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Connection between Flavor Mixing of Cosmologically Significant Neutrinos and Heavy Element Nucleosynthesis in Supernovae

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We use heavy element nucleosynthesis from supernovae to probe the mixing of ν_e with ν_τ (or ν_μ) possessing cosmologically significant masses (1 to 100 eV). We conclude that the $\nu_\tau(\nu_\mu)$ - ν_e vacuum mixing angle must satisfy $\sin^2 2\theta < 10^{-5}$, in order to ensure that r -process heavy elements can be produced in neutrino-heated supernova ejecta. Mixing at a level exceeding this limit precludes r -process nucleosynthesis in this site.

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In this Letter we show how the heavy element nucleosynthesis in the ejected material from the late stages of type II supernovae is sensitive to matter-enhanced neutrino flavor transformation. Rapid neutron capture process (r -process) nucleosynthesis may occur in the supernova environment. However, this process can operate only in neutron-rich conditions. Supernova ν_τ and ν_μ are more energetic than the ν_e , so that any $\nu_{\tau,\mu} \leftrightarrow \nu_e$ transformations result in higher energy electron neutrinos which are more readily captured by neutrons to produce protons. This implies that the neutron-to-proton ratio, and hence whether or not r -process nucleosynthesis is possible, is sensitive to neutrino flavor transformations. We exploit this sensitivity to probe the mixing between ν_e and a massive ν_τ or ν_μ .

The vacuum mass of the ν_τ (or ν_μ) which we can probe is between 1 and 100 eV. This corresponds to the mass required to give a mass-level crossing with a light ν_e in the region of the supernova where the relevant neutron-to-proton ratio for nucleosynthesis is determined. This region is located above the neutrinosphere and below the radius where weak nuclear reactions involving free nucleons freeze out [1]. Coincidentally, this is the same range of ν_τ (or ν_μ) mass which is relevant for dark matter [1-6]. The cosmological significance of this neutrino mass range, together with the lack of strict laboratory experimental constraints on ν_τ - ν_e and ν_μ - ν_e mixings in this mass range [7], makes this new probe of neutrino properties important.

Supernovae have long been implicated as the site for the r -process nucleosynthesis of heavy elements [8]. Recent calculations suggest that the neutrino-heated "hot bubble" which develops several seconds after the bounce of the core in a type II supernova is a likely site for r -process nucleosynthesis [9,10]. We therefore confine our subsequent discussion to the physics of the hot bubble region at times later than 1 s post-core bounce (or $t_{PB} > 1$ s).

The hot bubble epoch is between $t_{PB} \approx 3$ s and 15 s. The supernova explosion process has left a hot proto-neutron star with a relatively high-entropy (entropy per baryon $s/k > 400$) low-density electron-positron-pair dominated plasma above it. This region is heated primarily by absorption of neutrinos emitted from a neutrinosphere at a radius of approximately $r \gtrsim 10$ km [9,11]. Calculations show that the neutrino energy spectra and distribution functions at the neutrinosphere are roughly Fermi-Dirac with zero chemical potential [1].

During the hot bubble epoch, the surface layers of the proto-neutron star consist mostly of neutrons. This implies that the ν_e have a larger opacity (cross section per gram) than do the $\bar{\nu}_e$ because of the charged current capture reactions on free nucleons,

$$\nu_e + n \rightarrow p + e^-, \quad (1a)$$

$$\bar{\nu}_e + p \rightarrow n + e^+. \quad (1b)$$

This in turn implies that the $\bar{\nu}_e$ decouple deeper in the core and so have larger average energies than do the ν_e . Typical average energies for the $\bar{\nu}_e$ at this epoch are roughly 16 MeV while those for the ν_e are about 11 MeV. Neutrino species of all flavors have identical neutral current interactions but, due to energy threshold effects, the ν_τ , ν_μ , and their antiparticles lack the charged current capture reactions analogous to those in Eq. (1). The result is that the ν_τ , $\bar{\nu}_\tau$, ν_μ , and $\bar{\nu}_\mu$ have identical spectra with average energies of 25 MeV. Thus, the average neutrino energies in supernova models at this epoch will always satisfy $\langle E_{\nu_{\tau,\mu}} \rangle > \langle E_{\bar{\nu}_e} \rangle > \langle E_{\nu_e} \rangle$.

Close to the neutrinosphere the matter temperature is high enough that all strong and electromagnetic nuclear reactions are in equilibrium (nuclear statistical equilibrium, or NSE). When the temperature drops below about 0.5 MeV the nuclear reactions begin to freeze out (hereafter nuclear freeze-out)—the charged particle capture rates and photodissociation rates for nuclei fall below the expansion rate for the material. The nucleosynthesis pro-

duced in a mass element moving out with the wind is characterized by three physical quantities: the expansion time scale, the entropy per baryon, and the neutron-to-proton ratio. The neutron-to-proton ratio is $n/p = Y_e^{-1} - 1$, where Y_e is the number of electrons per baryon. Neutron-rich conditions obtain when $Y_e < 0.5$. The r process of nucleosynthesis is *only possible* when $Y_e < 0.5$ at freeze-out from NSE [10]. Freeze-out in proton-rich conditions at the relatively high entropies that obtain in the hot bubble would give an alpha-rich freeze-out or an rp process. These processes would produce some iron peak nuclei but no neutron-rich r -process nuclei with $A > 70$ [12].

The value of Y_e in the region above the neutrinosphere is determined by the interactions in Eq. (1). We can write the rate of change of Y_e with time (t) or radius (r) as

$$v(r) \frac{dY_e}{dr} = \frac{dY_e}{dt} = \lambda_1 - Y_e \lambda_2, \quad (2)$$

where $v(r)$ is the radial velocity field. In this equation, $\lambda_1 = \lambda_{\nu_e n} + \lambda_{e^+ n}$ and $\lambda_2 = \lambda_1 + \lambda_{\bar{\nu}_e p} + \lambda_{e^- p}$. We denote the rates of the reactions in Eqs. (1a) and (1b) as $\lambda_{\nu_e n}$ and $\lambda_{\bar{\nu}_e p}$, respectively. The rates for the reverse reactions associated with Eqs. (1a) and (1b) are denoted by $\lambda_{e^- p}$ and $\lambda_{e^+ n}$, respectively.

At some point the local material expansion rate will be faster than the rates of the reactions in Eq. (1). We call this the weak freeze-out point, since Y_e for the material flow above this point remains nearly constant in time and space. Near weak freeze-out the matter temperature is small compared to the effective temperatures for the ν_e and $\bar{\nu}_e$ distributions, so that $\lambda_{e^- p}$ and $\lambda_{e^+ n}$ are negligible compared to $\lambda_{\nu_e n}$ and $\lambda_{\bar{\nu}_e p}$. We can solve Eq. (2) to find

$$Y_e(r_{\text{NFO}}) \approx Y_e(r_{\text{WFO}}) \approx \frac{1}{1 + \lambda_{\bar{\nu}_e p}(r_{\text{WFO}})/\lambda_{\nu_e n}(r_{\text{WFO}})}, \quad (3a)$$

where r_{NFO} and r_{WFO} are the nuclear and weak freeze-out positions, respectively. Since the neutrino luminosities are about the same for all neutrino species at this epoch and the rates of the reactions in Eq. (1) are proportional to the product of neutrino luminosity and average energy, we can approximate Eq. (3a) as

$$Y_e(r_{\text{NFO}}) \approx \frac{1}{1 + \langle E_{\bar{\nu}_e} \rangle / \langle E_{\nu_e} \rangle}. \quad (3b)$$

In any supernova model where $\bar{\nu}_e$ are more energetic than ν_e Eq. (3b) predicts that the conditions in the NSE freeze-out zone will be neutron rich. We have performed detailed numerical supernova calculations which include a complete treatment of neutrino transport, hydrodynamics, and relevant weak and nuclear reaction rates [13]. Starting with a $20M_\odot$ stellar model matched to the SN1987A progenitor [14], the collapse of the core was followed until $t_{\text{PB}} \approx 15$ s. The calculated neutrino spectra and time evolution were in good agreement with the Kamiokande and IMB observations. The calculated explosion energy, 1.5×10^{51} erg, agrees with the energy inferred from observations. Numerical supernova models give $0.40 \lesssim Y_e \lesssim 0.46$ in the NSE freeze-out region, in good agreement with the simple estimates from Eqs. (3a) and (3b).

However, transformation of ν_τ or ν_μ with energies between 20 and 40 MeV into ν_e could drive $\lambda_{\nu_e n} > \lambda_{\bar{\nu}_e p}$ and, hence, $Y_e > 0.5$. A matter-enhanced level crossing between ν_τ or ν_μ and ν_e can occur if the vacuum masses for these species satisfy $m_{\nu_\tau, \mu} > m_{\nu_e}$. In this case there are no transformations among the antineutrinos.

As neutrinos propagate through the hot bubble material they acquire effective masses from forward scattering on leptons, nucleons, and nuclei. The ν_e acquire larger effective masses than ν_τ or ν_μ as a result of forward charged-current exchange scattering on e^- and e^+ and neutral-current exchange scattering on ν_e and $\bar{\nu}_e$ [15]. To first order in the weak coupling constant G_F , the difference in effective mass between ν_e and ν_τ (or ν_μ) from each of these processes can be written as

$$\delta m_{\text{eff}}^2 = 2E_\nu \langle \phi \rangle - 2\langle \mathbf{p}_\nu \cdot \mathbf{A} \rangle, \quad (4)$$

where $p_\nu^\mu = (E_\nu, \mathbf{p}_\nu)$ are the components of the energy-momentum 4-vector for the propagating neutrino, $A^\mu = (\phi, \mathbf{A})$ are the components of the effective weak potential generated by an exchange scattering process, and the brackets denote an average over the distribution function for the targets of this process [1,15]. Since the charged particles in the hot bubble have isotropic distributions, the second term in Eq. (4) vanishes for exchange scattering on e^- and e^+ . There would be no contribution to δm_{eff}^2 from exchange scattering on ν_e and $\bar{\nu}_e$ for radially free-streaming neutrinos, since the first and second terms in Eq. (4) would cancel. In fact, the neutrinos are not exactly radially free streaming. Therefore, we integrate over the actual ν_e and $\bar{\nu}_e$ fluxes to compute an effective weak charge density at radius r from exchange scattering on ν_e and $\bar{\nu}_e$,

$$n_{\bar{\nu}_e}^{\text{eff}} \approx \frac{1}{4\pi r^2} \left\{ \frac{L_{\nu_e}}{\langle E_{\nu_e} \rangle} - \frac{L_{\bar{\nu}_e}}{\langle E_{\bar{\nu}_e} \rangle} \right\} \left\{ \frac{R_\nu^2}{4r^2} \right\} \\ \approx 4.14 \times 10^{31} \text{ cm}^{-3} \frac{R_\nu^2}{r_6^4} \left\{ \frac{L_{\nu_e}}{10^{51} \text{ ergs}^{-1}} \frac{10 \text{ MeV}}{\langle E_{\nu_e} \rangle} - \frac{L_{\bar{\nu}_e}}{10^{51} \text{ ergs}^{-1}} \frac{10 \text{ MeV}}{\langle E_{\bar{\nu}_e} \rangle} \right\}, \quad (5)$$

where r_6 is the radius in units of 10^6 cm, R_ν is the neutrinosphere radius in units of 10^6 cm, $L_{\nu_e}(\bar{\nu}_e)$ denotes ν_e ($\bar{\nu}_e$)

luminosity, and $\langle E_{\nu_e}(\bar{\nu}_e) \rangle$ is the average ν_e ($\bar{\nu}_e$) energy. The effective mass difference between ν_e and ν_τ (or ν_μ) is then

$$\delta m_{\text{eff}}^2 = 2\sqrt{2}G_F \{ [n_e - n_{e^+}] + n_{\nu_e}^{\text{eff}} \} E_\nu, \quad (6)$$

where the net number density of electrons is $n_e = n_e - n_{e^+} = \rho N_A Y_e$, and N_A is Avogadro's number. The effective weak charge density is defined to be $n \equiv n_e + n_{\nu_e}^{\text{eff}}$. We find that the contribution to δm_{eff}^2 from exchange scattering on neutrinos is less than 10% for $\rho > 10^8 \text{ g cm}^{-3}$, and is only of order (20–30)% near weak freeze-out. Although we include these effects, our results are essentially unchanged if we leave out all $\nu\nu$ -exchange contributions to n [15].

A neutrino mass-level crossing, or resonance [16], between ν_e and ν_τ or ν_μ occurs when $\delta m_{\text{eff}}^2 = \delta m^2 \cos 2\theta$, where $\delta m^2 \equiv m_1^2 - m_2^2$ is the difference between the squares of the *vacuum* neutrino-mass eigenvalues, and θ is the *vacuum* mixing angle between neutrino flavor states. The oscillation length at resonance is

$$L_{\text{res}} = \frac{4\pi E_\nu}{\delta m^2 \sin 2\theta} \approx \frac{0.16 \text{ cm}}{\sin 2\theta} \left[\frac{1600 \text{ eV}^2}{\delta m^2} \right] \left[\frac{E_\nu}{1 \text{ MeV}} \right], \quad (7)$$

while the resonance “width” [16] is $\delta r = 2H \tan 2\theta$, where $H \equiv |d \ln n / dr|^{-1}$ is the effective weak charge density scale height. The matter density profiles in our numerical models are presented in Fig. 1 for two late times. The solid line is for $t_{\text{PB}} \approx 4 \text{ s}$ and the dotted line is for $t_{\text{PB}} \approx 6 \text{ s}$. The density profiles do not change appreciably over the time scale of interest. Also shown in Fig. 1 are the position of the neutrinosphere and the resonance positions of a $E_\nu = 25 \text{ MeV}$ neutrino for the cases where the heavier

$$\begin{aligned} P_{\nu_\tau, \mu \leftrightarrow \nu_e} &\approx 1 - \exp \left\{ -\frac{\pi^2}{2} \frac{\delta r}{L_{\text{res}}} \right\} \\ &\approx 1 - \exp \left\{ -0.04 \left[\frac{\delta m^2}{1 \text{ eV}^2} \right] \left[\frac{1 \text{ MeV}}{E_\nu} \right] \left[\frac{H}{1 \text{ cm}} \right] \sin^2 2\theta \right\}. \end{aligned} \quad (8)$$

The adiabatic conversion limit obtains when $L_{\text{res}} \ll \delta r$. Sound waves, turbulence, or other fluctuations could destroy adiabaticity at resonance when their amplitudes on the scale of the resonance width exceed the unperturbed variation of density across this region. Amplitudes of high frequency sound waves would have to exceed $(\delta \rho / \rho)_{\text{res}} > \tan 2\theta > 10^{-3}$ to have much effect. We do not expect such fluctuations on small scales in the relatively quiescent $t_{\text{PB}} > 3 \text{ s}$ environment where r -process nucleosynthesis occurs. Studies of convection in the hot bubble region do not appear to require alteration of the picture of neutrino transformation presented here [18,19]. High convective velocities are thought to be associated with spatial scales which are large compared to L_{res} [18,19].

We have followed the weak interactions in our numeri-

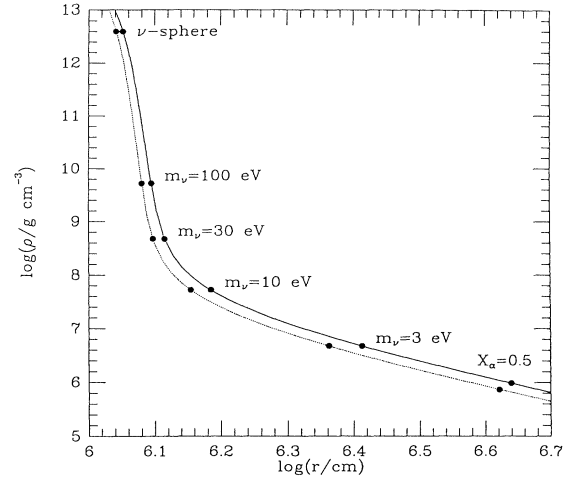


FIG. 1. Matter density profiles from numerical supernova models at late times. The solid line is for $t_{\text{PB}} \approx 4 \text{ s}$ and the dotted line is for $t_{\text{PB}} \approx 6 \text{ s}$. Also shown are the position of the neutrinosphere and the positions of neutrino mass-level crossings for a 25 MeV neutrino for the cases where the heavier vacuum neutrino mass is $m_\nu = 100, 30, 10,$ and 3 eV . The location where the alpha-particle mass fraction is 0.5 is shown.

neutrino mass is $m_\nu = 100, 30, 10,$ and 3 eV . The position where the alpha-particle mass fraction is 0.5 is shown in Fig. 1. We find that H ranges between 0.1 and 10 km over the region of interest and is of order 0.5 km when $\delta m^2 = 900 \text{ eV}^2$ and $E_\nu = 25 \text{ MeV}$.

We calculate neutrino transformation probabilities for our numerical supernova models using the Landau-Zener approximation [17] when the mixing angle is small ($\sin^2 2\theta \leq 0.01$). The probability for ν_τ (or ν_μ) conversion to ν_e , and vice versa, is

cal supernova model and our results are shown in Fig. 2, where we show the $Y_e = 0.5$ line at nuclear freeze-out on a plot of δm^2 against $\sin^2 2\theta$. Vacuum neutrino masses and mixing angles to the right of the $Y_e = 0.5$ line imply proton-rich conditions in the nuclear freeze-out zone and preclude heavy element r -process nucleosynthesis in the hot bubble. Theoretical models of neutrino masses and mixing angles could fall on either side of the $Y_e = 0.5$ line [20,21]. We note that mixing at a level exceeding $\sin^2 2\theta = 10^{-2}$ still gives $Y_e > 0.5$ and/or is ruled out by experiments [7,22].

If r -process nucleosynthesis is to come from the hot bubble region in supernovae then the neutrino parameter space to the right of the $Y_e = 0.5$ line is excluded. This conclusion is conservative, since the r -process nucleosyn-

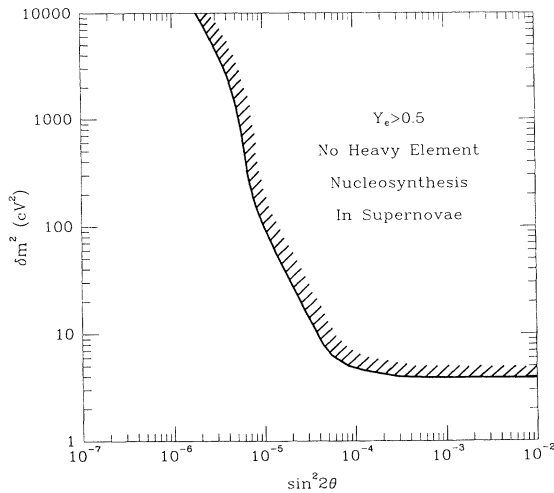


FIG. 2. The $Y_e=0.5$ line is shown on a plot of vacuum mass-square difference δm^2 versus $\sin^2 2\theta$. The region to the right of the $Y_e=0.5$ line is excluded if supernovae are to produce r -process heavy elements.

thesis from the marginally neutron-rich parameter region just to the left of this line will probably not give an acceptable nuclear abundance distribution [10]. If neutrinos have masses and mixing parameters which put nuclear freeze-out conditions on the proton-rich side of this line, then we must conclude that the r process cannot occur in the hot bubble region [23].

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