following proposition.

PROPOSITION 3. *For $\forall q_s \in (0,\ q_n)$, $w_n^{IH}(q_s) < w_n^{SM}(q_s) < w_n^{NM}(q_s)$ and $w_s^{IH}(q_s) < w_s^{SM}(q_s) < w_s^{NM}(q_s)$. However, the ordering between w_n^{IH} and w_n^{SM} is reversed when evaluated at the optimal quality levels. That is, $w_n^{SM}(q_s^{SM}) \leq w_n^{IH}(q_s^{IH}) < w_n^{NM}(q_s^{NM})$. We also have $w_s^{IH}(q_s^{IH}) \leq w_s^{SM}(q_s^{SM})$, but the relationship of $w_s^{NM}(q_s^{NM})$ to $w_s^{IH}(q_s^{IH})$ and $w_s^{SM}(q_s^{SM})$ depends on k. The ordering of w_s is the same across sourcing arrangements at the optimal quality levels. That is, $w_s^{IH}(q_s^{IH}) \leq w_s^{SM}(q_s^{SM}) < w_s^{NM}(q_s^{NM})$. Also, for $\forall q_s \in (0,\ q_n)$, $m_n^{IH}(q_s) > m_n^{SM}(q_s) > m_n^{NM}(q_s)$ and $m_s^{IH}(q_s) > m_s^{SM}(q_s) > m_s^{NM}(q_s)$. However, the ordering between m_n^{IH} and m_n^{SM}, and the ordering between m_s^{IH} and m_s^{SM} are reversed at the optimal quality levels. That is, $m_n^{NM}(q_s^{NM}) < m_n^{SM}(q_s^{SM}) \leq m_n^{IH}(q_s^{IH})$ and $m_s^{NM}(q_s^{NM}) < m_s^{SM}(q_s^{SM}) \leq m_s^{IH}(q_s^{IH})$. For $\forall q_s \in (0,\ q_n)$, $\pi_r^{IH}(q_s) \geq \pi_r^{SM}(q_s) > \pi_r^{NM}(q_s)$.*

Also, $\pi_r^{IH}(q_s^{IH}) > \pi_r^{SM}(q_s^{SM}) > \pi_r^{NM}(q_s^{NM})$. *All of the above inequalities hold strictly for* $k > \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$.

Proposition 3 states that, for any given level of q_s, the retailer's margins on both the store and national brand products are greater under IH than under SM. This is not surprising. However, the above ordering is reversed at the respective optimal quality levels. This is due to the fact that q_s is higher under SM than under IH. This not only allows the retailer to charge a higher price for the store brand, but it also forces both SM and IH to reduce their respective markups due to the increased competition between the products they produce. The increased retail profit margin partially compensates for the retailer's loss of power due to outsourcing store-brand production. However, by setting a higher q_s under SM than under IH, the retailer suffers from a reduction in total demand because the products are less differentiated. Consequently, she still obtains a lower profit than under IH.

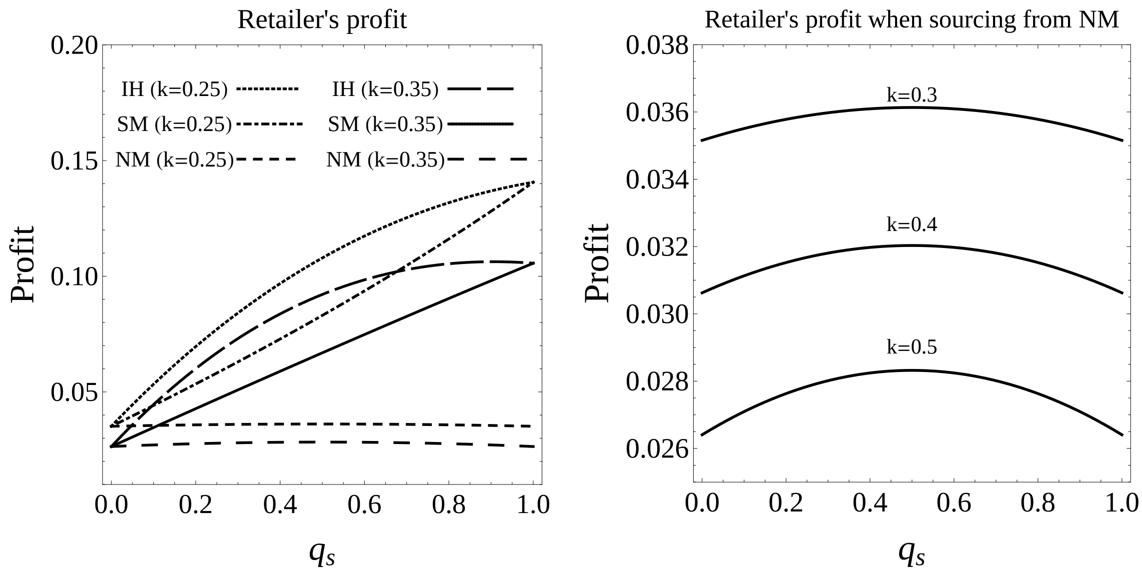


(A) $w_n(q_s)$ under three sourcing structures

(B) $w_s(q_s)$ under three sourcing structures

FIGURE 1. Comparison of equilibrium wholesale prices for a given q_s under Manufacturer-Stackelberg game ($q_n = 1$ and $\bar{\theta} = 1$)

In Figure 2(A), we compare the retailer's profit across the three store brand sourcing arrangements. The parameters used in this example are the same as those used for Figure 1. Although the retailer's profit when sourcing from NM appears to be constant in this figure, it is actually concave. Figure 2(B) displays the retailer's profit under outsourcing to NM using a different scale on the vertical axis and for different values of k, retailer's profit. From Figure 1, we can observe that, for a given k and a given quality level between 0 and q_n, the wholesale prices for both the store and the national brands are the lowest when the retailer is producing the store brand in-house, and the highest when the retailer sources the store brand from NM. This is consistent with the result in Figure 2, which shows that, for a given k and a given quality level between 0 and q_n, the retailer earns the highest profit when producing the store brand in-house, the next highest profit when she sources the store brand from SM, and the lowest profit when she sources the store brand from NM.



(A) Retailer's profit under three sourcing structures

(B) Retailer's profit when sourcing from NM

FIGURE 2. Comparison of retailer's profit for three sourcing structures under Manufacturer-Stackelberg game ($q_n = 1$ and $\bar{\theta} = 1$)

Another observation from Figure 1 is that, when the store brand and national brand quality are equal, the wholesale prices for both the store and the national brands are the same whether the retailer produces in-house or sources from SM. When the store brand quality level is zero (which represents a scenario in which the retailer does not carry a store brand at all), wholesale prices across all three sourcing scenarios are the same. These are consistent with the observation from Figure 2, which shows that, if the store brand quality level is q_n, the retailer earns the same profit whether she produces in-house or sources from SM. If the store brand quality level is 0, the retailer earns the same profit under all three sourcing structures.

3.2. Quality-Positioning under Three Sourcing Arrangements under RS

In this subsection, we study the retailer's store-brand quality decisions under IH, SM and NM when the retailer is the Stackelberg price leader. Lee and Staelin (1997) have pointed out that "the stability of the RS (retailer-Stackelberg) game might be debatable. This is because the price-following manufacturer sets its margin conditional upon the announced retail margin. However, once the product falls into the retailer's hand (and thus the retailer owns it), the retailer has the opportunity to charge a retail margin that is different from the one it previously announced ".....“the resulting retail margin will differ from the originally announced one and the RS game will fall apart, unless there exists some precommitment mechanism restricting the retailer's freedom to 'cheat'." Following Lee and Staelin (1997), we assume a precommitment mechanism exists (otherwise retailer price leadership could not be analyzed in our complex problem setting).

Under RS, the sequence of the game is as follows. The retailer first chooses a quality level. Then she sets her margins on the two products. Next, if the store brand is sourced in-house, NM sets the national-brand wholesale price. If the store brand is produced by NM, NM sets the wholesale prices for both the store and national brands. If the store brand is produced by SM, NM and SM engage in a Nash game to set wholesale prices for the products they produce. Finally, customers make purchases. For brevity, we omit derivations here and refer the reader to Appendix E for details.

PROPOSITION 4. *Under RS, when the retailer produces the store brand IH, the optimal quality level for the store brand is* $q_s^{IH} = \min\{\frac{1}{3k^2}[k^2 q_n + 2k\bar{\theta} - (-2k^4 q_n^2 + 4k^3 q_n \bar{\theta} + k^2\bar{\theta}^2)^{\frac{1}{2}}],\ q_n\}$. *When the retailer sources the store brand from SM, if* $k \leq \frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n} \approx 0.42\frac{\bar{\theta}}{q_n}$, *the optimal quality level for the store brand is* q_n, *but if* $k \in [\frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n},\ \frac{\bar{\theta}}{2q_n}]$, *the optimal store brand quality,* q_s^{SM}, *is the smallest solution to* $-2k^2(q_s^{SM})^3 + (13k^2 q_n + 2k\bar{\theta})(q_s^{SM})^2 - (8k^2 q_n^2 + 16kq_n\bar{\theta})q_s^{SM} + (6k^2 q_n^3 - 4kq_n^2\bar{\theta} + 6q_n\bar{\theta}^2) = 0$ *on* $(0,\ q_n)$. *There are three solutions to this cubic equation. When* $k \leq \frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n} \approx 0.42\frac{\bar{\theta}}{q_n}$, *all the three solutions are greater than* q_n, *in which case* q_n *is the optimal solution. Otherwise, if* $k > \frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n}$, *the smallest solution falls below* q_n *and is the optimal solution to the retailer's problem. When the retailer sources the store brand from NM, the optimal quality is* $q_s^{NM} = \frac{q_n}{2}$.

We defer the comparison of q_s^{IH}, q_s^{SM} and q_s^{NM} under RS to Section 3.4. Next, we discuss the retailer's quality setting strategy under another type of channel interaction: vertical Nash.

3.3. Quality-positioning under Three Sourcing Arrangements under VN

Under our Vertical Nash pricing games, the retailer first chooses her store brand quality. Then, the retailer and the manufacturer(s) engage in a Nash game in which the retailer sets the retail margins for both products, and the manufacturer(s) sets the wholesale price(s) for the product(s) they produce. Finally, customers make purchase decisions. Below, we present optimal quality levels for the three sourcing arrangements under VN. We refer the reader to the proof of Proposition 5 in Appendix F for a detailed derivations of these results.

PROPOSITION 5. *Under VN, when the retailer produces the store brand IH, the optimal quality level for the store brand is* $q_s^{IH} = \min\{q_n,\ \frac{1}{15k}[4kq_n + 10\bar{\theta} - (-44k^2 q_n^2 + 80kq_n\bar{\theta} + 25\bar{\theta}^2)^{\frac{1}{2}}]\}$. *When the retailer sources the store brand from SM,* $k \leq \frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n} \approx 0.424\frac{\bar{\theta}}{q_n}$ *the optimal quality level for the store brand is* q_n . *If* $k \in [\frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n},\ \frac{\bar{\theta}}{2q_n}]$, *the optimal store brand quality,* q_s^{SM}, *is the smallest solution to* $-5k^2 q_n^2(q_s^{SM})^3 + 135k^2 q_n^3(q_s^{SM})^2 + (-32k^2 q_n^4 - 218kq_n^3\bar{\theta} + 7q_n^2\bar{\theta}^2)q_s^{SM} + (54k^2 q_n^5 - 54kq_n^4\bar{\theta} + 81q_n^3\bar{\theta}^2) = 0$ *on* $(0,\ q_n)$. *There are three solutions to this cubic function. When* $k \leq \frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n} \approx 0.42\frac{\bar{\theta}}{q_n}$, *all the three solutions are greater than* q_n, *in which case* q_n *is the optimal solution. Otherwise, if* $k > \frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n}$, *the smallest solution falls below* q_n *and is the optimal solution to the retailer's problem. When the retailer sources the store brand from NM, the optimal quality is* $q_s^{NM} = \frac{q_n}{2}$.

3.4. Comparison of Quality Positioning Across Nine Scenarios

We have now derived the optimal quality levels for the nine combinations of game-and-sourcing structures and can proceed to compare them. We have the following proposition.

PROPOSITION 6. *When sourcing from NM under the three game structures, the optimal quality level is $\frac{q_n}{2}$, for all three game structures, and the retailer's profit levels have the ordering of MS+NM < VN+NM < RS+NM. Both the quality and profit levels are the lowest among the nine combinations. For the remaining six combinations of game-and-sourcing structures, the ordering of the retailer's optimal quality levels is MS+SM $\geq$ VN+SM $\geq$ RS+SM $\geq$ MS+IH $\geq$ VN+IH $\geq$ RS+IH, whereas her profit levels are in the reverse order. Furthermore, $\exists k_1 \in (0,\ \frac{\bar{\theta}}{2q_n})$ such that, for $\forall k \in (k_1,\ \frac{\bar{\theta}}{2q_n}]$, all of the inequalities above hold strictly.*

Proposition 6 presents results on the relation among the retailer's optimal quality levels and optimal profit levels across nine scenarios. We next discuss the intuition behind these results shortly. Before we do so, notice that from Proposition 6, if the retailer chooses the optimal quality level for each scenario, her profit levels across the nine scenarios have the ordering of $RS+IH \geq VN+IH \geq MS+IH \geq RS+SM \geq VN+SM \geq MS+SM > RS+NM > VN+NM > MS+NM$. One might ask whether this ordering holds if the retailer does not have the flexibility to change quality levels across different scenarios? The following corollary answers this question.

COROLLARY 1. *Suppose the quality level of the store brand is fixed and equal across scenarios. Then, given a sourcing structure, the retailer's profit level across the three game structures have the ordering of RS$\geq$VN$\geq$MS, with the inequality holding strictly for $q_s \in (0,\ q_n)$). Given a game structure, the retailer's profit across the three sourcing structures have the ordering of $IH \geq SM \geq NM$ with the inequalities holding strictly for $q_s \in (0,\ q_n)$). These orderings are consistent with the ordering of optimal profit levels stated in Proposition 6. However, different from the ordering stated in Proposition 6, the retailer's profit level under MS+IH can be smaller than her profit under RS+SM, and the retailer's profit under MS+SM can be smaller than her profit under RS+NM. These orderings arise if q_s is smaller than a threshold.*

We now discuss the intuition behind Proposition 6 and Corollary 1. Past literature on store-brand introduction has established that introducing a store brand helps a retailer elicit price concession from upstream NMs (Mills 1995 and 1999, Narasimhan and Wilcox 1998, Gabrielsen and Sørgard 2007, etc.). Proposition 6 states that this effect is not present if the retailer sources the store brand from NM, even if the retailer is the Stackelberg price leader. This occurs because if the retailer sources the store brand from NM, she confers pricing power over the wholesale prices of both the store and national brands to NM. As a result, NM chooses the same wholesale price on the national brand product irrespective of the store brand quality level, even if he is a Stackelberg follower when setting prices. Consequently, the retailer's only source of leverage provided by the store brand is product differentiation. As such, she chooses a low quality level that provides significant differentiation from the national brand product. The differentiation leads to an increase in overall profit for the retailer (versus having no store brand), so she adopts this low-quality choice. In this case, the retailer's optimal quality level is equal to that for the centralized supply chain consisting of the retailer and the national brand manufacturer, and the optimal quality level turns out to be lower than those in any of the other six scenarios. It appears that, in this case, the retailer is better off trying to appeal to the portion of the market that the national brand manufacturer cannot easily reach, rather than competing head-on with the national brand manufacturer when he has no incentive to offer the retailer a low price on her store brand product. The retailer's resulting profit is also the lowest when she sources the store brand from NM than under any other sourcing arrangement.

Under the remaining six scenarios (i.e., under combinations involving IH or SM), the retailer uses the store-brand quality level more aggressively as a lever to (partially) compensate for any reduction in leverage arising from a less advantageous sourcing arrangment or weaker pricing power. The optimal store-brand quality has a monotonic mapping to the game-and-sourcing combinations, with the optimal quality increasing along the following path: RS+IH $\longrightarrow$VN+IH $\longrightarrow$MS+IH $\longrightarrow$RS+SM $\longrightarrow$VN+SM $\longrightarrow$MS+SM. Stated another way, the optimal quality level for the centralized channel (consisting of the retailer and manufacturer(s)) is low. But as the retailer's sourcing arrangement or pricing power becomes less advantageous, the retailer can wrest a larger portion of the supply chain profit by deviating from the optimal quality level for the centralized system.

As mentioned in Section 3.1.4, the retailer sets a higher quality level when sourcing from SM than from IH under MS. We can also offer a stronger result: the retailer sets a higher quality level under SM than under IH, even if her pricing power differs between the two scenarios being compared. Surprisingly, this effect leads to the retailer even to choose a higher quality level under SM when she has pricing power (i.e., under RS) than she does under IH when NM is the Stackelberg price leader (i.e. under MS). The main reason is that when sourcing from SM, by increasing q_s, the retailer can heat up the competition between the national and the store brand manufacturers, which forces them to decrease their markups. This benefit is not available under IH.

Above, we compared the retailer's optimal quality levels across the nine scenarios. Next, we discuss the intuition for results regarding the retailer's profit levels in Proposition 6 and Corollary 1 which state that (i) given a game structure, the retailer prefers IH over SM over NM; and (ii) given a sourcing structure, the retailer prefers RS over VN over MS. Results (i) and (ii) hold both when the retailer's store brand quality level is fixed, and when the retailer chooses an optimal quality level for each of the scenarios. Although these results are intuitive, we are (to the best of out knowledge) the first to formally establish comparison across store brand sourcing arrangements.

We note that (ii) is consistent with the past literature on the value of being a price leader. For example, Choi (1991) and Lee and Staelin (1997) find that if the type of interaction in a channel is such that a channel member's best response is to reduce its margin when its channel partner increases its margin, then each channel member prefers having price leadership. This is exactly the type of channel interaction that arises in our model. In particular, if the national brand manufacturer experiences a cost increase (decrease) in our model and hence increases (decreases) its wholesale price, the retailer will not pass on the full amount of the change to customers. We are only aware of one paper involving store brand products that compares different vertical price leadership structures. Choi and Fredj (2013) study the interaction between a national brand manufacturer and two retailers (each offering a store brand), and show that each party earns more profit by having price leadership. Because the authors assume the retailers incur zero marginal cost on store brands, their model represents a special scenario in which the retailer produces the store brand IH and incurs no cost related to quality. We find that this result carries over to other sourcing arrangements as well. That is, given a sourcing structure, the retailer prefers RS over VN over MS.

We now turn to a comparison of the retailer's profit across scenarios when both pricing power and sourcing structure vary. Proposition 6 states that at the optimal quality level for store brand corresponding to each scenario, the retailer strictly prefers a scenario with a more preferable sourcing structure, even if she is a price follower in this scenario but a leader in the other. For example, retailer's profit under MS+IH is greater than her profit under RS+SM. This finding implies that, although past literature emphasizes how store brands give a retailer more pricing power versus the national brand manufacturer, a more preferable *sourcing* structure has greater value than a more favorable pricing power scenario to the retailer, i.e., the power to choose the source is more important than than pricing pwer. One key reason for this result is that sourcing from an outside party confers power over wholesale pricing even if that party is not the Stackelberg price leader. A less obvious reason is that sourcing from an outside party can cause the retailer to choose a lower quality for the store brand product than when it is produced in-house. This *does* occur when the retailer outsources to NM (but not when she outsources to SM). It is important to emphasize, however, that this result holds only when the retailer chooses the corresponding optimal quality level for each scenario. If the store brand quality level is fixed, it is possible that the retailer prefers RS+SM over MS+IH, or prefers RS+NM over MS+SM. Intuitively, if the store brand quality is too low, it does not pose enough of threat to the national brand manufacturer even if the retailer has a preferable sourcing arrangement for her store brand. In this case, the retailer does not gain much from having a preferable sourcing arrangement. Instead, she benefits more from having pricing leadership in the supply chain.

Next, we compare customer welfare at the optimal store brand quality levels, as well as equilibrium retail prices, across the nine combinations of game-and-sourcing structure.

PROPOSITION 7. *Across the nine game-and-sourcing structures, the ordering of the consumer welfare is VN+IH $\geq$MS+IH $\geq$ RS+IH $\geq$ VN+SM $\geq$ MS+SM $\geq$ RS+SM $\geq$ VN+NM $\geq$ RS+NM = MS+NM. Furthermore, $\exists k_2 \in (0,\ \frac{\bar{\theta}}{2q_n})$ such that, for $\forall k \in (k_2,\ \frac{\bar{\theta}}{2q_n}]$, all of the inequalities above hold strictly.*

COROLLARY 2. *Suppose quality level of the store brand is fixed. Then, given a sourcing structure, customer welfare across the three pricing power scenarios have the ordering of $VN \geq MS = RS$, and the inequalities hold strictly for $q_s \in (0,\ q_n)$). Given a pricing power scenario, customer welfare across the three sourcing structures have the ordering of $IH \geq SM \geq NM$ and the inequalities hold strictly for $q_s \in (0,\ q_n)$). These orderings are consistent with the ordering of the optimal quality levels stated in Proposition 7. However, different from the ordering stated in Proposition 7, customer welfare under RS+IH can be smaller than that under VN+SM, and can be smaller under RS+SM than that under VN+NM. These relationships arise when q_s is smaller than a threshold.*

COROLLARY 3. *Given a sourcing structure, the q_s that maximizes the retailer's profit under VN and MS also maximizes customer welfare under VN and MS, respectively. However, under RS, the retailer chooses a lower quality level than that which maximizes customer welfare.*

Proposition 7, and Corollaries 2 and 3 imply that:
(i) Given a game structure, customers prefer IH over SM over NM (just as the retailer does). This result holds both when the quality level is fixed and when the retailer chooses her optimal quality level for each scenario.
(ii) When quality level is fixed and equal across scenarios, given a sourcing structure, customers prefer VN over MS or RS, and are indifferent between MS and RS.
(iii) When the retailer chooses the quality level optimally for each scenario, the quality level

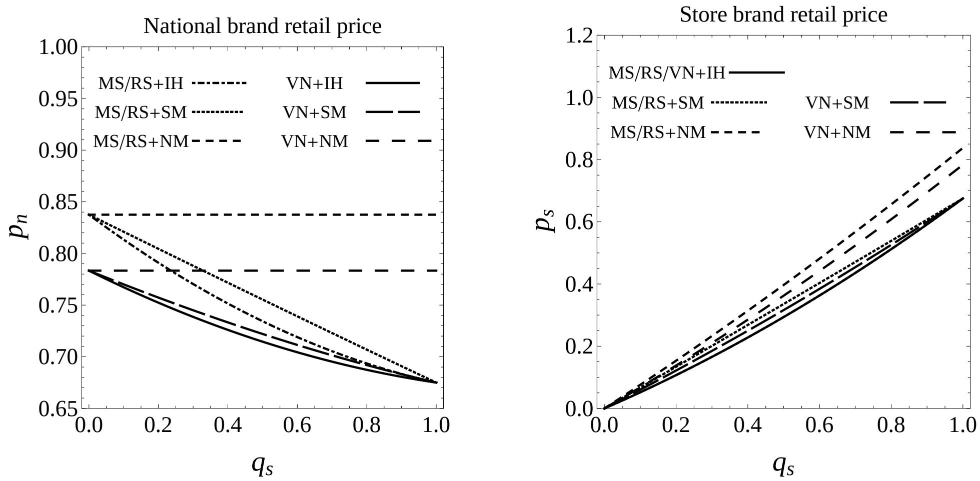
is also optimal for consumers when the pricing scenario is VN or MS. Under RS, she chooses a quality level lower than the optimal quality level for consumers.

The combination of (ii) and (iii) implies that:
(iv) when the retailer chooses her optimal quality level for each scenario, customers are better off under VN than they are under MS, and better off under MS than under RS. Finally, the results imply that:
(v) when the retailer chooses her optimal quality for each scenario, customers prefer a "better" sourcing arrangement irrespective of the pricing power scenario.
However, this result does not hold if the store brand quality level is fixed and equal across scenarios and is lower than a threshold. We elaborate each of these findings below.

Finding i) is consistent with the findings of Chen et al. (2010), who present results of an empirical study to estimate parameters of the demand and supply functions in a model of strategic interaction between two national brand manufacturers and two retailers for the fluid milk category in a major metropolitan area. They assume the type of competition between store brands upstream to be Cournot-Nash. That is, they assume that store brands are homogeneous at the wholesale level, producers (the national brand manufacturer and the retailers) simultaneously choose their production quantities, and price is determined by market clearing. On the other hand, they assume national brand products are differentiated upstream and compete in a Bertrand-Nash fashion. Downstream, all products are differentiated and compete in a Bertrand-Nash fashion. Their results indicate that consumers are better off when the retailer produces the store brand in-house than when she sources the store brand from a national brand manufacturer. This result would be expected in view of the effect of double marginalization.

Finding (ii) above is also consistent with results the literature. Choi (1991) and Lee and Staelin (1997) find that the equilibrium retail prices under MS and RS are the same and are higher than the retail prices under VN. This result implies that customers strictly prefer no channel leadership over the presence of channel leadership, and are indifferent as to who takes channel price leadership. However, our results (iii) and (iv) state that this is not true when the retailer chooses the optimal store brand quality level for each scenario. When she does so, the quality level she chooses under VN (MS) also maximizes consumer welfare under VN (MS), while she chooses a quality level that is suboptimal for consumers under RS. This means that customers strictly prefer MS over RS (even though they would have secured the same welfare under the two game scenarios if the quality levels were the same). Finally, our finding (v) says that when the retailer chooses her quality level optimally for each of the nine scenarios, customers prefer a preferable sourcing structure irrespective of the pricing power scenario. This result contributes to the store brand literature by again suggesting that managing sourcing is more important than gaining pricing power, because, not only the retailer, but also the customers, prefer the store brand being sourced in-house, irrespective of the pricing power scenario. However, one should understand that this result may not hold if the retailer does not have the flexibility to choose the optimal quality level for each scenario. If the quality level is lower than a threshold, the store brand does not pose enough of a threat to the national brand manufacturer, therefore a "good" sourcing structure is less valuable to both the retailer and customers.

While consumer welfare has the same ordering across the nine game-and-sourcing combinations for all k, the ordering of equilibrium retail prices across the nine combinations depends on the value of k. Indeed, for a given $q_s \in (0,\ q_n)$, the ordering of the equilibrium



(A) Equilibrium p_n for all values of q_s

(B) Equilibrium p_s for all values of q_s

FIGURE 3. Equilibrium retail prices for all values of q_s ($k = 0.35$, $q_n = 1$, $\bar{\theta} = 1$)

store brand retail price always has the ordering MS+NM=RS+NM > VN+NM > MS+SM = RS+SM > VN+SM > VN+IH = MS+IH = RS+IH. This relationship is shown in the numerical example in Figure 3(B). However, if the retailer chooses the optimal quality under each of the game-and-sourcing combinations, the ordering of the equilibrium retail price may vary with k. For example, for our numerical example, if $k = 0.50$, the ordering of the equilibrium store brand retail prices is MS+SM > VN+SM > RS+SM > MS+IH > VN+IH > RS+NM = MS+NM > RS+IH > VN+NM. If $k = 0.35$, the ordering of the equilibrium store brand retail price becomes MS+SM = RS+SM >VN+SM > MS+IH > VN+IH >RS+IH >RS+NM = MS+NM >VN+NM. Notice that the ordering of RS+SM and VN+SM, and that of RS+IH and RS+NM, are reversed for these two k values.

Before concluding this section, we note that Proposition 6 implies that a retailer earns less profit if she sources the store brand from NM than if she produces the store brand IH or sources from a SM. However, in reality, there are several factors that may cause a retailer to entrust store-brand production to NM. First, NM may have lower variable costs (due to economies of scale in procurement or processing). Second, there are fixed costs associated with owning and operating a factory. If the volume produced is not very high, it may not be worthwhile for the retailer to produce internally because the savings may not cover the fixed cost. Third, there may be incremental transportation or distribution costs associated with internal or third-party production. If NM is already delivering his product to the retailer's warehouses, the incremental cost of including the retailer's store brand may not be very high. On the other hand, SM needs to arrange for shipments to the retailer's warehouses, and these costs might be passed on to the retailer. It is straightforward to generalize our models to allow for differences in the variable cost across different manufacturers. The fixed costs of product introduction and/or for operating an in-house production facility can be accounted for by deriving equilibria with and without the new products and/or in-house facilities and determining whether the incremental profits cover the fixed costs.

In the next section, we study the effects of the retailer having alternate sourcing options. Even when considering the possibility of outsourcing to NM, the retailer might have the option of using a strategic third-party manufacturer or even producing in-house. We investigate NM's optimal strategy under threat of the retailer choosing an alternate source, as well as the retailer's optimal quality decisions in this environment.

4. National brand manufacturer's strategy and retailer's optimal quality level when the retailer has alternate sources

In this section, we study two variants of the game between the national brand manufacturer and the retailer. In the two variants, the retailer is asking the national brand manufacturer to produce both products under the implicit threat of using an alternate source, either IH or SM, to produce the store brand product. The retailer often has an alternate source, so these games represent realistic market environments.

We next describe the sequence of decisions in these games. First, the retailer sets the store brand quality level and asks the national brand manufacturer to quote wholesale prices for the national and store brand products, $(w_n,\ w_s)$. The retailer then decides whether to source both products from the national brand manufacturer. If she decides to do so, $(w_n,\ w_s)$ becomes effective. Otherwise, the retailer can source the store brand from her alternate source, in which case she will ask the national brand manufacturer to quote a wholesale price only on the national brand product. If SM is the alternate source for the store brand product, then NM's quote is determined via a Nash pricing game between SM and NM (as in the models analyzed earlier in this paper). Finally, the retailer sets retail prices and customer demands are realized. As before, we seek subgame-perfect Nash equilibria. We now proceed to analyze these two game variants using backward induction.

Given any q_s, the national brand manufacturer solves the following problem to set the wholesale prices if he is producing both products:

$$\max_{w_n,\ w_s} \quad \pi_m(w_n,\ w_s) \tag{10}$$

$$\pi_r(w_n, w_s) \geq \pi_{r0} \tag{11}$$

$$\frac{w_n - w_s}{q_n - q_s} \geq \frac{w_s}{q_s} \tag{12}$$

$$c_s \leq w_s \leq \frac{w_n}{q_n} q_s \tag{13}$$

where, given $(w_n,\ w_s)$ set by the national brand manufacturer, $p_n(w_n,\ w_s) = \frac{1}{2}(\bar{\theta}q_n + w_n)$ and $p_s(w_n,\ w_s) = \frac{1}{2}(\bar{\theta}q_s + w_s)$ are the retailer's optimal price response, and

$$\begin{aligned}\pi_m(w_n, w_s) \equiv&(w_n - c_n)D_n(p_n(w_n, w_s), p_s(w_n, w_s); q_s)\\ &+ (w_s - c_s)D_s(p_n(w_n, w_s), p_s(w_n, w_s); q_s)\end{aligned}$$

and

$$\begin{aligned}\pi_r(w_n, w_s) \equiv&(p_n(w_n, w_s) - w_n)D_n(p_n(w_n, w_s), p_s(w_n, w_s); q_s)\\ &+ (p_s(w_n, w_s) - w_s)D_s(p_n(w_n, w_s), p_s(w_n, w_s); q_s)\end{aligned}$$

are NM's and retailer's profit respectively. Define π_{r0} in (11) as the retailer's "reservation profit," which is the profit she would earn if she uses her alternate source to produce the store brand. Constraint (11) is a participation constraint that ensures the national brand manufacturer chooses wholesale prices so that the retailer earns at least her reservation profit. Thus, $\pi_{r0} \equiv (p_n(w_{n0},\ w_{s0}) - w_{n0})D_n(p_n(w_{n0}, w_{s0}), p_s(w_{n0}, w_{s0}); q_s) + (p_s(w_{n0}, w_{s0}) -$

$w_{s0})D_s(p_n(w_{n0}, w_{s0}), p_s(w_{n0}, w_{s0}); q_s)$, where w_{n0} denotes the national brand manufacturer's wholesale price and w_{s0} denotes the store brand supplier's wholesale price. More specifically, if the retailer's alternate source is IH, then $w_{n0} = \frac{1}{2}(\bar{\theta}q_n + c_n - (\bar{\theta}q_s - c_s))$ and $w_{s0} = C(q_s)$; and if the retailer's alternate source is SM, $w_{n0} = \frac{2q_n}{4q_n - q_s}\left(\bar{\theta}q_n - \bar{\theta}q_s + c_n + \frac{c_s}{2}\right)$ and $w_{s0} = \frac{q_s}{4q_n - q_s}\left(\bar{\theta}q_n - \bar{\theta}q_s + c_n + \frac{c_s}{2}\right) + \frac{c_s}{2}$, as derived in Section 3. Constraint (12) limits the range of $(w_n,\ w_s)$ so that the retailer carries the national brand. Similarly, constraint (13) limits the range of $(w_n,\ w_s)$ so that the retailer carries the store brand.

Whether the alternate source is IH or SM, $(w_{n0},\ w_{s0})$ is a feasible solution for (10). Therefore, when the national brand manufacturer supplies both products, he can do at least as well as when the retailer uses the alternate source for the store brand. Also, because π_m in (10) is jointly concave in $(w_n,\ w_s)$, and its unconstrained optimal solution does not satisfy (11), we know that, at the optimal solution of the manufacturer's problem, constraint (11) is binding. We can therefore solve both variants of the national brand manufacturer's problem by maximizing an "auxiliary" profit function:

$$\max_{w_n,\ w_s} \pi_m(w_n, w_s) + \lambda\left[\pi_r(w_n, w_s) - \pi_{r0}\right] \tag{14}$$

where λ is the Lagrange multipliers associated with constraint (11). The first order necessary conditions for maximizing this auxiliary profit function are:

$$\begin{cases} \frac{\lambda-2}{2\bar{\theta}(q_n-q_s)}(w_n - w_s) + \frac{1}{2\bar{\theta}}\left[k(q_n + q_s) + \bar{\theta}(1-\lambda)\right] = 0 \\ \frac{\lambda-2}{2\bar{\theta}(q_n-q_s)}\left(\frac{q_n}{q_s}w_s - w_n\right) - \frac{kq_n}{2\bar{\theta}} = 0 \\ \frac{w_n^2}{4\bar{\theta}(q_n-q_s)} + \frac{\frac{q_n}{q_s}w_s^2}{4\bar{\theta}(q_n-q_s)} - w_n\left[\frac{w_s}{2\bar{\theta}(q_n-q_s)} + \frac{1}{2}\right] + \frac{\bar{\theta}q_n}{4} - \pi_{r0} = 0 \end{cases} \tag{15}$$

Solving the first two equations simultaneously, we get $w_n = \frac{1}{2-\lambda}c_n + (1 - \frac{1}{2-\lambda})\bar{\theta}q_n$ and $w_s = \frac{1}{2-\lambda}c_s + (1 - \frac{1}{2-\lambda})\bar{\theta}q_s$. Substituting these expressions for w_n and w_s into the third equation, we obtain the following implicit expression for λ:

$$(2-\lambda)^2 = \frac{q_n}{4\pi_{r0}\bar{\theta}}\left[k^2\left(q_n^2 + q_nq_s - q_s^2\right) + \bar{\theta}(\bar{\theta} - 2kq_n)\right] \tag{16}$$

which has two solutions. We can verify that (12) and(13) are satisfied only at the smaller solution for λ, which is $\lambda = 2 - \sqrt{\frac{q_n}{4\pi_{r0}\bar{\theta}}\left[k^2\left(q_n^2 + q_nq_s - q_s^2\right) + \bar{\theta}(\bar{\theta} - 2kq_n)\right]}$). We can therefore conclude that the solution to (15) *is* the national brand manufacturer's optimal $(w_n,\ w_s)$ when the retailer has an alternate source for the store brand, whether it is IH or SM. Also, from the above analysis, we conclude that at the equilibrium, the manufacturer is strictly better off supplying both products than if he only supplies the national brand, although the retailer profit is the same as her reservation profit. Moreover, from (16), we know that as the retailer's reservation profit (i.e., π_{r0}) increases, λ increases, and therefore both w_n and w_s decrease. That is, not surprisingly, the national brand manufacturer is forced to lower both wholesale prices as the retailer's reservation profit increases, and the wholesale prices are lower when the alternate source is IH than when the alternate source is SM. Furthermore, both of these sets of prices are lower than when the retailer does not have an alternate source, as expected. At these wholesale prices, the retailer earns the same profit as she would if she directly chose her alternate source as the store-brand supplier.

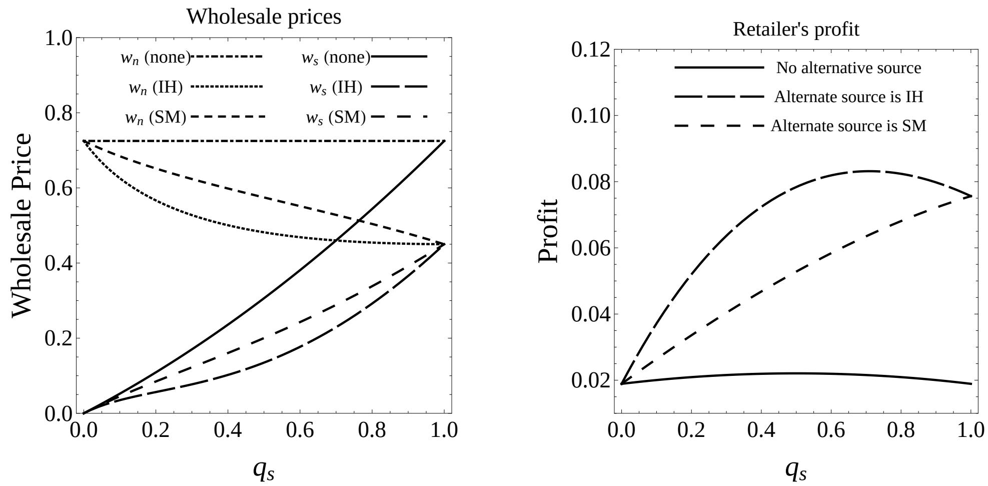
The analysis and discussion thus far in this section apply to all levels of store brand quality the retailer may choose. This includes the result that the retailer (when sourcing from NM while she has an alternate source) earns the same profit as she would if she directly chose her alternate source. Therefore, the optimal quality level the retailer chooses, under the modified scenario discussed in this section, equals the optimal quality level she would choose when using the alternate source.

COROLLARY 4. *Suppose the retailer sources both products from the national brand manufacturer while she has the option of using an alternate source, either IH or SM, to produce the store brand. Then the retailer makes the quality positioning decision in the same way as if she were securing the store brand from the alternate source.*

Although the retailer's optimal quality is the same as if she were sourcing the store brand from the alternate source, the equilibrium prices are **not** the same in the two settings. Taking the scenario in which the alternate source is IH as an example, when the retailer produces the store brand in-house, the "wholesale price" for the store brand is c_s and the wholesale price for the national brand is $w_{n0} = \frac{1}{2}c_n + \frac{1}{2}(\bar{\theta}q_n - (\bar{\theta}q_s - c_s))$. However, if the retailer were sourcing both products from the national brand manufacturer under threat of the retailer using IH, national brand manufacturer's wholesale prices would be $w_n = \frac{1}{2-\lambda}c_n + (1 - \frac{1}{2-\lambda})\bar{\theta}q_n$ and $w_s = \frac{1}{2-\lambda}c_s + (1 - \frac{1}{2-\lambda})\bar{\theta}q_s$. For any given value of q_s, w_s is greater than c_s and it is easy to show that w_n is less than w_{n0}.

Figure 4 shows the national brand manufacturer's pricing strategy and the retailer's equilibrium profit for different values of q_s in the presence and absence of an alternate source for the store brand. From the figure, we can observe that in the presence of an alternate source, the national brand manufacturer has an incentive to select the wholesale prices to secure his position as the supplier of both the national and store brands. These prices are chosen so as to make the retailer indifferent between the two sourcing options. The reason why the retailer chooses the same quality level under both scenarios can be explained as follows. Suppose the retailer's optimal quality level for the better alternate source is $\hat{q}$. The national brand manufacturer has selected wholesale prices to make the retailer indifferent between this option and NM, but the wholesale prices are different under the two scenarios. The question is whether the retailer can improve her profit by choosing $q \neq \hat{q}$ given the wholesale prices selected by NM. We claim that the retailer cannot do so. The logic is as follows. If the retailer could improve her profit by choosing $q \neq \hat{q}$ when sourcing the store brand from NM, then she could also improve her profit by choosing $q \neq \hat{q}$ when sourcing from her alternate source, as the retailer is indifferent between sourcing from NM and sourcing from her alternate source (for all values of store brand quality). But this contradicts the optimality of choosing $\hat{q}$ when the retailer is sourcing from her alternate source. In the proof of Corollary 4 (see Appendix I), we show that the retailer cannot increase her profit by deviating from $\hat{q}$ even though the wholesale prices are different under the two scenarios.

That the retailer is made indifferent between different sourcing structures under the modified scenario studied in this section partly explains why different retailers source store brand products in different ways, and a given retailer often sources the store brand differently for different categories of products. Indeed, the landscape of store-brand production has been changing. For example, Kroger has been attempting to purchase manufacturing capacity to produce more store-brand products in-house (Chen, 2010). Safeway, on the other hand, recently sold one of its manufacturing plants to a third-party manufacturer that will take over production of some of its store-brand products (Annie's, Inc., 2013). Moreover, while the store brand quality should be set differently under different sourcing structures when retailer does not have easy access to an alternate source, this is not true when she has such an option, in which case the retailer would set the quality level equal to the optimal quality level for the best of the available sourcing options.



(A) Manufacturer's pricing when the retailer does or does not have an alternate source (alternate source is shown in parentheses)

(B) Retailer's profit with and without an outside option

FIGURE 4. Manufacturer's pricing strategy and retailer's profit for all values of q_s ($\bar{\theta} = 1$, $q = 1$, $k = 0.45$)

5. Discussion

5.1. Outsourcing

In Section 3.4, we showed that retailers prefer producing store brands in-house over outsourcing production to outside parties, irrespective of the channel price leadership. However, as discussed in the Introduction, in practice, many retailers still outsource production of store brands to either large national brand manufacturers who also produce and offer competing products or to specialized store-brand manufacturers that have market power. One may ask why a retailer may choose to outsource production of its store brands, if producing them in-house is in her best interest. There are multiple answers to this question. First, as we showed in Section 4, when the retailer is asking the national brand manufacturer to produce both products under the implicit threat of producing the store brand in-house, the national brand manufacturer makes the retailer indifferent between IH and NM. In addition, there are several benefits of outsourcing not modeled in this paper that may explain a retailer's decision to outsource store brand production. These benefits have been identified in the extensive literature studying the operational advantages of a monopolist firm's outsourcing choice. For example, a firm may choose to outsource production if that leads to an overall reduction in transaction costs. Also, a firm may choose to outsource production to make use of the resources or knowledge possessed by a third-party. While we acknowledge the existence of these operational advantages, they are not the focus of this paper; we therefore refer the readers to Kroes and Ghosh (2010) for an extensive review.

There is also a stream of literature that investigates the benefit of outsourcing in competitive settings. These papers are related to our model to some extent, as we also study the benefit a retailer can derive from a preferable sourcing structure in its competitive interaction with a large national brand manufacturer. We briefly discuss main results from these papers and then relate these results to ours. In these papers, two firms compete in the same market, and each firm may choose either to produce its product in-house or outsource production. Cachon and Harker (2002) find that, in the presence of scale economies, outsourcing can benefit both firms by mitigating price competition between them, as outsourcing eliminates each firm's need to cut prices in order to enjoy the benefit of scale economies. In the absence of scale economies, Liu and Tyagi (2011) find that outsourcing also benefits firms by softening price competition between them. This happens when the product positioning decisions of the two firms are endogenized (i.e., each firm sets its location on a Hotelling line on which customers are uniformly distributed). With outsourcing, the two firms have more incentive to differentiate from each other so as to enjoy the benefit of a lowered wholesale price from the upstream supplier. Chen (2005) studies a game between two competing firms, Firm A and Firm B, where Firm A's upstream division supplies Firm A's downstream division, and it can also supply Firm B at a cost lower than what Firm B can obtain from its alternate source. Chen finds that Firm A has an incentive to disintegrate its upstream division from its downstream division. This is because the act of disintegrating implicitly commits to Firm B that Firm A's downstream division (with whom Firm B competes) cannot unilaterally enjoy the lower cost associated with scale economies at the upstream division. This implicit commitment gives Firm B more incentive to source from Firm A's upstream division compared to a scenario in which Firm A does not disintegrate.

Models in the research stream discussed above resemble our model to some extent. In our model, a retailer and NM compete, and the retailer might consider outsourcing production to SM. However, in the papers discussed above, both competing firms are assumed to be selling their products directly to consumers, whereas in our model, NM (i.e., one of the competing firms in our model) distributes its product through a retailer (i.e., the other competing firm in our model). Furthermore, in our model, the retailer carries another product and is considering whether to outsource its production to either NM or SM. In sum, there are considerable differences between the strategic interaction between a retailer and NM in the presence of a store brand (as modeled in this paper) and the strategic interaction between two competing firms downstream (as modeled in the stream of research investigating the benefit of outsourcing in competitive settings). This might explain why we obtain very different result from those in Cachon and Harker (2002), Liu and Tyagi (2011) and Chen (2005). Specifically, we find that, a retailer is worse off when she outsources production of her store brand to NM or SM than when she produces the store brand in-house.

5.2. Supply Chain Coordination

This section presents some comparisons of supply chain (SC) profit, i.e., the total profit of all parties in the supply chain, under the nine scenarios, Past literature suggests that a high quality store brand can be used as an implicit and successful mechanism for supply chain coordination (Corstjens and Lal 2000, Choi and Fredj 2013), when the quality level of the store brand is assumed exogenous. The results presented in this section complement the literature by investigating the effectiveness of store brand as a tool to coordinate the supply chain under different sourcing and price leadership scenarios when the retailer can choose the store brand quality level. We summarize our results below.

First, given a sourcing structure, SC profit under VN is greater than that under RS, which is greater than that under MS. Specifically, given a sourcing structure, if the quality of the store brand is fixed, SC profit under VN is greater than that under either RS or MS. Consequently, it is straightforward to show that SC profit under VN is also greater than that under RS or MS at their respective optimal store brand quality levels. The logic is as follows. SC profit is the same under MS and RS when the quality level of the store brand is fixed. But when the retailer can optimize her store brand quality, she chooses a quality level under MS which is higher than that under RS, whereas the optimal quality level that maximizes SC profit is lower than the quality she chooses under either MS or RS. Therefore, SC profit level under VN is greater than that under RS, which is greater than that under MS.

Second, given a game structure, when the quality level of the store brand is fixed, SC profit is the lowest under NM whereas the ordering between SC profit under SM and IH depends on the value of k. For small values of k, SC profit under SM is greater than that under IH (for all values of $q_s \in (0, q_n)$), whereas for large values of k, the ordering is reversed. This occurs because sourcing under SM introduces double marginalization on the store brand, which may hurt SC profit. But sourcing from SM may also increase SC profit as it increases the level of competition between NM and SM. The second effect leads to a reduction of NM's and SM's markups on their respective product, which increases SC profit. The second effect dominates when k is small. However, when the retailer chooses the optimal quality level of the store brand for each scenario, SC profit is always the greatest under IH and the lowest under NM. This is because, at the small values of k where SC profit is greater under SM than that under IH, the retailer sets q_s at q_n, which then makes SC profit the same under SM as that under IH.

Comparing the nine scenarios simultaneously, not surprisingly, SC profit is the highest under IH irrespective of the game structure. However, across the rest of the six scenarios involving SM and NM, it is not true that SC profit is higher under SM than under NM irrespective of the game structure. Specifically, for large values of k, SC profit is higher under VN+NM than it is under MS+SM. This is because, when k is large, the optimal quality level that maximizes SC profit under MS+SM is low, but the retailer, when optimizing her own objective, sets the highest possible quality level which has a significant adverse effect on SC profit, therefore, SC profit may be lower than that under VN+NM.

We conclude this section by highlighting that the supply chain is better coordinated when the store brand is produced in-house by the retailer, irrespective of the game structure. However, the supply chain is not always better coordinated when the store brand is sourced from SM than when it is sourced from NM. If the value of k is larger than a threshold, the supply chain is better coordinated when the store brand is sourced from NM under a desirable game structure for the supply chain (i.e., VN) than when it is sourced from SM under a less desirable game structure for the supply chain (i.e., MS).

6. Conclusions

In this paper, we study a retailer's equilibrium quality-positioning strategy under three sourcing structures, and for each sourcing structure, we consider three types of channel price leadership. The three sources are in-house (IH), a leading national brand manufacturer whose product the retailer also carries (NM), and a strategic third-party manufacturer (SM). The three types of channel price leadership are the ones most commonly seen in the literature: Manufacturer-Stackelberg (MS), Retailer-Stackelber (RS), and Vertical Nash (VN). Altogether, we examine nine combinations of sourcing and pricing power (or "game") scenarios, and compare the retailer's optimal quality positioning decision and other equilibrium results (including prices, retailer's profits, consumer welfare, and supply chain profits) across the nine scenarios. To the best of our knowledge, we are the first to study the interaction between store-brand sourcing and positioning decisions, and the interplay of these decisions with the retailer's pricing power; we are also the first to present a comparison of equilibria for the aforementioned nine realistic combinations of sourcing and pricing power in the store brand context.

We find that, when sourcing from the national brand manufacturer under the three game structures, the optimal quality level is the same for all three game structures, and both the quality and profit levels are the lowest among the nine combinations, respectively. The intuition is as follows. First, when sourcing the store brand from the national brand manufacturer, having a store brand product provides a retailer no additional leverage in dealing with the national brand manufacturer. (This is in contrast to the vast majority of the literature on store-brand introduction, which concludes that a retailer can use her store brand as a bargaining chip and thereby elicit price concessions from the national brand manufacturer.) Consequently, the retailer's only source of leverage provided by the store brand is product differentiation. As such, she chooses a low quality level that provides significant differentiation from the national brand product, which turns out to be lower than that in any of the other six scenarios. The differentiation leads to an increase in overall profit for the retailer (versus having no store brand), but the profit is still lower than when she sources the store brand from a strategic third-party or in-house.

Under the remaining six scenarios (i.e., under combinations involving in-house production or sourcing to a third-party manufacturer), the retailer uses the store-brand quality level more aggressively as a lever to (partially) compensate for any reduction in leverage arising from a less advantageous sourcing arrangement or weaker pricing power. The optimal store-brand quality has a monotonic mapping to the game-and-sourcing combinations, with the optimal quality increasing along the following path: RS+IH $\rightarrow$ VN+IH $\rightarrow$ MS+IH $\rightarrow$ RS+SM $\rightarrow$ VN+SM $\rightarrow$ MS+SM. However, an increase in the quality level only partially compensates for the retailer's reduction in leverage. Consequently, the retailer's profit levels under these six scenarios are in the reverse order of the optimal quality levels.

It is quite surprising that the retailer's optimal quality level is higher when she sources from a strategic third-party manufacturer than when she produces in-house, even if her pricing power differs between the two scenarios being compared. One might think that the introduction of double marginalization under production by the strategic third-party manufacturer would cause the retailer to compensate by lowering the quality (as she does when sourcing from the national brand manufacturer), thereby reducing the unit production cost incurred by the third-party manufacturer. Instead, increasing the quality level heats up the competition between the national brand manufacturer and the third-party manufacturer, which forces them to decrease their markups, and, as a result, the retailer earns a higher profit. Surprisingly, this effect leads to the retailer even to choose a higher quality level under SM when she has pricing power (i.e., under RS) than she does under IH when the national brand manufacturer is the Stackelberg leader (i.e., under MS).

The above results also imply that, at the optimal quality level for store brand corresponding to each scenario, the retailer strictly prefers a scenario with a more preferable sourcing structure, even if she is a price follower in this scenario but a leader in the other. This finding implies that, although past literature emphasizes how store brands give a retailer more pricing power versus the national brand manufacturer, a more preferable sourcing structure has greater value than a more favorable pricing power scenario to the retailer, i.e., the power to choose the source is more important than pricing power. One key reason for this result is that, sourcing from an outside party confers power over wholesale pricing even if that party is not the Stackelberg price leader. A less obvious reason is that sourcing from an outside party can cause the retailer to choose a lower quality for the store brand product than when it is produced in-house. It is important to emphasize, however, that this result holds only when the retailer chooses the corresponding optimal quality level for each scenario. If the store brand quality level if fixed, it is possible that the retailer prefers sourcing from a strategic third-party manufacturer under Retailer-Stackelberg over producing the store brand in-house under Manufacturer-Stackelberg.

We also provide a comparison of consumer welfare, equilibrium prices and supply chain coordination across the nine scenarios. We find that, given a game structure, consumers have the same ordering of preferences across the three store-brand sourcing structures as the retailer does (i.e., they prefer IH over SM over NM). When the retailer chooses the quality level optimally for each scenario, this preference ordering does not change even if the pricing power scenario changes across the scenarios being compared. In other words, for consumers, pricing power differences cannot compensate for sourcing differences. But this does not hold if the store brand quality level is fixed and equal across scenarios and is lower than a threshold. Regarding supply chain coordination, we find that, the supply chain is better coordinated (i.e., the supply chain profit is higher) when the store brand is produced in-house than when the production is outsourced, irrespective of the game structure. However, comparing the two arrangements involving outsourcing (i.e., SM and NM), differences in pricing power *can* compensate for differences between these two sourcing structures. This arises when the value of k is greater than a threshold.

One may ask why, in practice, a retailer may outsource store brand production when our model predicts otherwise. Note that the above result that the retailer prefers producing the store brand in house irrespective of the pricing power scenario relies on the retailer having the flexibility to choose or adjust the store brand quality. Retailers whose store brand quality level is fixed might still choose to outsource production to a strategic third-party or a national brand manufacturer. Apart from this, the literature studying firms' incentives for outsourcing identifies several operational benefits of outsourcing which we do not model in this paper. The most obvious is that in-house production incurs capital-related costs such as buildings and equipment in addition to fixed annual operating costs. Second, a firm may outsource production if outsourcing results in a reduction in the firm's size that leads to an overall reduction in transaction costs and/or if outsourcing brings in external resources or knowledge to the firm that can then become a firm's competitive advantage (Kroes and Ghosh 2010). However, the literature on outsourcing does not consider the specific setting in which store brands and national brands compete. In this paper, we show that, in the absence of overriding strategic considerations of core competence and transaction costs, or significant fixed costs, the retailer is strictly better off producing the store brand in-house.

We also study two variants of the game between the national brand manufacturer and the retailer. In the two variants, the retailer is asking the national brand manufacturer to produce both products under the implicit threat of using an alternate source, either in-house or a strategic third-party manufacturer, to produce the store brand product. We find that, when the national brand manufacturer supplies both products, he can do at least as well as when the retailer uses the alternate source for the store brand. Consequently, at the equilibrium, the national brand manufacturer lowers his wholesale prices on both the store and the national brands below what he would have charged in the absence of an alternate source. The national brand manufacture makes the retailer indifferent between the two sourcing options for the store brand product, and consequently is able to provide both products to the retailer. This result partly explains why different retailers source store brand products in different ways, and a given retailer often sources the store brand differently for different categories of products. This result also provides an explanation for why retailers may choose to outsource store-brand production even though our base model predicts that they prefer otherwise.

An interesting direction for further research would be to study whether the retailer's preference regarding store-brand sourcing may differ in the presence of retail competition. Past literature on outsourcing suggests that, in the absence of store-brand products. a firm may have an incentive to outsource production due to considerations of softening competition (Cachon and Harker 2002, Liu and Tyagi 2011, Chen 2005, Wu and Zhang 2014). Whether such an effect carries over to the context in which two retailers compete (each carrying the same national brand and a store brand of her own) might be an interesting line of research. It might be also interesting to study whether the main results in this paper apply under other contract forms between the retailer and the manufacturers or if the retailer and the manufacturers engage in a bargaining game.

APPENDICES

Appendix A: Proof of Proposition 1

As derived in the main text, $q_s^{IH} = \min\{q_n,\, f(k)\}$ where $f(k) \equiv \frac{1}{9k}[kq_n + 6\bar{\theta} - (-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2)^{\frac{1}{2}}]$, $q_s^{SM} = q_n$ and $q_s^{NM} = \frac{q_n}{2}$. Because $\min\{q_n,\ f(k)\} \leq q_n$, q_s^{IH} is less than or equal to q_s^{SM}. To show that it is strictly smaller than q_s^{SM} when $k \geq \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$, we first note that $f(k)$ is decreasing in k because $f'(k) = 2kq_n + 3\bar{\theta} - 2\sqrt{-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2} = -\frac{4kq_n(\bar{\theta}-kq_n)+3\bar{\theta}^2}{2kq_n+3\bar{\theta}+2\sqrt{-8k^2q_n^2+12kq_n\bar{\theta}+9\bar{\theta}^2}} < 0$. Second, $f(k) = q_n$ at $k = \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$. This can be easily verified by replacing k by $\frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$ in the expression for $f(k)$. These two properties together imply that $f(k) < q_n$ for $k \geq \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$. Because $q_s^{IH} = f(k)$ for k in this range, and $q_s^{SM} = q_n$, this proves that q_s^{IH} is strictly smaller than q_s^{SM} for $k \geq \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$. What remains to be shown is that $q_s^{NM} < q_s^{IH}$. Because $f(k)$ is decreasing in k, the minimum value of q_s^{IH} is $f(k)|_{k=\frac{\bar{\theta}}{2q_n}} = \frac{13-2\sqrt{13}}{9}q_n \approx 0.643q_n$, which is greater than $\frac{q_n}{2}$. Therefore, $q_s^{NM} < q_s^{IH}$.

Appendix B: Proof of Lemma 1

The optimal quality level for the store brand in a centralized supply chain is the solution to $\max_{q_s, p_n, p_s} \pi_0(q_s,\ p_n,\ p_s) \equiv D_n(p_n,\ p_s;\ q_s)(p_n - c_n) + D_s(p_n,\ p_s;\ q_s)(p_s - C(q_s))$. Substituting the expressions for $D_n(p_n,\ p_s;\ q_s)$ and $D_s(p_n,\ p_s;\ q_s)$ into the objective, and deriving the first order necessary conditions, we find that $(q_s^*,\ p_n^*,\ p_s^*) = (\frac{q_n}{2},\ \frac{1}{2}(c_n + \bar{\theta}q_n),\ \frac{1}{2}(C(\frac{q_n}{2}) + \bar{\theta}\frac{q_n}{2}))$ is the unique solution to the first-order necessary conditions. It can be easily verified that the Jacobian matrix of π_0 evaluated at $(q_s^*,\ p_n^*,\ p_s^*)$ is negative definite. Therefore $(q_s^*,\ p_n^*,\ p_s^*)$ *is* the solution maximizing π_0.

Appendix C: Proof of Proposition 2

First, we show that SM's markup is decreasing in q_s for $q_s \in [\frac{7}{13}q_n,\ q_n]$. Now $\frac{d\ w_s^{SM}(q_s)-C(q_s)}{d\ q_s}$ $=$ $\frac{2k(2q_n^3-8q_n^2q_s+7q_nq_s^2-q_s^3)+(4q_n^2-8q_nq_s+q_s^2)\bar{\theta}}{(q_s-4q_n)^2}$. Therefore we only need to show that the numerator of this expression, which we call $f(q_s)$, is negative for $q_s \in [\frac{7}{13}q_n,\ q_n]$. Observe that $f'(q_s) = 2k(-8q_n^2+14q_nq_s-3q_s^2)+(-8q_n+2q_s)\bar{\theta}$ is increasing in q_s. Moreover, $f'(q_n) = 6q_n(kq_n-\bar{\theta}) < 0$. Therefore, $f'(q_s) < 0$ for all q_s, so $f(q_s)$ is decreasing in q_s. Therefore we only need to show that $f(q_s) < 0$ at $q_s = \frac{7}{13}q_n$. But $f(q_s)|_{q_s=\frac{7}{13}q_n} = -\frac{3q_n^2(636kq_n+13\bar{\theta})}{2197}$, which is negative. This shows that $w_s^{SM}(q_s) - C(q_s)$ is decreasing in q_s on $q_s \in [\frac{7}{13}q_n,\ q_n]$. Second, we demonstrate that NM's incremental markup under SM above and beyond his markup under IH, is decreasing in q_s for $q_s \in [\frac{7}{13}q_n,\ q_n]$. Notice that $\frac{d\ w_n^{SM}(q_s)-w_n^{IH}(q_s)}{d\ q_s} = \frac{f(q_s)}{2(q_s-4q_n)^2}$ where $f(q_s)$ was defined earlier in this proof. Because $f(q_s) < 0$ for $q_s \in [\frac{7}{13}q_n,\ q_n]$, $w_n^{SM}(q_s) - w_n^{IH}(q_s)$ is decreasing in q_s on $q_s \in [\frac{7}{13}q_n,\ q_n]$. What remains to be shown is that $q_s^{IH} > \frac{7}{13}q_n$. This is true because the minimum value of q_s^{IH} is $f(k)|_{k=\frac{\bar{\theta}}{2q_n}} = \frac{13-2\sqrt{13}}{9}q_n \approx 0.643q_n$, which is greater than $\frac{7}{13}q_n \approx 0.538q_n$.

Appendix D: Proof of Proposition 3

We first establish the orderings among the wholesale prices and among the retail margins for the national brand product under the three sourcing structures for a given q_s. We have derived the following equilibrium wholesale prices in the main text: $w_n^{SM}(q_s) = \frac{2q_n}{4q_n-q_s}(\bar{\theta}q_n - \bar{\theta}q_s + c_n + \frac{c_s}{2})$, $w_n^{IH}(q_s) = \frac{1}{2}(\bar{\theta}q_n - \bar{\theta}q_s + c_n + c_s)$ and $w_n^{NM}(q_s) = \frac{1}{2}(\bar{\theta}q_n + c_n)$. We seek to show that $w_n^{IH}(q_s) < w_n^{SM}(q_s) < w_n^{NM}(q_s)$ for $\forall q_s \in (0,\ q_n)$. The first inequality is true because $w_n^{SM}(q_s) - w_n^{IH}(q_s) = \frac{q_s(q_n-q_s)[k(q_n-q_s)+\bar{\theta}]}{8q_n-2q_s} > 0$ for all $q_s \in (0,\ q_n)$, and the second inequality holds because $w_n^{SM}(q_s) - w_n^{NM}(q_s) = \frac{q_nq_s[k(q_n+2q_s)-3\bar{\theta}]}{8q_n-2q_s} < 0$ for all $q_s \in (0,\ q_n)$. Under all three sourcing structures, the retailer's profit margin is $m_n^x = \frac{1}{2}(w_n^x + \bar{\theta}q_n) - w_n^x = \frac{1}{2}(\bar{\theta}q_n - w_n^x)$ $(x \in \{IH,\ SM,\ NM\})$, which is declining in the wholesale price. Therefore, $m_n^{IH}(q_s) > m_n^{SM}(q_s) > m_n^{NM}(q_s)$ for $\forall q_s \in (0,\ q_n)$.

We now establish the ordering among the national brand wholesale prices and among the retail margins under the three sourcing structures at the corresponding optimal quality levels. First, we note $w_n^{IH}(q_n) = w_n^{SM}(q_n) = c_n$ and $w_n^{IH}(q_s)$,and $w_n^{SM}(q_s)$ are both strictly decreasing in q_s. Together with the fact that $q_s^{SM} = q_n$ and $q_s^{IH} \le q_n$, this implies that $w_n^{IH}(q_s^{IH}) \ge w_n^{SM}(q_s^{SM})$. Second, observe that $w_n^{NM}(0) = w_n^{IH}(0)$ and $w_n^{NM}(q_s)$ are invariant in q_s. This, combined with the fact that $w_n^{IH}(q_s)$ is strictly decreasing in q_s, implies that $w_n^{NM}(q_s^{NM}) > w_n^{IH}(q_s^{IH})$. Therefore, $w_n^{NM}(q_s^{NM}) > w_n^{IH}(q_s^{IH}) \ge w_n^{SM}(q_s^{SM})$. Because the retailer's profit margin is declining in the wholesale price for all three sourcing structures, this also implies that $m_n^{SM}(q_s^{SM}) \ge m_n^{IH}(q_s^{IH}) > m_n^{NM}(q_s^{NM})$.

Next, we characterize the ordering among the wholesale prices and among the retail margins for the store brand for three sourcing structures for a given q_s. We have shown in the main text of the paper that $w_s^{SM}(q_s) = \frac{q_s}{4q_n - q_s}(\bar{\theta}q_n - \bar{\theta}q_s + c_n + \frac{c_s}{2}) + \frac{c_s}{2}$, $w_s^{IH}(q_s) = c_s$ and $w_s^{NM}(q_s) = \frac{1}{2}(\bar{\theta}q_s + c_s)$. So $w_s^{SM}(q_s) - w_s^{IH}(q_s) = \frac{q_s(q_n - q_s)[k(q_n - q_s) + \bar{\theta}]}{4q_n - q_s} > 0$ and $w_s^{SM}(q_s) - w_s^{NM}(q_s) = \frac{q_s[k(2q_n^2 + q_s^2) - \bar{\theta}(2q_n + q_s)]}{8q_n - 2q_s} < 0$ for all $q_s \in (0,\ q_n)$. This proves that $w_s^{IH}(q_s) < w_s^{SM}(q_s) < w_s^{NM}(q_s)$. Under all three sourcing structures, the retailer's profit margin is declining in the wholesale price. Therefore, $m_s^{CF}(q_s) > m_s^{SM}(q_s) > m_s^{NM}(q_s)$.

We now analyze the ordering among store brand wholesale prices for the three sourcing structures at the respective optimal quality levels . Because $w_s^{SM}(q_s) > w_s^{IH}(q_s)\ \forall q_s < q_n$, $w_s^{SM}(q_s)$ is increasing in q_s and $q_s^{SM} \ge q_s^{IH}$, we know that $w_s^{SM}(q_s^{SM}) \ge w_s^{SM}(q_s^{IH}) \ge w_s^{IH}(q_s^{IH})$. We can also establish relationships among the profit margins. Because $m_s^{IH}(q_s^{IH}) = \frac{1}{2}(\bar{\theta}q_s^{IH} - C(q_s^{IH}))$ and $m_s^{SM}(q_s^{SM}) = \frac{1}{2}(\bar{\theta}q_n - C(q_n))$, we have $m_s^{SM}(q_s^{SM}) \ge m_s^{IH}(q_s^{IH})$. Moreover, $m_s^{NM}(q_s^{NM}) = \frac{1}{2}(\bar{\theta}q_s^{NM} - \frac{1}{2}(\bar{\theta}q_s^{NM} + C(q_s^{NM}))) = \frac{1}{4}(\frac{1}{2}\bar{\theta}q_n - \frac{1}{4}kq_n^2)$, which, after tedious mathematical manipulations, can be shown to be smaller than $m_s^{IH}(q_s^{IH})$.

Finally, we establish the statement regarding relationships among the retailer'r profit under the three sourcing structures. For $\forall q_s \in (0,\ q_n]$, $w_n^{IH}(q_s) \le w_n^{SM}(q_s) < w_n^{NM}(q_s)$ and $w_s^{IH}(q_s) \le w_s^{SM}(q_s) < w_s^{NM}(q_s)$, do $\pi_r^{IH}(q_s) \ge \pi_r^{SM}(q_s) > \pi_r^{NM}(q_s)$. This also implies that $\max_{q_s} \pi_r^{IH}(q_s) \ge \max_{q_s} \pi_r^{SM}(q_s) > \max_{q_s} \pi_r^{NM}(q_s)$, which proves the statement. All of the above inequalities hold strictly for $k > \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$ because $q_s^{IH} < q_s^{SM}$ for $k > \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n}$.

Appendix E: Proof of Proposition 4

In this proof, we derive the optimal quality level for the store brand under each of the sourcing structures by backward induction.

E.1. Producing the store brand IH under RS

E.1.1. Manufacturer's wholesale pricing problem

Given the retailer's margin, m_n, on the national brand, the retail price, p_s, on the store brand, NM's problem is:

$$\max_{w_n}\ (w_n - c_n)D_n(w_n + m_n,\ p_s;\ q_s) \tag{17}$$

For $m_n \le \bar{v}_n - c_n - (\bar{v}_s - p_s)$, from the first-order necessary conditions and second-order sufficient conditions, we obtain $w_n^*(m_n,\ p_s;\ q_s) = \frac{1}{2}\left[(\bar{\theta}q_n - m_n) + c_n - (\bar{v}_s - p_s)\right]$. Otherwise, it is not profitable for the national brand manufacturer to offer the national brand product to the retailer).

E.1.2 Retailer's retail pricing

Given a quality level for the store brand, the retailer solves the following problem anticipating the national brand manufacturer's response.

$$\max_{m_n,\ p_s} m_n D_n\left(w_n^*(m_n, p_s; q_s) + m_n, p_s; q_s\right) + (p_s - c_s) D_s\left(w_n^*(m_n, p_s; q_s) + m_n, p_s; q_s\right) \quad (18)$$

From the first-order necessary conditions for the objective in (18) and the corresponding second-order sufficient conditions, we obtain:

$$\begin{cases} m_n^*(q_s) & = \frac{1}{2}(\bar{\theta} q_n - c_n) \\ p_s^*(q_s) & = \frac{1}{2}(\bar{\theta} q_s + c_s) \end{cases} \quad (19)$$

Substituting for $m_n^*(q_s)$ and $p_s^*(q_s)$ in the expression for the national brand manufacturer's optimal wholesale price given a q_s (cf. Section E.1.1), we obtain:

$$\begin{aligned} w_n^*(m_n^*(q_s),\ p_s^*(q_s);\ q_s) &= \frac{1}{2}\left[(\bar{\theta} q_n - (\frac{1}{2}(\bar{\theta} q_n - c_n))) + c_n - (\bar{\theta} q_s - (\frac{1}{2}(\bar{\theta} q_s + c_s)))\right] \\ &= \frac{1}{4}(3c_n + c_s + q_n\bar{\theta} - q_s\bar{\theta}) \end{aligned}$$

With appropriate substitutions, we obtain the equilibrium retail price for the national brand as $p_n^*(q_s) \equiv w_n^*(m_n^*(q_s),\ p_s^*(q_s),\ q_s) + m_n^*(q_s) = \frac{1}{4}(c_n + c_s + 3q_n\bar{\theta} - q_s\bar{\theta})$.

E.1.3 Retailer's quality-setting problem

Substituting for $w_n^*(q_s)$ and $m_n^*(q_s)$ (as derived in Section E.1.2) in (18), the retailer's problem becomes

$$\max_{q_s} \frac{1}{2}(\bar{\theta} q_n - c_n) D_n\left(p_n^*(q_s), p_s^*(q_s); q_s\right) + (p_s^*(q_s) - c_s) D_s\left(p_n^*(q_s), p_s^*(q_s); q_s\right) \quad (20)$$

The first-order necessary condition for the above objective function is

$$\frac{c_n^2 + 2c_n k q_s(q_s - 2q_n) + k^2 q_s^2(6q_n^2 - 8q_n q_s + 3q_s^2) - 4k(q_n - q_s)^2 q_s \bar{\theta} + (q_n - q_s)^2 \bar{\theta}^2}{8(q_n - q_s)^2 \bar{\theta}} = 0 \quad (21)$$

Substituting kq_n^2 for c_n, (21) simplifies to

$$\frac{3k^2 q_s^2 - 2(k^2 q_n + 2k\bar{\theta}) q_s + k^2 q_n^2 + \bar{\theta}^2}{8\bar{\theta}} = 0 \quad (22)$$

Solving the above equality, we get $q_s^* = \frac{1}{3k^2}\left[k^2 q_n + 2k\bar{\theta} - \sqrt{-2k^4 q_n^2 + 4k^3 q_n \bar{\theta} + k^2\bar{\theta}^2}\right]$. We can easily verify that the second derivative of the retailer's profit function at q_s^* is negative. Therefore, if the retailer produces the store brand IH under RS, the optimal quality level of the store brand is: $\min\{\frac{1}{3k^2}[k^2 q_n + 2k\bar{\theta} - \sqrt{-2k^4 q_n^2 + 4k^3 q_n \bar{\theta} + k^2\bar{\theta}^2}],\ q_n\}$.

E.2. Sourcing SB from a strategic third-party manufacturer under RS

E.2.1 Manufacturers' wholesale pricing

Given the retailer's margins, m_n on the national brand and m_s on the store brand, as well as SM's wholesale price w_s, NM's problem is

$$\max_{w_n} (w_n - c_n) D_n(w_n + m_n,\ w_s + m_s;\ q_s) \quad (23)$$

From the first-order necessary and second-order sufficient conditions, we obtain $w_n^*(m_n, m_s, w_s; q_s) = \frac{1}{2}\left[(\bar{\theta}q_n - m_n) + c_n - (\bar{v}_s - m_s - w_s)\right]$ if $m_n \leq \bar{v}_n - c_n - (\bar{v}_s - m_s - w_s)$. Otherwise, it is not profitable for the national brand manufacturer to offer his product to the retailer.

The third-party manufacturer's problem is:

$$\max_{w_s} \; (w_s - c_s)D_s(w_n + m_n, \; w_s + m_s; \; q_s) \tag{24}$$

From the first-order necessary and second-order sufficient conditions, we obtain $w_s^*(m_n, m_s, w_n; q_s) = \frac{1}{2}\left[c_s + \frac{w_n + m_n}{q_n} q_s - m_s\right]$. The equilibrium wholesale prices can be obtained by simultaneously solving NM and SM's response functions:

$$\begin{cases} w_n = \frac{1}{2}\left[(\bar{\theta}q_n - m_n) + c_n - (\bar{\theta}q_s - m_s - w_s)\right] \\ w_s = \frac{1}{2}\left[c_s + \frac{w_n + m_n}{q_n} q_s - m_s\right] \end{cases} \tag{25}$$

The equilibrium is:

$$\begin{cases} w_n^*(m_n, \; m_s; \; q_s) = & \frac{2q_n(c_n - m_n) + q_n(c_s + m_s + 2q_n\bar{\theta} - 2q_s\bar{\theta}) + m_n q_s}{4q_n - q_s} \\ w_s^*(m_n, \; m_s; \; q_s) = & \frac{2q_n(c_s - m_s) + q_s(c_n + m_n + m_s + q_n\bar{\theta} - q_s\bar{\theta})}{4q_n - q_s} \end{cases} \tag{26}$$

E.2.2 Retailer's pricing problem

Given the two manufacturers' wholesale pricing strategies derived above, the retailer's pricing problem is:

$$\begin{aligned} \max_{m_n, \; m_s} \; & m_n D_n\left(w_n^*(m_n, m_s; q_s) + m_n, w_s^*(m_n, m_s, q_s) + m_s; q_s\right) \\ & + m_s D_s\left(w_n^*(m_n, m_s; q_s) + m_n, w_s^*(m_n, m_s, q_s) + m_s; q_s\right) \end{aligned} \tag{27}$$

From the first-order necessary and second-order sufficient conditions, we obtain the optimal margins, m_n and m_s, given q_s:

$$\begin{cases} m_n^*(q_s) = & \frac{1}{2}(\bar{\theta}q_n - c_n) \\ m_s^*(q_s) = & \frac{1}{2}(\bar{\theta}q_s - c_s(q_s)) \end{cases} \tag{28}$$

E.2.3 Retailer's quality-setting problem

Substituting for $m_n^*(q_s)$ and $m_s^*(q_s)$ in the retailer's objective (see Section E.1.2), the retailer's quality-setting problem becomes:

$$\begin{aligned} \max_{q_s} \quad & m_n^*(q_s) D_n\left(w_n^*(m_n^*(q_s), \; m_s^*(q_s), \; q_s) + m_n^*(q_s), \; w_s^*(m_n^*(q_s), \; m_s^*(q_s), \; q_s) + m_s^*(q_s)\right) \\ & + m_s^*(q_s) D_s\left(w_n^*(m_n^*(q_s), \; m_s^*(q_s), \; q_s) + m_n^*(q_s), \; w_s^*(m_n^*(q_s), \; m_s^*(q_s), \; q_s) + m_s^*(q_s)\right) \end{aligned} \tag{29}$$

The first derivative of the retailer's profit with respect to q_s is:

$$\frac{q_n}{4(4q_n - q_s)^2\bar{\theta}}\left[-2k^2q_s^3 + (13k^2q_n + 2k\bar{\theta})q_s^2 - (8k^2q_n^2 + 16kq_n\bar{\theta})q_s + (6k^2q_n^3 - 4kq_n^2\bar{\theta} + 6q_n\bar{\theta}^2)\right] \tag{30}$$

and the second derivative is $\frac{-2kq_n}{4(4q_n - q_s)\bar{\theta}}[k(q_n - 3q_s) + 2\bar{\theta}]$. Because $\bar{\theta} \geq 2kq_n$, the second derivative is negative on $q_s \in [0, \; q_n]$ (so the retailer's profit function is concave in q_s). Therefore the first derivative is decreasing in q_s. When $q_s = q_n$, the first derivative takes its smallest value, which is equal to $\frac{3q_n^2}{4(4q_n - q_s)^2\bar{\theta}}[(3k^2{q_n}^2 - 6kq_n\bar{\theta} + 2\bar{\theta}^2)] = \frac{9q_n^2}{4(4q_n - q_s)^2\bar{\theta}}[(\bar{\theta} - kq_n)^2 - \frac{\bar{\theta}^2}{3}]$, which is positive if $k \leq \frac{\bar{\theta}}{q_n} \cdot \frac{3-\sqrt{3}}{3} \approx 0.42\frac{\bar{\theta}}{q_n}$. This implies that the optimal quality level is q_n under these conditions. On the other hand, if $k \in [\frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n},\ \frac{\bar{\theta}}{2q_n}]$, the value of the first derivative of retailer's profit function is negative at $q_s = q_n$. Together with the concavity of the retailer's profit function, this implies that the optimal store brand quality is less than q_n.

E.3. Sourcing SB from NM under RS

E.3.1 Manufacturer's wholesale pricing problem

When the retailer sources its store brand from NM under RS, NM solves the following problem given m_n and m_s.

$$\max_{w_n} (w_n - c_n)D_n(w_n + m_n,\ w_s + m_s;\ q_s) + (w_s - c_s)D_s(w_n + m_n,\ w_s + m_s;\ q_s) \tag{31}$$

From the first-order necessary and second-order sufficient condition, we obtain the optimal solution:

$$\begin{cases} w_n(m_n,\ m_s;\ q_s) & = \frac{1}{2}(c_n + \bar{\theta}q_n - m_n) \\ w_s(m_n,\ m_s;\ q_s) & = \frac{1}{2}(c_s + \bar{\theta}q_s - m_s) \end{cases} \tag{32}$$

E.3.2 Retailer's pricing problem

Given the wholesale prices, the retailer solves the following problem for a given q_s:

$$\begin{aligned} \max_{m_n,\ m_s}\ & m_n D_n\left(w_n^*(m_n,\ m_s;\ q_s) + m_n,\ w_s^*(m_n,\ m_s,\ q_s) + m_s;\ q_s\right) \\ & + m_s D_s\left(w_n^*(m_n,\ m_s;\ q_s) + m_n,\ w_s^*(m_n,\ m_s,\ q_s) + m_s;\ q_s\right) \end{aligned} \tag{33}$$

From the first order necessary and second order sufficient conditions, we obtain:

$$\begin{cases} m_n^*(q_s) & = \frac{1}{2}(\bar{\theta}q_n - c_n) \\ m_s^*(q_s) & = \frac{1}{2}(\bar{\theta}q_s - c_s) \end{cases} \tag{34}$$

E.3.3 Retailer's quality-setting problem

Finally, the retailer solves the following problem to optimize the store brand quality:

$$\begin{aligned} \max_{q_s}\ & m_n^*(q_s)D_n\left(w_n^*(m_n^*(q_s),\ m_s^*(q_s),\ q_s) + m_n^*(q_s),\ w_s^*(m_n^*(q_s),\ m_s^*(q_s),\ q_s) + m_s^*(q_s)\right) \\ & + m_s^*(q_s)D_s\left(w_n^*(m_n^*(q_s),\ m_s^*(q_s),\ q_s) + m_n^*(q_s),\ w_s^*(m_n^*(q_s),\ m_s^*(q_s),\ q_s) + m_s^*(q_s)\right) \end{aligned} \tag{35}$$

The first-order necessary condition for the retailer's objective is:

$$\frac{c_n^2 + k^2 q_n(3q_n - 2q_s)q_s^2 + 2c_n k q_s(q_s - 2q_n)}{8\bar{\theta}(q_n - q_s)^2} = 0 \tag{36}$$

Substituting kq_n^2 for c_n, the first-order necessary condition simplifies to

$$\frac{k^2 q_n(q_n - 2q_s)}{8\bar{\theta}} = 0 \tag{37}$$

Solving the above equation, we obtain $q_s^* = \frac{q_n}{2}$. It is straightforward to show that second derivative of the retailer's profit is negative on $q_s \in [0,\ q_n]$ Therefore, under RS, the optimal quality level for store brand when the retailer sources its store brand from NM is $\frac{q_n}{2}$, which is the same as her optimal quality choice when she sources the store brand from NM under MS.

Appendix F: Proof of Proposition 5

In this proof, we derive the optimal quality level for the store brand under each of the sourcing structures using backward induction.

F.1. Producing the store brand IH under VN

For any q_s and w_n, the retailer's pricing problem is:

$$\max_{m_n, p_s} m_n D_n(m_n + w_n,\ p_s;\ q_s) + (p_s - c_s) D_s(m_n + w_n,\ p_s;\ q_s) \tag{38}$$

From the first-order neccessary and second-order sufficient conditions, we obtain the optimal solution as $m_n(w_n;\ q_s) = \frac{1}{2}(\bar{\theta} q_n - w_n)$ and $p_s(q_s) = \frac{1}{2}(\bar{\theta} q_s + c_s)$ if the following two conditions hold: $w_n \leq c_s + \bar{\theta}(q_n - q_s)$ and $\frac{w_n - c_s}{q_n - q_s} \geq \frac{c_s}{q_s}$. If the first condition does not hold, the retailer finds it unprofitable to sell the national brand, and if the second condition does not hold, the retailer finds it unprofitable to sell the store brand.

Given m_n and p_s, NM's pricing problem is

$$\max_{w_n} (w_n - c_n) D_n(m_n + w_n,\ p_s;\ q_s) \tag{39}$$

From the first-order necessary and second-order sufficient conditions, we obtain $w_n(m_n,\ p_s;\ q_s) = \frac{1}{2}[c_n - m_n + p_s + \bar{\theta} q_n - \bar{\theta} q_s]$. We solve the two following equations simultaneously

$$\begin{cases} m_n & = \frac{1}{2}(\bar{\theta} q_n - w_n) \\ w_n & = \frac{1}{2}[c_n - m_n + p_s + \bar{\theta} q_n - \bar{\theta} q_s] \end{cases}$$

to obtain the equilibrium wholesale price for the national brand: $m_n^*(q_s) = \frac{1}{6}[2(\bar{\theta} q_n - c_n) + (\bar{\theta} q_s - c_s)]$ which implies that $w_n^*(q_s) = \frac{1}{3}[\bar{\theta} q_n + 2c_n - (\bar{\theta} q_s - c_s)]$.

The retailer's quality-setting problem is therefore

$$\max_{q_s} m_n^*(q_s) D_n(m_n^*(q_s) + w_n^*(q_s),\ p_s^*; q_s) + (p_s^*(q_s) - c_s) D_s(m_n^*(q_s) + w_n^*(q_s),\ p_s^*(q_s)) \tag{40}$$

The first-order necessary condition is

$$\frac{1}{36\bar{\theta}}[15k^2 q_s^2 - \left(8k^2 q_n + 20k\bar{\theta}\right) q_s + \left(4k^2 q_n^2 + 5\bar{\theta}^2\right)] = 0 \tag{41}$$

Solving this, we get $q_s^* = \frac{1}{15k}\left[4k q_n + 10\bar{\theta} - \sqrt{-44k^2 q_n^2 + 80k q_n \bar{\theta} + 25\bar{\theta}^2}\right]$. It is straightforward to confirm that the retailer's objective is concave, so the optimal quality level for the store brand is $q_s = \min\{q_n,\ \frac{1}{15k}\left[4k q_n + 10\bar{\theta} - \sqrt{-44k^2 q_n^2 + 80k q_n \bar{\theta} + 25\bar{\theta}^2}\right]\}$.

F.2. Sourcing SB from a strategic third-party manufacturer under VN

For any given q_s, w_n and w_s, the retailer's pricing problem is:

$$\max_{m_n, m_s} \; m_n D_n(m_n + w_n,\; m_s + w_s;\; q_s) + m_s D_s(m_n + w_n,\; m_s + w_s;\; q_s) \tag{42}$$

From the first-order necessary and second-order sufficient conditions, we obtain $m_n(w_n,\; w_s;\; q_s) = \frac{1}{2}(\bar{\theta} q_n - w_n)$ and $m_s(w_n,\; w_s;\; q_s) = \frac{1}{2}(\bar{\theta} q_s - w_s)$.

For given q_s, m_n, m_s and w_s, the national brand manufacturer's problem is:

$$\max_{w_n} \; (w_n - c_n) D_n(m_n + w_n,\; m_s + w_s;\; q_s) \tag{43}$$

From the first-order necessary and second-order sufficient conditions, we obtain $w_n(m_n,\; m_s,\; w_s; q_s) = \frac{1}{2}[c_n - m_n + m_s + w_s + \bar{\theta} q_n - \bar{\theta} q_s]$.

SM's problem is:

$$\max_{w_s} \; (w_s - c_s) D_s(m_n + w_n,\; m_s + w_s;\; q_s) \tag{44}$$

From the first order necessary and second order sufficient conditions, we obtain $w_s(m_n,\; m_s,\; w_n;\; q_s) = \frac{1}{2}[c_s - m_s + \frac{q_s}{q_n}(m_n + w_n)]$.

The following equations, which constitute the three parties' optimal responses, must be solved simultaneously to find the equilibrium.

$$\begin{cases} m_n = \frac{1}{2}(\bar{\theta} q_n - w_n) \\ m_s = \frac{1}{2}(\bar{\theta} q_s - w_s) \\ w_n = \frac{1}{2}[c_n - m_n + m_s + w_s + \bar{\theta} q_n - \bar{\theta} q_s] \\ w_s = \frac{1}{2}[c_s - m_s + \frac{q_s}{q_n}(m_n + w_n)] \end{cases}$$

We obtain the equilibrium retail margins and wholesale prices as:

$$\begin{cases} m_n^*(q_s) = & \frac{q_n}{9q_n - q_s}\left[3q_n\bar{\theta} + q_s\bar{\theta} - 3c_n - c_s\right] \\ m_s^*(q_s) = & \frac{q_n}{9q_n - q_s}\left[4q_s\bar{\theta} - 3c_s - c_n\frac{q_s}{q_n}\right] \\ w_n^*(q_s) = & \frac{q_n}{9q_n - q_s}\left[6c_n + 2c_s + 3q_n\bar{\theta} - 3q_s\bar{\theta}\right] \\ w_s^*(q_s) = & \frac{q_n}{9q_n - q_s}\left[6c_s + \frac{q_s}{q_n}\left[2c_n + q_n\bar{\theta} - q_s\bar{\theta}\right]\right] \end{cases}$$

The retailer's quality positioning problem therefore becomes:

$$\max_{q_s} \; m_n^*(q_s) D_n\left(m_n^*(q_s) + w_n^*(q_s), m_s^*(q_s) + w_s^*(q_s); q_s\right) + m_s^*(q_s) D_s\left(m_n^*(q_s) + w_n^*(q_s), m_s^*(q_s) + w_s^*(q_s); q_s\right)$$

The first derivative of the retailer's objective is

$$\begin{aligned}\pi_r'(q_s) =& \frac{1}{(9q_n - q_s)^3\bar{\theta}}[-5k^2q_n^2q_s^3 + 135k^2q_n^3q_s^2 + (-32k^2q_n^4 - 218kq_n^3\bar{\theta} + 7q_n^2\bar{\theta}^2)q_s \\ &+ (54k^2q_n^5 - 54kq_n^4\bar{\theta} + 81q_n^3\bar{\theta}^2)]\end{aligned} \tag{45}$$

The second derivative is

$$\pi_r''(q_s) = \frac{-2q_n^2(7kq_n - \bar{\theta})}{(9q_n - q_s)^3\bar{\theta}}\left[(-169kq_n + 7\bar{\theta})q_s + (9kq_n^2 + 153\bar{\theta}q_n)\right] \tag{46}$$

If $k < \frac{\bar{\theta}}{7q_n}$, then $\pi_r''(q_s) > 0$ for all $q_s \in [0,\ q_n]$, which implies that $\pi_r'(q_s)$ is increasing on $q_s \in [0,\ q_n]$ and that $\pi_r(q_s)$ is strictly convex on $[0,\ q_n]$. We can easily confirm that $\pi_r'(0) > 0$. Therefore π_r is strictly increasing on $q_s \in [0,\ q_n]$, so $q_s^* = q_n$. On the other hand, if $k > \frac{\bar{\theta}}{7q_n}$, then $\pi_r''(q_s) < 0$ for all $q_s \in [0,\ q_n]$, which implies that $\pi_r'(q_s)$ is decreasing on $q_s \in [0,\ q_n]$ and that $\pi_r(q_s)$ is concave on $[0,\ q_n]$. Therefore, if $\pi_r'(q_n) \geq 0$, then $q_s^* = q_n$; and if $\pi_r'(q_n) < 0$, then $q_s^* < q_n$. Note that $\pi_r'(q_n) = \frac{1}{64\bar{\theta}}[19k^2 - 34kq_n\bar{\theta} + 11\bar{\theta}^2]$, which is greater or equal zero if $k \leq \frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n} \approx 0.424\frac{\bar{\theta}}{q_n}$. Therefore, when the retailer sources the store brand from SM under VN, the optimal quality for the store brand is q_n if $k \leq \frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n} \approx 0.424\frac{\bar{\theta}}{q_n}$. On the other hand, if $k \in [\frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n},\ \frac{\bar{\theta}}{2q_n}]$, then $\pi_r'(q_n) < 0$. Together with the concavity of $\pi_r(q_s)$ on $[0,\ q_n]$, this implies that the optimal store brand quality is less than q_n.

F.3. Sourcing the store brand from NM under VN

For any given q_s, w_n and w_s, the retailer's problem is:

$$\max_{m_n, m_s}\ m_n D_n(m_n + w_n,\ m_s + w_s;\ q_s) + m_s D_s(m_n + w_n,\ m_s + w_s;\ q_s) \tag{47}$$

From the first-order necessary and second-order sufficient conditions, we obtain $m_n(w_n,\ w_s;\ q_s) = \frac{1}{2}(\bar{\theta}q_n - w_n)$ and $m_s(w_n,\ w_s;\ q_s) = \frac{1}{2}(\bar{\theta}q_s - w_s)$.

Given m_n and m_s, NM's problem becomes:

$$\max_{w_n}\ (w_n - c_n)D_n(m_n + w_n,\ m_s + w_s;\ q_s) + (w_s - c_s)D_n(m_n + w_n,\ m_s + w_s;\ q_s) \tag{48}$$

From the first-order necessary and second-order sufficient conditions we obtain

$$\begin{cases} w_n &= \frac{1}{2}(c_n - m_n + q_n\bar{\theta}) \\ w_s &= \frac{1}{2}(c_s - m_s + q_s\bar{\theta}) \end{cases} \tag{49}$$

To identify the equilibrium prices, we need to solve the following equations simultaneously to ensure the prices and margins are mutually consistent:

$$\begin{cases} m_n &= \frac{1}{2}(\bar{\theta}q_n - w_n) \\ m_s &= \frac{1}{2}(\bar{\theta}q_s - w_s) \\ w_n &= \frac{1}{2}(c_n - m_n + q_n\bar{\theta}) \\ w_s &= \frac{1}{2}(c_s - m_s + q_s\bar{\theta}) \end{cases} \tag{50}$$

The solution is:

$$\begin{cases} m_n^*(q_s) &= \frac{1}{3}(\bar{\theta}q_n - c_n) \\ m_s^*(q_s) &= \frac{1}{3}(\bar{\theta}q_s - c_s) \\ w_n^*(q_s) &= \frac{1}{3}(\bar{\theta}q_n + 2c_n) \\ w_s^*(q_s) &= \frac{1}{3}(\bar{\theta}q_s + 2c_s) \end{cases} \tag{51}$$

Making appropriate substitutions, the retailer's quality positioning problem becomes:

$$\max_{q_s}\ m_n^*(q_s)D_n\left(m_n^*(q_s) + w_n^*(q_s),\ m_s^*(q_s) + w_s^*(q_s);\ q_s\right)$$
$$+ m_s^*(q_s)D_s\left(m_n^*(q_s) + w_n^*(q_s),\ m_s^*(q_s) + w_s^*(q_s);\ q_s\right)$$

The first derivative of the retailer's profit function above is

$$\pi_r'(q_s) = \frac{k^2 q_n}{9\bar{\theta}}(q_n - 2q_s) \tag{52}$$

The second derivative is negative. Therefore, if the retailer sources the store brand from NM under VN, the optimal quality level for the store brand is $\frac{1}{2}q_n$.

Appendix G: Proof of Proposition 6 and Corollary 1

In this proof, we first compare the optimal quality levels across the nine sourcing and pricing power scenarios. We first provide a detailed sketch of the proof; at the end of this section, we present the mathematical details.

To prove the proposition, we first show that when the retailer produces IH, we have $q_s^{RS} \leq q_s^{VN} \leq q_s^{MS}$. Then, we show that when the retailer sources from SM, $q_s^{RS} \leq q_s^{VN} \leq q_s^{MS}$. Third, we show that the optimal quality level under RS+SM is greater than the optimal quality level under MS+IH. Fourth, we show that the optimal quality level under RS+IH is greater than $\frac{q_n}{2}$ (which is the optimal quality level when the retailer sources from NM). The combination of the above results establishes that optimal quality levels for the nine scenarios satisfy the ordering stated in Proposition 6.

Then we establish relationships among the retailer's profits across the nine sourcing and pricing power scenarios. To achieve this, we first show that when the retailer produces IH, we have $\pi_r^{RS}(q_s) \geq \pi_r^{VN}(q_s) \geq \pi_r^{MS}(q_s)$ for any given $q_s \in [0,\ q_n]$. This implies that at the corresponding optimal quality levels under the three game structures, we also have $\pi_r^{RS}(q_s^{RS}) \geq \pi_r^{VN}(q_s^{VN}) \geq \pi_r^{MS}(q_s^{MS})$. Then we prove the same statement under sourcing from SM or NM. Next, we show that the retailer's optimal profit under MS+IH is greater than her optimal profit under RS+SM. Finally, we show that the retailer's optimal profit under MS+SM is greater than her optimal profit under RS+NM.

Ordering of quality levels across nine sourcing and pricing power scenarios (technical details)

We first show that when the retailer produces IH, we have $q_s^{RS} \leq q_s^{VN} \leq q_s^{MS}$. When the retailer produces IH, optimal quality levels under three game structures are, respectively, $q_s^{MS} = \min\{q_n,\ f_{MS}(k)\}$ where $f_{MS}(k) \equiv \frac{1}{9k}[kq_n + 6\bar{\theta} - (-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2)^{1/2}]$, $q_{RS} = \min\{q_n,\ f_{RS}(k)\}$ where $f_{RS}(k) \equiv \frac{1}{3k^2}[k^2q_n + 2k\bar{\theta} - (-2k^4q_n^2 + 4k^3q_n\bar{\theta} + k^2\bar{\theta}^2)^{1/2}]$ and $q_{VN} = \min\{q_n,\ f_{VN}(k)\}$ where $f_{VN}(k) \equiv \frac{1}{15k}[4kq_n + 10\bar{\theta} - (-44k^2q_n^2 + 80kq_n\bar{\theta} + 25\bar{\theta}^2)^{1/2}]$. We need to show that $f_{RS}(k) \leq f_{VN}(k) \leq f_{MS}(k)$. A comparison of $f_{RS}(k)$ and $f_{VN}(k)$ reveals that they are equal at $k = \frac{2\bar{\theta}}{3q_n}$ and $k = \frac{2\bar{\theta}}{q_n}$ and that $f_{RS}(k) \leq f_{VN}(k)$ if $k < \frac{2\bar{\theta}}{3q_n}$ or $k > \frac{2\bar{\theta}}{q_n}$. Because $k \leq \frac{\bar{\theta}}{3q_n} < \frac{2\bar{\theta}}{3q_n}$, we have $f_{RS}(k) \leq f_{VN}(k)$ for all k in the parameter space of interest. Similarly, a comparison of $f_{VN}(k)$ and $f_{MS}(k)$ reveals that they cross once and that $f_{VN}(k) \leq f_{MS}(k)$ if $k < \frac{2\bar{\theta}}{3q_n}$. Because $k \leq \frac{\bar{\theta}}{3q_n} < \frac{2\bar{\theta}}{3q_n}$, we have $f_{VN}(k) \leq f_{MS}(k)$ for all k in the parameter space of interest. Therefore $q_{RS} \leq q_{VN} \leq q_{MS}$.

Next, we show that when the retailer sources from SM, we have $q_s^{RS} \leq q_s^{VN} \leq q_s^{MS}$. When the retailer sources from SM, the optimal quality level under MS is $q_s^{MS} = q_n$. The optimal quality level under RS(+SM) is $q_s^{RS} = q_n$ if $k \leq \frac{3-\sqrt{3}}{3}\frac{\bar{\theta}}{q_n} \approx 0.42\frac{\bar{\theta}}{q_n}$. Otherwise, $q_s^{RS} < q_n$, where q_s^{RS} is the solution to $\pi'_{RS}(q_s) = -2k^2(q_s^{SM})^3 + (13k^2q_n + 2k\bar{\theta})(q_s^{SM})^2 - (8k^2q_n^2 + 16kq_n\bar{\theta})q_s^{SM} + (6k^2q_n^3 - 4kq_n^2\bar{\theta} + 6q_n\bar{\theta}^2) = 0$ on $(0,\ q_n)$. The optimal quality level under VN(+SM) is $q_s^{VN} = q_n$ if $k \leq \frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n} \approx 0.424\frac{\bar{\theta}}{q_n}$. If $k \in [\frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n},\ \frac{\bar{\theta}}{2q_n}]$, $q_s^{VN} < q_n$ where

q_s^{VN} is the solution to $\pi'_{VN}(q_s) = -5k^2q_n^2(q_s^{SM})^3 + 135k^2q_n^3(q_s^{SM})^2 + (-32k^2q_n^4 - 218kq_n^3\bar{\theta} + 7q_n^2\bar{\theta}^2)q_s^{SM} + (54k^2q_n^5 - 54kq_n^4\bar{\theta} + 81q_n^3\bar{\theta}^2) = 0$. The second derivatives of the two profit functions are $\pi''_{RS}(q_s) = \frac{-2kq_n}{4(4q_n-q_s)\bar{\theta}}[k(q_n - 3q_s) + 2\bar{\theta}]$ and $\pi''_{VN}(q_s) = \frac{-2q_n^2(7kq_n-\bar{\theta})}{(9q_n-q_s)^3\bar{\theta}}[(-169kq_n + 7\bar{\theta})q_s + (9kq_n^2 + 153\bar{\theta}q_n)]$. After tedious but straightforward mathematical manipulations, we can show that (1) $\pi''_{VN}(q_s) < \pi''_{RS}(q_s)\ \forall q_s$ if $k \geq \frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n}$ and (2) $\pi'_{VN}(q_s) > \pi'_{RS}(q_s)$ when $q_s = q_n$. The combination of (1) and (2) implies that $q_s^{RS} \leq q_s^{VN}$. Also, because $q_s^{MS} = q_n$, we have $q_s^{RS} \leq q_s^{VN} < q_s^{MS}$ when the retailer sources the store brand from SM.

Next, we show that the optimal quality level under RS+SM is greater than the optimal quality level under MS+IH. From our analysis in the main text and the proof of Proposition 4, the optimal q_s under MS+IH equals q_n for $k \leq \frac{3-\sqrt{3}}{4}\frac{\bar{\theta}}{q_n} \approx 0.317\frac{\bar{\theta}}{q_n}$ and equals $\frac{1}{9k}[kq_n + 6\bar{\theta} - \sqrt{-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2}]$ otherwise. The optimal q_s under RS+SM equals q_n for $k \leq \frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n} \approx 0.424\frac{\bar{\theta}}{q_n}$, and otherwise is the solution to $\pi'_{RS}(q_s) = -2k^2(q_s^{SM})^3 + (13k^2q_n + 2k\bar{\theta})(q_s^{SM})^2 - (8k^2q_n^2 + 16kq_n\bar{\theta})q_s^{SM} + (6k^2q_n^3 - 4kq_n^2\bar{\theta} + 6q_n\bar{\theta}^2) = 0$ on $(0,\ q_n)$. With these results, to establish that the optimal quality level under RS+SM is greater than that under MS+IH, we only need to show that when $k > \frac{17-4\sqrt{5}}{19}\frac{\bar{\theta}}{q_n}$, the solution to $\pi'_{RS}(q_s)$ is greater than $\frac{1}{9k}[kq_n + 6\bar{\theta} - \sqrt{-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2}]$. This can be shown by confirming that first, $\pi'_{RS}(q_s) < 0$ for all $q_s \in [0,\ q_n]$, and second, $\pi'_{RS}(\frac{1}{9k}[kq_n + 6\bar{\theta} - \sqrt{-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2}]) > 0$. It is straightforward to establish these two inequalities.

To complete the proof of the relationships among the optimal quality levels for the nine sourcing and pricing power scenarios, we now show that the optimal quality level under RS+IH is greater than $\frac{q_n}{2}$. Under RS+IH, the optimal quality level is $\min\{\frac{1}{3k^2}[k^2q_n + 2k\bar{\theta} - \sqrt{-2k^4q_n^2 + 4k^3q_n\bar{\theta} + k^2\bar{\theta}^2}],\ q_n\}$. After tedious mathematical manipulation, we can show that $\frac{1}{3k^2}[k^2q_n + 2k\bar{\theta} - (-2k^4q_n^2 + 4k^3q_n\bar{\theta} + k^2\bar{\theta}^2)^{\frac{1}{2}}]$ is decreasing in k, and is greater than $\frac{q_n}{2}$ at $k = \frac{\bar{\theta}}{2q_n}$.

Ordering of retailer's profit levels across nine sourcing and pricing power scenarios

We first show that when the retailer produces IH, $\pi_r^{RS}(q_s) \geq \pi_r^{VN}(q_s) \geq \pi_r^{MS}(q_s)$ for all $q_s \in [0,\ q_n]$. From our analysis in Section 3.1 and the proofs of Propositions 4 and 5 and a little algebra, we find that $\pi_r^{RS}(q_s) - \pi_r^{VN}(q_s) = \frac{1}{72\bar{\theta}}(q_n - q_s)(\bar{\theta} - k(q_n + q_s))^2$ and $\pi_r^{VN}(q_s) - \pi_r^{MS}(q_s) = \frac{1}{144\bar{\theta}}(q_n - q_s)(\bar{\theta} - k(q_n + q_s))^2$. Both of these differences are obviously equal to 0 if $q_s = q_n$. They are positive for all $q_s \in [0,\ q_n)$ because, for $q_s \in [0,\ q_n)$, we have $q_n > q_s$ and $\bar{\theta} - k(q_n + q_s) > 0$. (Recall that we have assumed that $k \leq \frac{\bar{\theta}}{2qn}$.) These results imply that $\max_{q_s} \pi_r^{RS}(q_s) \geq \max_{q_s} \pi_r^{VN}(q_s) \geq \max_{q_s} \pi_r^{MS}(q_s)$ when the retailer produces IH. We can show the same statement is true when the retailer is sourcing from SM or NM (details omitted). That is, $\pi_r^{RS}(q_s) \geq \pi_r^{VN}(q_s) \geq \pi_r^{MS}(q_s)$ for all $q_s \in [0,\ q_n]$, when the retailer sources from SM, or NM. This completes the proof of the relationships among the optimal quality levels under RS.

Next, we show that the retailer's *optimal* profit level under MS+IH is greater than her *optimal* profit level under RS+SM. We do so by showing the retailer's profit under MS+IH is greater than her profit under RS+SM for $\forall q_s \in [\min\{q_s^{MS+IH},\ q_s^{RS+SM}\},\ q_n]$. Notice that $\min\{q_s^{MS+IH},\ q_s^{RS+SM}\} = q_s^{MS+IH} = \min\{q_n,\ \frac{1}{9k}(kq_n + 6\bar{\theta} - \sqrt{-8k^2q_n^2 + 12kq_n\bar{\theta} + 9\bar{\theta}^2})\}$. The difference between the retailer's profit under MS+IH and her profit under RS+SM for a given q_s is $-\frac{(q_n-q_s)}{16(4q_n-q_s)\bar{\theta}}[k^2(4q_n^3+5q_n^2q_s+6q_nq_s^2-3q_s^3)-2k(4q_n^2+5q_nq_s-3q_s^2)\bar{\theta}+(4q_n-3q_s)\bar{\theta}^2]$, which can be shown (details omitted) to be strictly greater than zero for $\forall q_s \in [q_s^{MS+IH},\ q_n)$ and equal to zero for $q_s = q_n$. Now, because the retailer's profit under MS+IH at any given $q_s \in [q_s^{MS+IH},\ q_n)$ is greater than her profit under RS+SM, the retailer's optimal profit is also greater than or equal to her profit under RS+SM with q_s selected optimally on $q_s \in [q_s^{MS+IH},\ q_n]$ for each scenario. What remains is to show that the retailer's *optimal* profit under MS+SM is greater than her *optimal* profit under RS+NM. We do so by showing that the statement holds for any given q_s in $[\frac{q_n}{2},\ q_n]$. The difference between the retailer's profit under MS+SM and her profit under RS+NM, for a given q_s, is $-\frac{q_n(k^2(8q_n^4+6q_n^3q_s-25q_n^2q_s^2+3q_nq_s^3-q_s^4)+2kq_n(-8q_n^2+10q_nq_s+7q_s^2)\bar{\theta}+(8q_n^2-18q_nq_s+q_s^2)\bar{\theta}^2)}{8(q_s-4q_n)^2\bar{\theta}}$, which we can show (after tedious mathematical manipulations), is increasing with q_s on $[\frac{q_n}{2},\ q_n]$, and its value evaluated at $q_s = \frac{q_n}{2}$ is positive. Therefore, the retailer's profit under MS+SM is greater than her profit under RS+NM for any given q_s in $[\frac{q_n}{2},\ q_n]$, which implies that her *optimal* profit level under MS+SM is greater than her *optimal* profit level under RS+NM.

Appendix H: Proofs of Proposition 7, Corollaries 2 and 3

We first provide a detailed sketch of the proof. Mathematical details appear at the end of this section.

Customer welfare under a sourcing and pricing power scenario can be represented as $\int_{\frac{p_n-p_s}{q_n-q_s}}^{\bar{\theta}}(\theta q_n-p_n)f(\theta)d\theta+\int_{\frac{p_s}{q_s}}^{\frac{p_n-p_s}{q_n-q_s}}(\theta q_s-p_s)f(\theta)d\theta$. This function can be used to derive customer welfare under a sourcing and pricing power scenario and a q_s by substituting in the store brand quality level q_s and the retailer's equilibrium prices for that scenario. In this proof, we show that, (i) given a sourcing structure, customer welfare under VN is greater than that under MS for a given $q_s \in (0,\ q_n)$; and (ii) given a sourcing structure, the optimal quality level that maximizes customer welfare under VN and MS is the same as the quality level that maximizes retailer's profit under VN and MS, respectively. The combination of (i) and (ii) implies that, given a sourcing structure, customer welfare at the retailer's optimal store brand quality under VN is greater than or equal to that under MS. Then we show that (iii) given a sourcing structure, customer welfare under MS equals customer welfare under RS for a given $q_s \in [0,\ q_n]$. Together, (ii) and (iii) imply that, given a sourcing structure, customer welfare at the retailer's optimal store brand quality under MS is greater than or equal to that under RS. Specifically, if the sourcing structure is NM, the retailer's optimal quality level is the same under RS and MS, and therefore customer welfare under MS equals that under RS; whereas if the sourcing structure is IH or SM, the retailer's quality level under RS differs from her quality level under MS (see our analysis in the main text and the proof of Proposition 4). The fact that the retailer chooses the q_s that maximizes customer welfare under MS but chooses a different q_s under RS, together with (iii), implies that customer welfare under MS is greater or equal to that under RS. In sum, the combination of (i), (ii) and (iii) shows that customer welfare under a given sourcing structure (at the retailer's corresponding optimal quality) has the ordering of $VN + IH \geq MS + IH \geq RS + IH$, $VN+SM \geq MS+SM \geq RS+SM$, and furthermore, $VN+NM \geq MS+NM = RS+IH$. Finally, to complete the proof of the proposition, we show that (iv) customer welfare under $RS + IH$ is greater than that under $VN + SM$, and (v) customer welfare under $RS + SM$ is greater than that under $VN + NM$. This completes the detailed sketch of the proof. Mathematical details appear below.

Proofs of (i), (ii) and (iii). We provide detailed proofs of (i), (ii) and (iii) only for the IH sourcing structure, as the proofs for the SM and NM sourcing structures are similar. Under IH+VN, customer welfare for a given q_s is $\frac{1}{72\bar{\theta}}[k^2(4q_n^3+4q_n^2q_n-4q_nq_s^2+5q_s^3)-2k(4q_n^2+5q_s^2)\bar{\theta}+(4q_n+5q_s)\bar{\theta}^2]$. Under IH+MS, customer welfare for a given q_s is $\frac{1}{32\bar{\theta}}[k^2(q_n^3+q_n^2q_s-q_nq_s^2+3q_s^3)-2k(q_n^2+3q_s^2)\bar{\theta}+(q_n+3q_s)\bar{\theta}^2]$. The difference between these two functions representing customer welfare is $\frac{7(q_n-q_s)(\bar{\theta}-k(q_n+q_s))^2}{288\bar{\theta}}$, which is positive for $q_s \in [0,\ q_n)$ and is equal to zero for $q_s = q_n$. Also, both of these functions are concave in q_s. Taking the first derivative of each of them with respect to q_s and setting them equal to zero, we find that for each scenario, the same q_s maximizes both customer welfare and the retailer's profit. (The latter was derived in the main text of the paper.) This establishes (i) and (ii) for the IH sourcing structure. Finally, we derive the customer welfare for a given q_s under IH+RS as $\frac{1}{32\bar{\theta}}[k^2(q_n^3+q_n^2q_s-q_nq_s^2+3q_s^3)-2k(q_n^2+3q_s^2)\bar{\theta}+(q_n+3q_s)\bar{\theta}^2]$. This expression is exactly the same as customer welfare for a given q_s under IH+MS (which was derived earlier in this paragraph). This establishes (iii) for the IH sourcing structure. The proofs of (i), (ii) and (iii) for the SM and NM sourcing structures are similar.

Proof of (iv). As discussed above, customer welfare under RS+IH for a given q_s is $u_1(q_s) \equiv \frac{1}{32\bar{\theta}}[k^2(q_n^3+q_n^2q_s-q_nq_s^2+3q_s^3)-2k(q_n^2+3q_s^2)\bar{\theta}+(q_n+3q_s)\bar{\theta}^2]$. Customer welfare under VN+SM is $u_2(q_s) \equiv \frac{q_n^2}{2(q_s-9q_n)^2\bar{\theta}}[k^2(9q_n^3+4q_n^2q_s-2q_nq_s^2+5q_s^3)-2k(9q_n^2+q_nq_s+6q_s^2)\bar{\theta}+(9q_n+7q_s)\bar{\theta}^2]$. It is straightforward to show that $u_1(q_s)$ at the retailer's optimal q_s under RS+IH is greater than $u_2(q_s)$ for all q_s. This proves (iv).

Proof of (v). Customer welfare under RS+SM for a given q_s is $u_3(q_s) \equiv \frac{q_n^2}{8(4q_n-q_s)^2\bar{\theta}}[k^2(4q_n^3+q_n^2q_s+q_nq_s^2+3q_s^3)-2k(4q_n^2+q_nq_s+4q_s^2)\bar{\theta}+(4q_n+5q_s)\bar{\theta}^2)]$. Customer welfare under VN+NM for a given q_s is $u_4(q_s) \equiv \frac{1}{18\bar{\theta}}[q_n(k^2(q_n^2+q_nq_s-q_s^2)-2kq_n\bar{\theta}+\bar{\theta}^2)]$. It is straightforward to confirm that $u_3(q_s)$ at the retailer's optimal q_s under RS+SM is greater than $u_4(q_s)$ for all q_s. This proves (v).

Appendix I: Proof of Corollary 4

Our analysis in the main text of Section 4 has established that, for a given quality level, the national brand manufacturer makes the retailer indifferent between sourcing from NM and sourcing from her alternate source under the modified scenario studied in this section. In other words, for a given quality level, the retailer's profit when she is sourcing from NM under the modified scenario is the same as her profit when is sourcing from her alternate source. If we denote the retailer's profit function when she is sourcing from NM under the modified scenario as $\pi_r(q_s)$, and denote the retailer's profit function when is sourcing from her alternate source as $\pi_{r0}(q_s)$, we have $\pi_r(q_s) = \pi_{r0}(q_s)$ for $\forall q_s \in [0,\ q_n]$. From this, we immediately have $\underset{q_s}{argmax}\ \pi_r(q_s) = \underset{q_s}{argmax}\ \pi_{r0}(q_s)$. In other words, optimal quality level for the retailer is the same when she is sourcing from NM with alternate source available as her optimal quality level when sourcing from her alternate source.

CHAPTER 5

Conclusions

The goal of this dissertation is to develop optimal store brand strategies for retailers who have the option to carry a store brand product, a national brand product, or both. In Chapters 2 and 3, I investigate a retailer's product assortment, pricing, and quality-positioning problems when she faces competition from another retailer who may also offer the same national brand and a competing store brand. In Chapter 4, I study a retail monopolist's quality-positioning strategy under three sourcing structures, and for each sourcing structure, I consider three types of channel price leadership. The three sources are in-house (IH), a leading national brand manufacturer whose product the retailer also carries (NM), and a strategic third-party manufacturer (SM). The three types of channel price leadership are the ones most commonly seen in the literature: Manufacturer-Stackelberg (MS), Retailer-Stackelberg (RS), and Vertical Nash (VN).

In Chapter 2, I obtain a characterization of how each retailer's optimal assortment decision depends on the national brand's wholesale price. Past research has established that store brands help generate store brand traffic and help a store better differentiate itself. As a result, one would expect that a retailer would be more likely to introduce a store brand as a competitive strategy when she faces retail competition. In contrast, I find that, in the presence of retail competition, for any given wholesale price of the national brand product, a retailer makes the product assortment decision in the same way as if she were a downstream monopolist. The underlying reason for this result is that a retailer's assortment decision is determined by a comparison between the profitability of her store brand and the national brand products, which does not involve the other retailer. However, the presence of multiple retailers affects the national brand manufacturer's choice of a wholesale price, which then affects the ultimate assortments and prices at the retail level.

I characterize how each retailer's optimal assortment decision depends on the national brand's wholesale price. For wholesale prices below a lower threshold, the retailer carries only the national brand and for wholesale prices above an upper threshold, the retailer carries only the store brand. For wholesale prices between the lower and upper thresholds, the retailer carries both brands. Based on this characterization, I can infer that the national brand manufacturer needs to choose between two regimes: (1) selling through both retailers and optimizing the wholesale price within the interval in which both retailers choose to offer the national brand (along with their respective store brand), and (2) selling through only the retailer with the lower-quality store brand and optimizing the wholesale price within the relevant price interval. (In the third (high) price interval, neither retailer would choose to offer the national brand product, which is suboptimal for the national brand manufacturer.) My results indicate that the national brand manufacturer should consider distributing its product through only one retailer if the quality disparity between the two store brands is larger than a threshold. The rationale is that when the quality disparity is large, the manufacturer would prefer to set quite different wholesale prices for the two retailers if he were allowed to do so. But if he needs to compromise and set a single wholesale price, he may be better off just distributing through one retailer.

The threshold on the quality disparity decreases with the absolute quality level of the higher-quality store brand. If the quality of the better store brand is high, it poses strong competition for the national brand product. This makes it unattractive for the national brand manufacturer to sell through this retailer. Even if the quality disparity between the two store brands is low, the national brand manufacturer will choose to sell through the retailer with the lower-quality store brand because he otherwise will not sell anything at all, and if he sells through both retailers, the competition posed by their two strong store brands will force him to drop his wholesale price significantly. In the special case where the quality level of the higher-quality store brand is equal to that of the national brand, the threshold on the quality disparity falls to zero, meaning that the manufacturer will sell only through the retailer with the lower-quality store brand.

In Chapter 2, I also study the effects of customer loyalty on the structure of the retailer's optimal assortment decision. When there is no customer loyalty, both retailers stop carrying the national brand when the wholesale price exceeds the same threshold. Thus, the national brand manufacturer always distributes through both retailers. The retailers end up in a prisoner's dilemma at the equilibrium: both of them could have earned a higher profit if neither of them had carried the national brand, but they both end up carrying it. As the degree of customer loyalty increases, the national brand manufacturer may or may not distribute through both retailers depending upon the quality levels of the two store-brand products, as I discussed above. I also discuss how my model can be extended to handle different production cost functions (as a function of quality) for the two store-brand products as well as heterogeneity in customer loyalty to the retailers.

In Chapter 3, I find that the national brand manufacturer's wholesale pricing strategy (derived in Chapter 2) has important implications for how the retailer of interest should make quality-positioning decisions. The retailer knows that if the quality level of her store brand is much higher than that of the other store brand, the manufacturer will choose a (high) wholesale price that will make it no longer optimal for her to carry the national brand. If the quality level of her store brand is much lower than that of the other store brand, the manufacturer will choose a wholesale price that will cause the other retailer drop the national brand. If the store brand quality levels at the two retailers are not too disparate, the manufacturer will choose a wholesale price such that both retailers carry the national brand. It turns out that both retailers tend to be better off when they both carry their respective store brands as well as the national brand because together, each with a competing store-brand product available to sell, they are able to elicit greater price concessions from the national brand manufacturer while simultaneously reaching customers with a low willingness to pay per unit of quality via their store brands.

The finer details of the retailer's quality decision for her store-brand depend on how the unit production cost changes with the quality level. If the unit production cost of store brand is linearly increasing in the product quality, the retailer always sets the quality level of her store brand equal to that of the national brand. The underlying reason is that with a linear cost relationship, the retailer does not derive any value from differentiating her product from the national-brand product. Stated another way, the total number of units that she sells does not change with the quality level of the store brand. The retailer therefore increases the quality level of her store brand and thus earns a higher profit margin on each unit of the product sold. On the other hand, if the unit production cost of is strictly convex and increasing in the product quality, the retailer's quality decision becomes more complex, and it depends heavily in the quality level of the store brand at the other retailer. Indeed, the optimal store-brand quality level for the retailer of interest is non-monotonic and discontinuous as the quality level of the competing store brand increases. A small increase in the quality level of the competing store brand could lead to a jump in her optimal quality level. More specifically, if the quality level at the other retailer is low (high), the retailer will set the quality level of her store brand in the high (low) range. If the quality level of the other store brand is moderate, the retailer will set the quality level of her store brand in the moderate range. When the retailer's quality level is within the either the high or the low range, her optimal quality level is invariant with the quality level of the other store brand, whereas when her optimal quality level falls into the moderate range, the quality level is strictly decreasing with the quality level of the store brand at the other retailer. Intuitively, as the quality level of the other store brand increases, the national brand manufacturer is forced to reduce his wholesale price. The retailer of interest thus has less incentive to increase quality, as she can "free ride" to some extent on the price concessions that the competing retailer's high-quality store brand is able to elicit from the national brand manufacturer.

My results in Chapter 3 contribute to the literature by showing how the presence of retail competition leads to optimal store brand quality-positioning strategies that can be quite different from the common wisdom and also much more complex than what has been reported in past research. The past literature on store brand strategies has suggested that a monopolist retailer should set the quality level of her store brand as high as possible, as it can increase her bargaining power versus the national brand manufacturer. I find this is not the case if the retailer is facing competition. Under retail competition, setting a high quality level for the store brand can backfire. In particular, increasing the quality level of the store brand does not necessarily increase a retailer's bargaining power. If the retailer sets too high a quality level, the national brand manufacturer could charge a high wholesale price, effectively foreclosing the retailer of interest from selling the national brand (because it then becomes uneconomical for the retailer to offer the national brand product), although it would have been in the retailer's best interest to sell a moderate- or low-quality store brand product alongside the national brand product.

I also study how the intensity of competition affects the equilibrium. As retail competition becomes more heated, the retailer adjusts the quality level of her store brand in order to alleviate price competition between the product(s) sold at her store and the product(s) sold at the other retailer. More specifically, if it is optimal for the retailer to carry only the store brand at her store (which occurs if the quality of other store brand is low), she will increase the quality level of her store brand to become more differentiated from the store brand at the other retailer. If it is optimal for the retailer to carry both the store and the national brands at her store (which occurs if the other store brand is high or moderate in quality), she will decrease the quality level of her store brand.

In Chapter 3, I also generalize my model to allow the retailers and the national brand manufacturer to have different production cost parameters and show how this asymmetry affects the equilibria. I also explain how my results can be utilized to identify equilibria—for both symmetric and asymmetric production cost parameters—for a game in which the two retailers simultaneously set their store-brand quality levels.

In Chapter 4, I examine nine (three sourcing structures times three pricing power scenarios) combinations of sourcing and pricing power (or "game") scenarios that I mentioned at the beginning of this chapter, and compare the retailer's optimal quality positioning decision and other equilibrium results (including prices, retailer's profits, consumer welfare, and supply chain profits) across the nine scenarios. To the best of my knowledge, I am the first to study the interaction between store-brand sourcing and positioning decisions, and the interplay of these decisions with the retailer's pricing power; I am also the first to present a comparison of equilibria for the aforementioned nine realistic combinations of sourcing and pricing power in the store brand context.

I find that, when sourcing from the national brand manufacturer under the three game structures, the optimal quality level is the same for all three game structures, and both the quality and profit levels are the lowest among the nine combinations. The intuition is as follows. First, when sourcing the store brand from the national brand manufacturer, having a store brand product provides a retailer no additional leverage in dealing with the national brand manufacturer. This is in contrast to the vast majority of the literature on store-brand introduction, which concludes that a retailer can use her store brand as a bargaining chip and thereby elicit price concessions from the national brand manufacturer. Consequently, the retailer's only source of leverage provided by the store brand is product differentiation. As such, she chooses a low quality level that provides significant differentiation from the national brand product, which turns out to be lower than that in any of the other six scenarios. The differentiation leads to an increase in overall profit for the retailer (versus having no store brand), but is still lower than when she sources the store brand from a strategic third-party or produces in-house.

Under the remaining six scenarios (involving in-house production or sourcing from a third-party manufacturer), the retailer uses the store-brand quality level more aggressively as a lever to (partially) compensate for any reduction in leverage arising from a less advantageous sourcing arrangement or weaker pricing power. The optimal store-brand quality has a monotonic mapping to the game-and-sourcing combinations, with the optimal quality increasing along the following path: RS+IH → VN+IH → MS+IH → RS+SM → VN+SM → MS+SM. However, an increase in the quality level only partially compensates for the retailer's reduction in leverage. Consequently, the retailer's profit levels under these six scenarios are in the reverse order of the optimal quality levels.

It is quite surprising that the retailer's optimal quality level is higher when she sources from a strategic third-party manufacturer than when she produces in-house, even if her pricing power differs between the two scenarios being compared. One might think that the introduction of double marginalization under production by the strategic third-party manufacturer would cause the retailer to compensate by lowering the quality (as she does when sourcing from the national brand manufacturer), thereby reducing the unit production cost incurred by the third-party manufacturer. Instead, increasing the quality level heats up the competition between the national brand manufacturer and the third-party manufacturer, which forces them to decrease their markups, and, as a result, the retailer earns a higher profit. Surprisingly, this effect leads to the retailer to choose an even higher quality level under SM when she has pricing power (i.e., under RS) than she does under IH when the national brand manufacturer is the Stackelberg leader (i.e., under MS).

The above results also imply that, at the optimal quality level for store brand corresponding to each scenario, the retailer strictly prefers a scenario with a more preferable sourcing structure, even if she is a price follower in this scenario but a leader in the other. This finding implies that, although past literature emphasizes how store brands give a retailer more pricing power versus the national brand manufacturer, a more preferable sourcing structure has greater value than a more favorable pricing power scenario to the retailer, i.e., the power to choose the source is more important than pricing power. One key reason for this result is that, sourcing from an outside party confers power over wholesale pricing even if that party is not the Stackelberg price leader. A less obvious reason is that sourcing from an outside party can cause the retailer to choose a lower quality for the store brand product than when it is produced in-house. It is important to emphasize, however, that this result holds only when the retailer chooses the corresponding optimal quality level for each scenario. If the store brand quality level if fixed, it is possible that the retailer prefers sourcing from a strategic third-party manufacturer under Retailer-Stackelberg over producing the store brand in-house under Manufacturer-Stackelberg.

I also provide a comparison of consumer welfare, equilibrium prices and supply chain coordination across the nine scenarios. I find that, given a game structure, consumers have the same ordering of preferences across the three store-brand sourcing structures as the retailer does (i.e., they prefer IH over SM over NM). When the retailer chooses the quality level optimally for each scenario, this ordering preference does not change even if the pricing power scenario differs between the scenarios being compared. In other words, for consumers, pricing power differences cannot compensate for sourcing differences. But this does not hold if the store brand quality level is fixed and equal across scenarios and is lower than a threshold because the store brand does not pose enough of a threat to the national brand manufacturer under these conditions. Therefore a "good" sourcing structure is less valuable to both the retailer and customers in such an environment. I also find that the supply chain is better coordinated (i.e., the total supply chain profit is higher) when the store brand is produced in-house than when the production is outsourced, irrespective of the game structure. However, comparing the two arrangements involving outsourcing (i.e., SM and NM), differences in pricing power *can* compensate for differences between these two sourcing structures. This arises when the production cost parameter is greater than a threshold.

I also study two variants of the game between the national brand manufacturer and the retailer. In the two variants, the retailer is asking the national brand manufacturer to produce both products under the implicit threat of using an alternate source, either in-house or a strategic third-party manufacturer, to produce the store brand product. I find that when the national brand manufacturer supplies both products, he can do at least as well as when the retailer uses the alternate source for the store brand. Consequently, at the equilibrium, the national brand manufacturer lowers his wholesale prices on both the store and the national brands below what he would have charged in the absence of an alternate source. The national brand manufacture makes the retailer indifferent between the two sourcing options for the store brand product, and consequently is able to provide both products to the retailer. This result partly explains why different retailers source store brand products in different ways, and a given retailer often sources the store brand differently for different categories of products. This result also provides an explanation for why retailers may choose to outsource store-brand production even though my base model predicts they prefer otherwise.

An interesting direction for further research would be to study whether the retailer's preference regarding store-brand sourcing may differ in the presence of retail competition.

Past literature on outsourcing in the absence of store-brand products suggests that a firm may have an incentive to outsource production due to considerations of softening competition (Cachon and Harker 2002, Liu and Tyagi 2011, Chen 2005, Wu and Zhang 2014). Whether such an effect carries over to situations with retail competition and store brands might be an interesting line of research. It might be also interesting to study whether the main results in this paper apply under other contract forms between the retailer and the manufacturers or if the retailer and the manufacturers engage in a bargaining game.

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