

## **UC Merced**

# **Proceedings of the Annual Meeting of the Cognitive Science Society**

### **Title**

Efficiency and Minimal Change in Spatial Belief Revision

### **Permalink**

<https://escholarship.org/uc/item/5559r4zq>

### **Journal**

Proceedings of the Annual Meeting of the Cognitive Science Society, 33(33)

### **ISSN**

1069-7977

### **Authors**

Krumnack, Antje  
Bucher, Leandra  
Nejasmic, Jelica  
et al.

### **Publication Date**

2011

Peer reviewed

# Efficiency and Minimal Change in Spatial Belief Revision

Antje Krumnack (antje.krumnack@psychol.uni-giessen.de)

Leandra Bucher (leandra.bucher@psychol.uni-giessen.de)

Jelica Nejasmic (jelica.nejasmic@psychol.uni-giessen.de)

Markus Knauff (markus.knauff@psychol.uni-giessen.de)

Experimental Psychology and Cognitive Science, Justus Liebig University Giessen,  
Otto-Behaghel-Strasse 10F, 35394 Giessen, Germany

## Abstract

In the light of new information it is sometimes necessary to change existing beliefs regarding the state of the world. During such a belief revision reasoners have to decide which beliefs to retain and which ones to retract in order to regain consistency within their belief states.

Drawn from the conception that spatial belief revision is based on the alteration of mental models, we discuss the influence of cognitive effort and minimal change when constructing and revising mental models. We offer several possibilities how minimal change can be defined in this context and provide a computational model along with a cost function to describe the revision processes. Furthermore, we present empirical evidence for the suitability of the model. Results indicate that there is a significant influence of cognitive efficiency principles on spatial belief revision.

**Keywords:** Spatial reasoning; mental models; belief revision; mental models; computational framework; cost function.

## Introduction

Imagine two friends describe you the location of a new restaurant. The first one says: "The post office is to the left of the book store and the restaurant is to the right of the book store." The second friend interrupts: "No, the restaurant is to the left of the post office." At which point the first one agrees: "Yes, you are right, the restaurant is to the left of the post office." In which order do you think the three buildings are to be found? Most likely you had an initial belief about how the buildings were arranged (post office-book store- restaurant) which you then had to change in the light of the contradicting information. How reasoners change their beliefs when given new information as well as the underlying processes is topic of belief revision research. The investigation of spatial belief revision constitutes a new research topic (see also Bucher, Krumnack, Nejasmic, & Knauff, 2011). In this paper we want to discuss the question, what is the most efficient way to change a spatial belief?

assume that the major strategy employed in spatial reasoning is the successive construction of a "simulation" or "model" of the "state of affairs", which contains all the information given concerning the situation to be considered. New information, such as a reasoning problem's conclusion, is generated or evaluated by inspecting and varying the possible models (Johnson-Laird & Byrne, 1991). There exist numerous empirical findings that spatial reasoning in particular relies on the construction, inspection, and variation

of spatial mental models (Knauff, Rauh, & Schlieder, 1995; Rauh, Schlieder, & Knauff, 1997; Knauff, Rauh, Schlieder, & Strube, 1998). In the case of unambiguous descriptions of a spatial situation, from which only a unique spatial arrangement can be constructed (i.e. determinate descriptions), there is evidence that verbatim information from the premises describing a determinate arrangement is not reliably retrievable from memory (Mani & Johnson-Laird, 1982), which supports the assumption that mental models rather than sentences are stored in memory.

Based on these findings we propose the following three steps for the process of spatial belief revision. First, given a determinate description of a spatial arrangement, a mental model of the spatial arrangement is constructed according to the information provided. Second, given a statement that is in conflict with the information encoded in the mental model, this inconsistency has to be detected as a prerequisite for belief revision. We assume that inconsistency detection happens by model inspection (e.g. Knauff et al., 1995; Johnson-Laird, Legrenzi, & Girotto, 2004). Finally, the mental model is revised with the goal to create a model that is consistent with the new information. In the light of the above considerations we want to define belief revision as following:

*Definition Belief Revision:* Belief revision is a model change where a proposition  $x$ , inconsistent with a given model  $M$ , is integrated into  $M$  under the requirement that the revised model be consistent and represents the information of  $x$ .

Using this definition we consider the role of cognitive effort and minimal change during spatial belief revision. To that goal we discuss how minimal change can be defined in this context. We provide a computational model along with a cost function to describe the revision processes and present empirical evidence for the influence of cognitive efficiency principles on spatial belief revision and the suitability of the model.

## Construction, inspection and revision of spatial mental models

Consider the following two statements, also called premises:

1. A is to the left of B
2. C is to the right of B

From premises describing the spatial relations of objects a (mental) model of the arrangement of objects can be constructed, in this case  $A-B-C$ . This is the only model that is consistent with both premises. From the model we can derive conclusions or beliefs about the relations not explicitly stated in the premises, particularly about the relationship between A and C, for example: "A is to the left of C".

After having constructed the model, when presented with the above conclusion a reasoner can confirm that it is valid. However, when confronted with an invalid conclusion such as "C is to the left of A", that is inconsistent with the model but that is known to be true, the model has to be adapted to conform to this conclusion. One has to revise the model so that the conclusion is valid in the new model. The prerequisite for this process is, of course, that the reasoner is aware of the inconsistency between the statement and his mental model.

Given the model derived from the two above premises and confronted with the statement "C is to the left of A", there are two logically equivalent options to revise the model such that the conclusion holds in the new model:

$C-A-B$                       and                       $B-C-A$

In both of these models one of the original premises holds in addition to the new statement. This leads to the question which option is to be preferred?

### Informational Economy and Minimal Change

One principle that is widely regarded to guide belief revision is the concept of informational economy: unnecessary loss of information should be avoided. A belief revision should preserve as much information as possible; changes should be minimal (Harman, 1986; Gärdenfors, 1984). When we change our beliefs, we want to retain as much as possible of our old beliefs - we want to make a minimal change.

With regard to our definition of belief revision this principle can be expressed in that the revision of  $M$  with respect to  $x$  should represent the minimal change of  $M$  needed to accommodate  $x$  consistently. But what does minimal change mean in connection with beliefs based on spatial mental models? This leads to the question how we can measure the extent of a change in beliefs.

Harman (1986) suggested the following simple measure of change: Take the sum of the number of (explicit) new beliefs added plus the number of (explicit) old beliefs given up. How can this idea be applied to our definition of spatial belief revision? There are several aspects under which the extent of change in belief can be defined leading to different measures of change.

1. Number of changed propositions regarding the objects in the model (new propositions + given up propositions)
2. Number of necessary changes in the model (for example number of moved objects)
3. Number of steps necessary to revise the model

The first measure quantifies the change regarding informational aspects; there are no assumptions necessary how the model is realized. In contrast the last measure gauges the change under computational aspects which requires explicit assumptions about the model. The second measure scales the change with regard the model but does not need assumptions on how the model is realized.

In our above example there are two alternatives how to change the belief. How do they compare with regard to these measures? Are both alternatives equally efficient? In both of the alternatives one of the premises still holds (in addition to the conclusion). Since the change is symmetrical for the two alternatives, both alterations of the model conserve the same amount of information. The number of changed propositions is the same for both options, implying that there is no difference between the options with regard to the first measure.

For the second measure we have to consider the changes necessary in the model. In both alternatives one object has to be moved within the model, and in both alternatives the object has to be moved the same distance just in opposite directions. The change is symmetrical leading to the same amount of change with regard to the second measure. So according to the first two measures the two logical equivalent options of revising the model are also equivalent with regard to informational economy. This takes us to the last aspect of how the movement of an object is actually executed within the mental model. Are both options equivalent with regard to that measure as well? Or differently phrased: Are both options equally efficient?

To decide this we need to know how a spatial mental model can be conceived and which steps are necessary to revise such a mental model. This means we need a concept of what governs the construction and alteration of mental models.

### A model for spatial reasoning

To look at processes on a computational level we need a model for the mental spatial representation. In Krumnack, Bucher, Nejasmic, and Knauff (2010) and Krumnack, Bucher, Nejasmic, Nebel, and Knauff (in press) we introduced a model for one dimensional reasoning with a cost measure for calculating the cost of altering the model. The model reflects empirical evidence for directionality in spatial representation which we found in the processes of constructing and reasoning with mental models. This directionality can be formalized as follows:

We assume that models consist of a "queue" of objects and an interpretation what this queue represents. The queue describes in which order the objects are aligned but what this order represents depends on the relation that is considered. So while the order of a queue is implicit the interpretation of the order is not. The queue is defined by the following three assumptions:

- 1<sup>st</sup> There exists a starting point or first object.
- 2<sup>nd</sup> Each object is linked to the next object in the linear order. Only the last object is not linked to other objects.

3<sup>rd</sup> While this structure has an implicit direction, the interpretation of this direction depends on the context.

The queue is constructed by forming links between objects. The links signify which objects follow each other in that ordered arrangement. These links between the objects are one directional which means that, when inspecting the queue, we can move from one object to the next object in the queue but not to the preceding object. To access the queue one needs to access the first element of the queue, which is marked by a start pointer. From there all other objects in the mental model can be reached by following the links between objects. Due to the fact that one has to know how the queue starts in order to access it the starting point can also be considered a link, connecting the start of the queue to the first object.

How do reasoners construct a queue? Several studies suggest that the left end of a linear spatial arrangement is the preferred starting point resulting in a working direction from left to right reflecting the cultural bias to work in the same direction as reading and writing (Soto, London, & Handel, 1965; Chan & Bergen, 2005; Spalek & Hammad, 2005). We therefore assume that reasoners generally construct a queue with the implicit direction moving from the left to the right. So the beginning of the queue would represent the left side of a spatial arrangement and the end of the queue would represent the right side of a spatial arrangement.

Based on this the model from the above example can be portrayed as a queue the following way with the starting point marked by an asterisk:

$$* \rightarrow A \rightarrow B \rightarrow C$$

In this case the implicit direction would be interpreted as moving from the left to the right. This model can be easily implemented as a computer model using the data structure linked list.

While computer science provides standard complexity measures for the cost of operations, this measure is not cognitively adequate. We established a cost measure based on empirical data that leads to reasonable predictions of human behavior (Krumnack et al., 2010, in press). As a cost measure for the insertion of objects into the queue we use the number of links that need to be formed when inserting the object.

For any insertion the cost measure implies that it is most efficient to insert objects at the end of the queue. When inserting an object at the end of the queue only one new link needs to be created, linking that new object to the queue. If an object is inserted at the beginning of the queue, the new object has to be linked to the former first object of the queue and the start pointer has to be linked to the new object (see Figure 1. If an object is inserted in the middle of the queue two new links need to be formed, analogously to the insertion at the beginning of the queue.

However, there are still some open questions regarding this cost measure. The one that needs to be addressed for the revision of models is: Does this measure only hold for addition to the queue or also for removal from the queue?



Figure 1: Insertion of C in the queue containing A and B, solid arrows indicate existing links, dashed arrows indicate newly created links. Left: insertion at the end of the queue. Right: insertion at the beginning of the queue.

However, there are still some open questions regarding this cost measure. The one that needs to be addressed for the revision of models is: Does this measure only hold for addition to the queue or also for removal from the queue?

### Revising the model

When revising a model an object has to be removed from the queue before it can be inserted again. So far no research has been done on the difficulty of removing an object from a queue. However, using the cost measure we can make some assumptions. If an object is removed at the end of the queue no link needs to be created. The object is just detached from the queue. If an object is removed from the beginning or the middle of the queue, one link needs to be created, connecting the object preceding the removed object with the one following the removed object in the queue. This would mean higher cost for removing an object that is not at the end of the queue. If an object is removed from the beginning of the queue, the situation is the same as if inserting an object at the beginning of the queue: the starting point needs to be redefined which technically means a new link needs to be created (see Figure 2).



Figure 2: Left: Removal of C from a queue containing A, B and C. Right: Removal of A from a queue containing A, B, and C. Solid arrows indicate existing links, dashed arrows indicate newly created links.

If we add up the links formed while removing and reinserting an object during the revision process of the model we get the following cost for our two alternatives from the above example:

1. Moving C to the front of the queue: no cost for removing C and two links formed when inserting C at the beginning of the queue.
2. Moving A to the end of the queue: one new link (the starting point) redefined when removing A and one link formed when inserting A at the end of the queue.

In both alternatives two links need to be created which implies both alternatives produce the same cost. This leads to the prediction that there should be no difference between a revision to the right and a revision to the left. However, as

pointed out above, we do not know whether the cost measure of counting the number of formed links holds for the removal of objects from the queue.

For the removal process it is also conceivable that the link from the preceding object "merges" with the link to the following object since the order of the object is preserved. This might take less cognitive effort than creating a completely new link, i.e. linking objects that previously had no connection to each other. In a similar way the starting point might move somewhat "automatically" to the second object when the first object is removed, since this preserves the order of the objects. The starting point and the link to the second object could converge to become the new starting point. This would imply that the first option described above is more costly as two new links have to be formed during insertion while in the second option one of the new links is created by merging of two links during removal and only one completely new link is formed during insertion.

## Hypotheses

As stated above reasoners tend to construct a spatial arrangement from the left to the right and for this behavior we devise our hypotheses. Construction of the queue from the right to the left would lead to opposite predictions due to the asymmetric properties of the queue.

For the construction process we predict that an insertion on the left side of a spatial arrangement would create higher cost and therefore a higher cognitive effort than an insertion on the right side of a spatial arrangement. This should be noticeable in the reaction times and the number of correctly constructed models.

For the revision process we have two contradicting hypotheses:

1. The forming of links during the removal of an object from the queue takes as much mental effort as the links that need to be formed when reinserting the object into the queue. As outlined above this implies that a revision by moving an object from the right to the left should take the same time and happen with the same frequency as a revision by moving an object from the left to the right.
2. During the removal of an object from the beginning queue the two surrounding links merge to form a new link and this process takes less cognitive effort than forming a completely new link. This implies that a revision in the process of which an object is moved from the left to the right can be easier performed than a revision which requires an object to be moved from the right to the left. This would imply that a revision moving an object from the left to the right would take less time and possibly happen more often than a revision moving an object from the right to the left.

Please note that the prediction from the first hypothesis can also be derived from the two cost measures discussed in the section on minimal change which do not take processes of believe revision into account.

We tested these hypotheses in an experiment. The focus of the experiment was not which choice for revision is generally preferred but which revision is easier to perform under equal circumstances. Miller and Johnson-Laird (1976) suggested asymmetric roles of the two arguments of a binary spatial relation. We can define such a binary relation as collection of ordered pairs of objects  $r(X,Y)$ , where X is called the "to-be-located object" and Y the "reference object". In the sentence "A is to the left of B" the object A is the "to-be-located object" and B is the "reference object". In several experiments on spatial belief revision we found that reasoners generally relocate the "to-be-located object" while the "reference object" remains located at its initial position and different behavior normally led to longer decision times (Bucher, Krumnack, Nejasmic & Knauff, submitted). Therefore we will limit the data evaluation to trials in which reasoners acted according to that rule to ensure that semantic processes of sentence evaluation do not interfere with the processes we want to observe.

## Experimental evidence

### Method

**Participants** 23 participants (5 male; age:  $M = 22.30; SD = 2.18$ ) all students (among them 6 students of psychology) from the University of Giessen, gave written informed consent to participation. Subjects were tested individually and paid at a rate of 8 Euro per hour.

**Materials, procedure, and design** 64 items were randomly presented. The items followed a tripartite structure as follows (similar to the above example). In the *model construction* part two premises (presented sequentially in a self-paced manner) described a one-dimensional (linear) order of three (small, equal-sized, disyllabic-termed) objects, belonging to one of two categories (fruits or tools). The relations "left of" and "right of" were used in the premises which described one determined arrangement. The last of the three objects mentioned in the premises was added to the rightmost position in the arrangement in half of the items and to the leftmost position in the other half. Subsequently to premise presentation, participants were instructed to choose the correct order from two alternative orders (correct order and correct order mirrored) that were presented on the left and right side of the computer screen, indicating their choice by pressing a left or right response button with the left or right hand, accordingly. Left and right locations for correct and incorrect orders were counterbalanced across the experiment. The number of correct decisions and corresponding decision times were recorded. In the *inconsistency detection* a conclusive fact (font color red to contrast the fact with the premises presented in black) was presented that was either consistent (in half of the items) or inconsistent (in the other half of the items) with the conclusions that could be drawn from the premises, hence with the linear order of objects. As in the example above the relation stated in the conclusion was between the leftmost and the rightmost objects of the arrange-

ment that were not put in direct relation by a premise. Participants were instructed to decide whether the conclusive fact was consistent or inconsistent with the order of objects, indicating their decision by pressing the respective response button ("yes" or "no") with the left or right hand, accordingly. Successful inconsistency detection and corresponding detection times were recorded. In the *belief revision* part the participant's decision was "no" (i.e., decision that the fact was inconsistent with the information yielded by the premises), he or she was subsequently instructed to indicate how the initial order of the object would have to be revised in order to be consistent with the inconsistent fact. Participants were asked to mentally revise their belief about the order of objects and to press a button in order to indicate the end of the revision process. Immediately after, they indicated the revised order from two orders presented on the left and right side of the computer monitor by pressing the respective response button. Presentation locations of the two orders were counter-balanced across the experiment. The two orders presented reflected the two re-arrangements possible in order to regain consistency with the inconsistent fact, that is they reflected the two alternatives discussed above. Revised orders chosen and corresponding revision times as well as order selection duration was recorded. Four practice trials (not analyzed) preceded the experimental trials. All stimuli were generated and presented using Superlab 4.0 (Cedrus Corporation, San Pedro, CA, 1999) with an RB-530 response pad running on a standard personal computer connected to a 19"-monitor.

## Results and Discussion

**Model construction:** Reading times for the second premises were significantly longer when the third object had to be inserted at the left side of the arrangement ( $M = 5.87s, SD = 1.01$ ) compared to when the third object had to be inserted at the right side of the arrangement ( $M = 5.15s, SD = 1.48; t(22) = 2.982; p < .01$ ). The correct order of objects was chosen in 93.62% ( $SD = 7.02$ ) of the cases within 2.10s ( $SD = 0.65$ ). Erroneous trials were excluded from analysis.

**Inconsistency detection:** Inconsistency detection was successful in 90.98% ( $SD = 8.45$ ) of the trials and took 7.81s; ( $SD = 2.27$ ) on average. Erroneous trials were excluded from further analysis.

**Belief revision** In 72.81% ( $SD = 11.64$ ) of the trials the revisions were performed by relocation of the "to be located object". All other trials were excluded from further analysis for the reasons outlined above and because there were not enough cases in this condition for an analysis.

To evaluate whether revision times and frequency of preferred revisions ("relocating to be located object") were affected by location of insertion of the third object from the premises and/or fact relations and whether revision times for relocating "to be located objects" of inconsistent facts towards the left were comparable with relocating them to the right, ANOVAs with the factors location last inserted object (left end, right end)  $\times$  fact relation (left, right) were calculated, respectively.

ANOVA of revision time revealed a significant main effect fact relation [ $F(1, 21) = 8.83; p < .01; \eta^2 = .30$ ]. The main effect location of last inserted object ( $p = .462$ ) and the interaction ( $p = .665$ ) were non-significant. Relocating "to be located objects" to the left (i.e. according to the fact relation "left of": 6.82s;  $SD = 4.10$ ) took significantly longer than to the right (i.e. according to the fact relation "right of": 5.15s;  $SD = 3.08; t(22) = 2.70; p < .02$ ).

ANOVA of percentages of conducted preferred revision revealed no significant effect, but a marginally significant main effect fact relation [ $F(1, 22) = 4.03; p = .057; \eta^2 = .16$ ]. The interaction of factor fact relation and location of last inserted object was also marginally significant [ $F(1, 22) = 3.99; p = .058; \eta^2 = .15$ ]. The main effect location of last inserted object ( $p = .418$ ) was non-significant.

There was a tendency to perform revisions to the right ( $M = 79.76\%, SD = 25.45$ ) more often than revisions to the left ( $M = 66.24\%, SD = 17.44; t(22) = 1.78; p < .09$ ). A revision to the right when the last inserted object was also on the right ( $M = 74.82\%, SD = 26.80$ ) was conducted significantly more often than a revision to the left when the last inserted object was on the right ( $M = 57.18\%, SD = 16.70; t(22) = 2.75; p < .02$ ) as well as when the last object was inserted on the left ( $M = 59.58\%, SD = 20.22; t(22) = 2.10; p < .05$ ). A revision to the right when the last inserted object was also on the right ( $M = 74.82\%, SD = 26.80$ ) was conducted marginally significantly more often than a revision to the right when the last inserted object was on the left ( $M = 66.56\%, SD = 24.87; t(22) = 1.93; p = .067$ ). All other comparisons were non-significant ( $ps > .19$ ). Figure 3 depicts the results of the revision phase.

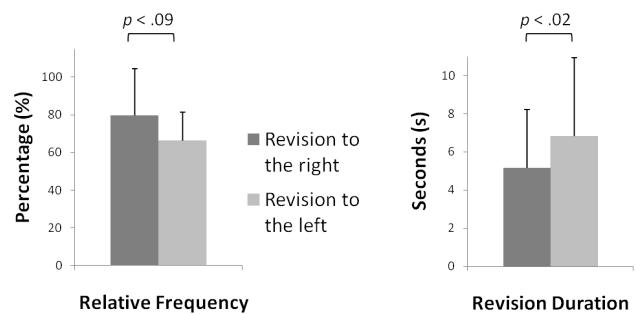


Figure 3: Relative frequency and decision times for revision processes.

Conforming to our prediction reading time for the second premise was longer, when the third object had to be inserted on the left than on the right. This can be seen as further support for our cost measure during construction process.

Independent of where the last object is inserted in the queue revision by moving an object from the left to the right is performed faster and more often than revision by moving an object from the right to the left. This suggests that it takes less cognitive effort to move an object from the left to the right

of the queue than the other way around. As a consequence we can assume in accordance with the second hypothesis that the merging of links during the removal of objects from the queue takes less effort than the creation of new links during construction.

## Discussion

We find evidence for an inherent directionality in the revision process that parallels the directionality of the construction process. Moving an object from the left to the right in the mental model seems to require less mental effort than moving an object from the right to the left. This indicates that there is an influence of the properties of the mental model on the revision process.

The fact that revisions by moving from the left to the right were not only performed faster but also slightly more often than those from the right to the left seems to indicate that human reasoners have indeed a preference for making revisions that require a smaller amount of cognitive effort. This preference is certainly not as dominant as the impulse to move the "to be located object", but the reduction of cognitive effort could be one of the factors that guide spatial belief revision. In a setting where there is no strong semantic incentive on behavior this factor might be more influential. The fact that the influence of cognitive effort is noticeable at all is particularly remarkable as many reasoners report they were not aware that there was more than one alternative for changing the model. This might imply that it is hard for them to adhere to strategies that minimize cognitive effort at least as long as there are also semantic considerations.

In addition the results clearly indicate that measures for minimal change that only take into account formal aspects of the change but not how the change is accomplished are not adequate for human behavior. While there are no differences between the described alternatives of model revision with regard to informational aspects there is a difference on the level of mental model manipulation. And if the aim of a minimal change is to minimize the cognitive effort, then the processing cost of a revision needs to be considered.

This work also constitutes a further development of the queue model and the connected cost measure. So far it mainly covered construction of models and reasoning with these models. With this study we are able to extend the cost measure to revision processes within a constructed model. The results suggest that forming links when removing an object from the queue takes less effort than creating a new link. This could be explained through a merging effect of the two links surrounding the object. An open question remains if this is a general effect of removal of an object from the middle of a queue or if this can only be observed when removing the first object.

## Acknowledgments

This work was supported by a grant of the Deutsche Forschungsgemeinschaft (DFG) to M. Knauff und B. Nebel

(KN 465/6-1). We thank Anja Gatzsche, Isabell Tapia-Leon, and Lilly Fehr for assistance with the experiment.

## References

- Bucher, L., Krumnack, A., Nejasmic, J., & Knauff, M. (2011). Cognitive processes underlying spatial belief revision. In L. Carlson, C. Hölscher, & T. Shipley (Eds.), *Proceedings of the 33rd annual conference of the cognitive science society*. Austin, TX: Cognitive Science Society.
- Chan, T. T., & Bergen, B. (2005). Writing direction influences spatial cognition. In *Proceedings of the 27th annual conference of the cognitive science society* (pp. 412–417).
- Gärdenfors, P. (1984). Epistemic importance and minimal changes of belief. *Australasian Journal of Philosophy*, *62*, 136–157.
- Harman, G. (1986). *Change in view: Principles of reasoning*. Cambridge, MA: MIT Press.
- Johnson-Laird, P. N., & Byrne, R. (1991). *Deduction*. Hove (UK): Erlbaum.
- Johnson-Laird, P. N., Legrenzi, P., & Girotto, V. (2004). How we detect logical inconsistencies. *Current Directions in Psychological Science*, *13*, 41–45.
- Knauff, M., Rauh, R., & Schlieder, C. (1995). Preferred mental models in qualitative spatial reasoning: A cognitive assessment of allen's calculus. In *Proceedings of the seventeenth annual conference of the cognitive science society* (pp. 200–205). Mahwah, NJ: Erlbaum.
- Knauff, M., Rauh, R., Schlieder, C., & Strube, G. (1998). Continuity effect and figural bias in spatial relational inference. In *Proceedings of the twentieth annual conference of the cognitive science society* (pp. 573–578). Mahwah, NJ: Erlbaum.
- Krumnack, A., Bucher, L., Nejasmic, J., & Knauff, M. (2010). Spatial reasoning as verbal reasoning. In *Proceedings of the 32nd annual conference of the cognitive science society* (pp. 1002–1008). Austin, TX: Cognitive Science Society.
- Krumnack, A., Bucher, L., Nejasmic, J., Nebel, B., & Knauff, M. (in press). A model for relational reasoning as verbal reasoning. *Cognitive Systems Research*.
- Mani, K., & Johnson-Laird, P. N. (1982). The mental representation of spatial description. *Memory and Cognition*, *10*, 181–187.
- Miller, G., & Johnson-Laird, P. N. (1976). *Language and perception*. Cambridge: Harvard University Press.
- Rauh, R., Schlieder, C., & Knauff, M. (1997). Präferierte mentale Modelle beim räumlich-relationalen schließen: Empirie und kognitive Modellierung. *Kognitionswissenschaft*, *6*, 21–34.
- Soto, C. D., London, M., & Handel, S. (1965). Social reasoning and spatial paralogic. *Journal of Personality and Social Psychology*, *2*, 513–521.
- Spalek, T. M., & Hammad, S. (2005). The left-to-right bias in inhibition of return is due to the direction of reading. *Psychological Science*, *16*, 15–18.