Title
GENERALIZED PARTICLE-CORE COUPLING

Author
Mathews, G.J.

Publication Date
1976-08-01

Submitted to Physics Letters

LBL-5076
Preprint c.1

GENERALIZED PARTICLE-CORE COUPLING

G. J. Mathews and J. D. Immele

August 1976

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48

LBL-5076 c.1

0 0 0 0 4 5 0 5 1 0 0

LBL-5076

Generalized Particle-Core Coupling†

G. J. Mathews

Department of Chemistry, University of Maryland
College Park, MD 20742
and
Nuclear Science Division, Lawrence Berkeley Laboratory
University of California, Berkeley, CA 94720

and

J. D. Immele

Lawrence Livermore Laboratory
University of California, Livermore, CA 94550

ABSTRACT

A general particle-core coupling scheme is proposed for the description of simultaneous shape and pairing correlations in odd-A nuclei of arbitrary deformation. This coupling scheme, which is similar to intermediate coupling in spherical vibrators, exactly reproduces the Coriolis decoupled bands and triaxial-rotor-plus-particle structure of deformed nuclei.

†This work was done with support from the U. S. Energy Research and Development Administration.

I. INTRODUCTION

In recent years, numerous theoretical models have been proposed to describe odd-A nuclei in the so-called transitional region, intermediate between regions of vibrational (spherical) and rotational (deformed) structure [1-6]. In this letter, we outline a general description of odd-A nuclei which accounts naturally for the coupling (weak, intermediate, or strong) of the odd nucleon to the nuclear core in all regions of interest (spherical, transitional, or deformed). This approach is significant for several reasons. First, the unified model Hamiltonian of Bohr and Mottelson [7] turns out to be easily soluable in a spherical basis both in the limits of weak and strong coupling. More generally, whatever the dynamic (shape) structure of the even-even core, as exhibited by its energies and transition moments, the particle-core coupling can be evaluated in a uniform scheme in which both the single particle and core states have good angular momentum. It is in this sense that we extend the idea of de-Shalit [8] to the transitional and deformed nuclei. A further development of the model presented here is the self-consistent treatment of pairing and shape correlations, that is, the occupation probabilities and the location of the Fermi surface are determined by the nuclear shape dynamics including static deformation, as well as the usual pairing potential.

The broad scope of this approach, which we shall call generalized particle-core coupling (or GPCC) is demonstrated by exact agreement with model computations in deformed basis sets, e.g., Coriolis decoupled bands [1] and triaxial-rotor-plus-particle spectra [2]. The connection to weak and intermediate coupling calculations [7], which are already routinely performed in a spherical basis, is also discussed.

II. THE MODEL

Most theories (macroscopic and microscopic) of odd-A collective motion can be developed with a Hamiltonian of the familiar form,

$$\mathcal{H} = \mathcal{H}_{sp} + \mathcal{H}_{core} + \mathcal{H}_{pair} + \mathcal{H}_{int} \tag{1}$$

In the present model, $\mathcal{H}_{sp}$ is the one-body operator for the kinetic and spherical potential energies of the odd nucleon. $\mathcal{H}_{core}$ describes the energy spectrum of the even-even core states. $\mathcal{H}_{pair}$ introduces J = 0 pairing. $\mathcal{H}_{int}$ accounts for the variation of the single particle potential energy with deformation of the core. This term is well known from the particle-vibration model [7],

$$\mathcal{H}_{int} = \frac{\partial\mathcal{H}_{sp}}{\partial\alpha} \cdot \alpha \quad , \tag{2}$$

where α is the deformation coordinate. Alternatively, in the particle-core coupling model [8]

$$\mathcal{H}_{int} = q \cdot Q \quad , \tag{3}$$

the scalar product of the quadrupole matrix elements of the particle and the core.

It is important to keep in mind that eqs. (2) and (3) are not limited to vibrational nuclei, but apply to quite general static and dynamic deformations. For the present, we consider only the quadrupole coupling term and assume that some theoretical or experimental description of the core, i.e., energies and transition moments, is available.

To obtain solutions to the Hamiltonian, eq. (1), for the states of the odd-mass system we employ an equations-of-motion method. This approach is similar to that of refs. [9] and [10]. However, in the present approach a schematic coupling term is employed, thus vastly simplifying the computational effort.

To construct the equations of motion we introduce an operator $O_I^\dagger$, which creates states of the odd-A nucleus by operation on the ground state of the even-even core, i.e.

$$O_I^\dagger = \sum_{j,S} \left(Y_{jS}^I a_j^\dagger + Z_{jS}^I \; a_{\bar{j}} \right) B_S^\dagger \quad . \tag{4}$$

The $a_j^\dagger (a_{\bar{j}})$ are quasiparticle creation (annihilation) operators, hence the basis is already diagonal in the pairing potential. The label j refers to the angular momentum of the spherical single quasiparticle state. For economy of notation, $\bar{j}$ denotes to the time reversal of j although we delete the explicit indexing of the magnetic substate and the Clebsch-Gordon coefficient. $B_S^\dagger$ is a boson operator which creates the rotationally symmetric core excited state, S, when applied to the core ground state. The core ground state is assumed to be a quasiparticle vacuum. The equations of motion can then be written in convenient matrix form [11],

$$\langle 0| \left[O_J \; , \; [H,O_I^\dagger] \right]_+ |0\rangle = (E_I - E_0) \langle 0| [O_J, O_I^\dagger]_+ |0\rangle \; , \tag{5}$$

to give,

$$\begin{bmatrix} (E_{j'} + E_{S'})\delta_{jj'}\delta_{SS'} + \mathcal{H}_{int}\xi_{jj'} & \mathcal{H}_{int}\eta_{jj'} \\ \mathcal{H}_{int}\eta_{jj'} & (-E_{j'} + E_{S'})\delta_{jj'}\delta_{SS'} - \mathcal{H}_{int}\xi_{jj'} \end{bmatrix} \begin{pmatrix} Y^I_{jS} \\ Z^I_{jS} \end{pmatrix}$$

$$E_I \begin{bmatrix} \langle 0| \left[B_{S'} a_{j'} , a^\dagger_j B^\dagger_S \right]_+ |0\rangle & \langle 0| \left[B_{S'} a_{j'} , a_{\bar{j}} B^\dagger_S \right]_+ |0\rangle \\ \langle 0| \left[B_{S'} a^\dagger_{\bar{j}'} , a^\dagger_j B^\dagger_S \right]_+ |0\rangle & \langle 0| \left[B_{S'} a^\dagger_{\bar{j}'} , a_{\bar{j}} B^\dagger_S \right]_+ |0\rangle \end{bmatrix} \begin{pmatrix} Y^I_{jS} \\ Z^I_{jS} \end{pmatrix} \quad (6)$$

where, $E_j^2 = (\varepsilon_j - \lambda)^2 + \Delta^2$ (7)

$$\xi_{jj'} = u_j u_{j'} - v_j v_{j'} \quad (8)$$

$$\eta_{jj'} = u_j v_{j'} + v_j u_{j'} \quad .$$

u_j and v_j are the pairing occupation amplitudes. E_j and E_S are the single quasiparticle and core energies respectively. The matrix element

$$\mathcal{H}_{int} \propto \langle j'\| q \| j\rangle \langle S'\| Q \| S\rangle \quad (3)$$

as mentioned previously, introduces the shift of the single particle strength due to quadrupole correlations.

If all terms involving the backward amplitude, Z^I_{jS}, are neglected, eq. (6) reduces to the familiar intermediate coupling Hamiltonian of vibrational nuclei [7], or in perturbation theory to weak coupling.

The terms containing the pairing amplitude, $\eta_{jj'}$, in eq. (6) introduce mixing of quasiparticle and quasihole configurations. Since $\eta_{jj'}$ is usually greater than $\xi_{jj'}$ in superfluid nuclei, these so-called backward-forward terms cannot be neglected. Because the backward-forward matrix elements do not conserve particle number they have the effect of altering the location of the Fermi surface and hence the occupation amplitudes, u_j and v_j, in eq. (8). We therefore demand that the pairing potential and Fermi energy be determined self-consistently in the GPCC equations. This can be achieved by relating the number operator and pairing potential to the GPCC amplitudes.

$$N = \sum_{j,m} \langle 0|c_j^\dagger c_j|0\rangle = \sum_{j,I} |\langle I|c_j^\dagger|0\rangle|^2 \,(2j+1) \qquad (9)$$

$$= \sum_{j,I} V_{jI}^2 (2j+1)$$

$$\Delta = -G \sum_{j,I} \langle 0|c_{\bar{j}}^\dagger|I\rangle\langle I|c_j^\dagger|0\rangle \,(2j+1)$$

$$= -G \sum_{j,I} U_{jI}V_{jI}(2j+1) \quad . \qquad (10)$$

The U_{jI} and V_{jI} in eqs. (9) and (10) can be related to the u_j and v_j of eq. (8) through the transformation,

$$\begin{pmatrix} U_{jI} \\ V_{jI} \end{pmatrix} = \begin{pmatrix} u_j & v_j \\ v_j & -u_j \end{pmatrix} \begin{pmatrix} Y_{j0}^I \\ Z_{j0}^I \end{pmatrix} \qquad (11)$$

The calculation then consists of locating values for λ and Δ such that the solutions of eqs. (6), (8) and (9) through (11) are self-consistent.

III. RESULTS

In this section, some comments on the equivalence of GPCC to the strong coupling models are presented. To simplify the comparisons the GPCC solutions are presented for several values of the parameters Δ and λ (rather than self-consistent solutions for a given interaction, G, and nucleon number, N). In the spirit of de-Shalit's core excitation model we assume a set of even-even energies and quadrupole matrix elements, namely those of the axially symmetric rotor.

$$E_S = (\hbar^2/2\mathcal{J})\ S(S+1)$$

and

$$\langle S'\|Q\|S\rangle = \sqrt{2S+1}\begin{bmatrix} S & 2 & S' \\ 0 & 0 & 0 \end{bmatrix}\beta \quad . \tag{12}$$

The normalization matrix on the right-hand side of eq. (6) is set equal to one. In the limit that $\Delta = \hbar^2/2\mathcal{J} = 0$, we have obtained a set of solutions degenerate with the Nilsson levels. The GPCC solutions are linear combinations of a nucleon in a <u>spherical</u> orbital coupled with <u>even-even core</u> states.

For $\Delta \neq 0$ and $\hbar^2/2\mathcal{J} \neq 0$, the results correspond to those obtained by diagonalizing the Coriolis interaction for deformed quasiparticles.* In the GPCC no explicit Coriolis correction is necessary since only the rotational energy of the core is included in the first place. The backward-forward terms ($n_{jj'} \neq 0$) occur if and only if Δ is non-zero. Their inclusion is crucial.

---

*The usual Coriolis solutions are incomplete. See ref. [2] for a discussion of the role of backward-forward matrix elements in Coriolis coupling.

It is particularly gratifying that for a single-j orbital $(h_{11/2})$, $\lambda << \varepsilon_{11/2}$, with rotational energies and coupling strength parameterized according to ref. [1], we reproduce Stephen's results for the decoupled band. This is shown in fig. 1. One can see the appropriate decoupled band behavior for positive deformation and the strongly coupled rotational limit approached for negative deformation.

Turning now to axially asymmetric shapes, one can write for $\gamma = 30°$ (maximum asymmetry),

$$E_{S,\sigma} = 3\hbar^2/8\mathcal{J}\ [4S(S + 1) - 3\sigma^2]$$

$$\langle S'\sigma' \| Q \| S\sigma \rangle = \beta \sqrt{\frac{(2S + 1)(2S' + 1)}{2(1 + \delta_{\sigma',0})(1 + \delta_{\sigma,0})}} \tag{13}$$

$$\times \left\{ \begin{bmatrix} S & S' & 2 \\ \sigma & \sigma' & 2 \end{bmatrix} + (-1)^{S'} \left(\begin{bmatrix} S' & S & 2 \\ \sigma' & \sigma & 2 \end{bmatrix} + \begin{bmatrix} S' & S & 2 \\ -\sigma' & \sigma & -2 \end{bmatrix} \right) \right\}$$

$$\sigma = 0, 2 \ldots, S \text{ (or } S - 1)$$

Substituting these terms into eq. (6) and taking $\Delta = 135/A$ and $\beta A^{2/3} = 7$ [2], the solutions shown in fig. 2 for the $h_{11/2}$ shell as a function of the position of the Fermi surface, λ, are obtained. These results are again in exact agreement with a complete triaxial-rotor-plus-particle calculation. The results given by Meyer-ter-Vehn [2] differ slightly from fig. 2.*

---

*See footnote on previous page.

IV. CONCLUSION

It has been shown here that a self-consistent equations-of-motion approach in a spherical basis is exactly equivalent to deformed-model calculations of odd-A nuclear structure. This conclusion is similar to that of Alaga and Parr [3] who have shown using perturbation theory that decoupled-band patterns can be obtained from a vibrational Hamiltonian, and to the results of Ikeda, Sheline, and Onishi [4] who have described decoupled bands by an angular-momentum projection method employing the intermediate coupling Hamiltonian, eq. (1). Here we have demonstrated that the rotor-plus-particle model and the particle-vibration model are in fact completely equivalent and only special cases of generalized particle-core-coupling. Further, a prescription has been formulated here for the simultaneous solution of the pairing field, thus rendering the model completely applicable to a more general class of problems, namely those in which the quadrupole and pairing potentials are "soft" and interdependent. Calculations for this regime are in progress.

REFERENCES

1. F. Stephens, Rev. Mod. Phys. 47 (1975) 43.

2. J. Meyer-ter-Vehn, Nucl. Phys. A249 (1973) 111.

3. G. Alaga and V. Parr, Phys. Letters 61B (1975) 129.

4. A. Ikeda, R.K. Sheline, and N. Onishi, Phys. Letters 44B (1973) 397.

5. P. Vogel, Phys. Letters 33B (1970) 400.

6. F. Iachello, Phys. Rev. C14 (in press).

7. A. Bohr and B.R. Mottelson, K. Dann. Vadensk. Selsk. Mat. Fys. Medd. 27, No. 16 (1957).

8. A. De-Shalit, Phys. Rev. 122 (1960) 1539.

9. G.J. Dreiss, R.M. Dreizler, A. Klein, and G. Do Dang, Phys. Rev. C6 (1976) 3.

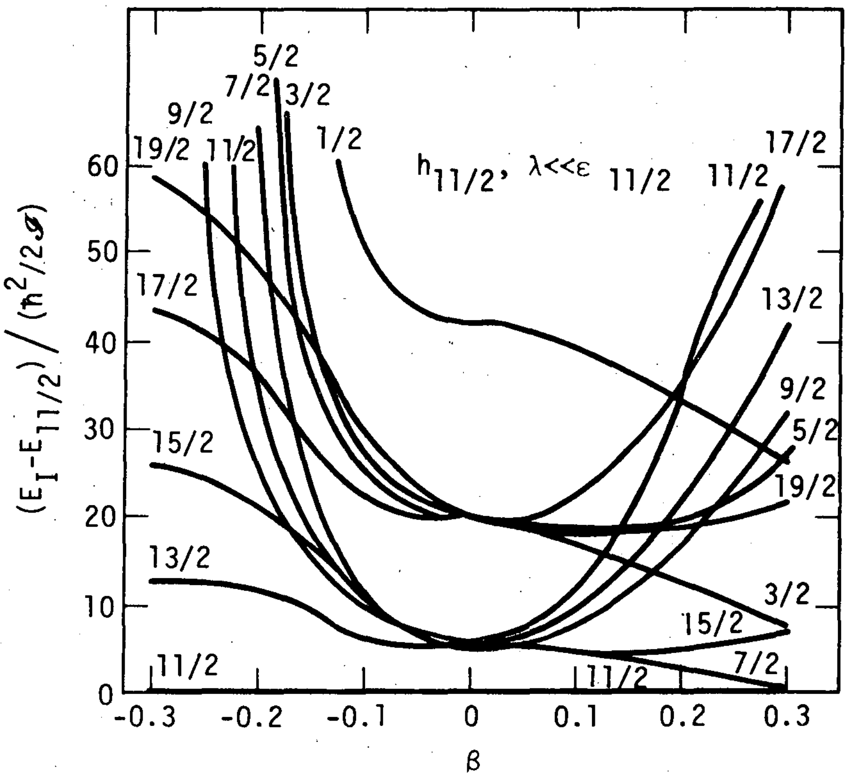
10. A. Goswami, O. Nalcioğlu, and A.I. Sherwood, Nucl. Phys. A153 (1970) 433.

11. D.J. Rowe, The Dynamic Structure of Nuclear States, D.J. Rowe, ed. (Univ. of Toronto Press, Toronto, 1972), pp. 101-153.

FIGURE CAPTIONS

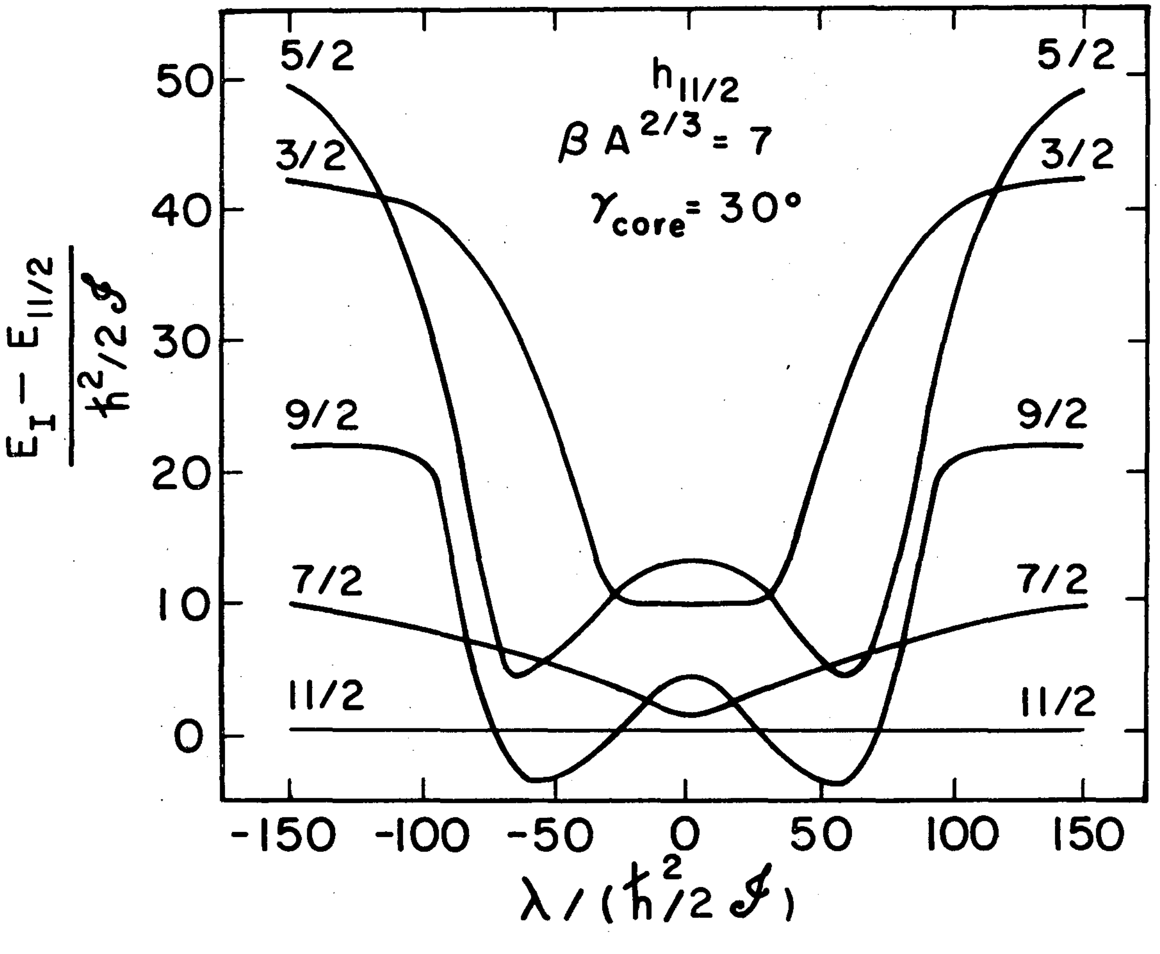
Fig. 1. GPCC solutions for the $h_{11/2}$ single-j shell ($\lambda \ll \epsilon_{11/2}$). The rotational energies of the core and the coupling strength are parameterized according to ref. [1]. Decoupled band behavior is seen to emerge for positive deformation, and the strongly coupled rotational limit is approached for negative deformation.

Fig. 2. GPCC solutions for the $h_{11/2}$ single-j shell as a function of the location of the Fermi energy for an asymmetric ($\gamma = 30°$) core. The parameterization is from ref. [2]. Only the lowest state for spins 3/2 - 11/2 are shown. These results are identical to those obtained from a complete triaxial-rotor-plus-particle calculation.



$h_{11/2}$, λ<<ε
$(E_I-E_{11/2})/(\hbar^2/2\mathcal{J})$
β
5/2
7/2
3/2
9/2
19/2
11/2
1/2
11/2
11/2
17/2
17/2
13/2
9/2
5/2
19/2
15/2
13/2
3/2
15/2
7/2
11/2
11/2
60
50
40
30
20
10
0
-0.3
-0.2
-0.1
0
0.1
0.2
0.3

Fig. 1



XBL768-3886

Fig. 2

This report was done with support from the United States Energy Research and Development Administration. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the United States Energy Research and Development Administration.

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720