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THE HIGHLY INELASTIC NEUTRINO REACTIONS PREDICTED FROM THE πN TOTAL CROSS SECTIONS*

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ABSTRACT

The scaling function $F_2(\omega)$ is calculated for the highly inelastic neutrino and antineutrino reactions from the πN total cross sections on the basis of the generalized scaling sum rules. With the optimum values for the parameters determined in a previous analysis the integral $\int_0^1 dx\,(F_2^{\nu p}(x) + F_2^{\bar{\nu} p}(x))$ saturates almost completely the inequality of the parton quark model. The ultra-precocious linear rise of the total cross sections for the neutrino and antineutrino reactions is a natural consequence of the generalized scaling sum rules.

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1. DYNAMICAL PICTURE OF SCALING

It has been pointed out that the scaling function $F_2(\omega)$ may be smoothly extrapolated from the deep inelastic limit to the small Q^2 region with a suitably modified scaling variable [1], [2]. We first review for a pedagogical purpose how this transition from the deep inelastic region to the shallow inelastic region takes place dynamically and what the parameters involved in such an extrapolation mean.

The matrix element we consider is

$$\langle n \mid \int J_0(\vec{x},t)\, e^{i\vec{q}\vec{x}}\, d^3x \mid \vec{p}\,\rangle , \tag{1.1}$$

where $\mid \vec{p}\,\rangle$ is the one-nucleon state with momentum $\vec{p}$, and $\mid n\,\rangle$ is an arbitrary state connected through the Fourier component of the current $J_0(\vec{x},t)$. To make the following argument definite and relevant to the calculations we will attempt later, let us choose the isovector axial vector current for $J_0(\vec{x},t)$. We usually go to the infinite momentum frame so that the dynamical statement may become relativistically invariant. When $\vec{q} = 0$ or $Q^2 (\equiv \vec{q}^{\,2}) = 0$, the transition matrix element

$$\langle\, n \mid \int J_0(\vec{x},t)\, d^3x \mid \vec{p}\,\rangle \tag{1.2}$$

is rewritten through the partially conserved axial vector current hypothesis in terms of that between the nucleon and the state $|n\rangle$ absorbing the pion,

$$(1.2) = (2\pi)^3\, \delta\, (\vec{p} - \vec{p}_n)\, \frac{f_\pi \quad T_{np}}{M_n^2 - m^2} , \tag{1.3}$$

where $p^2 = m^2$, M_n is the invariant mass of the state $|n\rangle$, $f_\pi \cos\theta_C$ is the pion decay constant defined through $\partial^\mu J_\mu^{\,i} = f_\pi m_\pi^2 \phi_\pi^{\,i}$

(i = 1, 2, and 3) with θ_C being the Cabibbo angle, and T_{np} is the invariant pionic transition matrix element. The absorption of the pion may lead the nucleon to the nucleon, the baryon resonances, and the continuum states. $|T_{np}|^2$ summed over n with fixed M_n^2 is, up to the incident pion energy, proportional to the πN total cross sections. We find from the πN scattering the shape of $\sum_n |T_{np}|^2 \,\delta(M_n^2 - s)$ as a function of s.

As Q^2 increases from zero to a nonzero (positive) value, the matrix element (1.1) varies in such a way that the transitions from the nucleon to the states $|n\rangle$ of small M_n^2 decrease to the amount that the axial vector form factors fall off, while the transitions to the continuum states of large M_n^2 would increase in a way compensatory to the decrease. In other words, the transitions are confined mainly to the lower excited states at small values of Q^2, but as Q^2 increase the transition "leaks" out to the higher continuum states. How fast this leakage takes place depends on how fast the axial vector form factors to the baryonic states $|n\rangle$ fall off. When the form factors become negligibly small at some large value of Q^2, the transition is almost completely thrown into the higher continuum states. In the deep inelastic limit $Q^2 \to \infty$, all the transitions to the states of finite invariant mass M_n vanish so that only the states of $M_n^2 = \mathcal{O}(Q^2) \to \infty$ contribute to the transition matrix elements. Constant core terms are not allowed for the form factors if the scaling functions approach zero as $\omega = 2m\nu/Q^2 \to 1$, for if there were core terms for a large number of the transition form factors, they would lead to $F_2(\omega = 1) \neq 0$. The asymptotic behavior of the form factors is thus closely related to the threshold behavior of the scaling functions [3], [4].

The generalized scaling means that the scaling function $F_2(\omega)$ smoothly extrapolated with the variable

$$\omega = \frac{2m\nu + M^2}{Q^2 + a^2} \tag{1.4}$$

should describe on average the function $\nu W_2(\nu, Q^2)$ all the way from $Q^2 = \infty$ down to $Q^2 = 0$. The parameter a^2 determines how fast or how violently the transitions to the lower baryonic states are blown to the higher continuum states as Q^2 increases. The smaller a^2 is, the farther the transitions are blown away. The other parameter M^2 causes the overall shift of the invariant masses of the states $|n\rangle$. This is significant only for the states of $M_n \approx M$. More quantitatively speaking, the transitions to the band of squared mass $M_1^2 \pm \delta_1$ at $Q^2 = Q_1^2$ are brought to the wider band of

$$M_2^2 \pm \delta_2 = \frac{Q_2^2 + a^2}{Q_1^2 + a^2}\left[M_1^2 \pm \delta_1\right] + \frac{Q_2^2 - Q_1^2}{Q_1^2 + a^2}\left(M^2 - m^2 - a^2\right), \tag{1.5}$$

when Q^2 increases from Q_1^2 to Q_2^2 ($> Q_1^2$). As Q^2 increases, the transitions to a narrow band in M_n^2 at lower energies get shifted to a broader band in M_n^2 at higher energies (see Fig. 1). The generalized scaling thus describes appropriately the "leakage" phenomenon for the matrix element of the Fourier component of the current [5]. The same argument goes through for the vector currents where the transitions to the excited states are completely forbidden at $Q^2 = 0$ because of the exact conservation of current.

2. ADLER'S NEUTRINO SUM RULE

The neutrino sum rule derived by Adler [6] is based upon the equal-time commutators of the time-components of the currents. It is far more fundamental than other sum rules derived in specific models [7]. Recently some people have cast doubt on the validity of Adler's sum rule in the light of the so far analyzed data on the neutrino reactions [8], [9], [10]. When one compares the sum rule with the experimentally observed scaling functions, the crucial point is how fast the sum rule converges in ω . A typical valence quark model [11] says that the 90% saturation is obtained only as high as at $\omega = 300 \sim 500$.

We will dispute this awfully slow convergence of the neutrino sum rule from the viewpoint of the generalized scaling. To do so, we remind you that the $Q^2 = 0$ limit of Adler's neutrino sum rule is the Adler-Weisberger sum rule. It is known from the measured πN total cross sections that the Adler-Weisberger sum rule is saturated up to 90% around the incident pion energy $\nu = 5$ GeV. If we relate the πN scattering and the highly inelastic neutrino reactions through the generalized scaling, the averaged curves of the πN total cross sections at the laboratory pion energy ν is equal to the scaling functions $F_2(\omega)$ of the neutrino and antineutrino reactions (strictly speaking, the strangeness-conserving parts only) at $\omega = (2m\nu + M^2)/a^2$ apart from a known proportional constant given by theory. For the neutrino sum rule to be saturated up to 90% at $\omega = 300$, the parameter a^2 must be as small as 0.03 GeV^2, provided that $M^2 = 0.5 \sim 2$ GeV^2. We would have correspondence between amplitudes with different Q^2 through the generalized scaling variable

$$\omega = \frac{2m\nu + M^2}{Q^2 + 0.03\ \text{GeV}^2} . \tag{2.1}$$

For $Q^2 \gtrsim 0.1$ GeV^2 and $\nu \gtrsim 2 \sim 3$ GeV, the variable ω would already be close enough to $2m\nu/Q^2$; the scaling would be reached as early as at $Q^2 \simeq 0.1$ GeV^2. In the region of Q^2 where the scaling is approximately realized, the vector and axial vector form factors must be negligibly small according to the argument in the previous Section. In the case discussed above the form factors would have to be almost zero at $Q^2 \gtrsim 0.1$ GeV^2. This clearly contradicts with the experimentally observed form factors [12], [13], [14]

$$G_V(Q^2) = \left[1 + \frac{Q^2}{0.71\ \text{GeV}^2} \right]^{-2} , \tag{2.2}$$

$$F_A(Q^2) = \left[1 + \frac{Q^2}{0.85\ \text{GeV}^2} \right]^{-2} . \tag{2.3}$$

We conclude that the very slow convergence of the neutrino sum rule is incompatible with the generalized scaling. Turning the argument around, we predict from the value for a^2 determined later in this paper and also through the independent analysis of the shallow inelastic electroproduction data [2] ($a^2 = 0.2 \sim 0.4$ GeV^2) that the 90% saturation is to be reached around $\omega = 30 \sim 50$ corresponding to $\nu = 5$ GeV through (1.4).

3. SUM RULES FOR $F_2(\omega)$ AND $\sigma_{tot}^{\pi p}(\nu)$

The generalized scaling makes the strong statement that $F_2(\omega)$ can be smoothly extrapolated by means of $(2m\nu + M^2)/(Q^2 + a^2)$ down to the small Q^2 region in the entire region of ω. According to the analysis based on the Deser-Gilbert-Sudarshan representation [15], [16], the generalized scaling is likely to hold for the Regge asymptotic amplitudes at $\omega \gg 1$ [17], [18]. We assume here that only the large ω region of $F_2(\omega)$ may be extrapolated to the small Q^2 region through the replacement of

$$\omega \to \omega' = \frac{2m\nu + M^2}{Q^2 + a^2} . \qquad (3.1)$$

This is a much weaker version of the generalized scaling, since the earlier version [1], [2] amounts to assuming that the parameters a^2 and M^2 are common to all the asymptotic powers. We now derive sum rules involving $F_2(\omega)$ and the πN total cross sections.

In the large ω region, we call it the Regge region from now on, $F_2(\omega)$ is shown in the analyses of the Bethe-Salpeter equation [19], the Deser-Gilbert-Sudarshan representation [20], and summation of a series of perturbation diagrams [21] to behave at $\omega \to \infty$ like

$$F_2(\omega) \sim \sum_i \beta_i \, \omega^{\alpha_i(0)-1} , \qquad (3.2)$$

where $\alpha_i(0)$ is the intercept of the ith Regge trajectory and β_i is related to its residue. The Regge asymptotic expansion with Q^2 fixed of the scattering amplitude $W_2(\nu,Q^2)$ is similarly written as

$$W_2(\nu,Q^2) \sim \sum_i \gamma_i(Q^2)\nu^{\alpha_i(0)-2} , \qquad (3.3)$$

where $\gamma_i(Q^2)$ behaves like $\beta_i(2m/Q^2)^{\alpha_i(0)-1}$ as $Q^2 \to \infty$. The generalized scaling weakened to the higher Regge asymptotic amplitudes only is therefore that $\beta_i \, \omega^{\alpha_i(0)-1}$ may be extrapolated smoothly on average to the small Q^2 region by the replacement of ω with $\omega' = (2m\nu + M^2)/(Q^2 + a^2)$. The parameter M^2 does not have much significance there since M^2 is related to the ratios of the leading power in ν to the Khuri satellite terms. The Regge residue $\gamma_i(Q^2)$ is related to β_i in (3.2) through

$$\gamma_i(Q^2) = \left(\frac{2m}{Q^2 + a^2}\right)^{\alpha_i(0)-1} \beta_i , \qquad (3.4)$$

where a^2 is a parameter to be determined from the experimental data. Although most generally a^2 may be different from one trajectory to another, we will later take a^2 dependent only on the height of the intercept $\alpha_i(0)$ for simplicity. In particular, a^2 will be set to the same value for the ρ and f trajectories. As was emphasized in the previous work [22], the ratio of the Pomeron contributions in $F_2(\omega)$ and $\nu W_2(\nu,Q^2)$ is independent of a^2 and M^2;

$$\lim_{\omega \to \infty} F_2(\omega) = \lim_{\nu \to \infty} \nu W_2(\nu,Q^2) . \qquad (3.5)$$

In fact, this relation is true for any correspondence between ω and ν so long as $\omega \to \infty$ at $\nu = \infty$. For the other trajectories,

$$\gamma_i(0) = \beta_i \left(\frac{2m}{a^2}\right)^{\alpha_i(0)-1} \qquad (3.6)$$

from (3.4), and therefore

$$F_2(\omega') \sim \sum_i \beta_i \, (\omega')^{\alpha_i(0)-1} , \qquad (3.7)$$

and

$$\nu W_2(\nu,Q^2) \sim \sum_i \beta_i \left(\frac{2m\nu}{Q^2+a^2}\right)^{\alpha_i(0)-1} . \qquad (3.8)$$

If we choose $\omega' = (2m\nu + M^2)/(Q^2+a^2)$, the difference $F_2(\omega') - \nu W_2(\nu,Q^2)$ falls off as fast as $\nu^{\alpha_i(0)-2}$. It looks that with M^2 chosen to be zero all the Regge asymptotic powers are exactly canceled so that $F_2(\omega')$ becomes the local average of $\nu W_2(\nu,Q^2)$ in the entire region of ω'. However, it would be too optimistic to expect that this should happen, for the parameter a^2 may well be dependent on the trajectories. It is more practical to shift $2m\nu$ by M^2 so as to absorb the first Khuri satellite as much as possible.

On the assumption that a^2 should depend only on the intercepts $\alpha_i(0)$, we are able to write down the sum rules for the crossing symmetric and antisymmetric amplitude;

$$\left[\int_1^\infty F_2^{(-)}(\omega)\frac{d\omega}{\omega} - \int_{\nu_0(Q^2)}^\infty \nu W_2^{(-)}(\nu,Q^2)\frac{d\nu}{\nu}\right] = 0 , \qquad (3.9)$$

and

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} F_2^{(+)}(\omega)\,d\omega - \frac{2m}{Q^2+a^2}\int_{\nu_0(Q^2)}^{\nu_{max}} \nu W_2^{(+)}(\nu,Q^2)d\nu\right] = 0, \qquad (3.10)$$

$$F_2^{(\pm)}(\omega) \equiv F_2^{\bar{\nu}p}(\omega) \pm F_2^{\nu p}(\omega) , \qquad (3.11)$$

where $\nu_0(Q^2)$ is to be below the nucleon pole, $F_2^{\bar{\nu}p}(\omega)$ and $F_2^{\nu p}(\omega)$ refer to the strangeness conserving axial vector parts of the scaling functions in the antineutrino and neutrino reactions. In Eq. (3.10) ω_{max} and ν_{max} are let to infinity, keeping the relation

$$\omega_{max} = \frac{2m\nu_{max} + M^2}{Q^2+a^2} . \qquad (3.12)$$

The first Khuri satellite term of the Pomeron does not contribute to the absorptive part, so the left-hand side converges when the leading powers of the Pomeron and the f-meson trajectory are canceled out. The sum rules of higher moments may be written on a stronger assumption. We will discuss them later in connection with the phenomenon referred to as the ultra-precocious scaling in the neutrino reactions.

The first sum rule (3.9) is nothing more than the content of Adler's sum rule. Dynamics is put in when we require that the left-hand side of (3.9) is saturated sufficiently well around ν = a few GeV. By numerical computation the 80% saturation is attained at ν = 2 GeV for the Adler-Weisberger sum rule

$$\left[\int_{\nu_0(0)}^{2\,\mathrm{GeV}} \nu W_2^{(-)}(\nu,0)d\nu\right] \Big/ \left[\int_{\nu_0(0)}^{\infty} \nu W_2^{(-)}(\nu,0)d\nu\right] \approx 0.8 . \qquad (3.13)$$

It is therefore natural to expect that the left-hand side of (3.9) is also saturated enough at ν_{max} = 2 GeV or equivalently at $\omega_{max} = [2m \times (2\,\mathrm{GeV}) + M^2]/a^2$. This is a dynamical input based on our knowledge in the πN total cross sections. It means that the averaged or smoothed $\nu W_2^{(-)}(\nu,0)$ coincides with $F_2(\omega)$ through the correspondence (1.4) semilocally in the resonance region ($\nu \lesssim$ 2 GeV) and the asymptotic region ($\nu \gtrsim$ 2 GeV) separately.

4. DETERMINATION OF $F_2(\omega)$ FROM $\sigma^{\pi p}_{tot}(\nu)$

The neutrino reactions involve both the vector and axial vector caling functions apart from the vector-axial vector interference term. ince the scaling function of the vector current is the same as that of he axial vector current in all of the existing models and theories, we ake it for granted in the following numerical analysis. The currents ontain small mixture of the strangeness changing components. We onsider the $\Delta S = 0$ parts only by either ignoring the small $\Delta S = 1$ arts or separating them out. The Cabibbo angle θ_C is to be factored ut. The partially conserved axial vector hypothesis relates as

$$\sigma^{\pi^+ p}_{tot}(\nu) = \frac{\pi}{4} f_\pi^{-2} \nu W_2^{\nu p}(\nu, 0) , \qquad (4.1)$$

$$\sigma^{\pi^- p}_{tot}(\nu) = \frac{\pi}{4} f_\pi^{-2} \nu W_2^{\bar{\nu} p}(\nu, 0) , \qquad (4.2)$$

here $\sqrt{2}\ f_\pi \cos\theta_C = 0.97\ m_\pi$ and νW_2's are the $\Delta S = 0$ parts nly, including both the vector and axial vector currents.

Let us look at the crossing antisymmetric amplitude first. The legge asymptotic behavior of the amplitude implies a term like $\omega^{-\frac{1}{2}}$ as $\to \infty$. Near the threshold $\omega = 1$ the scaling functions, especially he difference $F_2^{\nu p}(\omega) - F_2^{\bar{\nu} p}(\omega)$ should behave like $(\omega - 1)^3$ provided hat the isovector electromagnetic form factor of the nucleon fall off ike $(Q^2)^{-2}$ as $Q^2 \to \infty$ [3], [4], [23]. We therefore postulate the erm

$$A\ \omega^{-\frac{1}{2}} \left(1 - \frac{1}{\omega}\right)^3 \qquad . \qquad (4.3)$$

n addition to this term we introduce another term

$$B\ \omega^{-3/2} \left(1 - \frac{1}{\omega}\right)^3 \qquad , \qquad (4.4)$$

which represents the ρ' trajectory or the first Khuri satellite and everything else. The sum of (4.3) and (4.4)

$$F_2^{(-)}(\omega) = A\ \omega^{-\frac{1}{2}} \left(1 - \frac{1}{\omega}\right)^3 + B\ \omega^{-3/2} \left(1 - \frac{1}{\omega}\right)^3 \qquad (4.5)$$

must be subject to the scaling limit of Adler's sum rule

$$\int_1^\infty F_2^{(-)}(\omega) \frac{d\omega}{\omega} = 2 . \qquad (4.6)$$

From this Section on, the scaling function $F_2^{(\pm)}(\omega)$ denotes the sum of the vector and axial vector terms. Equation (4.6) sets a restriction

$$0.457\ A + 0.051\ B = 1 . \qquad (4.7)$$

Then we assume that (3.9) is saturated sufficiently well at $\nu = 2$ GeV which is just above the several conspicuous resonances. The sum rule reads

$$\int_1^{\omega_{max}} F_2^{(-)}(\omega) \frac{d\omega}{\omega} \simeq \int_{\nu_0}^{2\ \mathrm{GeV}} \nu W_2(\nu, 0) \frac{d\nu}{\nu}$$
$$= 2\ g_A^2 + \frac{4\ f_\pi^2}{\pi} \int_{m_\pi}^{2\ \mathrm{GeV}} \frac{p\left[\sigma^{\pi^- p}(\nu) - \sigma^{\pi^+ p}(\nu)\right]}{\nu^2} d\nu , \qquad (4.8)$$

where

$$\omega_{max} = [2m\nu \times (2\ \mathrm{GeV}) + M^2]/a^2 . \qquad (4.9)$$

It has been shown in the previous analysis [22] that the value of a^2 around 0.3 GeV^2 leads to the best fit to the $F_2^{(\gamma p)}(\omega) + F_2^{(\gamma n)}(\omega)$ in the large ω region. The amplitude is insensitive to the other parameter M^2 in the Regge region. The fit to the shallow inelastic electroproduction data in the small ω region suggests that M^2 is somewhere between 0.5 GeV^2 and 1.5 GeV^2 [1].

By substituting the πN cross sections we have found the right-hand side of Eq. (4.8),

$$\text{r.h.s. of (4.8)} = 1.53 , \tag{4.10}$$

which means that the Adler-Weisberger sum rule is saturated up to 76.5%. Substituting (4.5) into the left-hand side of (4.8) we obtain another restriction on A and B. Combining this restriction with (4.7), we are led to a set of values for A and B given in (4.5) as functions of a^2 and M^2. For $a^2 = 0.3\ GeV^2$ and $M^2 = 1.0\ GeV^2$ which are considered to be optimum, we obtain

$$A = 0.71 \quad \text{and} \quad B = 13.3 . \tag{4.11}$$

By changing M^2 from 0.5 GeV^2 to 1.5 GeV^2 we find that A and B change typically $\pm(5 \sim 10)\%$ for a^2 kept between 0.2 GeV^2 and 0.4 GeV^2. A and B are a little more sensitive to variation of a^2. As a^2 varies by $\pm$ 0.1 GeV^2 around 0.3 GeV^2, A and B are affected up to 50% and 30%, respectively. However, the variations of A and B are largely compensated when one takes the sum of the two terms in (4.5).

We next turn to the crossing symmetric amplitude. The functional form of the symmetric amplitude is suggested again by its Regge asymptotic behavior and the threshold behavior. We postulate

$$\begin{aligned} F_2^{(+)}(\omega) &= F_2^{\bar{\nu}p}(\omega) + F_2^{\nu p}(\omega) \\ &= C(1 - \frac{1}{\omega^2})^3 + D\,\omega^{-\frac{1}{2}}(1 - \frac{1}{\omega})^3 . \end{aligned} \tag{4.12}$$

The first term was proposed phenomenologically in the fit to the

existing data [24]. The coefficient C is the Regge residue of the Pomeron, which is determined independently of the values of a^2 and M^2 as [22]

$$\begin{aligned} C &= \frac{4 f_\pi^2}{\pi}\left[\sigma^{\pi^- p}(\infty) + \sigma^{\pi^+ p}(\infty)\right] \\ &= 1.59 \end{aligned} \tag{4.13}$$

for $\sigma^{\pi^- p}(\infty) = \sigma^{\pi^+ p}(\infty) = 25$ mb. The other coefficient D is going to be determined through the sum rule (3.10) with $\nu_{max} = 2$ GeV,

$$\begin{aligned} \int_1^{\omega_{max}} F_2^{(+)}(\omega)\, d\omega &\simeq \frac{2m}{a^2}\int_{\nu_0}^{2\ GeV} \nu\, W_2^{(+)}(\nu,0)\, d\nu \\ &= -\frac{2m_\pi^2}{a^2} g_A^2 + \frac{8mf_\pi^2}{\pi a^2}\int_{\nu_0}^{2\ GeV} \frac{p[\sigma^{\pi^- p}(\nu) + \sigma^{\pi^+ p}(\nu)]}{\nu} d\nu , \end{aligned} \tag{4.14}$$

where $\omega_{max} = [2m \times (2\ GeV) + M^2]/a^2$. The right-hand side of (4.14) turns out through the substitution of the experimental data to be

$$\text{r.h.s. of (4.14)} = 9.16\ GeV^2/a^2 . \tag{4.15}$$

A value of D is searched for in the left-hand side of (4.14) so as to achieve the equality with (4.15). Again for $a^2 = 0.3\ GeV^2$ and $M^2 = 1.0\ GeV^2$ we obtain

$$D = 2.86 .$$

It is interesting and important to evaluate

$$\int_0^1 F_2^{(+)}(x) dx \tag{4.16}$$

where $x = \omega^{-1}$. The CERN neutrino experiment has given [24]

$$\int_0^1 F_2^{(+)}(x)\,dx = 0.98 \pm 0.14 . \qquad (4.17)$$

With our form (4.12) where $C = 1.59$ and $D = 2.86$,

$$\int_0^1 F_2^{(+)}(x)\,dx = 1.02 . \qquad (4.18)$$

The agreement with experiment is very good. The right-hand side of (4.18) is not put in by hand nor derived from any other indirect information in the deep inelastic lepton reactions, but it has been calculated through the πN total cross sections. When we vary a^2 and M^2 as $a^2 = (0.3 \pm 0.1)\mathrm{GeV}^2$ and $M^2 = (1.0 \pm 0.5)\mathrm{GeV}^2$, we see fairly large variation in D. For instance, in the two extreme cases we find

$$D = 1.48 \quad \text{for} \quad a^2 = 0.2\ \mathrm{GeV}^2 \quad \text{and} \quad M^2 = 1.5\ \mathrm{GeV}^2 , \qquad (4.19)$$

$$D = 6.00 \quad \text{for} \quad a^2 = 0.4\ \mathrm{GeV}^2 \quad \text{and} \quad M^2 = 0.5\ \mathrm{GeV}^2 . \qquad (4.20)$$

The dependence on a^2 is particularly strong. We may therefore consider the experimental information (4.18) as another restriction imposed on the parameters a^2 and M^2. It is encouraging, however, that the values for a^2 and M^2 determined in the separate analysis [22] leads to a very good agreement with (4.17).

To summarize, we have obtained the scaling functions

$$F_2^{(-)}(\omega) = 0.71\,\omega^{-\frac{1}{2}}\left(1 - \frac{1}{\omega}\right)^3 + 13.3\,\omega^{-3/2}\left(1 - \frac{1}{\omega}\right)^3 , \qquad (4.21)$$

and

$$F_2^{(+)}(\omega) = 1.59\left(1 - \frac{1}{\omega^2}\right)^3 + 2.86\,\omega^{-\frac{1}{2}}\left(1 - \frac{1}{\omega}\right)^3 , \qquad (4.22)$$

where

$$F_2^{\bar{\nu}p}(\omega) = \frac{1}{2}\left[F_2^{(+)}(\omega) + F_2^{(-)}(\omega)\right] , \qquad (4.23)$$

$$F_2^{\nu p}(\omega) = \frac{1}{2}\left[F_2^{(+)}(\omega) - F_2^{(-)}(\omega)\right] . \qquad (4.24)$$

The optimum values for the parameters in the generalized scaling variable are:

$$a^2 = 0.3\ \mathrm{GeV}^2 \qquad (4.25)$$

$$M^2 = 1.0\ \mathrm{GeV}^2 . \qquad (4.26)$$

We have plotted in Fig. 2 the curves for $F_2^{\bar{\nu}p}(\omega)$ and $F_2^{\nu p}(\omega)$ given by (4.21) - (4.24) and also tabulated in Table 1 the scaling functions versus ω. Main differences from the curves obtained in Reference [10] are:

(i) The large ω limit $(x \to 0)$ is approximately 17% smaller than that in [10].

(ii) Both $F_2^{\bar{\nu}p}(\omega)$ and $F_2^{\nu p}(\omega)$ approach their $\omega \to \infty$ limits <u>from above</u>, just as $\sigma_{tot}^{\pi^- p}(\nu)$ and $\sigma_{tot}^{\pi^+ p}(\nu)$ do.

5. COMMENTS AND DISCUSSION

The two sum rules (4.8) and (4.14) have been derived, one for the crossing antisymmetric amplitude and one for the symmetric amplitude. If the generalized scaling variable $\omega = (2m\nu + M^2)/(Q^2 + a^2)$ works for all the Regge terms in ω with the same values for M^2 and a^2, we could write down more sum rules of higher moment. For the crossing antisymmetric amplitude

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} F_2^{(-)}(\omega)\,\omega^{2n-1}d\omega - \left(\frac{2m}{Q^2+a^2}\right)^{2n}\int_{\nu_0(Q^2)}^{\nu_{max}} \nu^{2n}W_2^{(-)}(\nu,Q^2)d\nu\right] = 0, \tag{5.1}$$

where $\omega_{max} = (2m\nu_{max} + M^2)/(Q^2 + a^2)$ and n is zero or any positive integer. The sum rules of even moment in ω depend on whether or not there exist nonsense wrong signature poles at $J = 0, -2, -4, \cdots$ in the complex J-plane, and whether or not their residues are Q^2 dependent. Although the nonsense right signature pole at $J = 1$ is proved in the current algebra to have the Q^2 independent residue, the proof does not apply to the wrong signature poles. The sum rules are written in the case of Q^2 independent residues as

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} F_2^{(-)}(\omega)\omega^{2n}d\omega - \left(\frac{2m}{Q^2+a^2}\right)^{2n+1}\int_{\nu_0(Q^2)}^{\nu_{max}} \nu^{2n+1}W_2^{(-)}(\nu,Q^2)d\nu\right] = 0. \tag{5.2}$$

With (5.1) and (5.2) combined together, the scaling function $F_2^{(-)}(\omega)$ is really the average of $\nu W_2^{(-)}(\nu,Q^2)$ locally. The local average suggested in [1] and [2] would hold accurately. The parallel argument is made for the crossing symmetric amplitude. Corresponding to (5.1),

there would exist the sum rules

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} F_2^{(+)}(\omega)\omega^{2n}d\omega - \left(\frac{2m}{Q^2+a^2}\right)^{2n+1}\int_{\nu_0(Q^2)}^{\nu_{max}} \nu^{2n+1}W_2^{(+)}(\nu,Q^2)d\nu\right] = 0\,. \tag{5.3}$$

The sum rules resulting from Q^2 independent residues of the wrong signature poles are written as

$$\lim_{\substack{\omega_{max}\to\infty \\ \nu_{max}\to\infty}} \left[\int_1^{\omega_{max}} F_2^{(+)}\omega^{2n-1}d\omega - \left(\frac{2m}{Q^2+a^2}\right)^{2n}\int_{\nu_0(Q^2)}^{\nu_{max}} \nu^{2n}W_2^{(+)}(\nu,Q^2)d\nu\right] = 0. \tag{5.4}$$

In the recent analysis of the CERN data [13] the total cross sections of the neutrino reactions, in particular of the antineutrino-nucleus reaction, rise linearly in the incident neutrino (antineutrino) energy E. It looks as if the scaling were reached ultra-precociously at $Q^2 \simeq 0.4$ GeV^2 as compared with $Q^2 \simeq 2$ GeV^2 in the deep inelastic electroproduction done by SLAC-MIT. At $Q^2 \simeq 0.4$ GeV^2, however, the vector and axial vector form factors are still about half the values at $Q^2 = 0$. Therefore, the scaling is <u>not</u> reached at $Q^2 \simeq 0.4$ GeV^2 or less. The neutrino cross sections look more like a series of peaks and humps due to the nucleon and the other baryon resonances. The linear rise of the total cross sections is indicative of the fact that the wiggly cross sections of the antineutrino reaction at lower energies may be integrated over ν and smoothed out into the scaling limit curve. We will look into this phenomenon a little more quantitatively. The double differential cross sections are written as

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{d\omega\, dy} = \frac{G^2 m E}{\pi \omega^2}\left[(1 - y - \frac{m y}{2\omega E})\nu W_2 + \frac{y^2 W_1}{\omega} \mp y(1 - \frac{y}{2})\frac{W_3}{\omega}\right], \qquad (5.5)$$

where $\omega = 2m\nu/Q^2$, $y = \nu/E$, E is the incident neutrino (antineutrino) energy, and W_1 and W_3 are the two others of the invariant functions of ν and Q^2. By integrating this over ν and Q^2, we get

$$\sigma^{\nu,\bar{\nu}}(E) = \frac{G^2}{2\pi}\int_0^{2mE} dQ^2 \int_{\nu_0(Q^2)}^{E-Q^2/4E} d\nu \left[(1 - \frac{Q^2}{4E^2})W_2(\nu,Q^2)\right.$$

$$\left. - \frac{\nu}{E} W_2(\nu,Q^2) + \frac{\nu Q^2}{2mE^2} W_1(\nu,Q^2) \mp \frac{Q^2}{2mE}(1 - \frac{\nu}{2E}) W_3(\nu,Q^2)\right]. \qquad (5.6)$$

The first term within the square bracket in the right-hand side may be rewritten through (5.1) with $n = 0$ and (5.4) with $n = 0$ as

$$2mE \int_0^1 dZ \int_1^{\omega_{max}} d\omega\, (1 - \frac{m}{2E}Z)\, F_2(\omega)/\omega\, , \qquad (5.7)$$

where

$$\omega_{max} = [1 - (m/2E) + (M^2/2mE)]/[Z + (a^2/2mE)]$$

and the new variable $Z = Q^2/2mE$ has been introduced. In the limit of $E \to \infty$ (5.7) reduces to

$$2mE \int_0^1 dZ \int_1^{Z^{-1}} \omega^{-2} d\omega\, F_2(\omega) = 2mE \int_0^1 dy \int_0^1 dx\, F_2(\omega). \qquad (5.8)$$

This is what we obtain by first going to the scaling limit. However,

the limit $E \to \infty$ really means that $\omega_{max} \to Z^{-1}$. It is sufficient to have

$$m/2E, \quad (M^2 - m^2)/2mE, \quad \text{and} \quad a^2/2mE \ll 1\, . \qquad (5.9)$$

According to the preceding analysis, $|M^2 - m^2|$ is between 0 and 0.5 GeV^2, and a^2 is around 0.3 GeV^2. Therefore $E = 2 \sim 3$ GeV is enough to satisfy (5.9). The first term is already close enough to its scaling limit at $E = 2 \sim 3$ GeV. The second term in the right-hand side of (5.6) is shown in an analogous way to be close enough to its scaling limit for E satisfying (5.9), namely $2 \sim 3$ GeV, if the sum rules (5.2) with $n = 0$ and (5.3) with $n = 0$ hold valid.

The third and fourth terms may be argued in parallel if we postulate the generalized scaling sum rules for $W_1(\nu,Q^2)$ and $\nu W_3(\nu,Q^2)$ corresponding to (5.1) to (5.4). We thus come to the conclusion that the total cross sections of the neutrino and anti-neutrino reactions should start rising linearly in E as early as at $E = 2 \sim 3$ GeV provided that the wrong-moment sum rules of the lowest order hold valid as well as the right-moment one. The observed linear rise of the total cross sections, referred to as the ultra-precocious scaling, supports strongly the generalized scaling sum rules of, at least, the lowest order.

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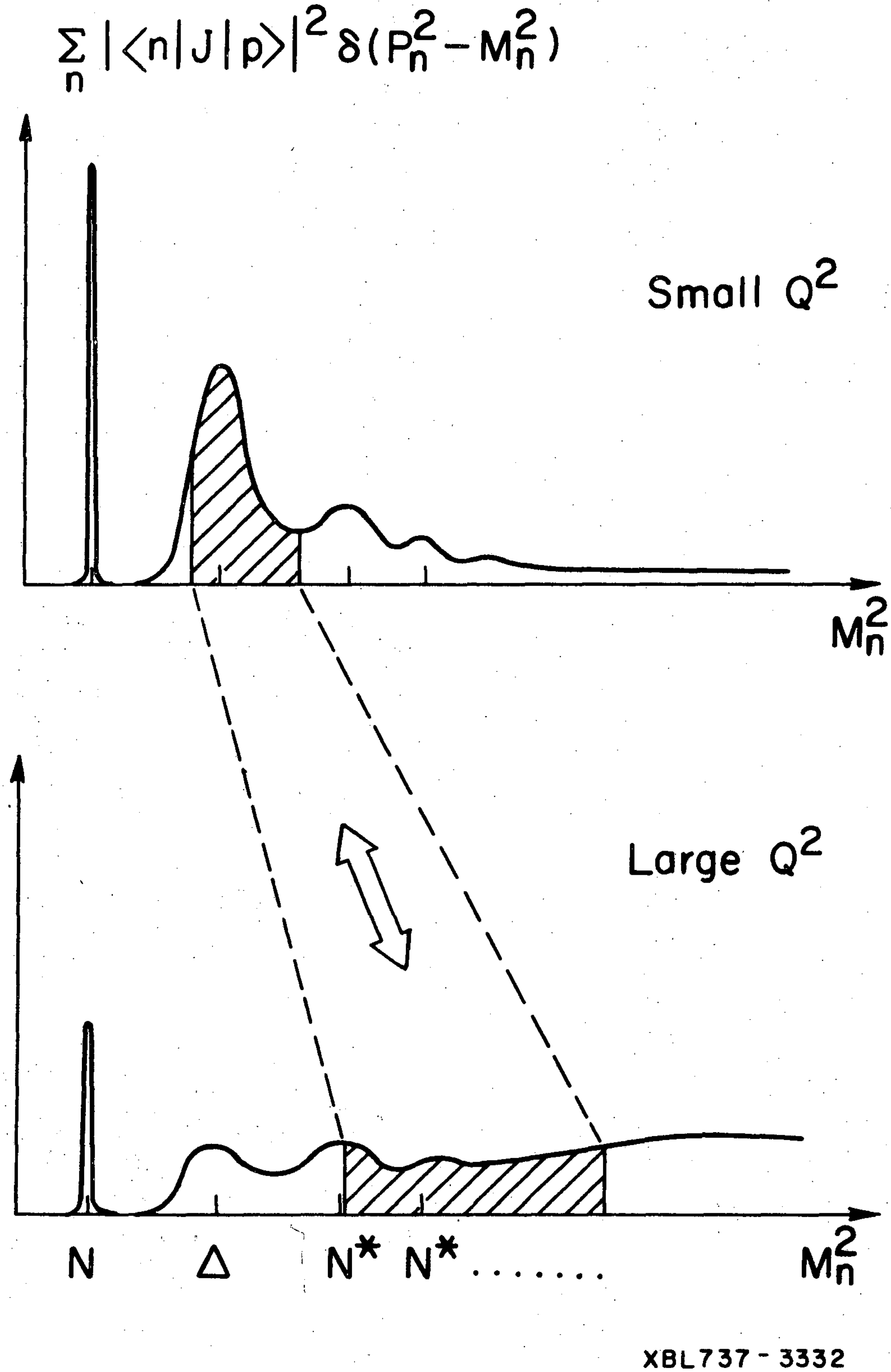
Table 1: The scaling functions $F_2^{\bar{\nu}p}(\omega)$ and $F_2^{\nu p}(\omega)$ of $\Delta S = 0$. See the formulae (4.21) - (4.24) in the text. The functions are tabulated in the variable $x \equiv 1/\omega$.

$x = 1/\omega$	$F_2^{\bar{\nu}p}(\omega)$	$F_2^{\nu p}(\omega)$
0	0.80	0.80
0.05	1.20	0.93
0.10	1.34	0.86
0.15	1.40	0.76
0.20	1.44	0.62
0.25	1.38	0.53
0.30	1.31	0.43
0.35	1.21	0.33
0.40	1.08	0.26
0.45	0.94	0.19
0.50	0.79	0.14
0.55	0.64	0.096
0.60	0.50	0.058
0.65	0.37	0.041
0.70	0.25	0.025
0.75	0.16	0.014
0.80	0.089	0.008
0.85	0.041	0.003
0.90	0.013	0.001
0.95	0.002	0.0005
1.00	0	0

FIGURE CAPTIONS

Fig. 1: Correspondence under the transform through the generalized scaling. The shaded areas in the top and bottom figures are to be the same.

Fig. 2: $F_2^{\bar{\nu}p}(\omega)$ and $F_2^{\nu p}(\omega)$ of $\Delta S = 0$ given by (4.21) - (4.24) in the text. They are plotted in $x \equiv 1/\omega$.



$\sum_n |\langle n|J|p\rangle|^2 \delta(P_n^2 - M_n^2)$
Small Q^2
M_n^2
Large Q^2
N
Δ
N*
N*
....
M_n^2
XBL737-3332

Fig. 1

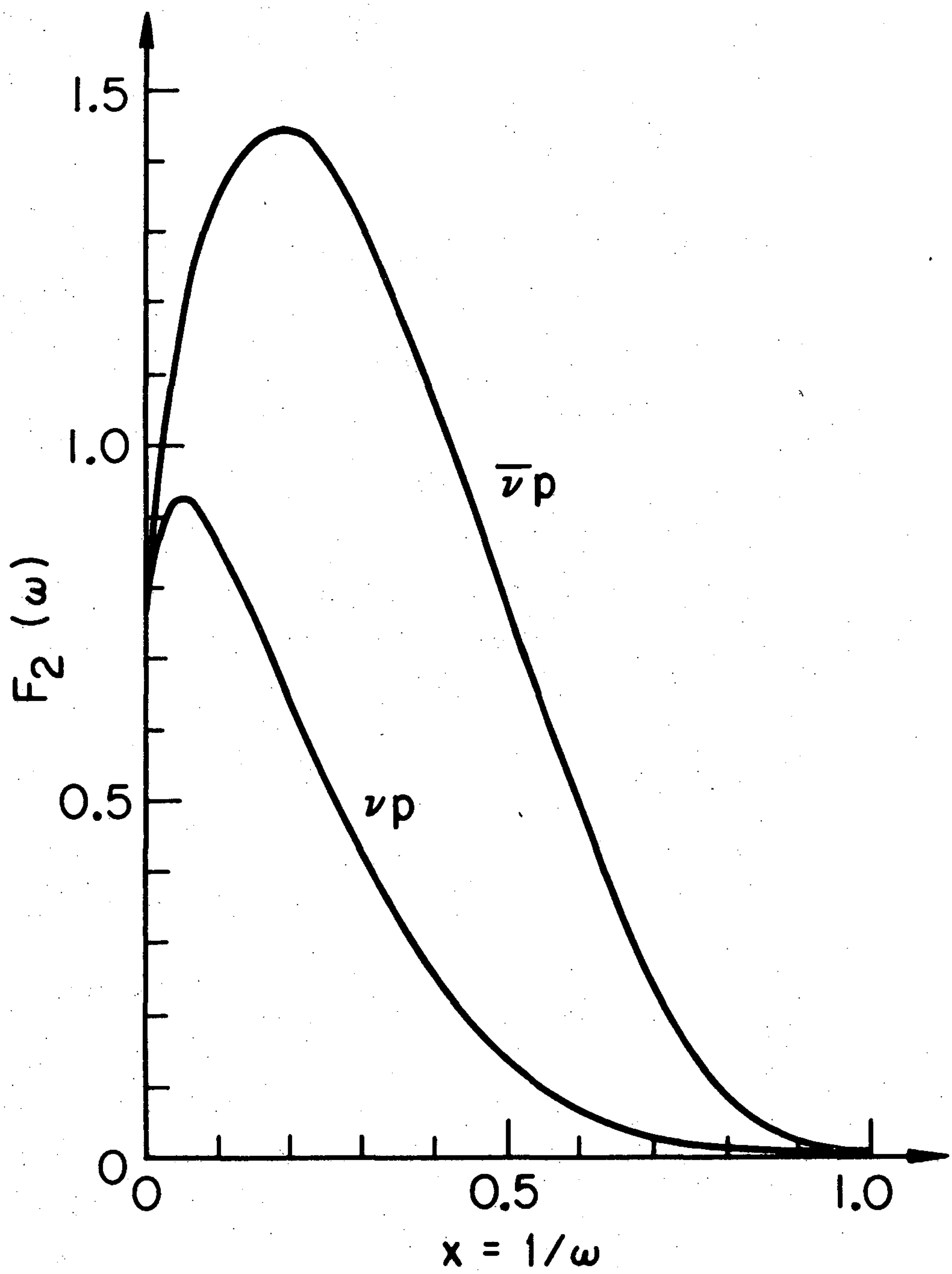


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Fig. 2

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