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EXTENDED BEAM LOSSES

J.B. McCaslin, W.P. Swanson and R.H. Thomas

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DISCLAIMER

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FOR POINT AND EXTENDED BEAM LOSSES

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"Then feed on thoughts, that voluntary move
Harmonious numbers"

From: Paradise Lost, Book iii, line 39

John Milton (1608-1676)

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ABSTRACT

The use of the empirical Moyer Model for the determination of transverse neutron shielding for high-energy proton accelerators is described and discussed. It is shown that an important advantage of the Moyer Model is the physical insight it offers towards understanding the complex interactions that comprise the shielding processes. Calculations for point-like and extended uniform beam loss distributions are discussed and their relationship to practical shielding conditions developed. The calculations required by the Model are readily performed on small programmable calculators and thus are widely accessible. Examples of program listings for practical calculations are given for a Hewlett-Packard HP-97 calculator.

1. INTRODUCTION

The development of the Moyer Model as an empirical method of determining transverse shield thicknesses for high-energy particle accelerators has been extensively described in the literature, to which the interested reader is referred (Mo61, Mo62, Pa73, Ri73, Ro69, St82).

Despite the development over the past decade of sophisticated high-energy radiation transport codes which may be applied to the detailed and accurate calculation of accelerator shielding (Ne80), Moyer Model calculations continue to be of great utility. As we shall show, since its first use in the early 1960s to calculate shielding for the improved Bevatron (Mo61, Mo62, We63) the Moyer Model has been used to design shielding for a large number of particle accelerators. A very recent example of its use is the design of the proposed Superconducting Super Collider (Th83, DOE84).

The reasons for this continuing interest in calculations utilizing the Moyer Model are threefold:

o The empirical Moyer Model is simple--employing only three parameters each of whose values are now well known (see Section 4). Calculations using the Model may be readily performed and yield physical insight into shielding that may be obscured when complex computer codes are used.

o The model is well adapted to the rapid, inexpensive estimation of accelerator and beam-line neutron shielding without recourse to complex computer codes. This is of great interest in industrially developing countries where large computers are of limited availabilty (Li81).

o Although under many conditions analogue Monte Carlo calculations are capable of great accuracy, they are themselves limited and may not permit direct calculation of shielding requirements for the occupational radiation intensities which exist at many operating high-intensity, high-energy accelerators e.g., typically $3x10^{-22}$ Sv/proton. Analogue calculations such as HETC (Ar72) and FLUKA (Ra72) can extend down to about $3x10^{-15}$ Sv/proton. Almost a factor of 10^{-2} may be obtained by particle splitting as is used, for example, in TRANKA (Ra74). Weighted Monte Carlo methods such as in CASIM (VaG75) may extend beyond FLUKA by a factor of 10^{-4} to $3x10^{-19}$ Sv/proton. Beyond the intensities reached by these calculations extrapolation is achieved via the Moyer Model (St82). It is therefore important to understand the Model and optimize the choice of its parameters.

In order that Monte Carlo methods for shielding calculation may be successfully employed it is important that the user have both detailed technical knowledge of the specific computer codes used and physical insight into the radiation transport processes involved. Without this physical insight it is possible to obtain results which have no practical value.

The use of transport codes as "black-boxes" is dangerous and can often lead to false information. Booth, for example, offers the following simple example to show how confusing results may be obtained:

"Consider particles trying to penetrate a thick
shield. If the shield is any good, very few of the
particles will penetrate the shield. Thus, it is
possible to simulate a huge number of particles without
any of the particles penetrating the shield. If none
of the simulated particles penetrate the shield, the
sample mean and the sample variance will both be zero.
However, even for the best shields the probability of
penetration is some number $\epsilon > 0$. The sample mean is
thus in error; at the same time the sample variance is
indicating zero error." (Bo79).

It is therefore extremely useful to have available phenomenological shielding
models which may be used as a rough check on the results of calculations made
by more sophisticated techniques. The Moyer Model is one such model providing
the physical insight useful for such comparisons and needed to fully
understand the results of Monte Carlo calculations.

2. SUMMARY OF PREVIOUS WORK

Since the early work of Moyer in designing shielding for the Bevatron (Mo61, Mo62) his method has been used, in whole or in part, to estimate shielding for many accelerators including the 20 GeV Stanford Linear Accelerator (deS62), the CERN SPS (CERN64), the Fermilab Proton Synchrotron (LRL65, URA68), the Stanford Positron-Electron Project [PEP] (McC73, Th77), the LBL Experimental Superconducting Accelerator Ring [ESCAR] (McC76) and the Chinese 50 GeV Proton Synchrotron (AS80, Ch80). Most recently this method was used in preliminary studies for the Superconducting Super Collider (Th83, DOE84).

The use of the Moyer Model in these designs has led to substantial experimental and theoretical improvement in the understanding of accelerator shielding phenomena and of the Model itself (Pa73, Ri73, St82).

Routti and Thomas addressed the application of the Moyer Model to uniform line sources and introduced the concept of Moyer Integrals (Ro69). McCaslin and Thomas, in unpublished notes, related finite uniform beam loss to loss at a point, anticipating similar work by Sullivan (Th69, McC76, Su81).

The Moyer Model utilizes three constants which are obtained empirically— the source strength term, $\mathcal{K}$, the angular distribution coefficient, β, and the attenuation length, λ.

During the design of the Stanford Positron Electron Project, de Staebler reviewed phenomenological shielding models and summarized them in a series of what he called "Moyer-like" equations convenient for rapid computation (deS77). In private communications, de Staebler drew attention to the conservative choice of the normalizing constant, $\mathcal{K}$, in the work of Routti and Thomas. This conservatism was a consequence of the assumption that there were no gaps between the ring magnets. The experimental data were taken from Gilbert et al. (Gi68, Gi69).

In design studies for the Beijing 50 GeV Proton Synchrotron, Stevenson and Liu (St80a,b) supported the conclusions of de Staebler (deS77) and as a result of their work, revised Moyer Model parameters were evaluated (Li82, St82). More recently the variation of the parameter $\mathcal{K}$ with primary proton energy has been investigated (the other two parameters β and λ are essentially invariant with energy—see Section 3) (Li84, Th84).

During the design of the shielding for PEP, McCaslin addressed the primary aspect of the de Staebler 1977 paper—that of simple computational techniques (deS77). Several programs were written for computers with limited memory. Examples of these programs are given in Appendices to this report.

3. SCOPE OF THIS PAPER

The design of high-energy accelerator shielding usually proceeds in two stages--first, an approximate calculation of shield thickness is made using semiphenomenological models for fairly simple geometries, and second, when accelerator parameters have been more closely defined, these simple calculations are verified by the use of more sophisticated numerical methods, usually involving Monte Carlo techniques to calculate electromagnetic and hadronic cascade phenomena in the shield (Ne80).

These numerical techniques are not necessarily more accurate than the empirical models in estimating the intensity of radiation fields outside shielding when the geometry is simple and the primary particle energy is in a region where good experimental data are available. Under these conditions both methods can predict radiation field intensities to within a factor of two or better. The numerical techniques are of greatest value in extrapolations to new energies or for calculations with complex geometries.

In the preliminary stages of shielding design, when simple but rapid methods suffice and are even to be preferred, it is necessary to idealize both the accelerator geometry and the pattern of beam loss in order to make the problem tractable.

It is common practice to assume two general types of beam loss which, although not necessarily realistic, serve to provide bounds to the shielding thickness:

o   Operational error beam loss (e.g., magnet failure, collimator slip-page, etc.), generally assumed to be located at a point. This assumption may be taken as determining the upper bound to the shielding requirement.

o   Randomly distributed beam loss resulting from normal operation which, when averaged over extended periods of time, may be assumed to be uniformly distributed along the beam path over a finite distance. This assumption may be taken as determining the lower bound to the shielding required.

Both the case of beam loss located at a point and of uniform beam loss along an infinite straight line are amenable to simple calculation. The case of uniform beam loss over a finite straight line is less tractable to analytical solution, and recourse to numerical techniques of integration is made.

The purpose of this paper is to explain the physical and geometrical aspects of the Moyer Model in such a way as to make the method more widely understood and useful to operational accelerator health physicists. The needed calculations are organized in such a way that they are within the capacity of many hand-held programmable calculators, without recourse to any supplementary material such as tables or graphs. Examples of specific programs written for the Hewlett-Packard HP-97 calculator are given (Appendices A2,A4,A5). The application of such programs to specific problems is demonstrated by the solution of several practical problems.

4. CALCULATIONS

4.1. Summary of the Moyer Model

In summary the Moyer Model expresses the dose equivalent $H(\theta)$, at some point, P, on the shield surface (Fig. 1), per proton interacting at a point on the beam axis, as:

$$H(\theta) = \mathcal{K}r^{-2} \exp(-\beta\theta) \exp(-\ell \csc \theta) \quad , \quad \frac{\pi}{3} < \theta < \frac{2\pi}{3} \qquad (1)$$

where:

θ is the angle subtended between the beam axis and a line joining the point of interaction and the point, P, of interest on the shield surface (see Fig. 1).

ℓ is the number of attenuation lengths in the shield (given by d/λ where d is the transverse shield thickness and λ the attenuation length of the shield material; see Fig. 1).

r is the distance from the point source to the point of interest on the shield surface (slant distance; see Fig. 1).

and the parameters $\mathcal{K}$, β and λ have been determined empirically.

With appropriate choices of the three Moyer Model parameters the model has been found to give good estimates of the dose equivalent at the shield surface—within the angular range indicated in Equation (1).

4.2. Values of the Moyer Model Parameters

Stevenson and his colleagues have recently summarized the published data which enable the parameters $\mathcal{K}$, β and λ to be calculated (St82, Li84, Th84). Experience has shown that both β and λ may be considered to be independent of energy for $E \geq 5$ GeV, and the best values obtained from the literature according to these authors are:

$\beta = 2.3 \pm 0.2$ radians^{-1}

λ (in earth) = 1170 ± 20 kg $\cdot$ m^{-2}

The attenuation length in a material of mass number, A, is given by:

$$\lambda = \left(\frac{A}{A_E}\right)^{1/3} \lambda_E \qquad (2)$$

where A_E is the effective mass number for earth ($A_E = 20.4$ for a 95%:5% mixture by weight of SiO_2 and H_2O) and λ_E is the attenuation length in earth.

The parameter $\mathcal{K}$ is a function of energy:

$$\mathcal{H} = H_o(E) \tag{3}$$

A recent review of published data extending up to 350 GeV by regression and analysis-of-variance techniques has suggested the variation of $\mathcal{H}$ with proton energy is expressed by the equation:

$$\mathcal{H} = H_o(E) = kE^m \tag{4}$$

with values of $m = 0.80 \pm 0.10$, $k = (2.84 \pm 0.14) \times 10^{-13}$ when $H_o(E)$ is in $Sv \cdot m^2$ and E in GeV (Th84).

The earlier work of Stevenson et al. (St82) had suggested that the Moyer Model Parameter, $\mathcal{H}$, was directly proportional to energy and given by:

$$\mathcal{H} = 1.60 \times 10^{-13} E \quad Sv \cdot m^2 \tag{4a}$$

when E is in GeV. This approximation is sufficiently accurate (± 20 percent) in the energy range from 5 to 30 GeV. However, at higher energies Equation 4a becomes increasingly inaccurate, overestimating $\mathcal{H}$ by a factor of two at 500 GeV and a factor of four at 20 TeV.

The discussion of this report is restricted to proton beam energies, E, above 5 GeV, where the above relationships hold. Tesch has recently studied the variation of λ and $\mathcal{H}$ for $0.05 < E < 1$ GeV (Te84).

4.3 Point Source Calculations

Although beam losses at high-energy particle accelerators are almost always of an extended nature, it is often the case that the region of high beam loss occurs over lengths small or comparable with the thickness of the accelerator shields and tunnel radius $R = (a + d)$ (see Fig. 1). Under such conditions the assumption of point loss may be used to determine the dose equivalent at the shield surface.

In calculating shield thickness it is often erroneously assumed that it is sufficient to determine the dose equivalent on the shield surface directly above the region of localized beam loss (i.e., $\theta = \pi/2$ radians). This section will explore the validity of this assumption and develop simple methods for the calculation of shielding which do not require it.

4.3.1. Dose Equivalent at the Shield Surface

Using the Moyer Model it is a simple matter to calculate the dose equivalent resulting on the shield surface at some point P (Fig. 1) subtending an angle θ to the beam direction, by the substitution of appropriate values for the parameters $\mathcal{H}$, β and λ into equation (1).

The dose equivalent at the shield surface directly above the point source ($\theta = \pi/2$) is given by:

$$H(\pi/2) = \frac{\mathcal{H} N}{R^2} \ \exp(-\beta\pi/2) \ \exp(-\ell) \qquad , \qquad (5)$$

where N is the number of protons stopped at the point. With $\mathcal{H}$ in $Sv \cdot m^2$, $H(\pi/2)$ will be in Sv. A calculation of $H(\pi/2)$ is shown in Fig. 2 for a point loss of N = 1 proton at E = 10 GeV. Values are normalized to a transverse distance of R = 1 m. As these data are for a point source, scaling to a different distance may be done by the inverse-square law. Scaling to another energy is done by means of Equation (4).

However, the maximum dose equivalent, H_m, occurs at a point downstream of the point source at angle θ_m; in general $\theta_m \leq \pi/2$. Figure 3 shows calculations of dose equivalent at the shield surface, normalized to that occurring at an angle of $\pi/2$ radians to the point source, as a function of shield thickness $(1 \leq \ell \leq 20)$ and subtended angle, θ $(50^\circ \leq \theta \leq 120^\circ)$. The figure plots the function $\overline{F}(\theta)$ defined as:

$$F(\theta) = \frac{H(\theta)}{H(\pi/2)}$$

$$= \frac{\exp(-\beta\theta) \ \exp[\ell(1 - \csc\theta)]}{\exp(-\beta\pi/2) \ \csc^2\theta} \qquad (6)$$

Substituting β = 2.3 radians^{-1}, Equation (6) becomes:

$$F(\theta) = 37.1 \ \sin^2\theta \ \exp(-2.3\ \theta) \ \exp[\ell(1 - \csc\theta)] \qquad . \qquad (6a)$$

[See Appendix A2 for details of a program to determine $F(\theta)$ using an HP-97 calculator.]

The maximum value of dose equivalent, H_m, is seen to occur at some angle, θ_m, in general less than $\pi/2$ radians. The thicker the shield the closer the value of θ_m approaches $\pi/2$ radians (see Fig. 4); as seen in Fig. 2, the relative and absolute difference between H_m and $H(\pi/2)$ diminishes with increasing shield thickness. But Figs. 2 and 3 show that, for thin shields, errors as much as a factor of two or more may be made by assuming that the maximum dose equivalent occurs at $\theta = \pi/2$ radians. For accurate calculations of shield thickness at regions of point loss it is therefore important to determine H_m. H_m may be calculated when the corresponding value of θ_m is known.

4.3.2. Location of Maximum Dose Equivalent—Determination of the Value of θ_m

The value of θ_m may be obtained by differentiating Equation (1) with respect to θ and setting the result equal to zero, yielding:

$$[\ell\cot\theta_m \ \csc\theta_m - \beta + 2\cot\theta_m] = 0 \qquad , \qquad (7)$$

where use is made of the relationship $r = R \csc\theta$ (Fig. 1). Figure 4 shows values of θ_m as a function of shield thickness, ℓ, for a value of the angular distribution coefficient β = 2.3.

As was to be expected θ_m increases with ℓ and approaches the value $\pi/2$ radians for very thick shields. Initially the rate of change of θ_m with ℓ is quite rapid, but slows with increasing shield thickness. At $\ell = 5$, θ_m has the value 1.3 radians (72°), and at $\ell = 10$ it has the value 1.4 radians (80°). For practical high-energy accelerator shields θ_m will, as we have already inferred, be less than $\pi/2$ radians (90°).

Figure 5 shows values of θ_m as a function of shield thickness for several values of beam spill length, η (η is defined in section 4.4.2).

4.3.3. Calculation of Maximum Dose Equivalent, H_m, on the Shield Surface

We define a function $C(\ell)$ such that:

$$H_m = C(\ell) \, H(\pi/2) \tag{8}$$

Substitution into Equation (5) and using $\beta = 2.3$ radians^{-1} gives:

$$C(\ell) = \exp[2.3(\pi/2 - \theta_m)] \, \exp[\ell(1 - \csc \theta_m)] \, \sin^2 \theta_m \tag{9}$$

Appendix A3 shows that a good numerical approximation for $C(\ell)$ is:

$$C(\ell) = 2.20 \, \ell^{-0.245} \tag{9a}$$

Combining Equations (5) and (9a) it follows that:

$$H_m = 0.0593 \, \frac{\mathcal{H} \, N \, \exp(-\ell)}{R^2} \, \ell^{-0.245} \quad , \tag{10}$$

with $\mathcal{H}$ in Sv·m^2 and the distance R measured in meters, H_m will be in Sv.

4.4. Extended Source Calculations

In practice, "point" sources do not generally occur in the operation of high-energy proton accelerators because the physical mechanisms resulting in beam loss usually spill beam over an extended region.

An approximation often useful in accelerator shield design is to assume that beam losses are uniform. This section discusses the calculation of dose equivalent for uniform beam loss both of infinite and finite extent.

4.4.1. Infinite Uniform Line Sources

Using the Moyer Model it is simply shown (Ro69) that for an infinite uniform line source of S protons per meter the dose equivalent on the shield surface is given by:

$$H_\infty = \frac{\mathcal{H} \, S}{R} \int_0^\pi \exp(-\beta\theta) \, \exp(-\ell \csc \theta) \, d\theta \tag{11}$$

In contrast to the point-source geometry just described, Equation (11) shows that the dose equivalent outside the shield produced by an infinite uniform line source diminishes as the inverse distance.

The integral of Equation (11) has been designated by $M(\beta,\ell)$ and is known as a Moyer Integral. Tabulations of Moyer Integrals for arguments in the range $0 \leq \beta \leq 10$; $0 \leq \ell \leq 40$ have been published (Ro69). For the calculation of high-energy accelerator shielding the Moyer Integral $M(2.3,\ell)$ is of particular interest. Appendix A4 gives a computer program for the calculation of this particular integral by Simpson's Rule using an HP-97 calculator.

Tesch has suggested that a suitably accurate approximation is given by:

$$M(2.3,\ell) = 0.065 \exp(-1.09 \; \ell) \quad , \tag{12}$$

valid for $\ell = 2 - 15$ (Te83). This expression is useful if tables of $M(2.3,\ell)$ or means of calculating the integral are not available. If $M(2.3,\ell)$ can be evaluated the dose equivalent on the shield surface $H(\ell,R)$ is then given by:

$$H_\infty(\ell,R) = \frac{\mathcal{H} S \; M(2.3,\ell)}{R} \tag{13}$$

As an example:

with $E = 10$ GeV, and

$$\mathcal{H} = 1.79 \times 10^{-12} \; Sv \cdot m^2,$$

then

$$H_\infty(\ell,R) = \frac{1.79 \times 10^{-12} \; S \; M(2.3,\ell)}{R} \; Sv \tag{14a}$$

when R is measured in meters and S in protons m^{-1}.

With the same parameters and using the approximation of Tesch:

$$H_\infty(\ell,R) = \frac{1.16 \times 10^{-13} \; S}{R} \exp(-1.09 \; \ell) \; Sv \quad . \tag{14b}$$

Stevenson et al. suggest that the use of Equation (14a) or (14b) should predict values of dose equivalent rate to within a factor of about two (St82).

4.4.2. Finite Uniform Line Sources

Here we consider the more practical case of the dose equivalent produced at the shield surface by a finite, but uniform, beam loss. To quantify the region of beam loss it is convenient to define the parameter, n, by the equation:

$$n = \frac{L}{R} \tag{15}$$

where L = distance over which beam is uniformly distributed--the "beam spill length" and R has previously been defined (see Fig. 6 and also Appendix A1).

The dose equivalent on the shield surface H(Z,L) per proton per meter lost along the beam axis is given by:

$$H(Z,L) = \frac{\mathcal{K}S}{R} \int_{\alpha_1}^{\alpha_2} \exp(-\beta\theta) \, \exp(-\ell \csc \theta) \, d\theta \tag{16}$$

where the angles α_1, α_2 limiting the integral are given by (see Fig. 6):

$$\alpha_1 = \tan^{-1} (R/Z) \tag{16a}$$

$$\alpha_2 = \tan^{-1} [R/(Z - L)] \tag{16b}$$

The integral of Equation (16) differs from that in Equation (11) only in the limits of integration and may be evaluated by numerical methods (see Appendix A5).

When N protons are lost uniformly over length, L,

$$\frac{dN}{dZ} = S = \frac{N}{L} \tag{17}$$

and since n = L/R [Equation (15)], we may rewrite Equation (16) as:

$$H(Z,L) = \frac{\mathcal{K}N}{nR^2} \int_{\alpha_1}^{\alpha_2} \exp(-\beta\theta) \, \exp(-\ell \csc \theta) \, d\theta \tag{18}$$

Designating the integral of Equation (18) as a restricted Moyer Integral: $M_{\alpha_1,\alpha_2}[\beta, \ell]$ we write:

$$H(Z,L) = \frac{\mathcal{K}N}{nR^2} M_{\alpha_1,\alpha_2}[\beta,\ell] \tag{18a}$$

The calculation of dose equivalent on the shield surface from Equation (18a) may be carried out by numerical means, and Appendix A4 gives a suitable program for use with an HP-97 calculator.

Figure 7 shows the results of an illustrative calculation of H(Z,L) using Equation (18a). A family of curves is shown for various spill lengths, n = L/R, and for the parameters:

$E = 10$ GeV

$\mathcal{H} = 1.79 \times 10^{-12}$ Sv m^2

$\ell = 5$

$R = 1$ m

$0.002 < \eta < 4$

Distances along the Z axis are measured in units of $\eta = L/R$, and the origin of coordinates is at the start of beam spill. The parameter $\mathcal{H} = 1.79 \times 10^{-12}$ Sv m^2, corresponding to 10 GeV, and $R = 1$ m, are chosen for convenience in normalization. As $N = 1$ is assumed for each spill length, the area under the curves in Fig. 6 is independent of η.

Figure 8 shows the behavior of the maximum dose equivalent, H_m, on the outside surface of the shield for a constant number of protons ($N = 1$ proton and $R = 1$ m), as a function of the beam spill length η. The limit for $\eta \gg 0$ at 4.7×10^{-16} Sv m^2 per proton is indicated. At the other extreme, as one increases η while holding N fixed, H_m approaches zero in a manner approximately proportional to the inverse spill length: $H_m \propto 1/\eta$. The approach to this limiting curve (hyperbola) is also indicated in Fig. 8. Values on the abscissa, η_p and η_∞, show where the true behavior deviates from either limiting assumption by 10 percent for this distance. The shaded area, bounded above by the two limiting assumptions, indicates the amount of overestimation that can be made by assuming the minimum of either the point-source or hyperbolic approximation.

In Fig. 9, the family of curves shows the distribution of dose equivalent outside of the shield for the case where the beam loss rate is held constant at $S = 1$ proton per meter. The dose equivalent is calculated using Equation (16), using the same parameters as for Fig. 7. In Fig. 9 the total number of protons, and therefore the area under the curves, is proportional to the length of the spill, L.

It can be seen from Figs. 7 and 9 that, for short spill lengths, $\eta \ll 1$, the source is nearly point-like and substitution of the appropriate value of N in Equation (10) will yield a good estimate of H_m. For values of $\eta > 1$ the source behaves more like an infinite uniform line source so that substitution of the appropriate value of S into Equation (14a) will provide an acceptable estimate of H_m.

An attempt is made in Fig. 10 to further define the boundary between the point-like and extended line-like sources in terms of the spill length parameter, η. As in Fig. 8, we define η_p as being that value of η at which H would be equal to 90 percent of that which would obtain if the same number of beam particles were deposited at a point. The reference distance of $R = 1$ m is assumed. In similar fashion, we define η_∞ as being that value of η at which H would be equal to 90 percent of that which would obtain if the source were an infinite uniform line source of the same loss per unit length, S.

The percentage overestimation resulting from use of either of the two limiting assumptions (Point source [constant N]) or (Infinite uniform line source [constant S]), is explicitly shown in Fig. 11. The percentage overestimation is defined as

$$100 \times \frac{H_A - H_m}{H_m} \qquad (19)$$

where H_m is the true maximum dose equivalent on the outside surface of the shield for the spill length in question, n, and H_A is the dose equivalent estimated using either of the two assumptions. Using 10 percent as an arbitrary criterion, we define the limits of applicability for the point source [Equation (10)] and for the infinite uniform line source [Equation (14a) or (14b)] as follows:

> For "short" spill lengths ($n < 0.6$), H_m may be calculated using the "point-source" Equation (10);
> For "long" spill lengths ($n > 1.4$), H_m may be calculated using the infinite line source Equation (14a or 14b).

The error associated with these procedures amounts to an overestimate of H_m which ranges from 0 to about 10 percent when $\ell = 5$. For the intermediate case $0.6 < n < 1.4$ the maximum dose rate must be calculated from Equation (18a) using the restricted Moyer integral. Figure 11 shows the amount of overestimation of H_m by either the point source equation for extended sources (keeping N constant) or by the infinite line source equation for finite line sources (keeping S constant). Percentage errors are plotted as functions of n for both of these assumptions.

4.5. Relation Between Point and Extended Uniform Sources

As we have suggested in Section 4.4, two simple assumptions are available to the accelerator shield designer if there exists some estimate of the total number of protons, N, lost to the accelerator. It may be assumed either that all the protons are lost at one location (point source) or that the beam loss is uniformly distributed along the accelerator structure of length, L.

The first assumption leads to an estimate of maximum dose equivalent for the number of protons lost, N, of:

$$H_m = \frac{\mathcal{H} \, N \, \exp(-\beta \theta_m) \, \exp(-\ell \csc \theta_m)}{R^2 \, \csc^2 \theta_m} \qquad (20)$$

where θ_m may be read from Fig. 4 and $\mathcal{H}$ obtained from Equation (4). Section 4.3 discusses the calculation of H_m in detail. Since "point-like" beam loss does not occur at high-energy proton accelerators this assumption will tend to overestimate the value of H_m and hence the shield thickness.

The second assumption proceeds by substituting the value N/L for S into Equation (14a) or (14b); these equations assume an infinite uniform source. In Section 4.4.2 we showed that this was a reasonable procedure for values of spill length, $n > 1.4$.

These two assumptions clearly lead to different estimates of dose equivalent, H_m and H_∞. The relationship between H_m and H_∞ may be obtained by combining Equations (11) and (20):

$$\frac{H_m}{H_\infty} = \frac{N}{SR} \frac{\exp(-\beta\theta_m)\exp(-\ell\csc\theta_m)}{M(\beta,\ell)\csc^2\theta_m} \quad . \tag{21}$$

Substituting $N = LS$ and $\eta = L/R$ yields

$$\frac{H_m}{H_\infty} = \eta \frac{\sin^2\theta_m \exp(-\beta\theta_m)\exp(-\ell\csc\theta_m)}{M(\beta,\ell)} \quad . \tag{22}$$

For convenience in expressing this relation, we define a function $B(\ell)$, such that

$$\frac{H_m}{H_\infty} = \eta B(\ell) \quad . \tag{23}$$

Figure 12 shows values of $B(\ell)$ calculated as a function of ℓ. It is seen that $B(\ell)$ may be approximated by a second order polynomial (dashed curve) which fits the curve within ±5 percent over the range $1 < \ell < 15$:

$$B(\ell) = 0.44 + 0.12\ell - 0.0028\ell^2 \quad . \tag{24}$$

Equation (24) facilitates the use of the Moyer Model for shielding calculations because H_m can be obtained from Equation (20) without too much difficulty, and the factor $B(\ell)$ can be evaluated from the polynomial [Equation (24)]. With these algorithms, Moyer Model calculations can be made with simple pocket calculators without having to solve the Moyer Integral directly.

5. Conclusions

The Moyer Model for high-energy neutron shielding has proved itself a very durable instrument since it was first developed in the early 1960's and has been utilized in the design of several important high-energy accelerator facilities. Its advantage over "sophisticated" methods lies in its simplicity; the algorithm contains only three parameters whose values are now well established over the energy range of at least 5 to 350 GeV. The figures presented and the sample codes for use with desk-top or hand-held calculators demonstrate the ease with which the Moyer Model can be applied to many shielding problems, some of which are beyond the present capability of well-established Monte-Carlo procedures. The procedure, which requires very modest computational power, is useful in its own right besides serving as a check on more elaborate, but less transparent, methods of shielding calculation.

ACKNOWLEDGMENTS

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Appendices

APPENDIX A1. Definitions of Symbols Used

Figures 1 and 6 of the text show the accelerator and beam loss geometries considered in this paper. The following symbols are used throughout:

Z   distance along the proton beam, measured from the point at which beam loss starts

a   radius of accelerator tunnel

d   transverse shield thickness

λ attenuation length of shield material

ℓ number of attenuation lengths in shield in transverse direction

R   distance normal to the proton beam axis to the shield outer surface

r   distance from point source to point of interest on shield surface (slant distance)

L   beam spill length

η beam spill length in units of R

θ angle subtended between beam axis and line joining source element and point of interest on shield outer surface

H   dose equivalent

H_m maximum dose equivalent on shield outer surface

θ_m angle θ at which H_m occurs

H_∞ dose equivalent due to infinite uniform line source

H_L dose equivalent due to extended uniform line source

$S = \dfrac{dN}{dZ}$ number of protons lost per unit length along beam axis

N   total number of protons lost

ρ density of shield material (mass per unit volume)

E   primary proton energy

The following relationships automatically follow from the definitions given above:

$$R = (a + d)$$

$$\ell = d/\lambda$$

$$\eta = \frac{L}{(a + d)} = \frac{L}{R}$$

$$r = (a + d)\, \csc\theta = R\, \csc\theta$$

$$r^2 = R^2 + Z^2$$

$$dZ = -(a + d)\, \csc^2\theta\, d\theta = -R\, \csc^2\theta\, d\theta$$

$$N = L\, \frac{dN}{dZ} = \eta R\, \frac{dN}{dZ} = \eta RS$$

APPENDIX A2. Hewlett-Packard HP-97 Code for Calculating F(θ)

This program, written for the Hewlett-Packard HP-97 calculator, determines values of F(θ), where

$$F(\theta) = H(\theta)/H(\pi/2)$$

$$= \frac{\exp(-\beta\theta) \, \exp[\ell(1 - \csc\theta)]}{\exp(-\beta\pi/2) \, \csc^2\theta} \tag{6}*$$

in increments of 5° for given values of shielding thickness in units of ℓ, assuming a point radiation source.

TO RUN THE PROGRAM: Store in the indicated registers the following:

Register A θ_{min}(deg), the smallest angle of concern;
       B θ_{max}(deg), the largest angle of concern;
       D β = 2.3, the angular distribution parameter;
       E ℓ, the number of attenuation lengths.

Press A.   The program will stop when the angular range from θ_{min} to θ_{max}, in steps of 5°, has been processed. New values of ℓ can then be stored in Reg E, and the process is repeated by pressing A.

PRINTOUT:

    Line 1 $g(\theta) = (\sin^2\theta) \, \exp(-2.3 \, \theta) \, \exp(-\ell\csc\theta)$

    Line 2 $F(\theta) = g(\theta)/[0.2697 \, \exp(-\ell)]$

    Line 3 θ, in degrees.

---

*Equation numbering is as in text.

001	*LBLA	21	11		032	RCLE	36	15
002	RCLB	36	12		033	CHS		-22
003	STOI	35	46		034	e^x		33
004	*LBL2	21	02		035	.		-62
005	RCLI	36	46		036	0		00
006	SIN		41		037	2		02
007	1/X		52		038	6		06
008	RCLE	36	15		039	9		09
009	x		-35		040	7		07
010	CHS		-22		041	5		05
011	e^x		33		042	x		-35
012	STO1	35	01		043	RCL1	36	01
013	RCLI	36	46		044	X⇌Y		-41
014	5		05		045	÷		-24
015	7		07		046	PRTX		-14
016	.		-62		047	RCLI	36	46
017	2		02		048	PRTX		-14
018	9		09		049	SPC	16	-11
019	6		06		050	RCLA	36	11
020	÷		-24		051	X=Y?	16	-33
021	RCLD	36	14		052	GTO4	22	04
022	x		-35		053	GTO3	22	03
023	CHS		-22		054	*LBL3	21	03
024	e^x		33		055	RCLI	36	46
025	STx1	35-35	01		056	5		05
026	RCLI	36	46		057	-		-45
027	SIN		41		058	STOI	35	46
028	X^2		53		059	GTO2	22	02
029	STx1	35-35	01		060	*LBL4	21	04
030	RCL1	36	01		061	RTN		24
031	PRTX		-14		062	R/S		51

APPENDIX A3. Hewlett-Packard HP-97 Code for Calculating C(ℓ) and
Numerical Approximations to the Function C(ℓ)

This program, written for the Hewlett-Packard HP-97 calculator, evaluates C(ℓ) = $H_m/H(\pi/2)$, i.e., the maximum dose equivalent at the shield outer surface for a point source, divided by the value at 90°. Values of the angle, θ_m, at which the maximum dose equivalent is observed at the shield outer surface, assuming a point source, have been determined from Equation (7) and plotted in Fig. 3*:

$$\ell\cot\,\theta_m\,\csc\,\theta_m - \beta + 2\cot\,\theta_m = 0 \qquad (7)$$

These values of θ_m, when substituted in Equation (9) shown below, yield values of C(ℓ) as a function of ℓ. These are shown as the solid line in Fig. A3-1.

$$C(\ell) = \exp[2.3(\pi/2 - \theta_m)]\,\exp[\ell(1 - \csc\theta_m)]\,\sin^2\theta_m \qquad (9)$$

Simple approximations to θ_m and C(ℓ) are given below. The angle θ_m can be approximated to better than ±5 percent from ℓ = 1 to ℓ = 15 by the simple expression

$$\theta_m = 55.9 + 9.8\,\ln(\ell)\ \text{(degrees)} \qquad \text{(A3-1)}$$

and C(ℓ) can be approximated by the following expression:

$$C(\ell) = 2.2\,\ell^{-0.245} \qquad (9a)$$

This is shown as the dashed line in Fig. A3-1. Although greater accuracy is not required, the following polynomial fits C(ℓ) very well as shown by the dots in Fig. A3-1:

$$C(\ell) = 2.652 - 0.589\ell + 0.116\ell^2 - 0.0124\ell^3$$

$$+ 6.588 \times 10^{-4}\ell^4 - 1.378 \times 10^{-5}\ell^5 \qquad \text{(A3-2)}$$

---

*Equation and figure numbering are as in text.

APPENDIX A4. Hewlett-Packard HP-97 Code for Calculating the
Moyer Integral and Dose Equivalent for Extended Uniform
Line Sources

This program uses Simpson's Rule to calculate the Moyer Integral for extended, finite and infinite, uniform line sources. It also calculates the dose-equivalent on the shield outer surface for unit beam loss rate [Equation (16)] and for unit beam loss [Equation (18a)] i.e., S = 1 proton per meter, and N = 1 proton lost along L:*

$$H = \frac{\mathcal{H}(E)S}{R} M_{\alpha_1,\alpha_2}[\beta,\ell] \qquad \text{and} \qquad (16)$$

$$H = \frac{\mathcal{H}(E)N}{\eta\,R^2} M_{\alpha_1,\alpha_2}[\beta,\ell] \qquad . \qquad (18a)$$

To use the program:

(1) Store the following in the indicated registers:

Register E ℓ the number of attenuation lengths along R;

 1 R (meters);

 2 L (meters);

 3 E proton beam energy (GeV);

 4 Z (>0, meters), the distance along the shield outer wall at which the dose equivalent is determined. Z = 0 coincides with the start of beam spill.

(2) Press B. This step performs the geometry routines which store the integration limits in registers A and B. This is followed by the Simpson's Rule integration to determine the Moyer integral.

(3) Press C. This calculates the loss per unit length case (S = 1 proton m^{-1}) from Equation (16).

(4) Press D. This calculates the unit loss case (N = 1 proton) from Equation (18a).

---

*Equation numbering is as in text.

001	*LBLA	21	11	053	PRTX		-14	105	GTOA		22	11
002	0		00	054	R/S		51	106	*LBL8		21	08
003	ST00	35	00	055	*LBL3	21	03	107	RCL4		36	04
004	RCLB	36	12	056	ST06	35	06	108	RCL2		36	02
005	RCLA	36	11	057	2		02	109	-			-45
006	ST09	35	09	058	.		-62	110	RCL1		36	01
007	-		-45	059	3		03	111	X⇌Y			-41
008	2		02	060	x		-35	112	÷			-24
009	0		00	061	CHS		-22	113	TAN⁻¹		16	43
010	÷		-24	062	eˣ		33	114	STOB		35	12
011	ST08	35	08	063	RCL6	36	06	115	GTOA		22	11
012	LSTX	16	-63	064	X⇌Y		-41	116	*LBLC		21	13
013	X⇌I	16	-41	065	ST06	35	06	117	SF1	16	21	01
014	DSZI	16 25	46	066	X⇌Y		-41	118	*LBLE		21	15
015	RCLA	36	11	067	SIN		41	119	RCL0		36	00
016	GSB3	23	03	068	X=0?	16	-43	120	STOI		35	46
017	ST+0	35-55	00	069	RTN		24	121	GSBb	23 16	12
018	*LBL2	21	02	070	1/X		52	122	RCLI		36	46
019	RAD	16	-22	071	RCLE	36	15	123	x			-35
020	RCL8	36	08	072	x		-35	124	RCL1		36	01
021	ST+9	35-55	09	073	CHS		-22	125	÷			-24
022	RCL9	36	09	074	eˣ		33	126	F1?	16 23	01
023	GSB3	23	03	075	RCL6	36	06	127	PRTX			-14
024	4		04	076	x		-35	128	RTN			24
025	x		-35	077	RTN		24	129	R/S			51
026	ST+0	35-55	00	078	*LBLB	21	12	130	*LBLb	21 16	12
027	DSZI	16 25	46	079	RCL1	36	01	131	RCL3		36	03
028	GT04	22	04	080	RCL4	36	04	132	.			-62
029	GT06	22	06	081	÷		-24	133	8			08
030	*LBL4	21	04	082	TAN⁻¹	16	43	134	Yˣ			31
031	DSZI	16 25	46	083	STOA	35	11	135	2			02
032	GT09	22	09	084	RCL2	36	02	136	.			-62
033	GT06	22	06	085	RCL4	36	04	137	8			08
034	*LBL9	21	09	086	X=Y?	16	-33	138	4			04
035	RCL8	36	08	087	GT07	22	07	139	EEX			-23
036	ST+9	35-55	09	088	X>Y?	16	-34	140	CHS			-22
037	RCL9	36	09	089	GT08	22	08	141	1			01
038	GSB3	23	03	090	-		-45	142	3			03
039	2		02	091	RCL1	36	01	143	x			-35
040	x		-35	092	X⇌Y		-41	144	RTN			24
041	ST+0	35-55	00	093	÷		-24	145	R/S			51
042	GT02	22	02	094	TAN⁻¹	16	43	146	*LBLD		21	14
043	*LBL6	21	06	095	CHS		-22	147	CF1	16	22	01
044	RCLB	36	12	096	Pi	16	-24	148	GSBE		23	15
045	GSB3	23	03	097	+		-55	149	RCL2		36	02
046	ST+0	35-55	00	098	STOB	35	12	150	÷			-24
047	RCL8	36	08	099	GTOA	22	11	151	PRTX			-14
048	STx0	35-35	00	100	*LBL7	21	07	152	RTN			24
049	3		03	101	Pi	16	-24	153	R/S			51
050	ST÷0	35-24	00	102	2		02					
051	RCL0	36	00	103	÷		-24					
052	DSP3	-63	03	104	STOB	35	12					

EXAMPLES:

Example 1. Calculate the Moyer integral, $M(2.3, \ell)$, for an infinite line source. Approximate the infinite line source by a 10 km line ($L = 10,000$ m) with the point of measurement at 5 km ($Z = 5000$ m). Let $\ell = 5$, $R = 1$ m and $E = 10$ GeV. Press B. Read $M(2.3, 5) = 2.63 \times 10^{-4}$.

Example 2. Calculate the dose equivalent under the above conditions for a proton beam energy of 25 GeV when the loss rate is 1 proton per meter. Press C. Read 9.82×10^{-16} Sv.

Example 3. Repeat the 2nd example for a loss of 1 proton along L. Press D. Read 9.82×10^{-20} Sv.

Example 4. Calculate the restricted Moyer integral $M_{\alpha_1, \alpha_2}[\beta, \ell]$ at $Z = 2$m for a spill length, L, of 4 meters where $R = 1$ m ($\eta = 4$). Let $\ell = 5$. Press B. Read $M_{\alpha_1, \alpha_2}[\beta, \ell] = 2.63 \times 10^{-4}$.

Example 5. Calculate the dose equivalent under the above conditions when $E = 25$ GeV for a loss rate of 1 proton per meter. Press C. Read $H_L = 9.81 \times 10^{-16}$ Sv.

Example 6. Repeat Example 5 for a loss of 1 proton along L. Press D. Read $H_L = 2.42 \times 10^{-16}$ Sv.

APPENDIX A5. Hewlett-Packard HP-97 Code for Calculating Dose
Equivalent from Point, Extended and Infinite Uniform Line
Sources by Approximate Methods

This program, written for the Hewlett-Packard HP-97 calculator, solves the
following equations. First, assuming a point source:

1. $H(\pi/2) = \mathcal{H} R^{-2} \exp(-2.3\pi/2) \exp(-\ell)$ (Sv per proton);   (A5-1)

2. $H(\theta) = \mathcal{H} R^{-2} \sin^2\theta \exp(-2.3\ \theta) \exp(-\ell \csc\theta)$ (Sv per proton); and   (A5-2)

3. $H(\theta_m) = C(\ell)\ H(\pi/2)$ (Sv per proton), where   (A5-3)

$C(\ell) = 2.2\ \ell^{-0.245}$. The expression $\theta_m = (55.9 + \ln \ell)$ (degrees) is used
to estimate θ_m, and $Z(\theta_m) = (R/\tan\theta_m)$ (meters) is used to define the distance
along the outer surface of the shield at which the maximum dose equivalent,
$H(\theta_m)$, is observed relative to the position at 90° to the loss point. Then,
assuming an extended uniform line source of length L:

4. $H_L = \dfrac{S\ L\ H(\theta_m)}{\eta\ B(\ell)} = \dfrac{R\ H(\theta_m)}{B(\ell)}$ (Sv per proton per meter) .   (A5-4)

Here, $H(\theta_m)$ is multiplied by S L so that the number of protons lost is the
same for $H(\theta_m)$ as for H_L. S is assumed in this program always to be equal
to one proton per meter along L. The function, $B(\ell)$, is approximated by

$B(\ell) = 0.44 + 0.12\ \ell - 0.0028\ \ell^2$.   (A5-5)

This calculation should be restricted to values of $\eta > \eta_L$, where η_L is
dependent on ℓ. For values of η in the intermediate region, the restricted
Moyer integral solution should be used (Appendix A4).

To Use the Program

Store in the following registers:

Register D θ(degrees), any angle of interest
   E $\ell = d/\lambda$;
   1   R(m);
   2   L(m);
   3   E, proton beam energy (GeV);
   4   Z(m).

Press A to calculate $H(\pi/2)$;
   B $H(\theta)$;
   C $H(\theta_m)$;
   D H_L.

Examples:

1. Calculate $H(\pi/2)$, $H(\theta_m)$ and $H(60°)$ for the following parameters:

    ℓ = 5 (dimensionless);
    R = 2 m;
    E = 10 GeV.

    Find θ_m and $Z(\theta_m)$. Also find H_L when L = 8 m and Z = 6 m:

$H(\pi/2)$	= 8.147964468-17	Sv/proton
$H(60°)$	= 9.399262676-17	Sv/proton
$H(\theta_m)$	= 1.208437395-16	Sv/proton
$Z(\theta_m)$	= 0.663	meters
θ_m	= 71.672	degrees
H_L	= 2.491623494-16	Sv per proton per meter.

2. Calculate the dose equivalent for an "infinite" uniform source of L = 10,000 m.

    Let R = 1 m, ℓ = 5, and the point of measurement be halfway along the spill, Z = 5000 m. The proton beam energy is 25 GeV.

    H_L = 1.037204566-15   Sv per proton per meter.

001	*LBLA	21	11	053	RCL7	36	07	105	8		08
002	9		09	054	SIN		41	106	x		-35
003	0		00	055	1/X		52	107	5		05
004	ST07	35	07	056	RCLE	36	15	108	5		05
005	GSB1	23	01	057	x		-35	109	.		-62
006	PRTX		-14	058	CHS		-22	110	9		09
007	SPC	16	-11	059	e^x		33	111	+		-55
008	R/S		51	060	ST08	35	08	112	STOI	35	46
009	*LBLB	21	12	061	RCL7	36	07	113	TAN		43
010	RCLD	36	14	062	.		-62	114	1/X		52
011	ST07	35	07	063	0		00	115	RCL1	36	01
012	GSB1	23	01	064	1		01	116	x		-35
013	PRTX		-14	065	7		07	117	RTN		24
014	SPC	16	-11	066	4		04	118	*LBL4	21	04
015	R/S		51	067	5		05	119	RCLE	36	15
016	*LBLC	21	13	068	x		-35	120	x^2		53
017	9		09	069	2		02	121	.		-62
018	0		00	070	.		-62	122	0		00
019	ST07	35	07	071	3		03	123	0		00
020	GSB1	23	01	072	x		-35	124	2		02
021	STOI	35	46	073	CHS		-22	125	8		08
022	GSB2	23	02	074	e^x		33	126	x		-35
023	RCLI	36	46	075	STx8	35-35	08	127	CHS		-22
024	x		-35	076	RCL7	36	07	128	ST09	35	09
025	PRTX		-14	077	SIN		41	129	RCLE	36	15
026	GSB3	23	03	078	x^2		53	130	.		-62
027	PRTX		-14	079	STx8	35-35	08	131	1		01
028	RCLI	36	46	080	RCL8	36	08	132	2		02
029	PRTX		-14	081	RCL9	36	09	133	x		-35
030	SPC	16	-11	082	x		-35	134	ST+9	35-55	09
031	R/S		51	083	RCL1	36	01	135	.		-62
032	*LBLD	21	14	084	x^2		53	136	4		04
033	9		09	085	÷		-24	137	4		04
034	0		00	086	RTN		24	138	ST+9	35-55	09
035	ST07	35	07	087	*LBL2	21	02	139	RTN		24
036	GSB1	23	01	088	RCLE	36	15	140	*LBL5	21	05
037	STOI	35	46	089	.		-62	141	RCL3	36	03
038	GSB2	23	02	090	2		02	142	.		-62
039	RCLI	36	46	091	4		04	143	8		08
040	x		-35	092	5		05	144	Y^x		31
041	STOI	35	46	093	CHS		-22	145	2		02
042	GSB4	23	04	094	Y^x		31	146	.		-62
043	RCLI	36	46	095	2		02	147	8		08
044	RCL9	36	09	096	.		-62	148	4		04
045	÷		-24	097	2		02	149	EEX		-23
046	RCL1	36	01	098	x		-35	150	CHS		-22
047	x		-35	099	RTN		24	151	1		01
048	PRTX		-14	100	*LBL3	21	03	152	3		03
049	R/S		51	101	RCLE	36	15	153	x		-35
050	*LBL1	21	01	102	LN		32	154	RTN		24
051	GSB5	23	05	103	9		09	155	R/S		51
052	ST09	35	09	104	.		-62				

LIST OF FIGURES

Fig. 10.   Maximum dose equivalent, H_m, on the shield outer surface for a
            finite uniform beam loss of 1 proton/m over a spill length $\eta = L/R$,
            as a function of η. Curve is calculated with the parameters shown
            using restricted Moyer Integrals. The rising straight line is the
            asymptote obtained by assuming all beam loss occurs at a point.
            The horizontal dashed line is the limit for an infinite uniform
            line source. Intermediate region shows where actual prediction
            deviates from either the point-source or infinite-line-source
            approximations by more than 10 percent when $\ell = 5$. Shaded region
            indicates amount of overestimation that can be made by assuming the
            minimum of either the point source ($N = S \times L$) or infinite uniform
            line source assumptions.

Fig. 11.   Percent overestimation of true maximum dose equivalent, H_m, on
            the shield outer surface, as a function of beam spill length
            $\eta = L/R$ (for $\ell = 5$), if either the infinite-line source or
            point-source approximation is assumed instead. Values η_p, η_∞
            indicate where 10 percent overestimate occurs. Curves are based on
            the data of Fig. 9.

Fig. 12.   Function $B(\ell) = \eta^{-1} H_m/H_\infty$, which relates the actual maximum dose
            equivalent, H_m, to that given by an infinitely extended uniform
            source having the same beam loss per unit length $S = N/L$. Solid
            curve: calculation; dashed curve: polynomial fit to calculated
            values.

Fig. A3-1. Function $C(\ell) = H_m/H(\pi/2)$, which relates the maximum dose equiva-
            lent on the shield outer surface to that at 90°, is shown as a
            solid curve, the power law as a dashed curve, and the 5th order
            polynomial, practically indistinguishable from $C(\ell)$, is shown by
            the points.
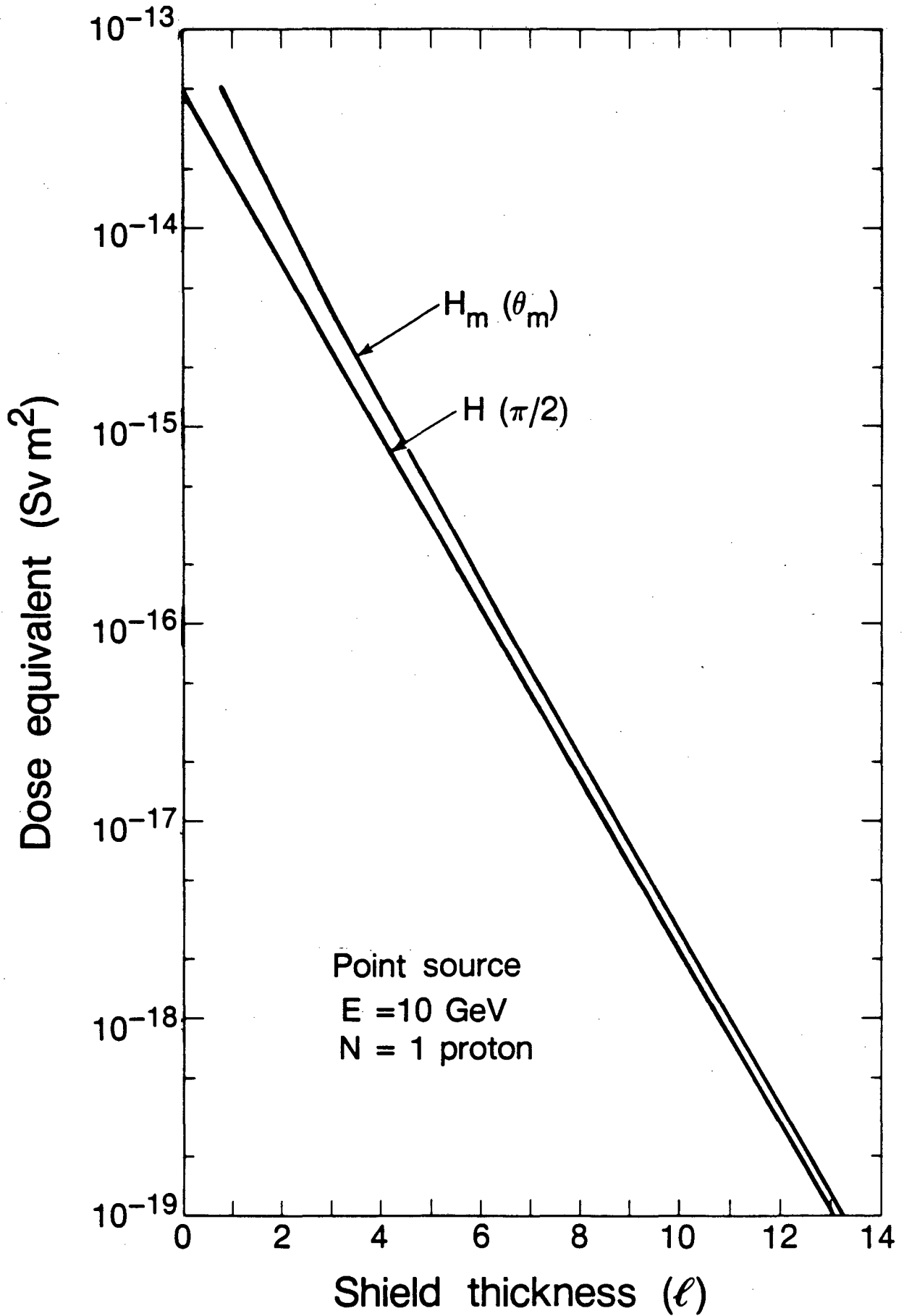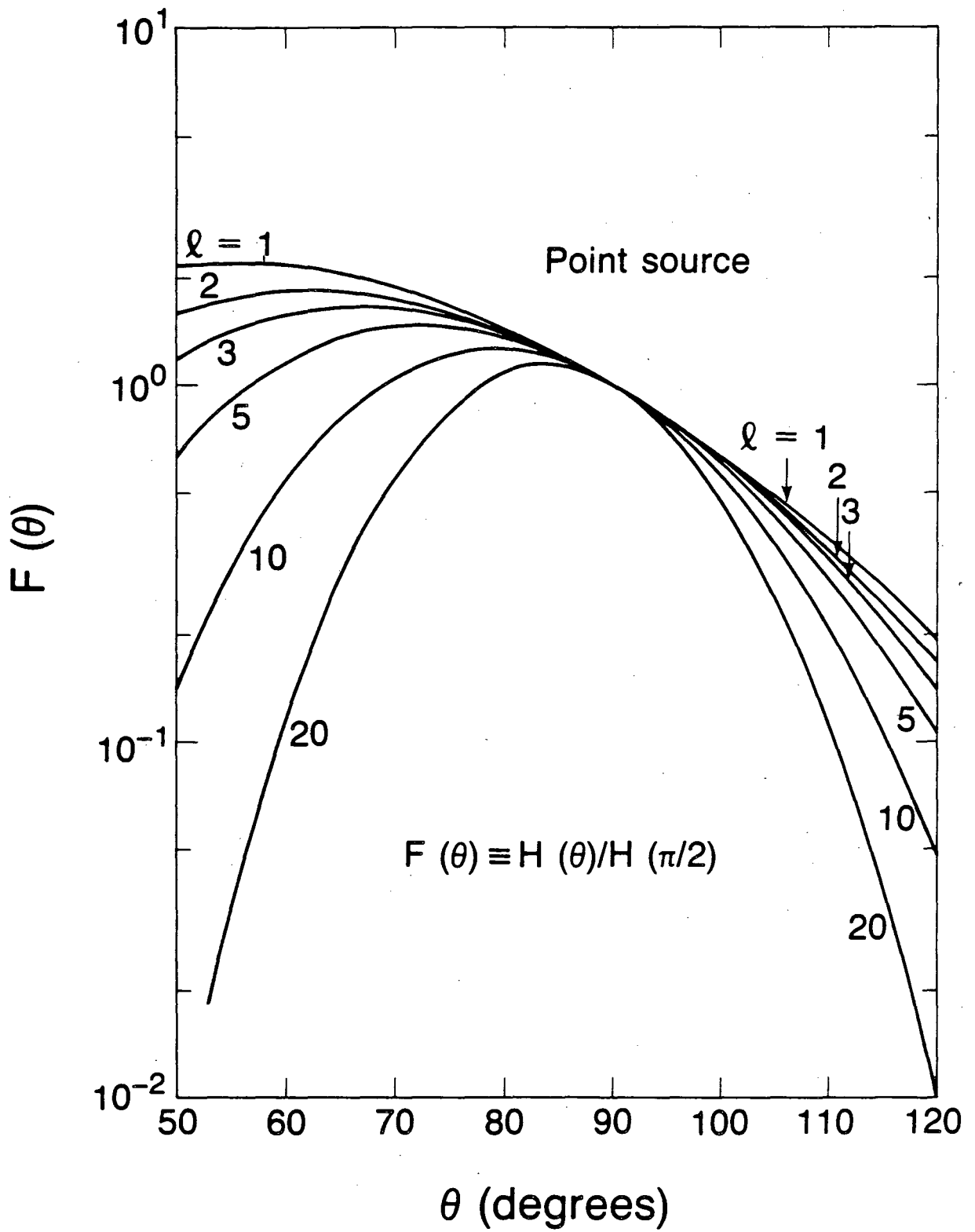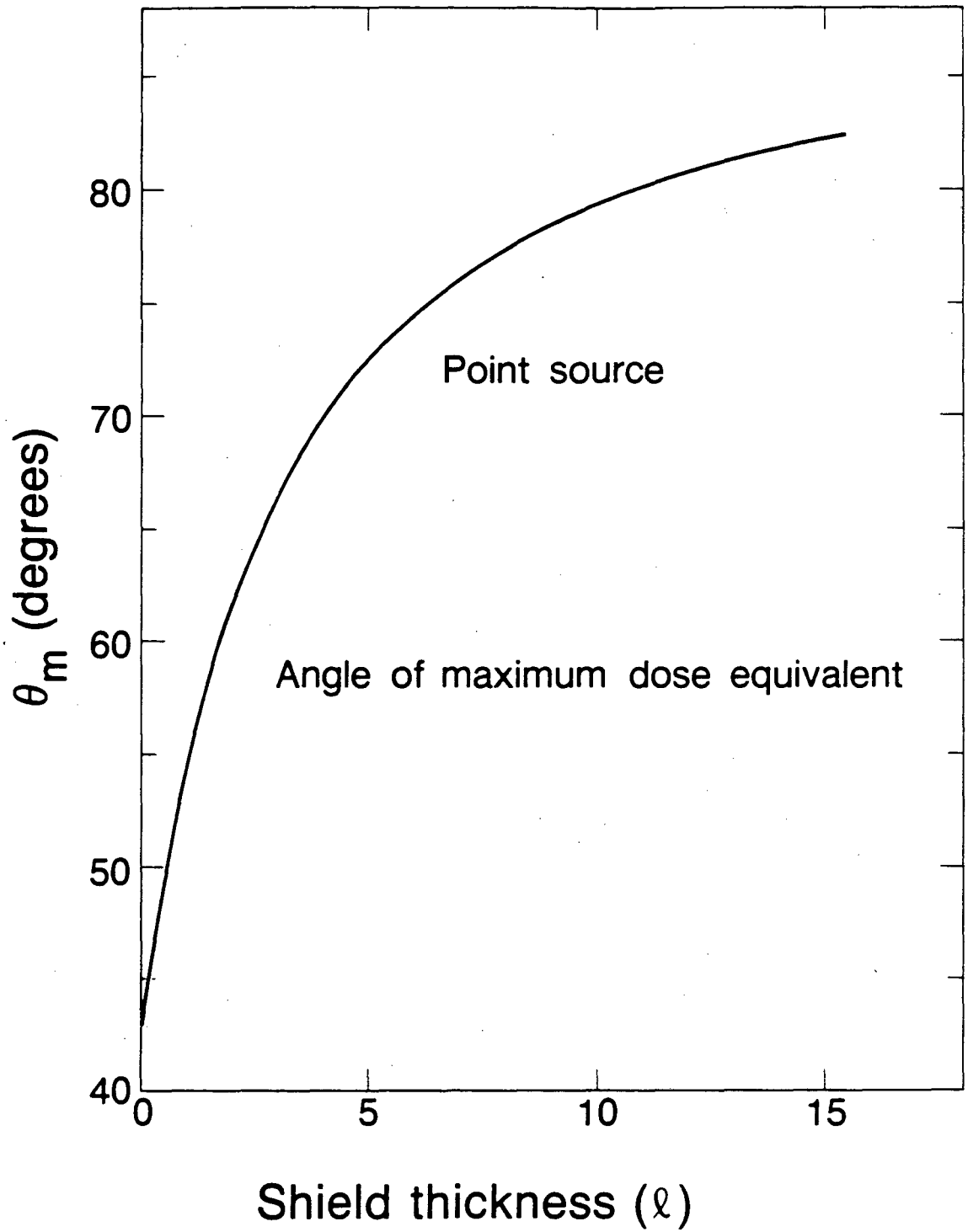
(a)

P(π/2), H(π/2) $P_m(\theta_m)$, $H_m(\theta_m)$ P(θ), H(θ)

Ringtop

Shield

d

R

a

Point loss target

r θ_m θ

Beam line

Tunnel

Z

(b)

Shield

R

d

a

Tunnel
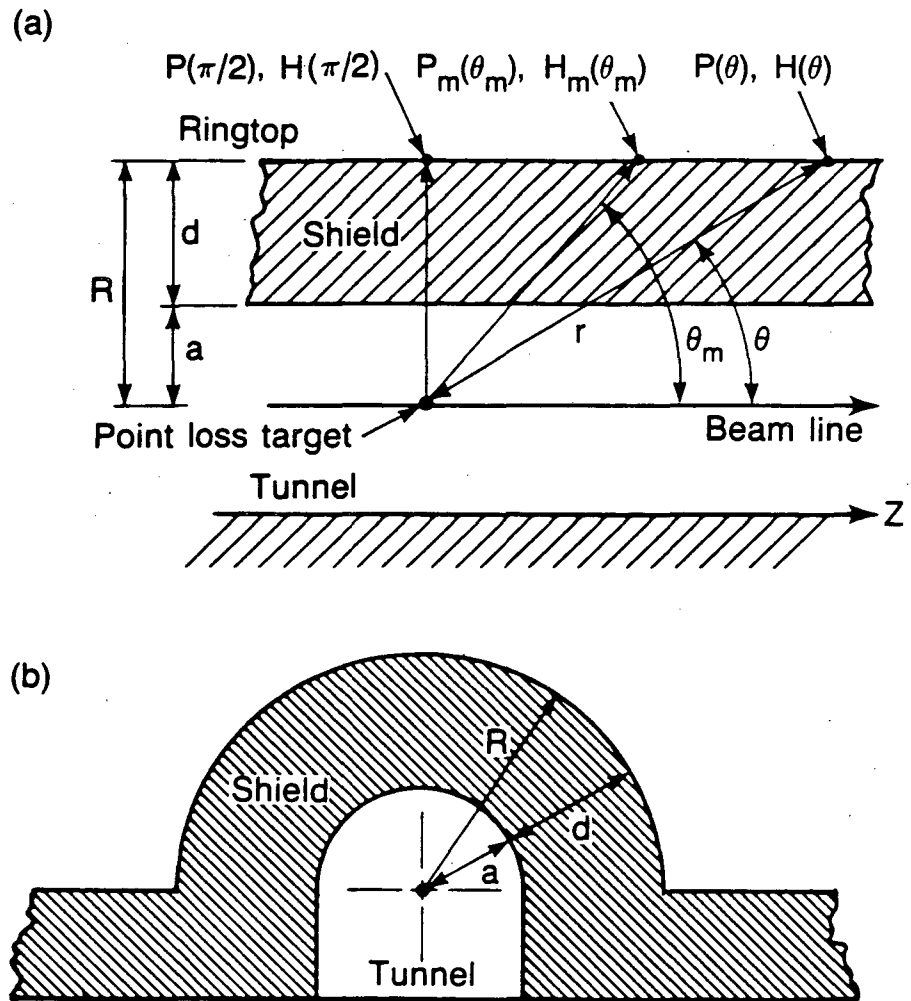
XBL 8411-6346

Fig. 1

Fig. 2

Fig. 3

XBL 8411-6352

Point source

Angle of maximum dose equivalent

Shield thickness (ℓ)

Fig. 4

XBL 8411-6348

Fig. 5

XBL 8411-6351

Finite uniform line source

Fig. 6   XBL 8411-6355

N = 1 proton
ℓ = 5

η = 0.002
0.2
0.4
0.6
0.8
1.0
1.5
2.0
4.0

Start of beam loss

Dose equivalent at 1m (Sv)

Distance from start of beam loss (Z/R)
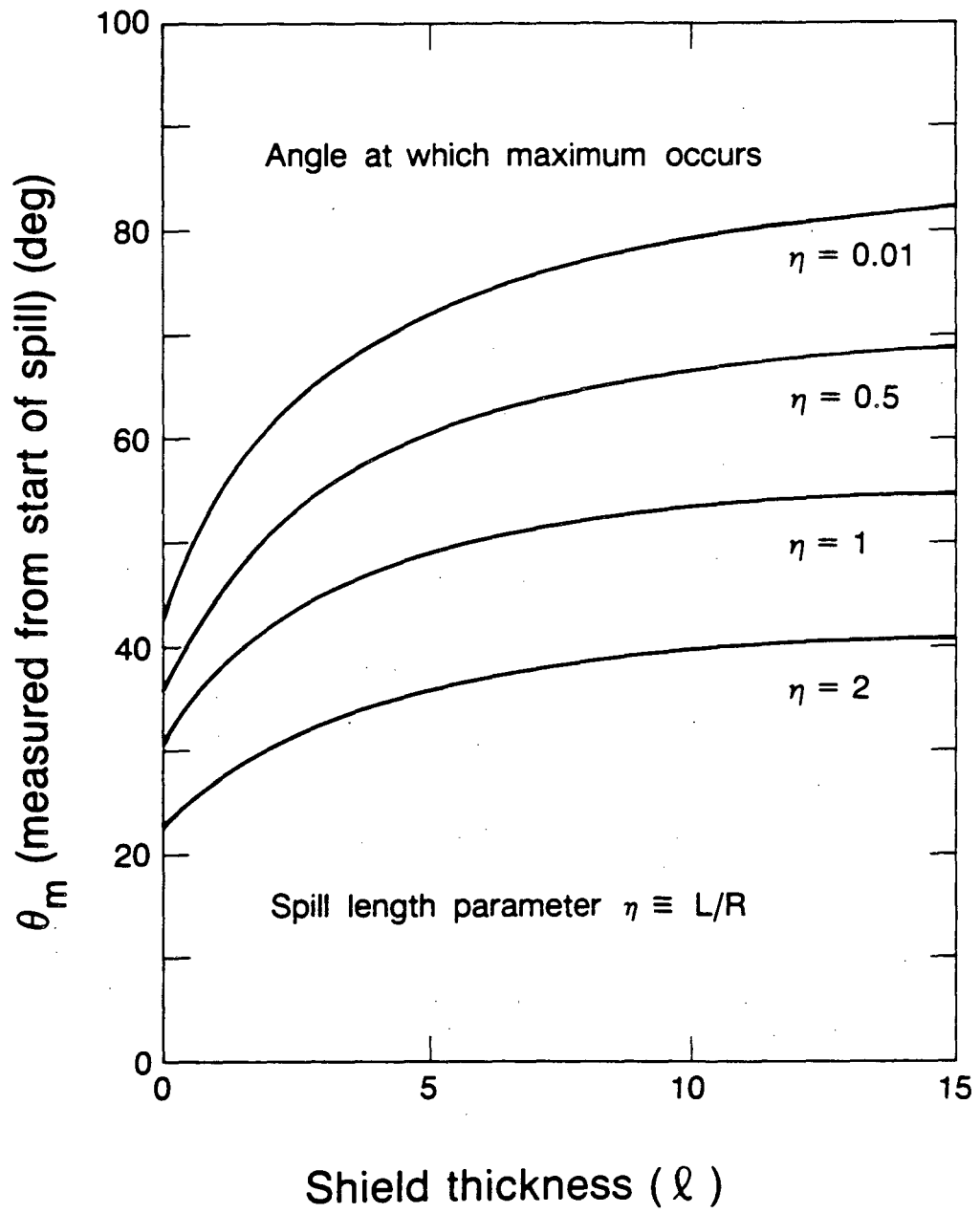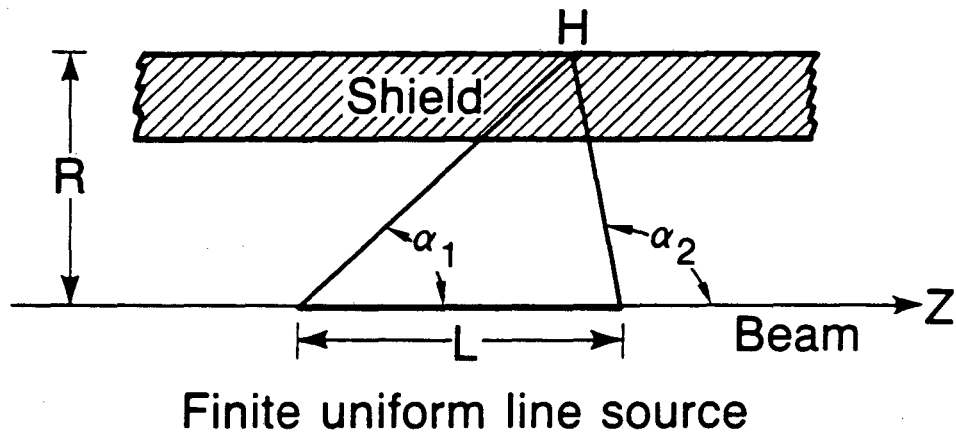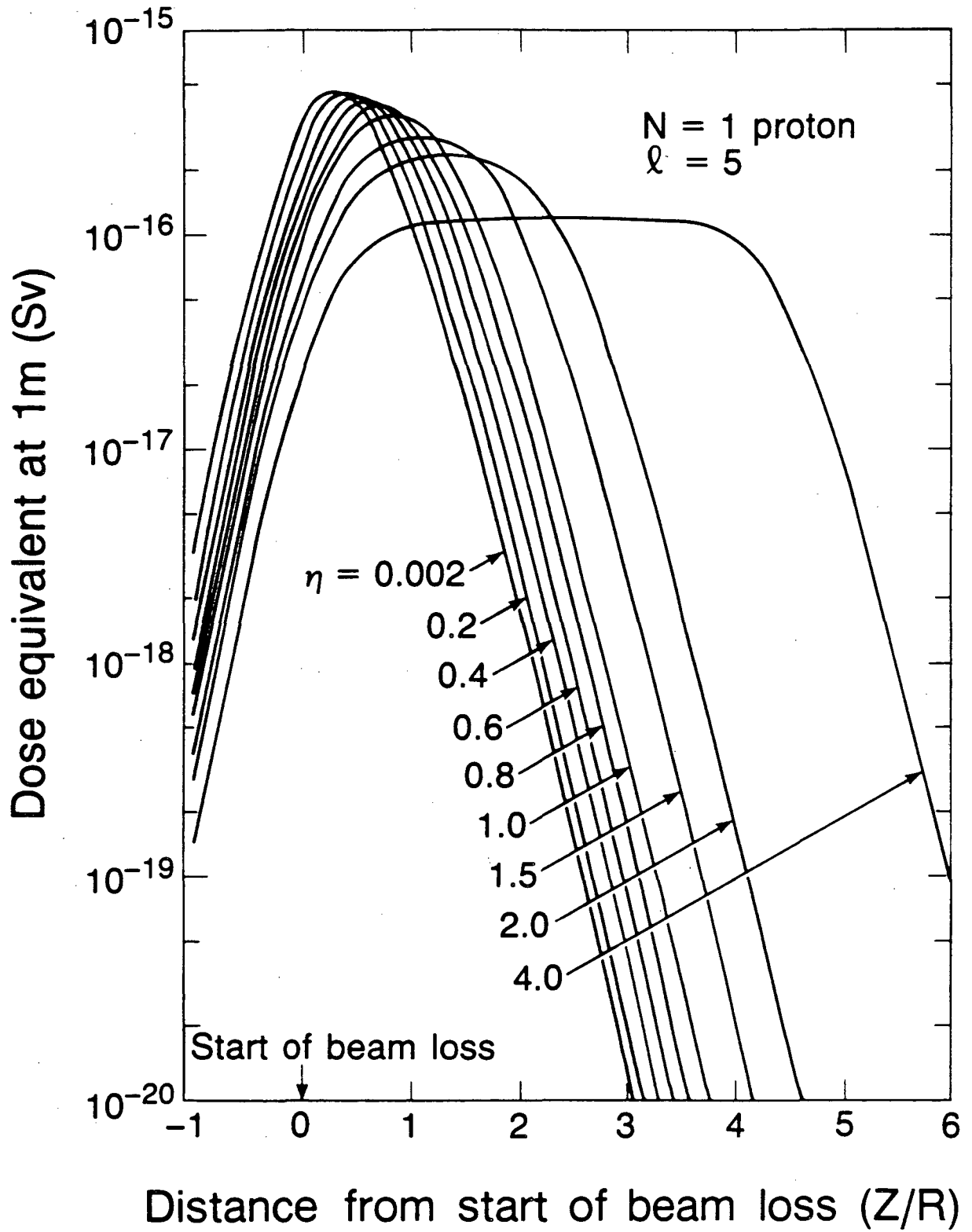
Fig. 7
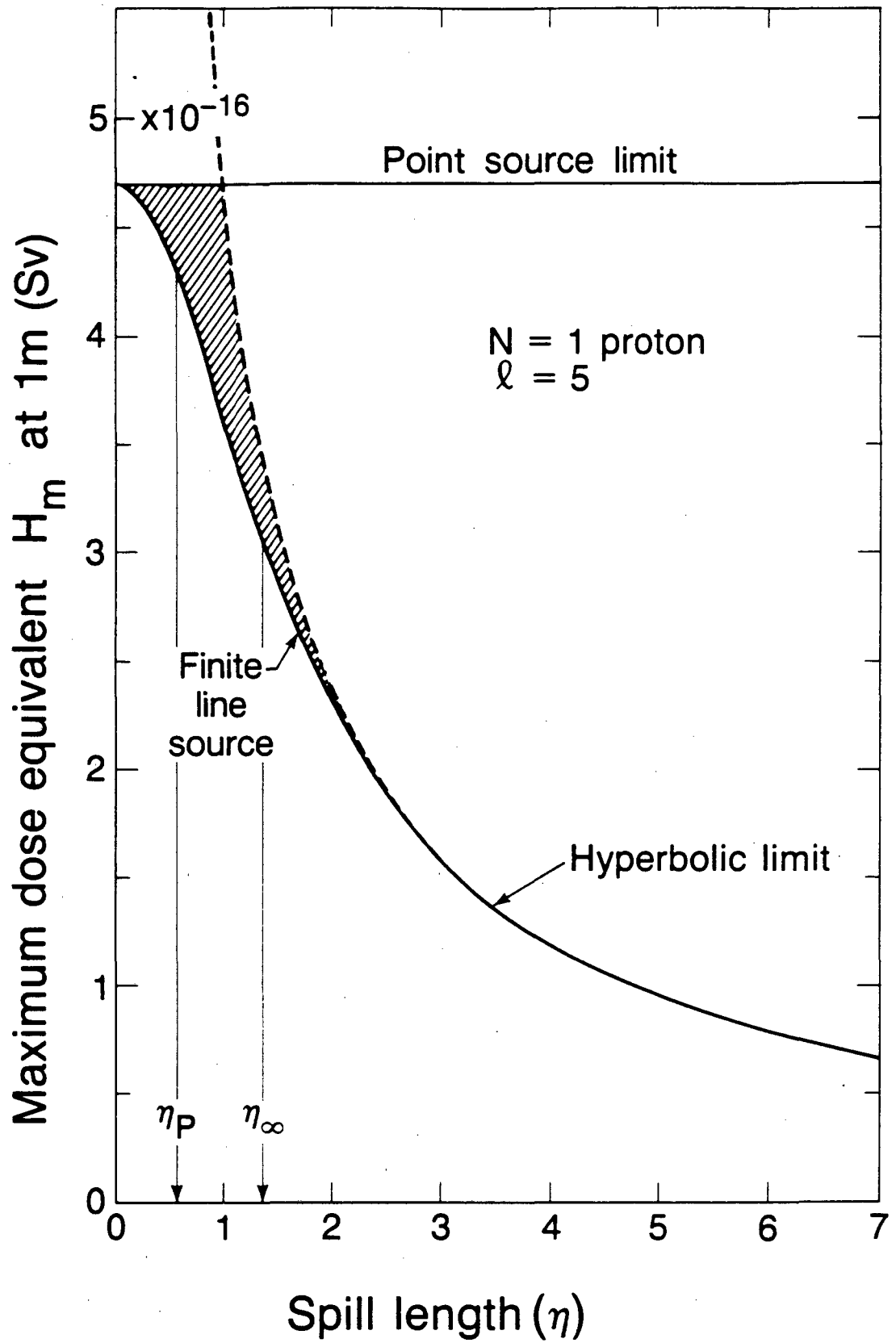
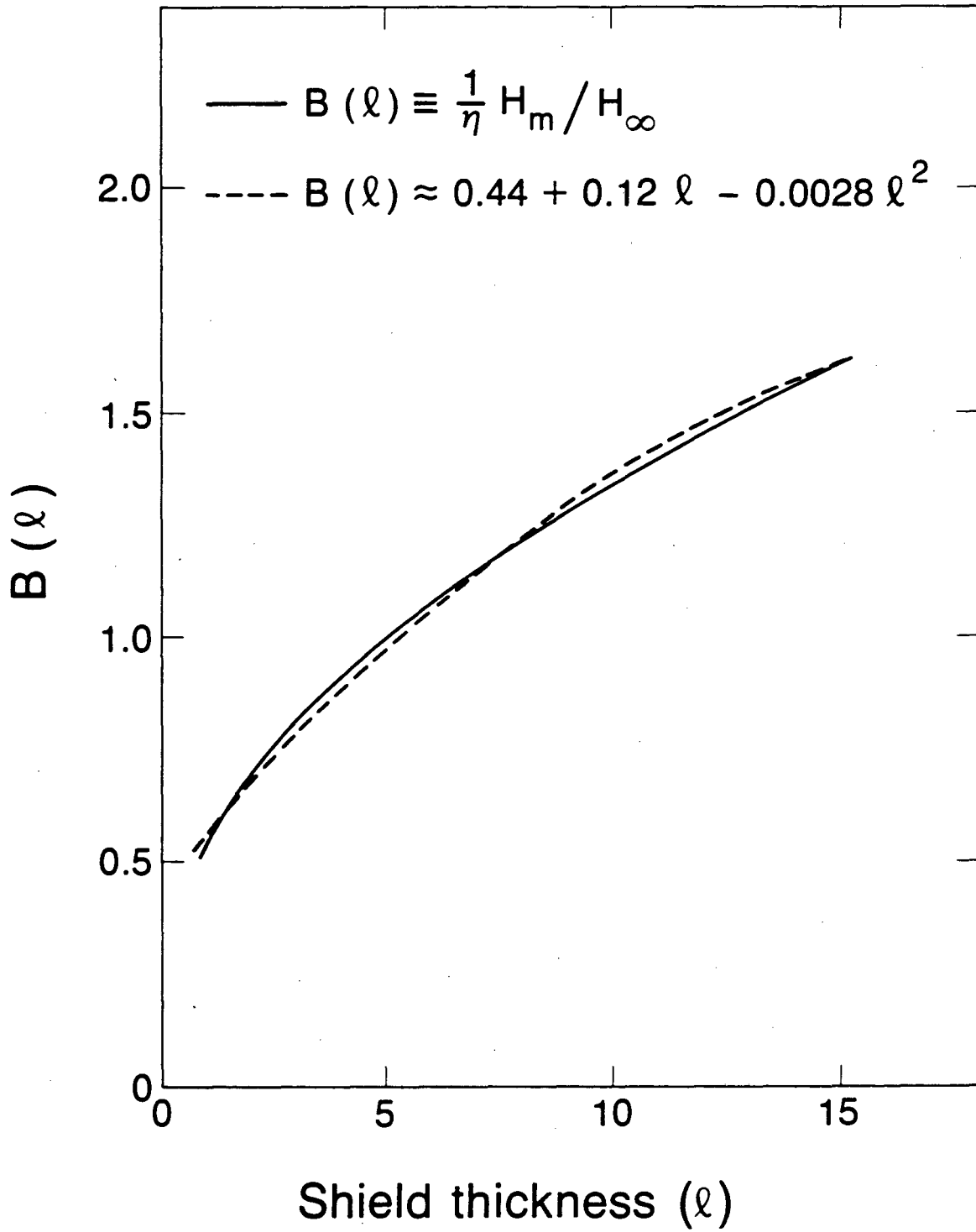XBL 8411-6345

Fig. 8

XBL 8411-6354

Fig. 9

XBL 8411-6342

Fig. 10

XBL 8411-6358

$$100 \times \left[\frac{H_A - H_m}{H_m} \right]$$

Point source assumption (constant N)

Infinite line source assumption (constant S)

η_P

η_∞

Percent overestimate of H_m

Spill length (η)
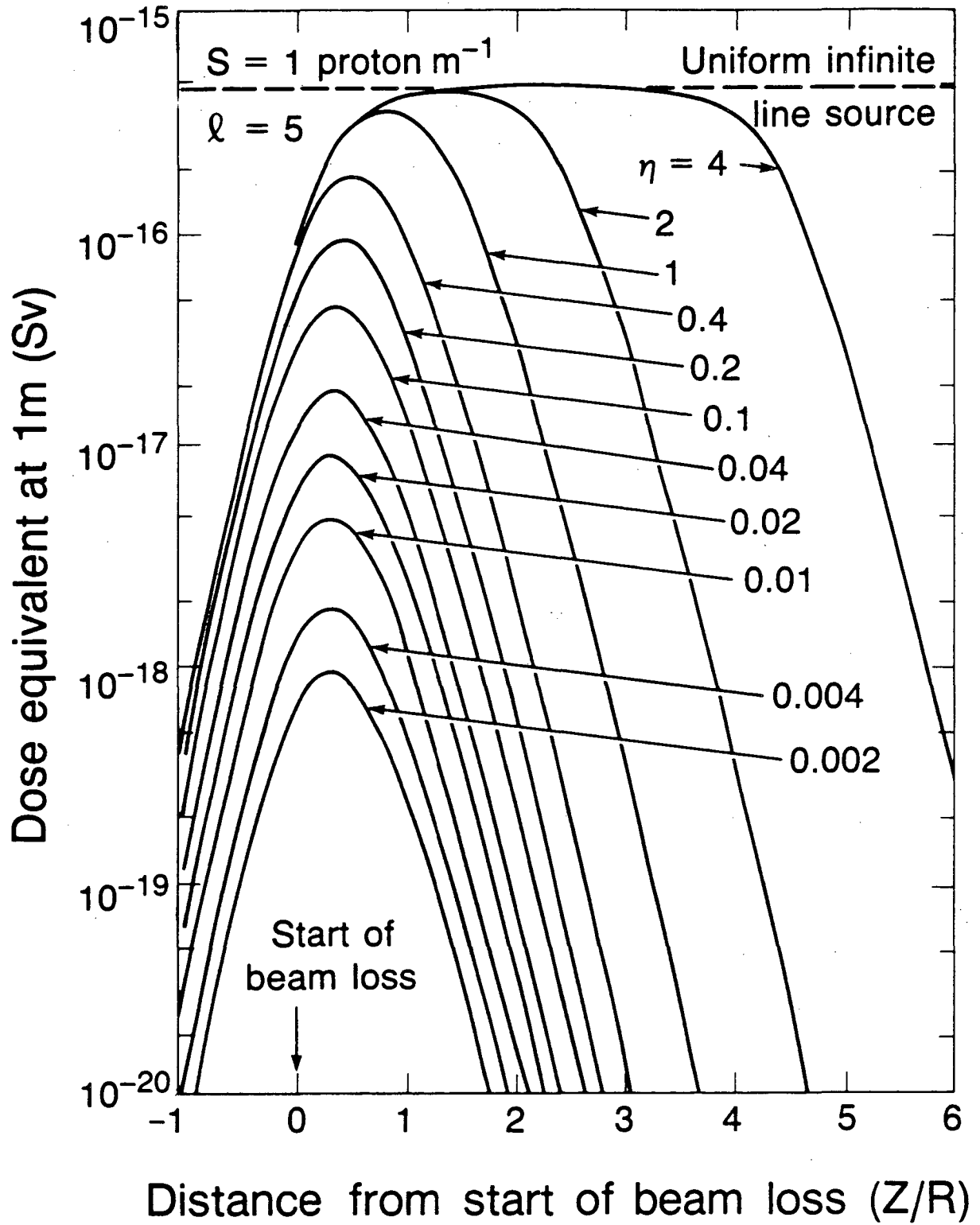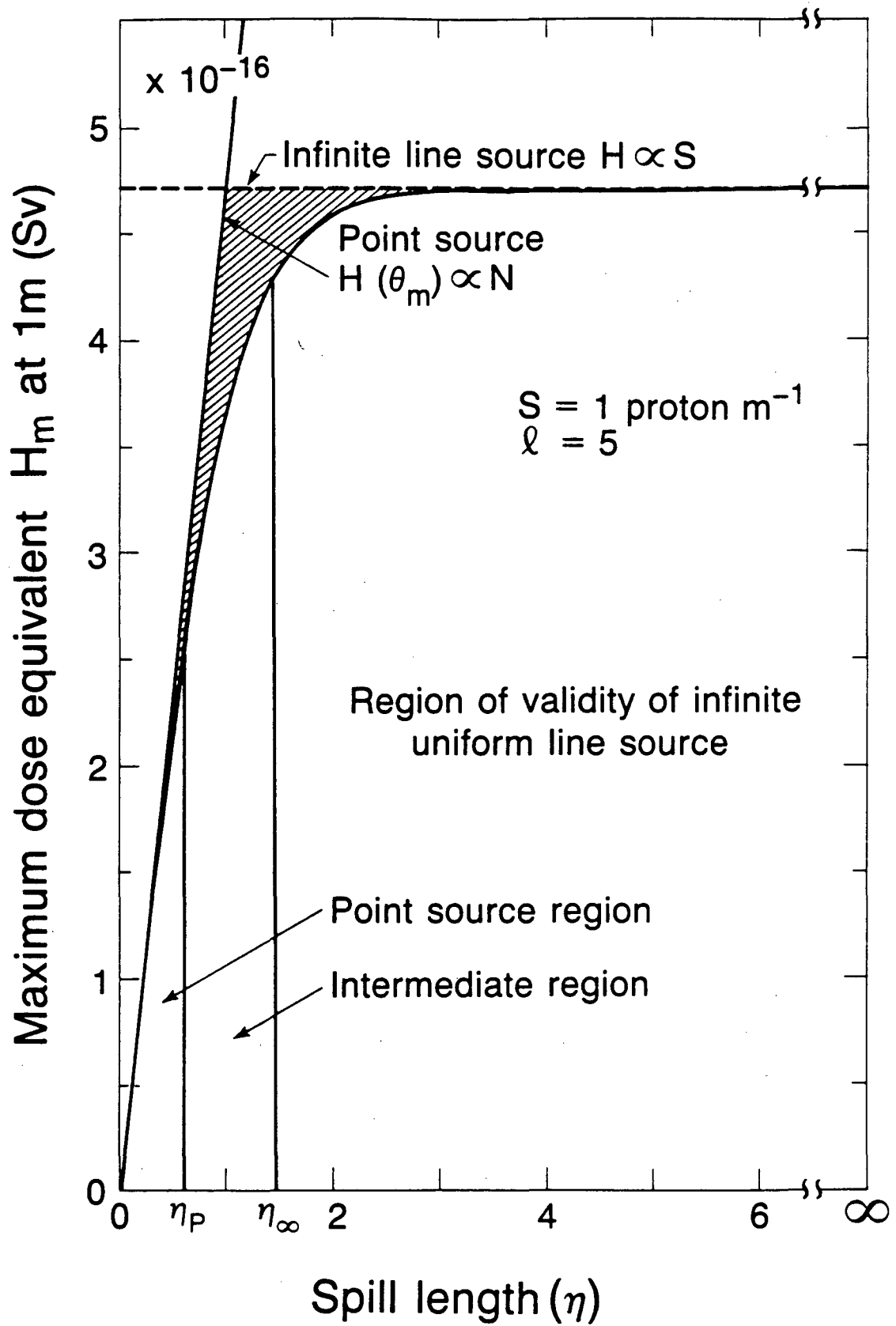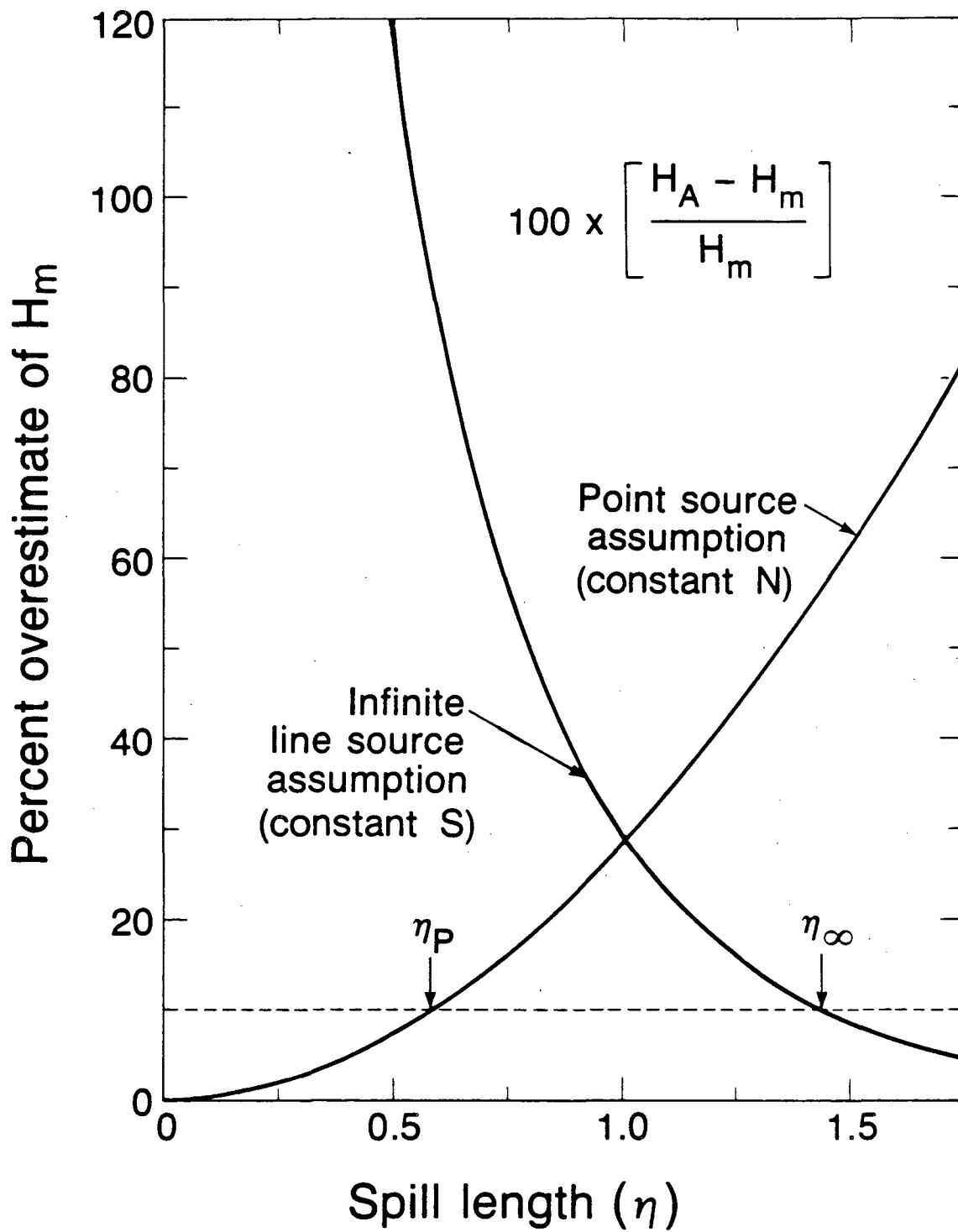
Fig. 11

XBL 8411-6357

$$B(\ell) \equiv \frac{1}{\eta} H_m / H_\infty$$

$$B(\ell) \approx 0.44 + 0.12\,\ell - 0.0028\,\ell^2$$

Fig. 12

XBL 8411-6353

Fig. A-3.1

XBL 8411-6356