Title
PROPOSED F.F. BUNCHER FOR A-12.

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Publication Date
1952-06-25

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PROPOSED R.F. BUNCHER FOR A-12

S. A. Colgate

June 25, 1952

Berkeley, California

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PROPOSED R.F. BUNCHER FOR A-12

S. A. Colgate

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

June 25, 1952

OUTLINE

FIGURES

ABSTRACT

A buncher and pre-accelerator for A-12 is described that has 360° phase acceptance and could handle a current of one ampere from a low voltage source.

INTRODUCTION

The present design of A-12 calls for d.c. injection of about one ampere of deuterons at 94 kv. Of this one ampere nearly one half is lost in the first four cells. A significant portion of the lost ions strike drift tube surfaces under conditions favorable to secondary emission ratios of about ten. The resulting high flux of charge may provide such favorable conditions for sparking that either the injection current will have to be reduced appreciably or frequent replacement of the first few drift tubes will be required. If a feasible buncher can be made with reasonable economy, it would obviously be worthwhile. The buncher described here appears to both feasible and practical.

The requirements for bunched injection into an accelerator are that the particles be bunched both in space and in velocity. It is not possible to achieve both of these conditions in one operation[1], but it is possible with a series of quasi-static operations, such as in a linear accelerator. This proposed buncher is a modified linear accelerator which solves the usual focusing problems of linear accelerators in a new way.

The basic concept is to establish a stable rotating beam around a central electrode. A d.c. potential between an outer cylindrical conductor and the central wire (conductor) establishes an attractive field proportional to 1/r towards the central wire. If a beam particle has a

[1]Garren, A., UCRL-1394

given angular momentum around the central wire, then its centrifugal force appears as a repulsive field proportional to $1/r^3$. The addition of the static d.c. field and the repulsive dynamic field gives a stable potential well that will confine the limits of radial motion of a particle. The particle orbit around the central wire is simply a Kepler orbit in a $1/r$ field.

The Z motion along the length of the "coax line" is independent of the potential well and can be modulated rf wise to bunch and accelerate the particle as in a Sloan-Lawrence type accelerator. The outside cylindrical electrode is divided into drift tube sections that are rf modulated. (See Fig. 1)

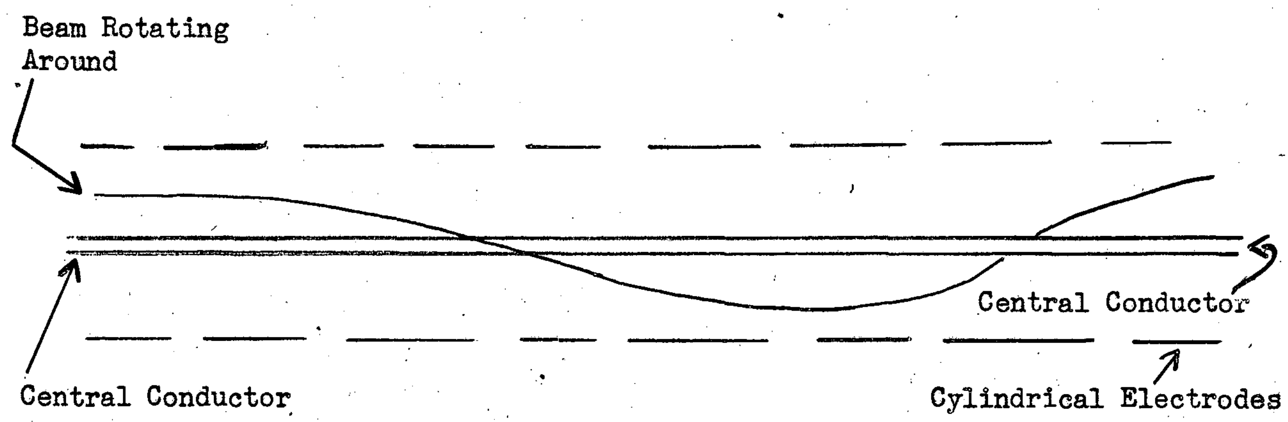

Fig. 1

The advantage of this type of configuration is that by introducing charge (central wire) into the center of the beam, the radial space charge forces can be overcome, and the resulting stability allows a relatively long time to operate on the beam with other (bunching) fields. This long time is necessary because in order to bunch a beam into discrete bunches in velocity and space the forces applied must be relatively weak and quasi-static. A sudden strong kick will always give a velocity modulated bunch which will have only partial acceptance by the linear accelerator.

In order to describe the general characteristics of this type of buncher, the theory and equations relative to the means of focusing, bunching and accelerating are developed. A set of practical numbers are then chosen to illustrate the magnitude of both the advantages and feasibility of the bunching method.

II THE D.C. GENERAL EQUATIONS

Consider first a particle with a given angular momentum Ω in a 1/r field. This would be the electric field in a coaxial system of central wire and outer cylindrical electrode. One calculates the potentials due to the centrifugal force of the angular momentum and due to the d.c. potential of the coax system. From observing the resulting potential well one gains a certain physical insight into the dynamics of a particle in the well.

1. Definitions (Fig. 2)

Ω = angular momentum of particle around central wire

a = radius of central wire b = radius of outer electrode

f = radius of injectron

r = generalized radial coordinate

m = mass of particle

V = generalized potential energy

V_o = d.c. potential between inner and outer conductors

ω = angular frequency about center conductor

ρ = space charge density

2. For centrifugal force $F_{cen} = mr\omega^2$

3.
$$\Omega = mr^2\omega$$

$$F_{cen} = \frac{mr\Omega^2}{m^2r^4} = \frac{\Omega^2}{mr^3}$$

$$F_{cen} \sim 1/r^3$$

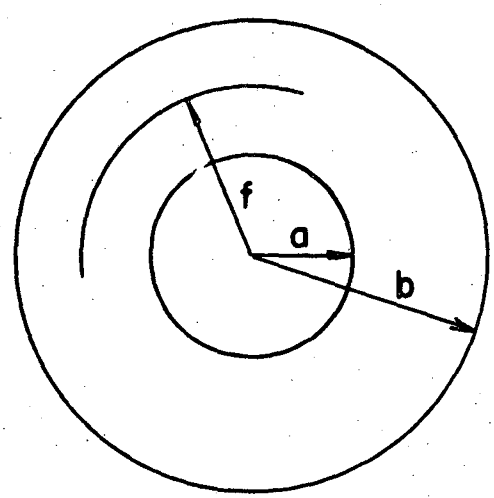

FIG. 2

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4. $$V_{cen} = -\int_r^a F_c\, dr = \int_a^r \frac{\Omega^2}{mr^3}\, dr$$

$$V_{cen} = \frac{\Omega^2}{2m}\left(\frac{1}{a^2} - \frac{1}{r^2}\right)$$

The d.c. potential $V_o = \int_a^b E_r\, dr$

5. $$V_o = -\int_a^b \frac{E_o}{r}\, dr = E_o \ln\frac{a}{b}, \quad E_o = \frac{V_o}{\ln\frac{a}{b}}$$

$$\therefore V_{DC} = -\int_a^r \frac{V_o}{\ln\frac{a}{b}} \frac{1}{r}\, dr = \frac{V_o}{\ln\frac{a}{b}} (\ln a - \ln r)$$

$$V_{DC} = V_o \frac{\ln a/r}{\ln a/b}$$

Fig. 1, Curves I = V_{cen}, is drawn for suitable values of the parameters and II = V_{DC}, and III = $V_{cen} + V_{DC}$, indicates the type of potential well the particle sees. It is evident that the centrifugal repulsive potential is a "hard core" type repulsive potential, namely that a particle with a given angular momentum will be very strongly repulsed at radii less than a certain minimum. Practically, this means that if the central electrode is small enough, the beam loss to it will be negligible. Later beam loss will be discussed as the change in minimum orbit radius for a given change in angular momentum.

Considering the potential well of Fig. 3 further, suppose one injects a particle at 2 cm radius. Then the minimum total radial energy, corresponding to this point of injection, is the potential energy corresponding to this point of the well. The particle will then oscillate back and forth in the well at the level W with its maximum radius = 2 cm and minimum radius = 0.35 cm. If, however, the particle injected at radius 2 cm were given some additional kinetic energy in the form of $(m/2)\dot{r}^2$, then a higher level W' of oscillation in the well will be established such that $W' - W = (m/2)\dot{r}^2$. This means that the total energy E of injection (exclusive of Z direction) determines the minimum and maximum orbit radii.

From the standpoint of a practical device one wants the difference between minimum and maximum orbit radii to be small, i.e. as close to a circular orbit as possible. This would tend to minimize beam loss to the central electrode and keep a well confined beam. On the other hand one wants a large source injection area in order to get a large current. However, the maximum orbit radius is at <u>least</u> as great as the injection radius. This least value is for zero $\dot{r}$ at injection. Therefore the equation of orbit motion will be derived under the assumption of $\dot{r} = 0$ at injection. Then the perturbation to the orbit by a small $\dot{r}$ at injection will be investigated.

The differential equations of motion cannot be integrated exactly but all the desirable features can be derived from an energy analysis of the problem.

Let W = total energy

V = potential energy

T = kinetic energy

Then in general

6. $$T + V = W \qquad \Omega^2 = m^2 r^4 \omega^2$$

7. $$T = T_r + T_\varphi = \frac{m}{2}\dot{r}^2 + \frac{\Omega^2}{2mr^2}$$

$$V = \frac{V_o \ln a/r}{\ln a/b} \qquad \text{from (5)}$$

8. W = potential + kinetic energy at injection radius f (i.e. we have a conservative system)

$$\therefore W = \frac{V_o \ln a/f}{\ln a/b} + T\,(r = f)$$

At injection let $\dot{r} = 0$

i.e. $\dot{r}(r=f) = 0$

$$\therefore W = V_o \frac{\ln a/f}{\ln a/b} + \frac{\Omega^2}{2mf^2}$$

Therefore the general equation becomes

$$9.\quad \frac{m}{2}\dot{r}^2 + \frac{\Omega^2}{2mr^2} + \frac{V_o \ln a/r}{\ln a/b} = \frac{V_o \ln a/f}{\ln a/b} + \frac{\Omega^2}{2mf^2}$$

$$\text{or}\quad \frac{m}{2}\dot{r}^2 + \frac{\Omega^2}{2m}\left(\frac{1}{r^2} - \frac{1}{f^2}\right) = V_o \frac{\ln r/f}{\ln a/b}$$

To find the minimum and maximum orbit radii, one knows that at $r_{\text{min or max}}$ $(\dot{r} = 0)$

$\therefore$ the two roots of the equation

$$10.\quad \frac{\Omega^2}{2m}\left(\frac{1}{r^2} - \frac{1}{f^2}\right) = V_o \frac{\ln r/f}{\ln a/b}$$

determine the minimum and maximum orbit radii. If one lets $r = f$, one sees immediately that the injection radius is the maximum orbit radius (provided there is a bound orbit). This is identically a consequence of letting $\dot{r} = 0$ at $r = f$ (injection). The minimum orbit radius can be found for any given condition by iterating a trial radius $a < r < f$ and it will converge quite rapidly.

To find what the charge distribution in the beam will look like, one notes that the probability of a particle being at a given radius is inversely proportional to the radial velocity. Let the probability of a particle being between r and $r + dr$ be $P(r)dr$. Then $P(r)dr = dt$, $P(r) = dt/dr = Pr$. $Pr \sim \text{prob}(r) \sim 1/\dot{r}$.

11. From 9, then one has

$$Pr \sim \frac{1}{\dot{r}} \sim \frac{1}{\sqrt{\frac{2}{m}\left[V_o \frac{\ell n\ r/f}{\ell n\ a/b} + \frac{\Omega^2}{2m}\left(\frac{1}{f^2} - \frac{1}{r^2}\right)\right]}}.$$

For an example of this sort of distribution see Fig. 4.

Another quantity one would like to know is the apse angle or the angle between the minimum and maximum radii.

$$\text{If } \alpha = \text{apse} \angle = \int_{r_{min}}^{r_{max}} d\theta = \int_{r_{min}}^{r_{max}} \frac{d\theta}{dr}\, dr$$

$$\frac{d\theta}{dr} = \frac{d\theta}{dt}\frac{dt}{dr} = \frac{d\theta}{dt}\Big/\dot{r} = \frac{\omega}{\dot{r}}$$

$$\text{but } \Omega = mr^2\omega, \quad \therefore \quad \frac{d\theta}{dr} = \frac{\Omega}{mr^2}\,\frac{1}{\dot{r}}$$

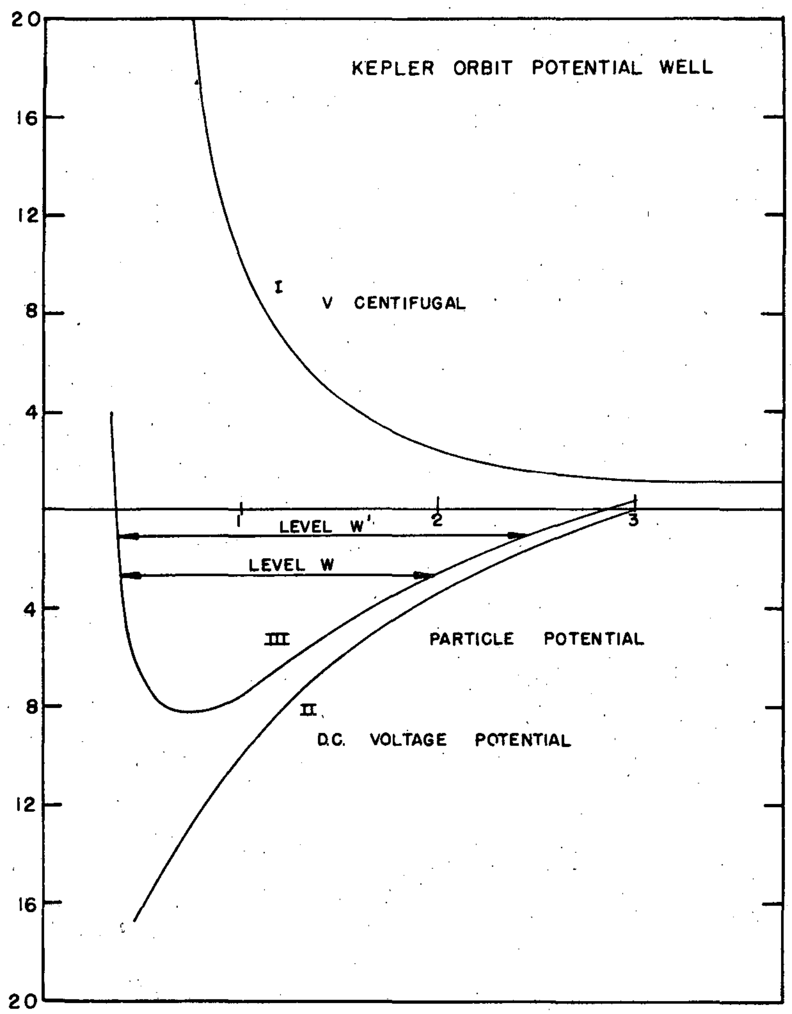
12. $$\alpha = \int_{r_{min}}^{f} \frac{\Omega}{mr^2} \frac{dr}{\sqrt{\frac{2}{m}\left[V_o \frac{\ell n\ r/f}{\ell n\ a/b} + \frac{\Omega^2}{2m}\left(1/f^2 - 1/r^2\right)\right]}} =$$

$$\int_{r_{min}}^{f} \frac{f\ dr}{r^2\sqrt{\frac{2mf^2}{\Omega^2}\left[V_o \frac{\ell n\ r/f}{\ell n\ a/b} + \frac{\Omega^2}{2mf^2}\left(1 - f^2/r^2\right)\right]}}$$

α will vary between 90° (straight line oscillation through center $\Omega = 0$) for a bound orbit to $\theta = \infty$ for an orbit which is not quite bound.

It would now be well to find out the change in minimum and maximum radii for small changes in applied voltage V_{DC}. The major case of interest will be for the injection potential constant and rf changes of the radial field.

Finding the change in maximum radius $r_{max} = f + \Delta$



20
16
12
8
4
4
8
12
16
20
KEPLER ORBIT POTENTIAL WELL
I
V CENTIFUGAL
1
2
3
LEVEL W'
LEVEL W
III
PARTICLE POTENTIAL
II
D.C. VOLTAGE POTENTIAL

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FIG. 3

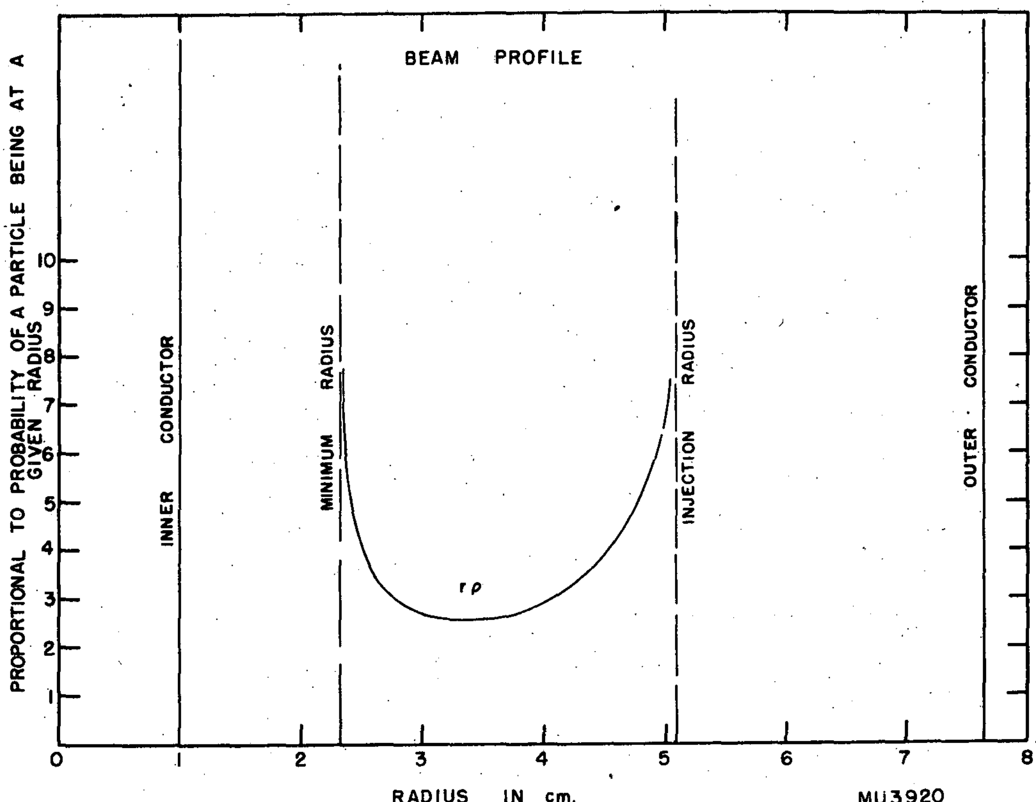

FIG. 4

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$$V_r = (V_o + \partial)\ \frac{\ell n\ a/v}{\ell n\ a/b}$$

Let $\dot{r} = 0$, potential energy at injection is still $V_o \frac{\ell n\ a/f}{\ell n\ a/b}$

Substituting in (9) one has, $\frac{\Omega^2}{2M}\left(\frac{1}{(F+\Delta)^2} - \frac{1}{f^2}\right) = V_o\ \frac{\ell n\ a/f}{\ell n\ a/b} - (V_o + \partial)\frac{\ell n\ a/f+\Delta}{\ell n\ a/b}$

rearranging and neglecting terms in Δ^2 one has:

$$\frac{\Omega^2}{2mf^2}\left(\frac{f^2 - f^2 - 2\Delta f}{f^2 + 2\Delta f}\right) = \frac{V_o\ \ell n\ \frac{f+\Delta}{f}}{\ell n\ a/b} + \frac{\partial\, \ell n\ \frac{f+\Delta}{f}}{\ell n\ a/b} - \partial\ \frac{\ell n\ a/f}{\ell n\ a/b}$$

or: $$-\frac{\Omega^2}{2mf^2}\left(\frac{2\Delta}{f}\right) = \frac{V_o}{\ell n\ a/b}\ \frac{\Delta}{f} + \partial\,\frac{\Delta}{f}\Big/\ell n\ a/b - \frac{\partial\ \ell n\ a/f}{\ell n\ a/b}$$

13. or: $$\frac{2\left(\frac{2}{2mf^2}\right) + \frac{V_o}{\ell n\ a/b}}{V_o\ \frac{\ell n\ a/f}{\ell n\ a/b}}\left(\frac{\Delta}{f}\right) = \frac{\partial}{V_o}$$

To find the change in minimum radius one has for a change in potential after injection $r = r_{min} + \Delta$

$$\frac{\Omega^2}{2mf^2}\left(\frac{f^2}{(r_{min}+\Delta)^2} - 1\right) = V_o\ \frac{\ell n\ a/f}{\ell n\ a/b} - (V_o + \partial)\ \frac{\ell n\ \frac{a}{r_{min}+\Delta}}{\ell n\ a/b}$$

or: $$\frac{\Omega^2}{2mf^2}\left(\frac{f^2 - r_{min}^2}{r_{min}^2} - \frac{2\Delta}{r_{min}}\right) = V_o\ \frac{\ell n\ a/f}{\ell n\ a/b} - V_o\frac{\ell n\ a/r_{min}}{\ell n\ a/b} - \frac{\partial\ \ell n\ \frac{a}{r\ min}}{\ell n\ a/b} -$$

$$\frac{V_o\ \ell n\ \frac{r_{min}}{r_{min}+\Delta}}{\ell n\ a/b} - \frac{\partial\ \ell n\ \frac{r_{min}}{r_{min}+\Delta}}{\ell n\ a/b}$$

The terms that satisfy the quadratic for r_{min} drop out identically by definition or r_{min} and one has:

$$\frac{\Omega^2}{2mf^2}\left(\frac{2\Delta}{r_{min}}\right) = \frac{V_o\ \frac{\Delta}{r\ min}}{\ell n\ a/b} - \frac{\partial\ \ell n\ \frac{a}{r_{min}}}{\ell n\ a/b}$$

or: $$-\frac{2\left(\frac{\Omega^2}{2mf^2}\right) + \frac{V_o}{\ell n\ a/b}}{V_o\ \frac{\ell n\ a/r_{min}}{\ell n\ a/b}}\left(\frac{\Delta}{r_{min}}\right) = \frac{\partial}{V_o}$$

One finds the change in maximum radius for a change in injection total energy E. This corresponds to an addition of $m/2\ (\dot{r})^2$ energy.

Analogous to Equation 13, one has:

$$-\frac{\Omega^2}{2\,mf^2}\left(\frac{2\Delta}{f}\right)+\frac{m}{2}\ (\dot{r}^2)=Vo\ \frac{\ln f+\frac{\Delta}{f}}{\ln {}^a/b}$$

15. or: $$\frac{\Delta}{f}=\frac{m/2\ (\dot{r})^2}{2\left(\frac{\Omega^2}{2mf^2}\right)+\frac{Vo}{\ln {}^a/b}}$$

Each of these Perturbation Formulae Equations, 13, 14, 15 will be applied in the case of "The Specific Proposal", Part VI, and in general they can be described qualitatively as:

13. The maximum orbit radius is "soft" to potential changes; i.e., the percentage change in radius is about twice the percentage change in voltage.

14. The minimum orbit radius is "hard" to potential changes, i.e. $\frac{\Delta r_{min}}{r_{min}}$ = 1/2 the percentage change in voltage.

15. The maximum radius is relatively "hard" to injection ($\dot{r}$), i.e. $\frac{\frac{m}{2}\dot{r}^2}{Vo/2}$ is about the fractional change in r_{max} so that the injected beam does not have to be exactly parallel to the axis in the r, Z plane.

III. R. F. BUNCHING

Consider a stable beam rotating around a central conductor at an average radius r with a forward or Z velocity = $\dot{Z}$. Now if one divides the outside conductor into rings or drift tubes and modulates them rf wise, one considers the field components present.

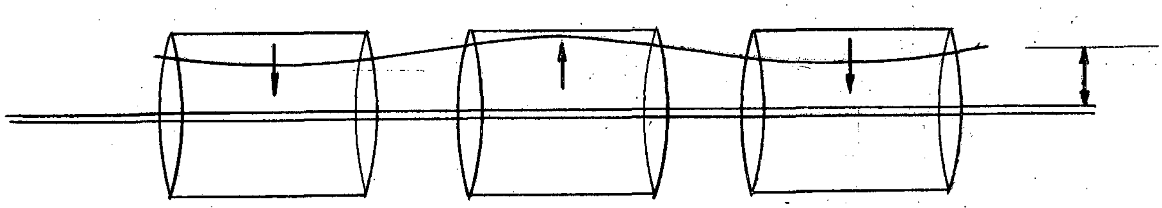

Fig. 5

The general directions of the major rf field components are indicated by the arrows for one instant of rf phase.

We note that the radial rf components are such as to neck down the outside envelope of the beam in the (+) drift tubes and expand the envelope in the (-) drift tubes. The longitudinal fields E_z are those one would expect in any linear accelerator modified by the central conductor. Namely, they are directed towards bunching the beam in the (-) electrode. The radial fields have already expanded the diameter or volume here and necked it down under the (+) drift tubes so that the space charge forces will add an additional component that will also tend to bunch the beam in the region of the negative electrode.

These are the forces at only one instant in rf phase. One must also satisfy the further condition as in any linear accelerator that the beam and the rf be in phase.

Letting

16. $\beta C \equiv$ the $\mathring{Z}$ velocity of the beam

$\lambda =$ wave length in free space of the rf

$\ell =$ length of 1/2 cell = drift tube length + gap

$g =$ gap length

then $\ell = \beta \lambda/2$ for beam and rf in phase.

Consider the bunching action further. In Fig. 5 there is a phase stable region in the region of the negative electrode. This phase stable region will move in the Z direction with the beam with $\mathring{Z} \equiv \beta C$. This phase stability can be represented by a phase potential well-moving with velocity βC. A particle will oscillate in this phase potential well with a constant amplitude dependent upon its original phase of injection. All particles therefore will oscillate through the center of the potential well giving a logarithmic central spike distribution of phase about the mean stable phase for each particle

A = amplitude of phase oscillation

Z = distance in phase from center of phase stable potential well

17. Probability of being at Z, $p(Z) \sim \dfrac{1}{\sqrt{A^2 - Z^2}}$,

see page 28, UCRL-1095, Panofsky.

18. Then, density at $Z = \sigma(Z) \sim \displaystyle\int_Z^{\ell} \frac{dA}{\sqrt{A^2 - Z^2}}$

$$= \ln\left(\frac{Z + \sqrt{\ell^2 - Z^2}}{Z}\right)$$

for small Z $\longrightarrow$ $\ln(\ell/Z)$

This is not of itself a very well confined bunch. However, the amplitude of a phase oscillation is damped if the beam is accelerated. It can be accelerated, as in any linear accelerator, by increasing the length of each cell. The damping of the phase oscillations is given by Panofsky, UCRL-1095, p. 19 as $N^{-.64}$

N = number of drift tubes, 0.64 is the approximate exponent of damping for $g/L = 0.25$.

This means for N = 10 the beam bunch will be approximately 1/4.5 as long, which is good enough bunching for injection into the big machine. This indicates that acceleration is a necessary part of the bunching process.

The gain in energy that a particle gets in crossing an rf gap at near constant β is:

19. $$\Delta W = \int_{-\ell/2}^{+\ell/2} E_Z \cos(\phi + \delta)\, dZ$$

where ΔW = change in energy

E_Z = Z component of the electric field

ϕ = rf phase angle of particle = $\frac{\ell/2 - Z}{\ell/2}\,\frac{\pi}{2}$

δ = equilibrium phase angle.

This can be rewritten (dropping out terms in sin ϕ by odd-even symmetry) as

20. $$\Delta W = \int_{-\ell/2}^{\ell/2} E_Z \cos\left[\left(\frac{\ell/2 - Z}{\ell/2}\right)\frac{\pi}{2}\right] dZ \cos\delta$$

Since β will be small ~ 0.01 for a practical case, the rf fields can be approximated by the d.c. case and electrolytic tank measurements can be made with a double probe lined up in the Z direction to give E_Z directly. This can be done at various radii to give transit time figures as a function of radius.

Transit time is a term to describe the fraction of the maximum rf voltage across a gap that a particle can pick up.

t = transit time

21. $$t(r) = \frac{\int_{-\ell/2}^{\ell/2} E_Z(r) \cos\phi\, dZ}{\text{max voltage}}$$

In electrolytic tank measurements the maximum voltage is defined by

$$\int_{-g/2}^{g/2} E_Z\,(r = b)\,dZ \quad , \quad \text{where} \quad b = \text{radius of outer conductor}$$

at $r = b$, $E_r = 0$ for all Z so that the integral of the field at this radius gives the voltage across the gap.

These transit time measurements have been made with an electrolytic tank for various r for the specific proposal.

$\beta = 0.01$ and 0.01732

$\ell = 4.88$ in. and 8.46 in.

$a = 3/8$ in., inner conductor radius, $b = 3$ in., outer conductor radius

$g/\ell = 0.20$

See Fig. 6.

At the large radius of the stable beam it can be seen that the transit time factors are quite large -- approximately 50 percent - so that the acceleration is quite efficient.

IV SPACE CHARGE EFFECTS

The additional bunching forces derived from space charge by the necking of the diameter of the beam in the region of the positive rf electrodes has already been mentioned in the last section. In this section we are principally interested in the perturbation to the beam profile by the radial space charge forces.

The exact calculation of the space charge effects consists of an iterating procedure. The unperturbed beam profile as given by Equation 11 gives rise to a new potential V(r) that should be used in determining V in Equation 11 and the procedure repeated until satisfactory convergence. The convergence should be rapid because in general the space charge is a small perturbation to the d.c. potentials, nevertheless this procedure is long and gives little physical picture of what is going on.

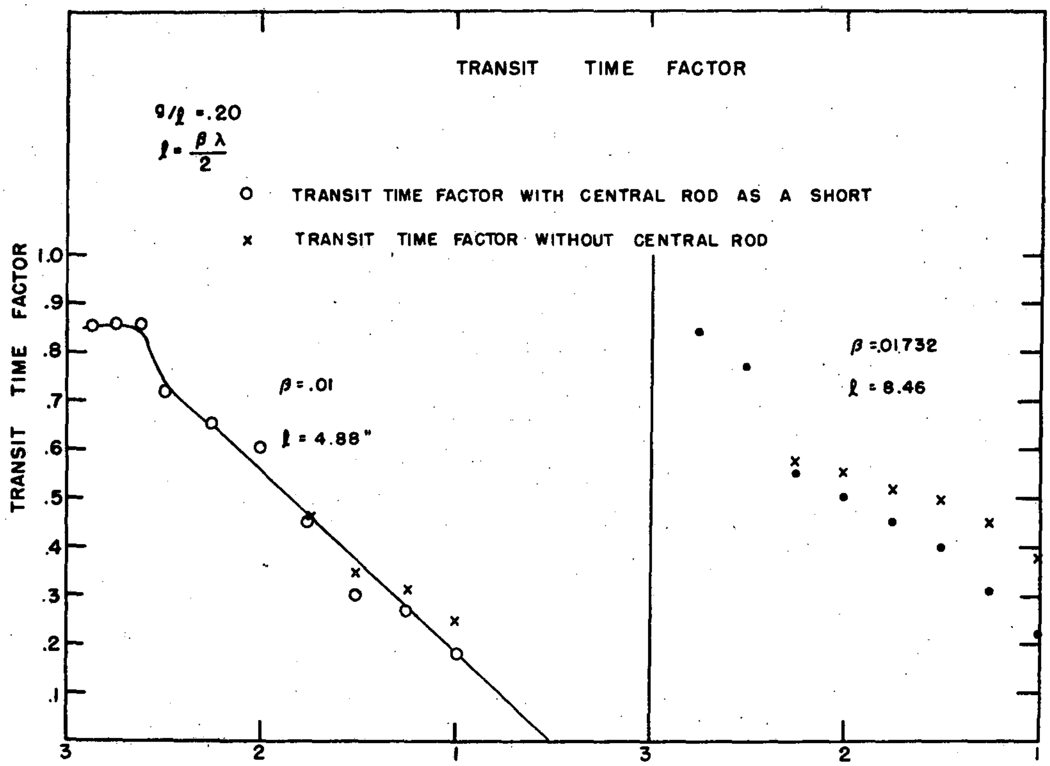
TRANSIT TIME FACTOR
g/ℓ =.20
ℓ = βλ/2
○ TRANSIT TIME FACTOR WITH CENTRAL ROD AS A SHORT
× TRANSIT TIME FACTOR WITHOUT CENTRAL ROD
TRANSIT TIME FACTOR
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
β = .01
ℓ = 4.88"
β =.01732
ℓ = 8.46
3
2
1
3
2
1
RADIUS IN INCHES
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FIG. 6

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A simpler approximation can be made to show directly that the space charge perturbation is small. The radial electric field outside the beam will be calculated and this will be compared to the d.c. applied field. Since both fields will be radial ~ 1/r we can say that outside the beam there is a change in potential proportional to the ratio of these fields. The change in potential at smaller radii will be less than this because at a radius of the inner conductor the potential will be the same as the unperturbed value. The net effect of this potential distribution is that the maximum or outside diameter of the beam will remain the same. This is because a particle injected at a given radius ($\dot{r} = 0$) and into a given potential (this will <u>not</u> be strictly a 1/r field because of space charge) will oscillate in the potential well at the level of its injection. The inside radius on the other hand will be smaller by Δ/r_{min} determined by ∂/V_o by Equation 14.

Let I = current in beam in amperes

f = its maximum radius

E_f = radial field at f due to space charge

βC = velocity of beam

Q = charge/cm in coulombs

22. By Gauss's theorem $\int_s \bar{\bar{E}}\cdot\vec{n}\, ds = 4\pi Q$

Since all the E flux is radial through an area $2\pi f$, then

$2\pi f E_f = 4\pi Q$

$E_f = \dfrac{2Q}{f}$

Q x velocity = Q βC = I coulombs/sec = I c/10 esu/sec

or Q = $1/10\beta$ esu/cm

23. $\therefore\ E_f = (2I/\beta f)\dfrac{300}{10}$ practical volts/cm

$$E_f = 60I/\beta f$$

E (at f) from d.c. potential $= \dfrac{V_o}{f \ln \frac{a}{b}}$ from Equation 5.

24. $$\frac{E_f \text{ space charge}}{E_f \text{ d.c. potential}} = \frac{60\ I/\beta f}{V_o/f \ln a/b} = \frac{60\ I \ln a/b}{\beta\ V_o}$$

To indicate the order of magnitude we take

$V_o = 10^5$ volts

$I = 1$ amp

$\beta = 0.01$

$\ln a/b = 1.6$

$$\frac{E_f \text{ space charge}}{E_f \text{ d.c. potential}} = 0.1$$

This will mean a change in r_{min} of only about $0.05 = \Delta/r_{min}$ which is insignificant as a change in beam profile.

V MEANS OF INJECTING ANGULAR MOMENTUM

Angular momentum can be injected by two reasonable means:

1. Injecting a beam at an angle θ to the axis and perpendicular to a radius r.

2. Having the beam originate in an axially symmetric magnetic field at a radius r.

For Scheme 1, θ = angle of injection relative to the axis perpendicular to radius f.

Then $\Omega = mf^2\dot{\phi}$

if βC = total velocity of injection

25. $\Omega = mf\ \beta C \sin \theta$

If the angular momentum is expressed as the energy in the radial direction at injection we have

26. $T_{radial(f)} = \frac{\Omega^2}{2mf^2} = m/2\ \beta^2 C^2 \sin^2 \theta = T_{injection} \sin^2 \theta$

For Scheme 2, if there is a radial symmetric axial magnetic field, a particle leaving that field will gain angular momentum depending upon its radius in the magnetic field and upon the magnetic field strength. Physically it can be seen that the angular momentum comes from the curvature relative to the axis that the particle gets in going through the radial return field of the magnet.

This can be expressed in an invariant form as the conservation of canonical angular momentum.

27. $\frac{d}{dt}(mr^2 \dot{\phi} - er A\phi) = 0$

$A\phi$ = The ϕ component of the magnetic vector potential.

In an axial magnetic field of strength B_z

28. $A\phi = 1/2\ r B_z$

∴ substituting into Equation 27 we have

$\Omega = mr^2\dot{\phi} = \frac{e\ r^2}{2} B_z$

$$T_{\substack{\text{angular}\\\text{momentum}}} = \frac{\Omega^2}{2mf^2} = \frac{e^2f^2B_z^2}{8m} = 6.4 \times 10^{-8} f^2B_z^2 \text{ electron kilovolts for deuterons}$$

This gives $B_z = 7900$ gauss for f = 5.1 cm and $T_{radial} = 10$ kilovolts which are reasonable conditions. The magnetic field method of injecting angular momentum has the further advantage of decreasing the injected angular momentum as the radius of injection decreases. This tends to keep the orbit dynamics constant for a finite range of injection radius.

The reason 10 KV is chosen for the angular momentum injection in the specific proposal is that it is the maximum amount compatible with the allowable defocusing forces (of angular momentum) in the big machine. This defocusing force goes as $1/r^3$ so that if we let a 2 in. radius beam expand to 6 in. radius, this gives a reduction in radial force of 1/27 which gives a radial force less than the planned over-focusing force for A-12 as required.

VI A SPECIFIC PROPOSAL FOR A-12

30. Beam current = 1 ampere

referring to (1)

$a = 3/8$ in.

$b = 3$ in.

$f = 2$ in. $= 5.1$ cm

$V_o = 100$ KV

$\beta_z = 0.01$

$\beta_\phi = 0.00316$

$T_\phi = \frac{\Omega^2}{2mf^2} = 10$ KV , i.e., 10 kilovolt angular momentum at r = f

$\ln a/b = -2.03$

$\ell n\ a/f = 1.63$

$g/\ell = 0.2$

Peak rf = ± 30 KV to 50 KV

number of cells = 10

over-all length = 85 in.

To find beam profile and limits:

From Equation 10

31. $10(f^2/r^2 - 1) = -49.2\ \ell n\ r/5.1$

$r_{min} = 2.31$ cm

The beam profile is given in Fig. 4.

From Equation 13 the change in maximum radius is

32. $\Delta/f = -2.74\ \partial/V_o$

i.e. the maximum radius f is quite soft to a change in radial potential.

From Equation 14 the change in minimum radius

33. $\Delta/r_{min} = 0.60\ \partial/V_o$

i.e. the minimum radius is quite hard to a change in radial potential.

The change in maximum radius for a radial injection energy $\mathring{T}r$ is given by Equation 15.

$$\Delta/f = \frac{\mathring{T}r}{20 + 49.2} \qquad \mathring{T}r \text{ in kilovolts.}$$

I.e., 10 kilovolts radial $\mathring{T}r$ would give 1/7 change in radius and would imply a radial direction to the axis of 18.5°.

By integrating Equation 12 numerically, we get an apse angle $\alpha = 120°$. This gives an approximate orbit shape (see Fig. 7).

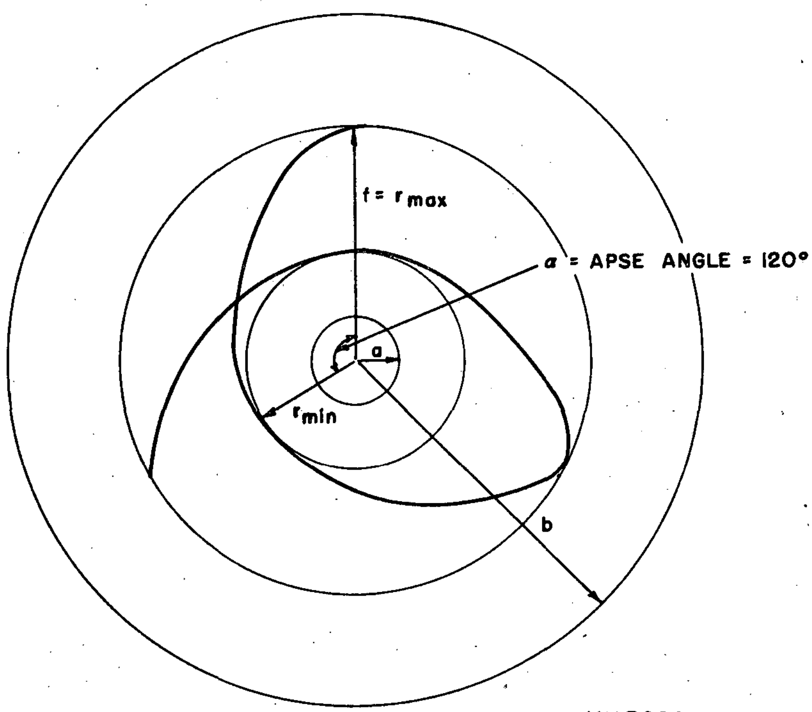

FIG. 7

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ORBIT SHAPE

We now modulate the beam with rf with the outer 6 inch diameter conductor separated into drift tubes with

$$\frac{\text{gap}}{\text{cell length}} = 0.2$$

$$\text{cell length} = \beta \lambda/2$$

If ± 30 KV rf voltage is used in the electrodes, an estimate can be made of the maximum modulated beam diameter.

The apse angle is 120°, so that the path length for a complete orbit (r_{max} to r_{max}) will be about 7 inches at an average β of ~0.0045. This means that the beam will go forward 15 inches or 3 cell lengths per orbit cycle which would give a maximum orbital energy gain per orbit cycle of about ± 10 KV. From Equation (32) this expands the inner radius of the beam from 2.25 inches to 2.9 inches, i.e., inside the bore of 3 inches diameter. Actually this is an exaggeration of the radial energy gain, because field penetration will cut this down just as the transit time factor for acceleration is less than unity. This would have to be investigated on a full scale model. Also the radial energy gain can be greatly reduced by inductive loading of the central electrode, cell by cell.

With the beam profile given by Fig. 4 one gets a mean transit time factor of about 40 percent. If the rf voltage is graded from ± 30 KV at the beginning of the buncher to ± 50 KV at the end (i.e. constant gap gradient), then the energy gain for 10 cells will be

$$\Delta W = 10 \times 0.40 \times 2 \times 40 = 320 \text{ KV}$$

Fig. 8 is a sketch of the proposed buncher. The 100 KV dc injection energy is lost by the time when the particle gets into the big machine, i.e. it starts from ground potential in the plasma and returns to ground potential in the machine so that the net energy gain is just rf wise = 320 KV.

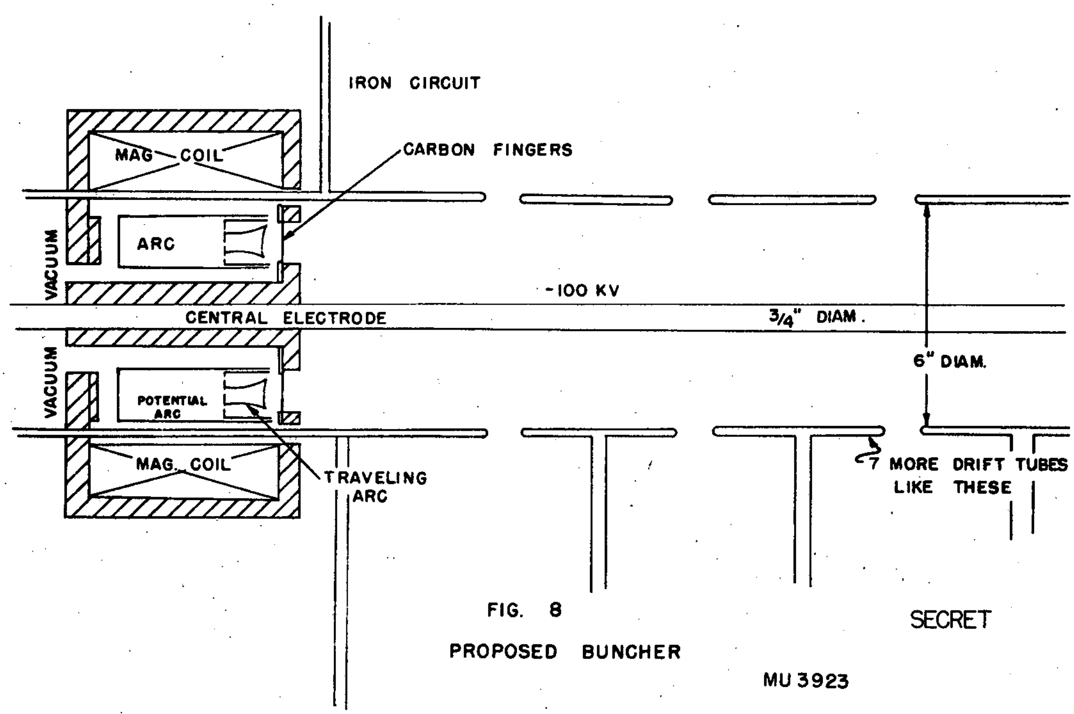

FIG. 8

PROPOSED BUNCHER

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Over-all length = 85 inches.

VII DRAIN TO INNER CONDUCTOR

The current drain to the central conductor is made up of two parts

1. Direct beam loss due to change in orbit dynamics.
2. Ionization of residual gas at 10^{-5} mm Hg pressure.

The direct beam loss due to change in orbit dynamics is estimated to be negligible for the following reason: If the change in angular momentum is calculated necessary to reduce the minimum orbit radius to that of the central electrode, it is found that the injected angular momentum must be reduced by 3/4. It is hard to conceive of any forces or statistical fluctuation that could remove the 10 KV of injected angular momentum.

The ionization of residual gas will not perturb the primary beam particle in the collision (the average energy of collision is 32 ev), but the ions made will go to the central electrode with, in general, a major fraction of the d.c. potential.

To calculate the total current of these ions the energy loss per particle in going two meters at maximum ionization density will be calculated,

For protons or deuterons, energy loss

$$(dE/dx)_{max} \quad 600 \text{ Mev/g/cm}^2 - \text{air}$$

$$E = 0.2 \text{ Mev}$$

For hydrogen

$$(dE/dx)_{max} = 1000 \text{ Mev/g/cm}^2$$

two meters at 10^{-5} mm Hg of hydrogen

$$= 1.3 \times 10^{-8} \text{ at} \times \frac{200 \text{ cm} \times 2}{22.4 \times 10^3} \quad \text{g/cm}^2$$

$$= 2.4 \times 10^{-10} \text{ g/cm}^2$$

$$(dE/dx) \times 2.4 \times 10^{-10} = 2.4 \times 10^{-1} \text{ ev/traversal}$$

If one considers it takes 32 ev per ion pair, each particle going two meters in 10^{-5} pressure will make

$$\frac{2.4 \times 10^{-1}}{32} \quad \text{ion pairs per deuteron}$$

$$= 7.5 \times 10^{-3} \text{ ions per deuteron}$$

For 1 amp of deuterons this would be a current drain to the central electrode of about 10 milliamp. There will probably be about 10 times this drain in electrons but this will be to the large area outside the electrode. The heating problems of both these current drains should be relatively minor since they are generally over large areas.

VIII BIGGER IDEAS

The major limitation to the beam current in this proposal is the current available from a source with plasma area of a ring 1-2/4 inches to 2-1/4 inches in radius. If this ring could be greatly expanded in diameter and at the same time keep the output beam small in diameter, the useful current to A-12 could be greatly increased.

In a conservative potential the maximum orbit radius is always at least the injection radius as pointed out in the introduction of this report. However, the potentials can be made to appear non-conservative by taking advantage of the Z motion. If one applies forces to the beam that convert a radial motion into a Z or longitudinal motion, then, insofar as orbits are concerned, these forces will look non-conservative, i.e. they will convert orbital energy into Z direction energy. The simplest way to do this for a converging beam is to have it enter an axial magnetic field, Fig. 9. Then $\mathring{r} \longrightarrow \mathring{Z}$ and a new stable orbit can be established dependent upon the residual $(\mathring{r})^2$ at the radius in question. Such a scheme would allow as large a source area as desired and the beam would be limited by space charge effects and current drain heating.

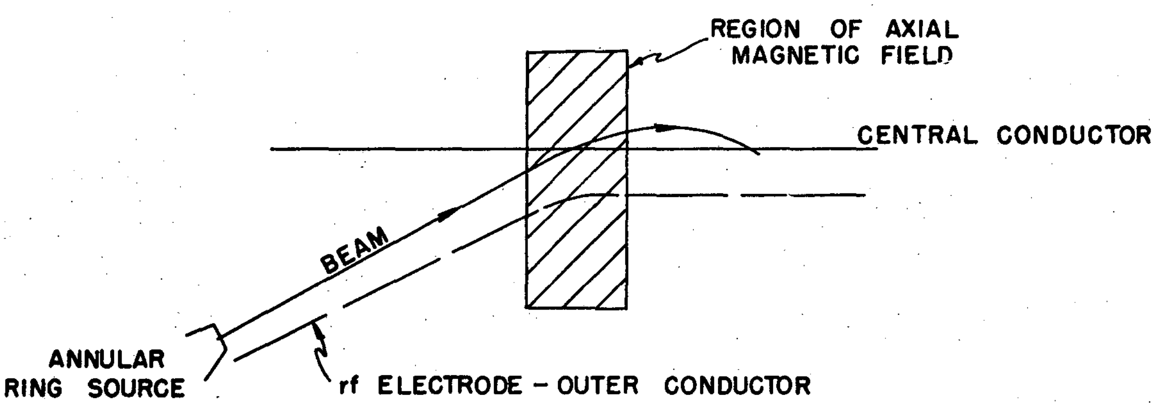

FIG. 9

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