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Spatial Reasoning: the Effect of Training for Adults and Children

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Abstract

The mental models theory of relational reasoning postulates that individuals reason by constructing the possible models of the situation described by the premises. The present article reports two experiments about spatial relational reasoning and focuses on the possibility of training In Experiment 1, we compared two different training methods, one in line with the mental models theory and one in line with the rule-based account Both accuracy and training data supported the mental models theory. In Experiment 2, we compared different training methods for children. Again, results were in line with the mental models theory.

Keywords: mental models theory; spatial reasoning; training

Introduction

Many daily deductions depend on relations between things. Suppose you want to pile up some boxes on a shelf. Your partner gives you the following information:

Box A is heavier than Box B.
Box C is lighter than Box B.

On the basis of this information, you can infer that Box A is heavier than Box C. For some problems, people can easily make such relational inferences, but for some other problems they regularly make mistakes. This paper reports two experiments, one with adults and one with children. The focus is on a specific form of relational reasoning, that is spatial reasoning, and on how we can improve spatial reasoning by training.

There has been considerable controversy over how people deal with spatial deductions. Byrne and Johnson-Laird (1989) contrasted experimentally a rule-based theory and the mental models theory. According to the account based on the *mental models theory* (Johnson-Laird, 1983), one first constructs a model of the meaning of the premises (the model construction stage). Next, one formulates a putative conclusion on the basis of this model (the conclusion construction stage). Finally, one searches for a falsifying model, that is, a model that is consistent with the information in the premises, but inconsistent with the putative conclusion (the conclusion validation stage). If one finds such a model, one returns to the second stage. If one doesn't find such a model, the putative conclusion is accepted as a valid conclusion.

Consider the following problem:

Problem 1:

A is to the left of B
B is to the left of C
D is in front of A
E is in front of C
What is the relation between D and E?

According to the mental models theory (Byrne & Johnson-Laird, 1989), first one should construct the following model:

A	B	C
D		E

On the basis of this model, one can infer that "D is to the left of E" (or "E is to the right of D"). Next, one tries to falsify this initial conclusion by attempting to build another model compatible with the premises. Because there is not such a model, *Problem 1* is called a one-model problem. The initial conclusion can be considered as the final conclusion.

Consider now *Problems 2* and *3* with the same question as for *Problem 1*:

Problem 2:	*Problem 3:*
A is to the left of B	A is to the left of B
C is to the left of B	C is to the left of B
D is in front of C	D is in front of C
E is in front of B	E is in front of A

For *Problem 2*, a first model can be built,

C	A	B
D		E

which supports the conclusion "D is to the left of E". In contrast with *Problem 1*, another model is compatible with the premises:

A	C	B
	D	E

However, both models support the same conclusion "D is to the left of E". *Problem 2* is a multiple-model-problem. For *Problem 3*, there are also two models, but now these two models lead to contradictory conclusions:

C	A	B
D	E	
A	C	B
E	D	

Consequently, there is no determinate answer and *Problem 3* is called a problem-with-no-valid-answer. According to the mental models theory, *Problem 2* should be more difficult than *Problem 1* because it is harder to deal with two models than with one model. Moreover, *Problem 3* should be more difficult than *Problem 2* because it necessarily calls for the construction of two models in order to reach the correct answer.

A *rule-based approach* as framed in Hagert (1984) should make the opposite prediction with respect to one-model and multiple-model-problems. In order to solve *Problem 1*, one must infer the relation between the pair of items to which the two items in the question (*D* and *E*) are directly related. To make this inference, one must use a meaning postulate that captures the transitivity of the relations in the premises: *If x is to the left of y, and y is to the left of z, then x is to the left of z*. Multiple-model-problems, such as *Problem 2*, do not require the use of such a meaning postulate. The first premise is irrelevant, and the second explicitly shows the relation between the pair of items to which D and E are related. Therefore, according to the rule-based theory of Hagert (1984), *Problem 2* should be easier than *Problem 1*.
No evidence was found for the latter prediction. Instead, the predictions of the mental models theory have been supported in a number of other studies using different types of relational premises, that is, spatial (Carreiras & Santamaria, 1997; Roberts, 2000, Vandierendonck & De Vooght, 1996), temporal (Schaeken, Johnson-Laird & d'Ydewalle, 1996a; 1996b; Schaeken, Girotto & Johnson-Laird, 1998; Schaeken & Johnson-Laird, 2000; Vandierendonck & De Vooght, 1996) and abstract relational premises (Carreiras & Santamaria, 1997). Van der Henst (1999, 2002; Van der Henst & Schaeken, 2005; see also Schaeken, Van der Henst, & Schroyens, 2007) presented somewhat mixed results.

It is one thing to investigate the original performance of reasoners. It is, however, another thing to investigate whether their original accuracy can be increased. In a review, Klauer and Meiser (2007) convincingly showed that deductive reasoning can be trained. They reported training gains in propositional and syllogistic reasoning. Successful training was often focused on improving the semantic understanding, whereas syntactic approaches showed in general less evidence for training gains. In a semantic training, one tries to support or improve the construction and explication of the representation of the possible situations. In a syntactic training, one trains one or more syntactic inference rules, which would drive the underlying reasoning processes.

Johnson-Laird (2006) developed a practical training method for the mental models theory that takes only a few minutes to learn. This model method consists of one command: *Try to construct all the possibilities consistent with the given information.* Participants learned how to operationalize this command by drawing a diagram designed to keep track of the different possibilities.

The experiments in Johnson-Laird (2006) with conditionals, biconditionals and disjunctions show that the model method, even though it takes only a few minutes to teach, has robust effects on both the accuracy and speed of reasoning. The crucial aspect seems to be that it helps individuals to bear in mind the alternative possibilities compatible with the premises. To our knowledge, however, no training methods for spatial reasoning have been tested. In this article, we aim to compare different methods and to include a developmental dimension.

Experiment 1

We developed two different trainings, one in line with the mental models theory and one in line with the rule-based account. Klauer and Meiser (2007) argued that training conditions for the mental models theory might practice the construction of appropriate models of the premises and the training conditions might aim at optimizing the required operations that must be performed on the constructed mental models. The method of Johnson-Laird (2006) is perfectly in line with this. Therefore, we developed a variant of this successful training method, which we focused on spatial reasoning problems. For the formal rules theory, training conditions might support the interpretative process and they might enhance the accessibility and application of required inference rules (Klauer & Meiser, 2007). With a formal training, we opted to enhance the accessibility of the meaning postulate that captures the transitivity of the relations in the premises.

Method

Participants A total of 48 adults participated in the experiment. They were all psychology students at the University of Leuven and participated as part of a course requirement. They had not received any training in logic.

Design The crucial and between-subjects manipulation consisted in the variation of the training. One group received a spatial mental models training, one group a formal training and one group received no training.

Material and Procedure The participants were tested in three different groups, one for each training condition and one for the control group. The participants carried out three sorts of problems. The three sorts of problem were as follows: one-model-problems, multiple-model-problems, and problems-with-no-valid-answers.

The participants carried out four versions of each of the three different sorts of problems, making a total of 12 trials. The four versions were constructed in the following way: The first two premises contained either the spatial relation 'to the left of' or else the spatial relation 'to the right of'; for half the problems, the first object in the question was to the left of the second object and for half the problems, this was the opposite. All the problems had a different content (although all problems were about fruits on a table), and they were presented in a different random order to each participant. The instructions were presented on the first page of the booklet. They explained that the participants' task was to answer a question based on the information in the preceding assertions, and that the answers should be those that must be true given the truth of the previous assertions. If the participants thought that there was no definite answer to the question, they had to write that as their conclusion. Each problem was on a separate page of the booklet and the experiment was conducted in Dutch.

There were two sorts of training. In the *spatial model training*, an instructor solved two problems in front of the participants. The first problem was a one-model-problem with a valid conclusion based on two premises:

> The Apple is to the right of the Banana.
> The Pear is to the right of the Apple.
> What is the spatial relation between the Banana and the Pear?

The instructor read the first premise and drew the spatial relation on the blackboard, using the first letters of the pieces of fruit. Next, she read the second premise and added the piece of fruit. This resulted in the following drawing:

B A P

After this, the instructor read the question and concluded: "therefore, the banana is to the left of the Pear". Finally, she asked if everyone agreed and she waited until everyone said "yes".

Next, she started with the second training problem, a problem-with-no-valid-answer:

> The Kiwi is to the right of the Cherry.
> The Orange is to the right of the Cherry.
> What is the spatial relation between the Kiwi and the Orange?

She read the first premise and drew the spatial relation. Then, she read the second premise, placed the Orange to the right of the Kiwi and explicitly said: "Oh, is this the only place I can place the Orange? Oh no, there is a second possibility, I can place the Orange also in between the Kiwi and the Cherry." After that, she drew these two possibilities:

C K O

and

C O K

The instructor read the question, showed that the participants had to look to the two possibilities and pointed out that the correct answer is "you cannot know what the relation is between the Kiwi and the Orange". Next she asked if everyone agreed and she waited until everyone said "yes".

In the *formal training*, participants were confronted with the two same training problems. When they read the first problem, the instructor said "we know spatial relations are transitive, that is, if A is to the left of B, and B is to the left of C, then A is to the left of C. Let's look if we can use this transitivity rule for the current problem." The instructor showed that it could be applied, formulated the correct answer and asked if everyone agreed. Next, she presented the second problem, showed that the transitivity rule could not be applied and produced the correct answer that "you cannot know what the relation is between the Kiwi and the Orange". Again she asked if everyone agreed.

In the *no-training* group participants were just handed the booklets without being given any further information, apart from the regular instructions.

Results and Discussion

Table 1 presents the percentages of correct responses to the different sorts of problems. We performed a repeated measures ANOVA which resulted in a significant main effect of the within variable Problem Type ($F(2, 90) = 12.296$, $p < .0005$). Fisher LSD Post-Hoc Tests show that one-model-problems are significantly easier than multiple-model-problems (90.3% versus 63.2%, $p < .00001$) and problems-with-no-valid-answers (90.3% versus 68.8%, $p < .0005$). The two sorts of multiple-model-problems did not differ significantly from each other.

Table 1: The percentages of correct responses in Experiment 1 for the three sorts of problems and the three conditions.

	1M	**MM**	**NVC**
Control	90	58	54
Spatial Training	94	83	77
Formal Training	88	48	75

There was a significant main effect of the between variable Training ($F(2,45) = 5.6276$, $p < .01$). Fisher LSD Post-Hoc Tests show that in the condition with spatial model training participants solved more problems correct than in the control condition (84.7% versus 67%, $p < .005$) and in the condition with the formal training (84.7% versus 70.1%, $p < .05$). The control condition and the formal training condition did not differ from each other.

The interaction between Training and Problem Type was marginally significant ($F(4, 90) = 2.4147$, $p = .05458$). Fisher LSD Post-Hoc Tests show that there was no difference between the three conditions for the one-model-problems. For the multiple-model-problems, however, participants in the spatial model training condition were significantly more accurate than participants in the control condition and the formal training condition (83.3% vs 58.3%; $p < .0005$; 83.3% vs 48.2, $p < .0005$). The difference between the control condition and the formal training condition was not significant. For the problems-with-no-valid-answer, participants in both training conditions were significantly more accurate than those in the control condition: 77.1% vs 54.2% for the spatial model training ($p < .05$) and 75% vs 54.2% for the formal training ($p < .05$).

Hence, regarding the difference between the three sorts of problems, we confirmed the mental models theory prediction that one-model-problems are the easiest. We did not observe a difference between multiple-model-problems and problems-with-no-valid-answer. However, this lack of a difference is also observed in some other studies and is not really problematic for the mental models theory. Multiple-model-problems are especially easier than problems-with-no-valid-answer if reasoners construct only one of the models. In that situation, they can still formulate the correct response for a multiple-model-problem, but they will draw an erroneous conclusion for a problem-with-no-valid-answer. Of course, when reasoners construct both models for the two sorts of problems, the difference is not expected to be big.

Regarding the training, the spatial models training led to the best performance. This observation extends the beneficial effect of the model training (Johnson-Laird, 2004) to spatial reasoning. The spatial model training led to an overall improvement and in more detail, after the model training participants were more accurate on both the multiple-model-problems and the problems-with-no-valid-answer compared to the control condition. The latter problems were solved better in the formal training condition. However, for the multiple-model-problems the trend was in the opposite direction: after formal training participants were less accurate. It seems that they correctly observed that they could not use the transitivity meaning postulate for these multiple-model-problems, but next inferred incorrectly from this that there was no valid conclusion. One could argue that the formal training was a bit ambiguous: by stressing the importance of the transitivity meaning postulate, it disfavoured the multiple-model-problems. However, if that's the case, one could say the same for the spatial model training: Indeed constructing two models is only vital for the problems-with-no-valid-answer.

Experiment 2

Experiment 1 established that a mental model training improved spatial reasoning for adults. Would such a training have a similar effect with children, who have less working memory capacities? That is the aim of Experiment 2. Moreover, we wanted to test the boundaries of successful training in more detail. Therefore we developed three versions of the mental model training: (1) the one used in Experiment 1, (2) a very short version and (3) one that did not have spatial problems as the training problems. The short version is one where the children were only told that they would perform better when they would draw all possibilities for the problems. In the more distant training, the children were still trained in thinking of all possibilities but the specific problems presented were disjunctions instead of spatial problems. Finally, we dropped the formal training. This type of training only had a weak effect in Experiment 1, that is, only an effect on the problems-with-no-valid-answer. Moreover, we believed that a term as "transitivity" would be too difficult for children to understand and because of the lack of clear effects in Experiment 1. We acknowledge that we cannot rule out that this might partly explain the lack of other formal training effects in Experiment 1, although none of the participants mentioned this.

Method

Participants A total of 179 children from an elementary school participated in Experiment 2. Eighty-three children were nine years old (M: 9.1, SD: 0.3) and 86 were 11 years old (M: 10.9, SD: .27).

Design, Material and Procedure The participants were tested in four different groups, one for each training condition and one for the control group. They solved the same problems as the adults in Experiment 1. However, the question was a little bit different ("which object was the most to the left/right?") and the children did not have to produce their answer themselves but could select the correct answer between four options: the correct fruit, the wrong one, you cannot decide, and I don't know.

The *spatial training* was almost the same as in Experiment 1, except that the instructor read the question and showed the children where they had to look for the answer (i.e., to the left or to the right of the drawing).

In the *short-tip training*, the children simply read on the first page of the booklet the following: "before you start, we give you a tip. It's a simple one, but we know the tip works if you use it well: try to draw all possible solutions for the problems. This will help you to find the correct answer."

In the *more distant training*, the children received the training developed by Johnson-Laird (2004). The first problem was:

> Mister Adams is teaching or mister Peters is teaching or both are teaching.
> If Mister Peters is not teaching, then Mister Jones is teaching.
> Mister Jones is not teaching.

As in Johnson-Laird (2004) all possibilities were drawn sequentially on the blackboard. This is the result after Premise 1 and 2:

1	2	3
Mister A	Mister P	Mister A & P
Mister J		

Next, the instructor focused on Premise 3 and explained that this premise rules out the first possibility. She therefore crossed possibility 1 and said that one could conclude that "Mister Peters definitely is teaching and that Mister Adams might be teaching".

Results

Table 2: The percentages of correct responses in Experiment 2 for the three sorts of problems, the four conditions and the two age groups.

	1M		MM		NVC	
	9y	*11y*	*9y*	*11y*	*9y*	*11y*
Control	60	85	62	82	12	56
Spatial training	61	80	69	80	36	70
Short tip	63	74	64	75	34	28
More distant	43	81	52	66	29	54

Table 2 presents the percentages of correct responses to the different sorts of problems.We performed We performed a repeated measures ANOVA which resulted in a significant main effect of the between variable age ($F(1, 172) = 37.270$, $p < .000001$): the 11-year-olds were significantly more accurate than the 9-year-olds (69% versus 49%). There was also a significant main effect of the within variable Problem type ($F(2, 344) = 57.902$, $p < .000001$). Fisher LSD Post-Hoc Tests show that problems-with-no-valid-answer are significantly more difficult than multiple-model-problems (41% versus 69%, $p < .000001$) and one-model-problems (41% versus 69%, $p < .000001$).

There was no significant main effect of the between variable Training, but there was a significant interaction between Training and Problem Type ($F(6, 344) = 2.9194$, $p < .01$). Fisher LSD Post-Hoc Tests show that there is no difference between the different training conditions for the one-model and the multiple-model-problems. However, there is a significant difference for the problems-with-no-valid-answer: The children performed significantly better when they received spatial training in comparison with no training (54% versus 36%; $p = .05$) and in comparison with a short tip (54% versus 36%; $p < .01$).

Finally, there was a three-way interaction between Age, Problem Type and Training ($F(6, 344) = 2.2727$, $< .05$). Fisher LSD Post-Hoc Tests show that for the nine-year old children, there is only a significant difference for the problems-with-no-valid-answer: the condition with spatial training is significantly better than the condition with no training (36% vs 12%; $p < .05$). For the eleven-year old children, we observed a bad performance for the problems-with-no-valid-answer in the short tip condition: this condition is significantly worse than the condition with no training (28% versus 56%, $p < .05$), spatial training (28% versus 70%, $p < .0005$) and with more distant-training (28% versus 54%, $p < .05$). When one looks at the condition with no training, the nine-year-olds perform worse than the eleven-year-olds on the one-model-problems and the problems-with-no-valid-answer (respectively 60% versus 85%, $p = .026263$ and 12% versus 56%, $p < .0005$). For the multiple-model-problems, the results are in the same direction, but only marginally significant (62% versus 82%, $p = .079343$).

Hence, regarding the difference between the three sorts of problems, we confirmed the mental models theory prediction that problems-with-no-valid-answer are the most difficult ones. We did not observe a difference between one-model-problems and multiple-model-problems. These findings seem to indicate that children just construct one model: That's enough for solving the one-model and multiple-model-problems correctly, but not for the problems-with-no-valid-answer. This is in line with other developmental findings (see e.g; Markovits & Barrouillet, 2002).

Regarding age, it was also observed that eleven-year-olds performed better, especially on the one-model-problems and the problems with no-valid-conclusion. One explanation might be more working memory capacity (see). However, increased fluency, processing fluency or some other executive function might cause this effect. Further research should elucidate this effect.

Regarding the training, as for the adults, the spatial models training led to the best performance, especially on the problems-with-no-valid-answer. The other trainings were less effective. For the eleven-year old children, we observed an unexpected bad performance for the problems-with-no-valid-answer in the short tip condition. Further research has to clarify why this was a worse training condition for the eleven-year-olds in comparison with the nine-year-olds.

General Discussion

The aim of the present paper was twofold: First we wanted to shed light on the debate between the mental models theory and the rule-based approach on the way people reason with spatial deductions. Secondly, it was tested whether the accuracy on spatial reasoning problems can be enhanced by training. More specifically, the efficiency of the "model method" was compared to other training methods.

The results are supportive for the mental models theory, whereas no real support for the rule-based approach is observed: One-model-problems are easier than multiple-model-problems and older children perform better than younger children. Moreover, the experiments show the beneficial effect of model training over other types of training on spatial reasoning for adults. In Experiment 2 it was shown that the spatial mental model training is also beneficial for children.

Nevertheless, we admit that more research is definitely necessary. As we mentioned already, a term as "transitivity" might have been too difficult to understand, even for the university students in the first experiment. Moreover, some rule-based theorists do not claim that mental rules are accessible consciously. Rips (1994), for instance, argues that mental rules operate automatically once certain conditions are met. Therefore, we definitely cannot rule out that other formal trainings would produce better results.

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