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Innovative Applications of O.R.

A queueing model for managing small projects under uncertainties

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ABSTRACT

We consider a situation in which a home improvement project contractor has a team of regular crew members who receive compensation even when they are idle. Because both projects arrivals and the completion time of each project are uncertain, the contractor needs to manage the utilization of his crews carefully. One common approach adopted by many home improvement contractors is to accept multiple projects to keep his crew members busy working on projects to generate positive cash flows. However, this approach has a major drawback because it causes "intentional" (or foreseeable) project delays. Intentional project delays can inflict explicit and implicit costs on the contractor when frustrating customers abandon their projects and/or file complaints or lawsuits. In this paper, we present a queueing model to capture uncertain customer (or project) arrivals and departures, along with the possibility of customer abandonment. Also, associated with each admission policy (i.e., the maximum number of projects that the contractor will accept), we model the underlying tradeoff between accepting too many projects (that can increase customer dissatisfaction) and accepting too few projects (that can reduce crew utilization). We examine this tradeoff analytically so as to determine the optimal admission policy and the optimal number of crew members. We further apply our model to analyze other issues including worker productivity and project pricing. Finally, our model can be extended to allow for multiple classes of projects with different types of crew members.

1. Introduction

Project delays are common. Boeing experienced a series of delays (4 years) and a $6 billion cost overrun when managing the 787 development project (Gates, 2008; Greising & Johnsson, 2007). The delays associated with this complex development project were expected due to many uncertain elements such as unproven technologies (due to the use of composite materials), unprecedented outsourcing of design (in addition to the traditional outsourcing of manufacturing), untested multi-tier supply chain (as opposed to the traditional one-tier supply chain), and unprecedented risk-sharing contract (as opposed to the traditional contract under which each supplier will receive his payment once he completes his own development task).[1] The reader is referred to Tang and Zimmerman (2009) and Kwon, Lippman, McCardle, and Tang (2010a) for a detailed description of the Boeing 787 development project and the analysis that examines the impact of risk-sharing contracts on the project completion time.

Not only large and complex projects face major delays, well-defined projects such as construction projects can experience long delays. For example, Al-Momani (2000) examined 130 public construction projects (such as residential or office buildings, school buildings, medical centers, etc.) in Jordan. Out of these 130 projects, 106 projects were completed late due to various reasons ranging from poor design, poor planning, change orders (by customers), etc. Similar reasons were identified in various surveys of contractors who worked on large public and private construction projects; see Odeh and Battaineh (2002).

Many small and well-defined home improvement projects (e.g., flooring installation, counter/cabinet installation, basic home remodeling, etc.) also tend to complete later than the completion time quoted by the contractor. Unlike large projects, there is no academic research study that examines the underlying causes. However, there are many online forums and blogs commenting that project delay are usually caused by (a) contractor's poor planning; (b) unforeseen conditions; and (c) change orders (by customers).

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[1] Under the risk-sharing contract, each supplier will receive his payment only after *all* suppliers have completed their development tasks.

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Contractor's poor planning can be unintentional or intentional. *Unintentional* poor planning is usually due to contractor's inexperience or poor execution. For examples, the contractor fails to check with vendors about the availability of required materials for the project or to schedule the project tasks properly. Through our discussions with a number of home improvement contractors, we learned of the notion of *intentional* "poor planning" under which the contractor takes on multiple projects to keep his crews busy in generating revenue. This intentional act, however, causes project delays and frustrates customers.

It is common practice among small project contractors to accept multiple projects intentionally, but this practice has not been examined in the research literature. Contractors adopt this practice because, in many instances, they have to pay their crew members when they are between projects.[2] As both arrival times between projects and project completion times are uncertain, a small contractor has strong incentives to take on multiple projects in order to keep his crew busy to generates revenue. The contractor then "rotate or spread" his crew members among these different projects so as to *string along* multiple customers.

This intentional act, however, can cause damages to the contractor. When a contractor accepts too many projects, project delays ensue, causing frustrated customers to forfeit their deposits and abandon their projects. In some cases, instead of filing a claim against the contractor, unhappy customers can damage the contractor's reputation by filing complaints with the Best Business Bureau (www.bbb.org), State government (www.dca.ca.gov), Small Business Association (www.sba.gov), or giving bad ratings at various online forums such as Yelp (www.yelp.com) or Angie's list (www.angieslist.com).[3]

The above considerations create the following dilemma for the contractor. If the contractor accepts too few projects, he faces a higher risk of idling his crew members and incurring unnecessary labor cost. If the contractor accepts too many projects, he faces a higher risk of customers abandoning projects and damaging his reputation. These two competing forces motivate us to develop a stylized model to examine the underlying tradeoff.

To capture the uncertain customer arrivals and the inherent uncertain processing times (due to unforeseen conditions), we present a simple queueing model to examine the tradeoff encountered by a contractor with K identical regular crew members. We capture the contractor's dilemma through his *project admission policy* N, where N is a decision variable that represents the maximum number of projects (or customers) he will accept at any point in time. In our model, these K crew members are the *servers* who process the accepted projects by the contractor as a team simultaneously at any point in time. For instance, if there are i ($i \leq N$) customers in the system, the contractor will rotate (or spread) these K servers among all i projects in order to string along the customers.[4] Therefore, servers are idle only when $i = 0$. (This setting differs from the traditional queueing models in which $K - i$ servers will become idle when $i \leq K$.)

[2] For certain types of projects, it is difficult to adjust part-time crew members dynamically, especially for those who are trained to perform certain tasks. Without a stable team of crew members, many contractors and regular crew members find it difficult to communicate and coordinate with part-time crew members.

[3] While customers may add liquidated damages (due to delay) in a project contract agreed by the contractor, some legal experts commented that liquidated damages clauses are generally considered unenforceable in certain jurisdictions. For small home improvement projects, it is uncommon for customers to hire lawyers to draw up such a legal contract. Even if the customers want to take legal actions against the contractor, the claim may not be high enough to justify the time and effort to do so.

[4] As we shall explain later, this assumption can be relaxed and the analysis remains the same as long as all K crew members are kept busy when $i > 0$.

Under an admission policy N, the contractor will reject any potential customer who arrives at the time when there are already N customers in the system. In this case, he forgoes the revenue associated with this potential customer. At the same time, each customer already in the system may abandon her project due to waiting. In this case, he cannot collect the remaining revenue (except the upfront project deposit) associated with this project. Also, he may incur reputation damages when an abandoned customer launches a complaint in a public forum.

By applying the standard queuing theory approach, we use the steady state analysis to examine the following research questions:

1. Suppose the contractor has K crew members and accepts up to N projects at any point in time. What is the expected project completion time? How often will he turn customers away? How often will a customer abandon her project?
2. Taking into account the revenue loss and reputation damage due to project abandonment, what is the optimal N^* (i.e., the maximum number of projects that the contractor should accept)? What is the optimal K^* (i.e., the optimal crew size)?
3. Should the contractor offer a higher payment to his crew members to entice them to work faster?
4. Should the contractor charge a higher project price and/or require a higher deposit?

Our steady state analysis enables us to obtain closed form expressions for various performance measures including the expected project completion time (i.e., expected waiting time in system), the rejection rate (i.e., the percentage of customers being turned away), and the abandonment rate (i.e., the percentage of customers in the system who abandon their projects). We use these steady state performance measures to examine the impact of the admission policy, arrival rate, process rate, and crew size on these performance measures analytically. We also determine the optimal admission policy N^* and the optimal number of crew members K^*.

Using an extensive numerical analysis, we illustrate how the contractor should manage his crew size in addition to selecting the optimal admission policy. For example, our numerical results show that the contractor should adjust his optimal admission policy N^* to cope with small increases in the arrival rate. However, to cope with major increases in the arrival rate, the contractor should adjust the optimal number of crew members K^* instead. Therefore, our analysis enables us to gain a deeper understanding about the underlying tradeoff faced by the contractor. We further demonstrate how our model can be used to examine other issues of managerial importance including worker productivity and project pricing. Our overall analysis indicates that, to strike a balance optimally, the contractor needs to select his admission policy and manage his crew size carefully.

Finally, we show how our model can be extended to allow for two classes of projects with two types of crew members. Due to numerical complexity, we develop a simple approximation for solving this extended model and provide numerical results to show that our approximation scheme can be used for obtaining near-optimal crew size and admission policy efficiently.

This paper is organized as follows. We review relevant literature in Section 2. We describe the contractor's problem in Section 3. Section 4 presents our queueing model that captures the underlying uncertainties and customer's abandonment dynamics. We use the steady state analysis to determine various performance measures that would enable us to evaluate the tradeoff faced by the contractor. In Section 5 we provide an extensive set of numerical to illustrate our model results and generate useful managerial insights for managing crew size, admission policy, worker productivity and project pricing. Section 6 presents an extension to the base model to allow for two classes of projects with two types of crew members. We conclude the paper in Section 7. All proofs are provided in the Appendix.

2. Literature review

Our paper complements the existing project management literature that deals with uncertain project arrivals and uncertain project completion times. While Critical Path Method (CPM), Project Evaluation and Review Techniques (PERT), and cost-time tradeoff analysis are effective tools for managing projects with little uncertainty in project completion times and/or operating costs, relatively little is known about ways to manage projects with considerable uncertainty in customer arrivals and project completion time (Klastorin, 2004).

Research literature that deals with project management under uncertainty can be divided into two types. The first type deals with issues arising from the customer's perspective. Customers are concerned about project cost and project completion time. Because customers do not have complete information nor complete control about project contractor's cost structure and his operations, designing incentive contracts for the contractor to complete the project on time and on budget is critical. While incentive contract theory has been widely studied in the economics literature (e.g., Weitzman, 1980), the application of contract theory to project management has been limited. Most analytical models in this stream assume that the customer and the project contractor engage in a Stackelberg game in which the customer acts as the leader who determines the incentive contract and the project contractor acts as the follower who determines his effort level.

By using a game-theoretic framework, Bayiz and Corbett (2005) examined the use of a linear incentive contract to coordinate the efforts of different subcontractors under asymmetric information. By comparing the equilibrium outcomes, they found that contracts that offer incentive for early completion are weakly superior to the fixed price contracts in terms of expected project completion time. Kwon et al. (2010a) showed that time-based and cost sharing contracts are optimal incentive contracts when the project completion time is exponentially distributed and when the contractor's effort cost is a quadratic function of the work rate. When the project is comprised of parallel tasks, Kwon, Lippman, and Tang (2010b) examined delayed payment contracts under which homogeneous subcontractors will receive their payments only after all subcontractors complete their tasks. Chen, Klastorin, and Wagner (2015) examined delayed payment contracts in a different context that deals with sequential tasks with non-homogeneous subcontractors. In contrast to the results found by Kwon et al. (2010b) that delayed payment projects may be more profitable for the customer under some conditions in equilibrium, Chen et al. (2015) found that delayed payment contracts can never be more profitable. Kerkhove and Vanhoucke (2015) provided a comprehensive review of different types of incentive project contracts including fixed price, cost-plus, piece-wise linear price, non-linear price contracts, as well as incentive for early (and dis-incentive for late) project completion. Through extensive computational experiments, they found that contracts that include non-linear incentives for cost and project completion time are more efficient. Besides the use of contract theory to examine incentive contracts specified by the customers, Gupta, Snir, and Chen (2015) and Tang, Zhang, and Zhou (2015) used different auction theoretic models to examine a situation where multiple contractors compete for a project with uncertain amount of work by submitting bids that contain two elements: completion time and contract price.

Our paper belongs to the second type of project management literature that deals with issues arising from the project contractor's perspective. To manage multiple projects under different contracts, the project contractor needs to effectively manage his resources (crew members, subcontractors, materials deliveries, equipment rentals), especially when the completion time of each project is inherently uncertain. Due to the underlying complexity, most researchers used simulation models to estimate various performance metrics. For example, Antoniol, Cimitile, Lucca, and Penta (2004) developed simulation models to examine the impact of various staff planning and project scheduling rules on the probability of meeting the project deadline. Anavi-Isakow and Golany (2003) conducted simulation experiments to examine the system performance (workload of various resources, project completion time, etc.) when the project manager keeps the crew busy with a constant number of projects at all time, i.e., constant number of project in process (ConPIP). Cohen, Mandelbaum, and Shtub (2004) extended the work of Anavi-Isakow and Golany (2003) by considering other multi-project scheduling rules including: (a) no control (first come first serve), (b) ConPiP, and (c) a dynamic project admission policy in which a project is admitted to the system only when the queue length of the bottleneck stage is below a pre-determined threshold.

This second type of project management literature is based on the assumption that there is a long list of available projects in the backlog. In contrast, our paper considers the case when the projects arrivals are uncertain. Also, by focusing on single-stage projects (instead of a network of projects), we develop a simple queueing model to examine the optimal project admission policy by using the steady state analysis instead of simulation analysis. We capture the dynamics of project arrivals, crew management, and project abandonment in a simple queueing model and apply the standard queuing theory approach to conduct the steady state analysis of the system. This steady state analysis enables us to examine the issue of project delay when the contractor can accept multiple projects intentionally. More importantly, it allows us to determine the optimal project admission policy and optimal crew size. To our knowledge, our paper represents a first attempt in the project management literature that uses a simple queuing model to examine the project admission policy and project delay when the project arrivals and the project completion times are uncertain.

3. Problem description

Consider a project contractor who manages a crew comprising of K regular members. For tractability, we assume that customers arrive randomly according to a Poisson process with rate λ and that each project processing time is Exponentially distributed with rate μ. The exponential completion time assumption is commonly used in the project management literature (e.g., Adler, Mandelbaum, Nguyen, and Schwerer, 1995; Magott and Skudlarski, 1993; Pennings and Lint, 1997. Dean, Merterl, and Roepke (1969) also argue that an exponential completion time is more realistic in the context of project management than the commonly used Normally distributed completion times (e.g., Bayiz & Corbett, 2005), as supported by empirical evidence in the project management literature (e.g., Cohen et al., 2004). As small home improvement contractors usually focus on a specific type of projects (e.g., flooring installation for residential homes), it is reasonable to assume that the service rate μ is not project-specific.

We focus on the class of admission policies under which the contractor will admit only up to N customers at any point in time, where $N \geq 1$ is a decision variable. Specifically, under an admission policy N, an arriving customer will be rejected when there are already N customers in the system.

3.1. Customer payments

The contractor pays each of his K crew members c per unit time regardless of whether the crew member is busy or idle. Also, the contractor charges each admitted customer r for conducting the project. As customary in project management, each admitted customer is required to pay (a) a non-refundable upfront deposit $d \in [0, r]$ at the time of admission; and (b) the remaining payment of $(r - d)$ upon project completion.

We assume that all K crew members will be shared among all existing customers in the system. (We shall explain ways to allocate crew members to existing customers later.) As sharing the crew members among all existing customers could possibly lengthen the project completion time if there are too many existing customers in the system, we also assume that some impatient customers would abandon their projects before completion. If this happen, the customer forfeits her deposit d, but is not liable to pay the remaining payment $(r - d)$. In this case, the contractor loses $(r - d)$ and also incurs a goodwill cost g due to customer complaints.

3.2. Customer abandonment

There are different ways to model the customer abandonment behavior in the queueing literature. One possible approach to allow a customer to abandon if her waiting time exceeds a pre-specified deadline. Unfortunately, the approach is analytically intractable in general. (Gromoll, Zwart, Robert, and Bakker (2006) manage to get tractable steady state behavior by approximating the system using a fluid model. However, their results are highly complex, which are not suitable for examining the aforementioned tradeoff analytically.) Instead, we assume that each customer will randomly abandon the project at a constant rate α, where α is a measure of customer's impatient level. (Assaf and Haviv (1990) examine the situation where customer cares about her utility as well as the utilities of all other customers. They determine an abandonment strategy that exhibits this property.) This assumption of modeling customer abandonment behavior allows us to develop tractable results in our analysis.

3.3. The contractor's problem

For any given K crew members and for any admission policy N, let R be the rejection probability that an incoming customer is being rejected (which occurs when there are N customers in the system). Therefore, the effective admission rate of customers to the system is equal to $\lambda(1 - R)$, as the system has a Poisson arrival rate of λ.

Also, let A be the abandonment probability that an admitted customer will abandon her project before completion. Then, for each admitted customer who did not abandon the project before project completion, the contractor receives an expected revenue of $(1 - A)r$. On the other hand, for each admitted customer who abandoned the project, the contractor receives an expected revenue of $A(d - g)$, where d is the upfront deposit and g is the goodwill cost due to customer abandonment.

By accounting for the payment to all K crew members at rate c, it is easy to see that the contractor's expected profit per unit time can be written as:

$$\pi(N) = [\lambda(1 - R)]\{(1 - A)r + A(d - g)\} - cK. \tag{1}$$

Therefore, the contractor's problem is to determine the optimal N^* that maximizes his expected profit $\pi(N)$, i.e., the contractor solves the problem: $Max_N\{\pi(N)\}$.

The above contractor's profit function $\pi(N)$ captures the underlying tradeoff between the rejection probability R and the abandonment probability A. To elaborate, suppose that the contractor sets a high value of N. In this case, he admits more customers so that the rejection probability R is smaller. At the same time, more admitted customers will abandon their projects because the abandonment probability A is higher due to system congestion. On the other hand, when the contractor sets a low value of N, he rejects more arriving customers (i.e., the rejection probability R is higher), but few admitted customers will abandon their projects (i.e., the abandonment probability A is smaller).

4. A queueing model with server sharing

To solve the contractor's problem for finding the optimal admission policy N^*, we need to determine the rejection probability R and the abandonment probability A in steady state when a contractor uses a team of K crew members and adopts an admission policy N. To do so, we analyze a queueing model with server sharing, customer rejection and abandonment. Once we determine R and A, we can find the optimal admission policy N^* that maximizes the contractor's expected profit $\pi(N)$ in the following section.

Under Poisson arrival rate λ and exponential service time, we model the admission policy N under which the project contractor will admit up to N projects at any point in time as a queueing system with K identical servers and a finite waiting room capacity N. At any point in time, there will be i customers in the system, where $i = 0, 1, \ldots, N$. If $i = 0$, then all K crew members are idle. If $1 \le i \le N$, then all K crew members are rotated or spread among all i customers. To capture the spirit of sharing all K servers among all i customers, the effective service rate for each of the i customers in the system is equal to $\frac{K\mu}{i}$.[5] In this case, the effective *departure rate* associated with project completion is equal to $\frac{K\mu}{i} \cdot i = K\mu$ when there are i customers in the system.

For ease of exposition, we assume a specific allocation rule under which all K crew members are shared equally among all i customers in the system, where $1 \le i \le N$. As it turns out, our analysis remains the same for all other allocation rules under which all K crew members are assigned to work on any j customers in the system, where $1 \le j \le i$. For example, if $j = 1$, then all K members will work on one project at a time. In this case, when there are i customers in the system, and the effective *departure rate* associated with any allocation rule associated with j is equal to $\frac{K\mu}{j} \cdot j = K\mu$. Hence, from the system perspective, any allocation rule will yield the same system performance.

For each of the i customers in the system with $1 \le i \le N$, we assume she may decide to abandon her project before completion at rate α.[6] For tractability, we further assume that the abandonment times are exponentially distributed and are independent of the service times.

4.1. Steady state analysis

We can model the queueing system described above as a Markovian queuing system with the following transition rate diagram associated with the number of customers in the system $i = 0, 1, 2, \ldots, N$. For example, the transition rate from state $(i + 1)$ to i is given by $(\frac{K\mu}{i+1} + \alpha) \cdot (i + 1) = K\mu + (i + 1)\alpha$ for $i = 1, 2, \ldots, N$, as there are $(i + 1)$ customers in the system, and each of these $(i + 1)$ customers has an independent exponential service and abandon times with rates of $\frac{K\mu}{i+1}$ and α, respectively.

Let p_i denote the steady-state probability when there are i customers in the system, $i = 0, 1, 2, \ldots, N$. From the transition rate di-

[5] Our analysis can be easily extended to allow for a more general setting in which the service rate of each server may depend on the state; i.e., the number of customers in the system i. We ignore the details here for a simpler exposition.

[6] Our model can also be extended in a straightforward manner to allow for state-dependent abandonment rate, which can be used to analyze situations where customers become more anxious and more likely to abandon the project when the system is overly crowded.

agram given in Fig. 1, the steady-state probabilities p_i satisfy the following set of equations:

$$\lambda p_0 = (K\mu + \alpha)p_1,$$
$$(\lambda + K\mu + i\alpha)p_i = \lambda p_{i-1} + (K\mu + (i+1)\alpha)p_{i+1} \quad for \quad i = 1, 2, \ldots, N-1,$$
$$(K\mu + N\alpha)p_N = \lambda p_{N-1},$$
$$\sum_{i=0}^{i} p_i = 1.$$

We solve the above set of linear equations and obtain the following steady state probabilities:

$$p_0 = \left\{1 + \sum_{n=1}^{N}\prod_{j=1}^{n}\frac{\lambda}{(K\mu + j\alpha)}\right\}^{-1}, \tag{2}$$

$$p_i = p_0\prod_{j=1}^{i}\frac{\lambda}{(K\mu + j\alpha)}, \quad for \quad i = 1, 2, \ldots, N. \tag{3}$$

By using the steady-state probabilities p_i given in (2) and (3), we can determine the traditional system performance measures as well as the rejection probability R and the abandonment probability A.

4.2. Average number of customers and waiting time in system

For any admission policy N, we can use the steady-state probabilities p_i given in (2) and (3) to determine the average number of customers in the system is given by $L = \sum_{i=1}^{N} ip_i$. Also, an arriving customer will be admitted by the contractor when $i < N$. Hence, the effective admission rate (or the effective customer arrival rate entering the queueing system) is equal to $\lambda(1 - p_N)$. Therefore, we can apply the Little's Law to determine W, the average waiting time in system experienced by each customer admitted to the system (i.e., project completion time), where:

$$W = \frac{L}{\lambda(1 - p_N)} = \frac{\sum_{i=1}^{N} ip_i}{\lambda(1 - p_N)}. \tag{4}$$

4.3. Idle, rejection and abandonment probabilities

We now use the steady-state probabilities p_i given in (2) and (3) to determine three different probabilities that would enable us to calculate the profit function $\pi(N)$ given in (1). First, for a system in which all K crew members are shared among i customers at any point in time, either all K members are busy when $N \geq i > 0$ or all K members are idle only when $i = 0$. Therefore, the idle probability of all crew members I satisfies:

$$I = p_0 = \left\{1 + \sum_{n=1}^{N}\prod_{j=1}^{n}\frac{\lambda}{(K\mu + j\alpha)}\right\}^{-1}. \tag{5}$$

Second, for any admission policy N, the contractor will admit an arriving customer as long as $i < N$. Therefore, an arriving customer will be rejected for admission only when $i = N$. Hence, the rejection probability R satisfies:

$$R = p_N = \prod_{j=1}^{N}\frac{\lambda}{(K\mu + j\alpha)} \cdot p_0. \tag{6}$$

Third, we can determine the abandonment probability A for any admitted customer by analyzing the average *inflow* and *outflow* rate of customers abandoning the system in steady-state. Observe that the average inflow rate of admitted customers who will abandon the system is equal to $A \cdot \lambda(1 - p_N)$, where $\lambda(1 - p_N)$ is the effective customer admission rate for any admission policy N. Also, the average outflow rate of customers leaving the system at any state i is equal to $(K\mu + i\alpha)$. Since the processing time and the abandonment time are independent exponential random variables, we can apply the result developed by Ross (1983) to show that $\frac{i\alpha}{K\mu + i\alpha}$ portion of the outflow rate is due to abandonment, while the remaining portion is due to project completion. Therefore, the average outflow rate of customers abandoning the system at state i is equal to $(K\mu + i\alpha)\frac{i\alpha}{K\mu + i\alpha} = i\alpha$. By equating the average inflow rate and outflow rate of abandoning customers in steady-state, we have the following relationship:

$$A \cdot \lambda(1 - p_N) = \sum_{i=1}^{N} p_i \cdot i\alpha$$

Therefore, the abandonment probability A is equal to

$$A = \frac{\alpha\sum_{i=1}^{N} ip_i}{\lambda(1 - p_N)}. \tag{7}$$

We next examine the properties of the idle, rejection, and abandonment probabilities I, R, A given in (5)–(7), respectively.

Proposition 1. *The idle, rejection, and abandonment probabilities I, R, and A possess the following properties:*

1. *Both the idle probability I and the rejection probability R are decreasing in N. Furthermore, when the arrival rate is small so that $\lambda < K\mu + \alpha$, the idle probability is higher than the rejection probability for any N; i.e., $I > R$. However, when the arrival rate is large so that $\lambda \geq K\mu + \alpha$, there exists a threshold $\tau > 0$ such that $I < R$ if $N < \tau$ and $I > R$ if $N \geq \tau$.*
2. *The abandonment probability A is increasing in N. The average project completion time W is also increasing in N.*
3. *For any fixed N, the idle probability I is decreasing in λ and is increasing in μ and K. However, both the rejection probability R and the abandonment probability A are increasing in λ and are decreasing in μ and K.*

Proposition 1 exhibits the underlying tradeoff associated with any admission policy N. When the contractor increase the value of N, he admits more customers into the system (by reducing the rejection probability R), keeps his crew members busier (by reducing the idle probability I), but causes more admitted customers to abandon their projects (by increasing the abandonment probability A). Proposition 1 further illustrates the impact of the admission policy N on the relative difference between I and R, and shows an intuitive result that the average project completion time (i.e., average waiting time in system) W given in (4) is increasing in N.

Proposition 1 also demonstrates the impact of the market demand λ as well as the available system capacity (as captured by model parameters μ or K) on these probabilities I, R and A. For example, it illustrates the intuitive result that the idle probability I decreases as the market demand increases or the available system capacity decreases. Similarly, both the rejection probability R and the abandonment probability A increase as the market demand increases or the available system capacity decreases.

4.4. Profit function

We apply the expressions for the idle, rejection, and abandonment probabilities I, R, A given in (5)–(7) to analyze the contractor's profit function $\pi(N)$ given in (1). Observe from (1) that the profit function $\pi(N)$ involves the term $\lambda(1 - R) \cdot A$, i.e., the effective rate of admitted customers who abandons their project before completion. In this case, we can apply (6) and (7), and the steady state analysis equations to show that

$$\lambda(1 - R) \cdot A = \sum_{i=1}^{N} p_i i\alpha = \sum_{i=1}^{N} p_{i-1}\frac{i\alpha\lambda}{K\mu + i\alpha}$$

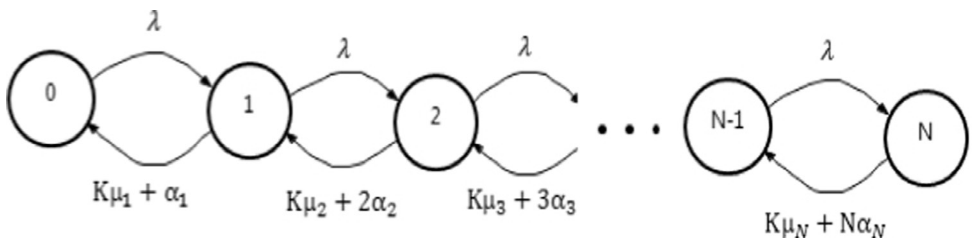

Fig. 1. Transition rate diagram.

$$
\begin{aligned}
&= \lambda \sum_{i=1}^{N} p_{i-1}\left(1 - \frac{K\mu}{K\mu + i\alpha}\right) \\
&= \lambda(1 - p_N) - K\mu \sum_{i=1}^{N} p_i \\
&= \lambda(1 - p_N) - K\mu(1 - p_0) \\
&= \lambda(1 - R) - K\mu(1 - I). \qquad (8)
\end{aligned}
$$

The above expression has a simple interpretation. Essentially, it states that effective rate of admitted customers who abandons their project before completion $\lambda(1-R)\cdot A$ is equal to the customer admission rate $\lambda(1-R)$ minus the effective project completion rate $K\mu(1-I)$.

Using expression (8), the profit function (1) can be simplified as

$$\pi(N) = (d-g)\lambda(1-R) + (r-d+g)K\mu(1-I) - cK. \qquad (9)$$

The above simplified expression has a nice interpretation. Besides the operating cost at rate cK, it states that the contractor earns the upfront deposit d according to the effective admission rate $\lambda(1-R)$, and earns the remaining value $(r-d)$ according to the project completion rate $K\mu(1-I)$. However, the contractor also incurs a goodwill cost g according to the effective abandonment rate $\lambda(1-R) - K\mu(1-I)$ as given in (8). Therefore, the first term of the profit function (9) is based on the "contribution margin" $(d-g)$ associated with the admission rate, the second term is based on the "contribution margin" $(r-d+g)$ associated with the project completion rate, and the third term is based on the operation cost of the crew. Thus, in addition to the tradeoff between admitting more customers to keep the crew busy versus reducing customer abandonment, we need to take the different contribution margins into consideration in determining the optimal admission policy N^*.

We next provide some analytical properties of the optimal admission policy N^* that maximizes the contractor's profit function as given in (9).

Proposition 2. *The optimal admission policy N^* has the following properties:*

1. *When the goodwill cost is lower than the deposit so that $g \le d$, the optimal admission policy $N^* = \infty$.*
2. *When the goodwill cost is higher than the deposit so that $g > d$, the profit function $\pi(N)$ given in (9) is unimodal so that the optimal N^* is unique. Furthermore, the cost and system parameters have the following impact on N^*: (i) N^* is decreasing in the goodwill cost g, but is increasing in the deposit d and price r; and (ii) N^* is decreasing in the arrival rate λ, but is increasing in the service rate μ and crew size K.*

When the goodwill cost g (that would only incur when a customer abandons her project) is lower than the non-refundable deposit d, Statement 1 of Proposition 2 reveals that the contractor can afford to admit all arriving customers. When the goodwill cost is higher than the non-refundable deposit d, there exists no analytical expression for the optimal N^* that maximizes the profit function (9), as the rejection probability R given in (6) and the idle probability I given in (5) are complex functions of N. Statement 2 of Proposition 2, however, ensures in this case that the optimal admission policy N^* is unique and possesses some intuitive properties that allow us to efficiently compute the optimal N^* numerically.

Statement 2(i) of Proposition 2 illustrates the impact of the cost parameters (d, g and r) on the optimal N^* when $g > d$. First, it is clear that the two cost parameters, d and g, have the opposite effect on N^*. As the deposit d increases (or the goodwill cost g decreases), the contractor is less concerned about customer abandonment, and thus can accept more customers in the system to maximize the expected profit. Similarly, as the project price r increases, the contractor should admit more customers into the system.

Statement 2(ii) of Proposition 2 demonstrates the impact of the system parameters (λ, μ and K) on the optimal N^* when $g > d$. First, the contractor should accept fewer customers when the arrival rate λ increases, as a higher market demand reduces the risk of having his crew members idle and he would be more concerned about customer abandonment. On the other hand, the impact of μ (or K) on N^* has the opposite effect of λ: the contractor should admit more customers when μ (or K) increases, as a higher worker productivity (or a larger crew size) increases the risk of idling his crew members and he would be less concerned about customer abandonment.

5. Numerical illustrations and managerial insights

We conducted a comprehensive set of numerical experiments to illustrate how our model can be used to assist small project contractors to accept new projects and to manage their available resources effectively. For example, a contractor can tackle short-term seasonal demand fluctuations by simply adjusting the number of projects to be accepted. We illustrate this tactical decision in Section 5.1. For long-term demand changes, the contractor would need to further adjust his crew size and/or improve the worker productivity. Such strategic decisions need to be carefully analyzed, as it is difficult and expensive to hire or lay off workers, especially in situations where skilled workers are in short supply and the contractor would want to maintain a loyal and competent team of crew members. We illustrate these types of strategic decisions in Sections 5.2 and 5.3. Finally, Section 5.4 illustrates the situation where the contractor can utilize different pricing strategies, in addition to optimal admission policy and crew size management, to cope with the underlying market uncertainty.

5.1. Optimal admission policy

In our base case, we set $K = 5$, $\lambda = 5$, $\mu = 1$, $\alpha = 0.5$, $r = 10$, $d = 1$, $g = 5$ and $c = 1$. We shall use this base case for all the numerical results reported for the remainder of this section.

Table 1 illustrates the impact of the admission policy N on different performance measures. As given in Proposition 1, the idle

Table 1
Numerical results for the base case example.

N	$I = p_0$	$R = p_N$	A	W	$\pi(N, K)$
1	0.5238	0.4762	0.0909	0.1818	17.8571
2	0.3750	0.2841	0.1270	0.2540	24.4318
3	0.3077	0.1793	0.1565	0.3129	27.0445
4	0.2728	0.1136	0.1796	0.3593	28.1750
5	0.2536	0.0704	0.1971	0.3942	28.6553
6	0.2429	0.0421	0.2096	0.4192	28.8383
7	0.2370	0.0242	0.2181	0.4363	28.8906
8	0.2339	0.0133	0.2236	0.4472	28.8921
9	0.2323	0.0069	0.2269	0.4538	28.8789
10	0.2315	0.0035	0.2288	0.4576	28.8655
11	0.2311	0.0016	0.2298	0.4597	28.8559
12	0.2309	0.0007	0.2304	0.4607	28.8501
13	0.2309	0.0003	0.2306	0.4612	28.8469
14	0.2308	0.0001	0.2307	0.4614	28.8453
15	0.2308	0.0001	0.2308	0.4615	28.8445
16	0.2308	0.0000	0.2308	0.4616	28.8442
17	0.2308	0.0000	0.2308	0.4616	28.8441
18	0.2308	0.0000	0.2308	0.4616	28.8440
19	0.2308	0.0000	0.2308	0.4616	28.8440
20	0.2308	0.0000	0.2308	0.4616	28.8440

Table 2
Impact of λ on K^*, N^* and π^*.

λ	K^*	N^*	π^*
0.2	1	4	0.003
0.4	1	4	0.872
0.6	2	7	2.023
0.8	2	7	3.214
1.0	3	11	4.496
1.5	4	14	7.890
2.0	5	16	11.444
2.5	5	14	15.120
3.0	6	17	18.906

Table 3
Impact of μ on K^*, N^* and π^*.

μ	K^*	N^*	π^*
0.1	1	1	−1.893
0.11	1	1	−1.802
0.12	3	1	−1.635
0.13	19	3	−1.171
0.14	23	4	0.317
0.15	25	5	1.896
0.2	24	7	8.938
0.3	20	11	17.745
0.4	16	12	22.908
0.5	14	14	26.345
0.6	13	18	28.828
0.7	12	20	30.701
0.8	11	22	32.196
0.9	10	23	33.421
1.0	9	23	34.421

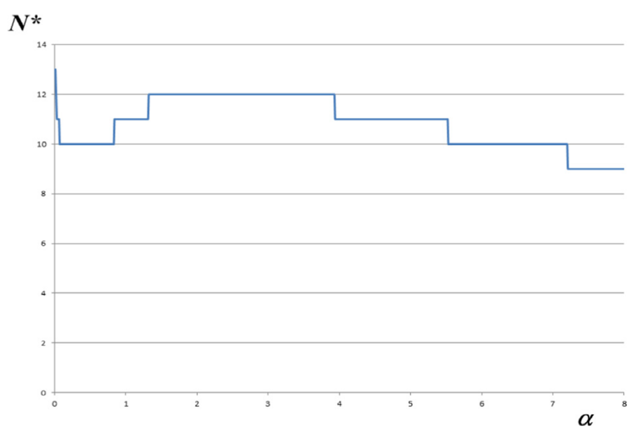

Fig. 2. Impact of α on N^*.

probability $I = p_0$ and the rejection probability $R = p_N$ are decreasing in N, while the abandonment probability A and the project completion time W are increasing in N. Also, the profit function $\pi(N)$ is unimodal in N as stated in Proposition 2, with the optimal admission policy $N^* = 8$ and the optimal profit $\pi^* = 28.8921$. We note that the various system performance measures converge when $N > 15$, while the profit stays essentially the same when $N > 15$.

Proposition 2 shows that the optimal admission policy N^* (i) is decreasing in the goodwill cost g, but is increasing in the deposit d and price r; and (ii) is decreasing in the arrival rate λ, but is increasing in the service rate μ and crew size K. However, we are unable to establish the monotonicity of the abandonment rate α on N^*. Fig. 2 provides a numerical example for which N^* is not monotone (and not even unimodal) in α. (In this numerical example, we set $K = 5$, $\lambda = 10$, $\mu = 1$, $r = 32$, $d = 1$, $g = 2$ and $c = 1$.) One possible explanation of this non-monotone behavior is that a higher value of α increases both the idle probability I and the abandon probability A, where an increase in idle probability causes the contractor to increase N, while an increase in abandonment probability requires the contractor to decrease N. Our numerical results indicate that N^* would decrease in α when α is sufficiently large, which suggests that the second effect tends to dominate the first effect when the abandonment rate is high. In all cases, the optimal profit π^* decreases as α increases, showing the intuitive result that a higher customer abandonment rate would always hurt the contractor's profit.

5.2. Optimal crew size

We next analyze the optimal crew size. Because the crew size decision K is dependent on the admission policy N, it is necessary to consider joint optimal policy under which the contractor selects the optimal crew size K^* and the optimal admission policy N^* that maximizes his profit $\pi(N)$ as given in (9). Essentially, we solve the contractor's problem: $\max_{K,N} \pi(N, K)$. We next illustrate how the different model parameters affect the optimal K^* and N^*, and the resulting optimal profit π^*.

Table 2 illustrates how the arrival rate λ affects the optimal values of K^*, N^* and π^*. To cope with increasing customer demand, the contractor can now adjust both his crew size K and admission policy N. Our result shows that it is optimal for the contractor to use these two levers iteratively to cope with increasing arrival rate. To cope with small arrival rate increases, the contractor should adjust his admission policy N while keeping the same crew size. Hence, when λ lies within a small range, the crew size K^* is kept constant and the optimal admission policy N^* decreases in λ as depicted in Statement 2(ii) of Proposition 2. However, to cope with large arrival rate increases, the contractor needs to also increase the crew size K^*. Observe that the optimal profit π^* always increases as λ increases.

Table 3 illustrates the impact of productivity change μ on K^*, N^* and π^*. The impact of μ on the joint optimal policy K^* and N^* is consistent with that of λ as shown in Table 2. Specifically, when μ lies within a small range, the crew size K^* remains constant, while the optimal admission policy N^* increases in μ as depicted in Statement 2(ii) of Proposition 2. When the process rate μ increases significantly (i.e., when crew members become very productive), the contractor will also need to adjust his crew size K^*, but the change of K^* is not monotone in μ. When μ is small

Table 4
Impact of α on K^*, N^* and π^*.

α	K^*	N^*	π^*
0.1	7	57	40.165
0.2	8	42	38.207
0.3	8	29	36.706
0.4	9	28	35.506
0.5	9	23	34.421
0.6	10	23	33.459
0.7	10	20	32.596
0.8	10	18	31.775
0.9	11	17	31.013
1.0	11	17	30.319
5.0	16	6	14.684
10.0	18	3	4.828
20.0	16	1	−6.242
30.0	13	1	−11.958

Table 5
Impact of d on K^*, N^* and π^*.

d	K^*	N^*	π^*
0	10	22	33.968
1	9	23	34.421
2	9	30	34.891
3	9	45	35.361
4	9	90	35.831

Table 6
Impact of g on K^*, N^* and π^*.

g	K^*	N^*	π^*
2	9	90	35.831
3	9	45	35.361
4	9	30	34.891
5	9	23	34.421
6	10	22	33.968
7	10	18	33.566
8	10	16	33.164
9	10	14	32.762
10	10	12	32.359

(relative to the market demand λ), the contractor would increase K^* as a higher productivity justifies a large crew size to meet the high market demand. However, when μ is high, the contractor would instead reduce K^* to cut labor cost, as the market demand no longer justifies the large crew size. The optimal profit π^* always increases as μ increases.

Table 4 illustrates the impact of abandonment rate α on K^*, N^* and π^*. We observe that the contractor can cope with a small increase in abandonment rate α by simply adjusting N^* while keeping K^* constant. For large increases in α, the contractor would generally need to increase his crew size K^* to reduce customer abandonment, except at very high levels of abandonment rate in which the contractor would instead reduce K^* as it is no longer profitable for him to maintain a large crew size. The optimal profit π^* decreases as α increases, showing again the intuitive result that a higher customer abandonment rate would always hurt the contractor's profit.

We next examine how the cost parameters d and g affect the joint optimal policy, and the results are summarized in Tables 5 and 6. Here, we observe the same pattern as before. To cope with small changes in the parameter value (d or g), the contractor could simply adjust his optimal admission policy N^* while keeping K^* constant; and to cope with large changes in the parameter value, the contractor also needs to adjust his crew size K^*. Furthermore, as long as the optimal value of K^* is the same within the same range of parameter values, the results provided in Tables 5 and 6 are consistent with Statement 2 of Proposition 2 for a fixed value of K: the contractor should increase his admission policy N^* as d increases (or g decreases). As expected, the optimal profit π^* always increases as d increases or g decreases.

Table 7
Impact of c on K^*, N^* and π^*.

c	K^*	N^*	π^*
0.2	17	61	43.880
0.4	13	41	40.917
0.6	11	32	38.495
0.8	10	27	36.370
1.0	9	23	34.421
1.2	9	23	32.621
1.4	8	18	30.939
1.6	8	18	29.339
1.8	7	14	27.745
2.0	7	14	26.345

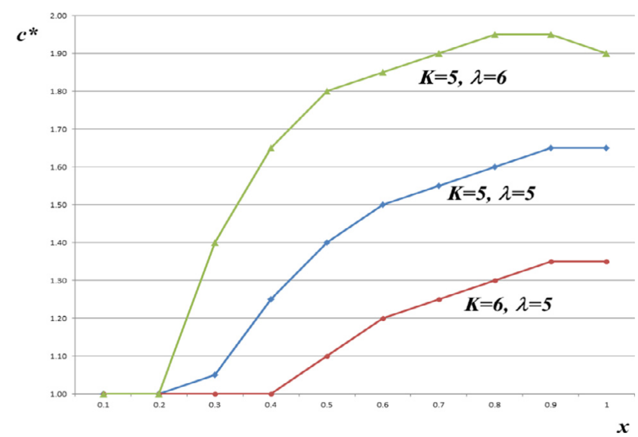

Fig. 3. Impact of x on w^*.

Finally, Table 7 illustrates the impact of the crew cost c on K^*, N^* and π^*. We observe that as c increases, the contractor needs to reduce the crew size K and then adjust the admission policy N accordingly. Also, the optimal profit π^* decreases as c increases.

5.3. Worker productivity

In practice, the contractor can increase the wage c to entice the workers to increase μ, i.e., when the service rate μ is a function of c. We conducted a numerical experiment by considering the case when $\mu(c) = \mu_0 c^x$, where wage $c \geq 1$ and $x \geq 0$ denotes the productivity index. Notice that the case $x = 0$ corresponds to the base case when $\mu(c) = \mu_0$ for any wage c so that the service rate μ is independent of wage c.

In the first set of numerical experiments, we consider a fixed crew size K. However, we consider the case when the contractor to choose the optimal admission policy N^* and the optimal wage c^*. Essentially, we solve the contractor's problem: $\max_{c,N} \pi(N, c)$, where $\pi(N, c)$ as given in (9) with $\mu(c) = \mu_0 c^x$. Fig. 3 illustrates how the productivity index x affects the optimal wage c^*. Based on our numerical results, we have the following observations:

- For fixed K and λ, the optimal c^* is non-monotone in the productivity index x. In other words, it is not always optimal for the contractor to offer a higher wage c^* for a higher productivity index x.
- For fixed λ (e.g., $\lambda = 5$) and for any x, the optimal wage c^* decreases as crew size K increases. When there is sufficient capacity (i.e., crew size) to meet existing market demand, it is unnecessary for the contractor to offer a higher wage to increase the

Table 8
Impact of x on c^*, K^* and π^*.

x	c^*	K^*	π^*
0.1	1.00	9	34.421
0.9	1.00	9	34.421
0.95	1.01	9	34.423
0.99	1.03	9	34.432
1.00	1.03	9	34.435
1.00	9.30	1	34.435
1.01	9.21	1	34.640
1.1	8.43	1	36.277
1.5	6.16	1	40.718
2.0	4.72	1	43.341

crew's service rate. Instead, it is optimal for the contractor to reduce the optimal wage c^*.

- For any fixed K (e.g., $K = 5$) and for any x, the optimal wage c^* increases as λ increases, which shows the intuitive result that an increase in worker productivity becomes more attractive as market demand increases, resulting in a higher optimal wage c^*.

In the second set of numerical experiment, we allow the contractor to choose the optimal crew size K^*, the optimal admission policy N^*, and the optimal wage c^* by solving $\max_{c,\,N,\,K} \pi(N, K, c)$. The results are summarized in Table 8. For $x < 1$, the optimal crew size K^* is equal to 9, while the optimal c^* remains at the minimum value of 1 except when x is very close to 1. This suggests that the contractor should deploy the strategy of keeping a low wage while maintaining a high crew size when the marginal increase in worker productivity μ is decreasing in wage c (i.e., when $x < 1$). When $x > 1$, the optimal crew size K is always equal to the minimum value of one, while the optimal c^* is above 1. In other words, the contractor should deploy the strategy of keeping the minimum crew size while increasing the crew wage when the marginal increase in worker productivity is increasing in wage c (i.e., when $x > 1$). The contractor is indifferent to these two strategies when $x = 1$. Also, observe that the expected profit always increases as the productivity index x increases.

5.4. Pricing decisions

We next analyze the impact of project pricing in terms of the project price r and the upfront deposit d, and consider the situation where the market demand λ is dependent on both r and d. To capture the notion that a higher price r or a higher deposit d could lead to a lower customer demand, we consider the case when the market demand $\lambda(r, d) = \lambda_0 e^{-\epsilon_1 r - \epsilon_2 d}$, where $\epsilon_1 \geq 0$ and $\epsilon_2 \geq 0$ measure the demand elasticity with respect to project price r and upfront deposit fraction d, respectively. Also, to model the fact that a higher deposit fraction $\frac{d}{r}$ would deter customers from abandoning their project, we shall consider the case when the abandonment rate $\alpha(d, r) = \alpha_0 e^{-\epsilon_3 (\frac{d}{r})}$, where $\epsilon_3 \geq 0$ measures the abandonment elasticity with respect to deposit fraction $\frac{d}{r}$.

5.4.1. Project price r

We first study how the demand elasticity ϵ_1 affects the optimal price r^* by solving the contractor's problem: $\max_{r,N,K} \pi(N, K, r)$ with market demand $\lambda(r, d)$ and abandonment rate $\alpha(r, d)$ as stated above. To isolate the impact of r in this study, we keep $\frac{d}{r}$ and $\frac{g}{r}$ constant so that $\frac{d}{r} = 0.1$, $\frac{g}{r} = 0.5$, and we set $\mu = 1$, $c = 1$, $\lambda_0 = 5$, $\epsilon_2 = 0$, $\alpha_0 = 0.5$, and $\epsilon_3 = 0$. Also, we consider the case when the project price r lies within the range [1, 10]. Table 9 shows the impact of demand elasticity ϵ_1 on the optimal values of r^* and K^*, and the optimal profit π^*. Based on our numerical results, we have the following observations:

Table 9
Impact of ϵ_1 on r^*, K^* and π^*.

ϵ_1	r^*	K^*	π^*
0.0	10.00	9	34.421
0.1	10.00	4	10.301
0.2	5.87	3	3.486
0.24	4.89	3	2.405
0.25	5.30	2	2.205
0.3	4.42	2	1.504
0.4	3.31	2	0.628
0.45	2.95	2	0.338
0.5	3.32	1	0.188
0.6	2.77	1	−0.010
0.7	2.37	1	−0.151

Table 10
Joint impact of ϵ_2 and ϵ_3 on d^*, K^* and π^*.

Optimal d^*						
ϵ_2	$\epsilon_3 = 0$	1	2	3	4	5
0	10.0	10.0	10.0	10.0	10.0	10.0
0.01	1.0	10.0	10.0	9.2	8.0	7.2
0.02	1.0	4.6	5.8	5.2	4.8	4.4
0.03	1.0	1.0	2.0	3.0	3.7	3.5
0.04	1.0	1.0	1.0	1.5	2.3	2.2
0.05	1.0	1.0	1.0	1.0	1.2	1.8
Optimal K^*						
ϵ_2	$\epsilon_3 = 0$	1	2	3	4	5
0	6	6	6	6	6	6
0.01	6	5	5	5	5	5
0.02	9	7	6	6	6	6
0.03	9	9	8	7	6	6
0.04	9	9	9	8	7	7
0.05	9	9	9	8	8	7
Optimal π^*						
ϵ_2	$\epsilon_3 = 0$	1	2	3	4	5
0	39.62	41.65	42.85	43.48	43.77	43.89
0.01	35.62	37.51	38.76	39.52	40.10	40.55
0.02	33.65	34.29	35.66	36.76	37.59	38.25
0.03	33.27	33.75	34.29	35.17	35.97	36.71
0.04	32.89	33.37	33.82	34.31	34.95	35.58
0.05	32.51	32.98	33.43	33.86	34.32	34.81

- The optimal project price r^* is decreasing in ϵ_1 when K^* is constant. For a small increase in price sensitivity ϵ_1, the contractor can keep the crew size the same by simply lowering the project price r.
- The optimal crew size K^* is decreasing in ϵ_1. Besides the optimal admission policy N^*, the contractor has two basic levers to manage profit: 1) lower the project price r to boost demand, or 2) reduce the crew size K to contain cost. When customers become more price sensitive (as ϵ_1 increases significantly), it is no longer sufficient for the contractor to simply reduce project price r, he now needs to reduce the crew size K to contain cost as well.
- The optimal profit π^* is decreasing in ϵ_1. This result is intuitive: the profit decreases as customers become more price sensitive.

5.4.2. Upfront deposit d

To examine the issue of deposit d that affects the market demand $\lambda(r, d)$ and the abandonment rate $\alpha(r, d)$, we fix $r = 10$ and $\epsilon_1 = 0$, and solve the problem $\max_{d,N,K} \pi(N, K, d)$. Here, we consider the case when d lies within the range [1, 10] so that the deposit ratio d/r lies within the range [0.1, 1]. Table 10 illustrates the joint impact of demand elasticity ϵ_2 and the abandonment elasticity ϵ_3 on the optimal values of d^* and K^*, and the optimal profit π^*.

Table 11
Joint impact of ϵ_1, ϵ_2 and ϵ_3 on r^*, d^* and K^*.

		$\epsilon_3=1$			$\epsilon_3=3$			$\epsilon_3=5$		
ϵ_1	ϵ_2	r^*	$\frac{d^*}{r^*}$	K^*	r^*	$\frac{d^*}{r^*}$	K^*	r^*	$\frac{d^*}{r^*}$	K^*
0.1	0.01	10.0	1.00	2	9.8	1.00	2	9.9	0.81	2
	0.03	8.5	1.00	2	9.3	0.71	2	9.7	0.56	2
	0.05	10.0	0.10	4	9.4	0.34	3	9.5	0.45	2
0.2	0.01	5.2	1.00	2	5.1	1.00	2	5.1	0.93	2
	0.03	5.4	1.00	1	4.8	0.94	2	4.9	0.72	2
	0.05	5.0	1.00	1	4.7	0.77	2	4.8	0.61	2
0.3	0.01	4.0	1.00	1	4.6	1.00	1	5.1	1.00	1
	0.03	3.8	1.00	1	4.3	1.00	1	4.8	0.91	1
	0.05	3.6	1.00	1	4.1	1.00	1	4.6	0.84	1
0.5	0.01	2.4	1.00	1	2.8	1.00	1	3.1	1.00	1
	0.03	2.4	1.00	1	2.7	1.00	1	3.0	0.97	1
	0.05	2.3	1.00	1	2.6	1.00	1	2.9	0.91	1

First, when $\epsilon_2 = 0$, the market demand λ is independent of the deposit d. Therefore, the contractor should always choose $d^* = r$ because the abandonment rate α is decreasing in d. Second, for any fixed $\epsilon_2 > 0$, Table 10 shows that the optimal d^* is not monotone in ϵ_3. The optimal deposit d^* first increases as ϵ_3 increases from zero, which indicates that the impact of d^* on abandonment rate dominates the impact on market demand. However, the optimal d^* then decreases as ϵ_3 increases further, suggesting that the marginal reduction in abandonment rate no longer justifies the corresponding decrease in market demand after ϵ_3 exceeds a certain threshold.

While the optimal d^* is not monotone in ϵ_3 for any fixed $\epsilon_2 > 0$, the optimal crew size K^* is decreasing in ϵ_3 whenever there is an increase in the optimal deposit d^*. This result can be explained as follows. When the contractor charges a higher deposit d^*, the market demand $\lambda(r=10, d^*)$ and the abandon rate $\alpha(r=10, d^*)$ decrease. To contain cost, it is optimal for the contractor to reduce his crew size K^*. Also, the optimal profit π^* always increases as ϵ_3 increases, which shows that the contractor would always benefit when the customers become less inclined to abandon their project due to the deposit.

We next consider the impact of ϵ_2 when ϵ_3 is fixed. When $\epsilon_3 = 0$, the deposit d does not affect the abandonment rate but it does affect market demand. For this case, we observe that the optimal deposit d^* decreases and the optimal crew size K^* increases as ϵ_2 increases. This result is intuitive as the market demand becomes more sensitive to the deposit, the contractor needs to reduce the deposit to increase customer demand and also increase his crew size to cope with the increased market demand. Next, for any $\epsilon_3 > 0$ where the deposit d also affects the abandonment rate, it remains valid that the optimal deposit d^* decreases and the optimal crew size K^* increases as ϵ_2 increases for $\epsilon_2 > 0$. Also, the optimal profit π^* always decreases as ϵ_2 increases, which shows that the contractor would always suffer when market demand becomes more sensitive to the deposit.

5.4.3. Joint decisions of r and d

We now study the impact of ϵ_1, ϵ_2 and ϵ_3 on the joint optimal pricing decisions of r^* and d^*. To do so, we solve $\max_{r,d,N,K} \pi(N, K, d, r)$. Table 11 shows the optimal price r^*, the optimal deposit ratio $\frac{d^*}{r^*}$ and the optimal crew size K^* for different values of ϵ_1, ϵ_2 and ϵ_3.

Based on our numerical results, we have the following observations:

- Consider the set of parameter values under which the optimal crew size K^* remains the same. We observe that the optimal price r^* decreases as ϵ_1 or ϵ_2 increases for any fixed value of ϵ_3. This result is intuitive. When ϵ_1 increases (i.e., customers become more price sensitive), the contractor should reduce the project price r^*. When ϵ_2 increases (i.e., customers become sensitive towards deposit d), the contractor should reduce the deposit d^* and the project price r^* to stabilize demand λ and the abandonment rate α. This also explains why the optimal deposit ratio $\frac{d^*}{r^*}$ decreases as ϵ_2 or ϵ_3 increases for any fixed value of ϵ_1.
- Consider large changes in the parameter value under which the optimal crew size K^* also changes. We observe that K^* decreases as ϵ_1 increases. This suggests that for a large increase in ϵ_1, it is insufficient for the contractor to simply reduce price r, he now needs to reduce the crew size K to contain cost. This result is consistent with those given in Section 5.4.1. However, K^* does not necessarily increase or decrease as ϵ_2 or ϵ_3 increases. For example, K^* increases as ϵ_2 increases when $\epsilon_1 = 0.1$ and $\epsilon_3 = 1$, while K^* decreases as ϵ_2 increases when $\epsilon_1 = 0.2$ and $\epsilon_3 = 1$. Similarly, K^* increases as ϵ_3 increases when $\epsilon_1 = 0.2$ and $\epsilon_2 = 0.03$, while K^* decreases as ϵ_3 increases when $\epsilon_1 = 0.1$ and $\epsilon_2 = 0.2$.

6. Extension to two-class systems

We can extend our model to analyze a system with multiple project classes. Below we provide a discussion for the two-class system, but the analysis can be extended to multiple classes in a straightforward manner. Suppose that there are two project classes with (possibly) different arrival, service and abandon rates. We use the subscript i to denote the parameters associated with class i. Also, the project contractor has two types of team members, where Type-1 members can handle both Class-1 and Class-2 projects, while Type-2 members can handle Class-2 projects only. As such, Type-1 members can be considered as more skillful, whereas Class-1 projects require more skilled labor. It is thus reasonable to assume that the labor rates $c_1 \geq c_2$, and that the revenues $r_1 \geq r_2$, although these assumptions are not required in our analysis. Let K_i denote the available number of Type-i members, and define $K = K_1 + K_2$.

As for the single-class model, we assume that all K_i Type-i members are shared equally among all Class-i projects in the system, $i = 1, 2$. Furthermore, when there is no available Class-1 project in the system, all K_1 Type-1 members will also be shared equally, together with K_2 Type-2 members, among all available Class-2 projects. All K_1 Type-1 members will be immediately switched back to process any new arriving Class-1 project in the system, i.e., a preemptive two-priority class system.

Suppose that the contractor uses an admission policy (N_1, N_2) under which the contractor will admit up to N_1 Class-1 projects and N_2 Class-2 projects in the system at any time point. Following a similar approach as for the single-class system, we can perform a steady-state analysis of the associated Markovian queueing system by deriving the following state transition equations, where the state (i, j) represents the number of Class-1 projects and Class-2 projects in the system:

$$
\begin{aligned}
&(\lambda_1+\lambda_2)p_{0,0} = (K_1\mu_1+\alpha_1)p_{1,0} + (K\mu_2+\alpha_2)p_{0,1}\\
&(\lambda_1+\lambda_2+K\mu_2+j\alpha_2)p_{0,j} = \lambda_2 p_{0,j-1} + (K_1\mu_1+\alpha_1)p_{1,j}\\
&\quad + (K\mu_2+(j+1)\alpha_2)p_{0,j+1}, \quad 0<j<N_2\\
&(\lambda_1+\lambda_2+K_1\mu_1+i\alpha_1)p_{i,0} = \lambda_1 p_{i-1,0}\\
&\quad + (K_1\mu_1+(i+1)\alpha_1)p_{i+1,0} + (K_2\mu_2+\alpha_2)p_{i,1}, \quad 0<i<N_1\\
&(\lambda_1+\lambda_2+K_1\mu_1+K_2\mu_2+i\alpha_1+j\alpha_2)p_{i,j} = \lambda_1 p_{i-1,j} + \lambda_2 p_{i,j-1}\\
&\quad +(K_1\mu_1+(i+1)\alpha_1)p_{i+1,j} + (K_2\mu_2+(j+1)\alpha_2)p_{i,j+1},\\
&\quad 0<i<N_1, 0<j<N_2\\
&(\lambda_2+K_1\mu_1+K_2\mu_2+N_1\alpha_1+j\alpha_2)p_{N_1,j} = \lambda_1 p_{N_1-1,j} + \lambda_2 p_{N_1,j-1}\\
&\quad + (K_2\mu_2+(j+1)\alpha_2)p_{N_1,j+1}, \quad 0<j<N_2
\end{aligned}
$$

$$(\lambda_1 + K_1\mu_1 + K_2\mu_2 + i\alpha_1 + N_2\alpha_2)p_{i,N_2} = \lambda_1 p_{i-1,N_2} + \lambda_2 p_{i,N_2-1} + (K_1\mu_1 + (i+1)\alpha_1)p_{i+1,N_2}, \quad 0 < i < N_1$$
$$(\lambda_2 + K_1\mu_1 + N_1\alpha_1)p_{N_1,0} = \lambda_1 p_{N_1-1,0} + (K_2\mu_2 + \alpha_2)p_{N_1,1}$$
$$(\lambda_1 + K\mu_2 + N_2\alpha_2)p_{0,N_2} = \lambda_2 p_{0,N_2-1} + (K_1\mu_1 + \alpha_1)p_{1,N_2}$$
$$(K_1\mu_1 + K_2\mu_2 + N_1\alpha_1 + N_2\alpha_2)p_{N_1,N_2} = \lambda_1 p_{N_1-1,N_2} + \lambda_2 p_{N_1,N_2-1} \quad (10)$$

and

$$\sum_{i=0}^{N_1}\sum_{j=0}^{N_2} p_{i,j} = 1.$$

Unfortunately, there does not exist a product-form solution for the steady-state probabilities $p_{i,j}$ given in (10) from which we can derive analytical expressions for the various system performance measures. Although it is feasible to solve the above set of linear equations numerically to compute the steady-state probabilities $p_{i,j}$ and to derive the associated system performance measures, this numerical approach poses challenges for the purpose of searching for the optimal values of (N_1, N_2) and (K_1, K_2). Therefore, we propose the following efficient scheme for approximating the system performance measures.

First, observe that the system performance for Class-1 projects is the same as that for the single-class system with parameters (λ_1, μ_1, α_1, K_1, N_1), as all K_1 Type-1 members will be assigned to work on any available Class-1 projects at any point in time. In particular, the respective idle probability and rejection probability for Class-1 projects are given by

$$I_1 = \left\{1 + \sum_{n=1}^{N_1}\prod_{j=1}^{n}\frac{\lambda_1}{(K_1\mu_1 + j\alpha_1)}\right\}^{-1}, \quad (11)$$

and

$$R_1 = \prod_{j=1}^{N_1}\frac{\lambda_1}{(K_1\mu_1 + j\alpha_1)} \cdot I_1. \quad (12)$$

Using (8), the corresponding abandonment probability for Class-1 projects are given by

$$A_1 = 1 - \frac{K_1\mu_1(1 - I_1)}{\lambda_1(1 - R_1)}. \quad (13)$$

Second, the system performance for Class-2 projects is similar to that for the single-class system with parameters (λ_2, μ_2, α_2, K_2, N_2) except that when there is no available Class-1 project in the system, all K_1 Type-1 members will also be used to handle the available Class-2 projects. Thus, we will approximate the system performance for Class-2 projects by a single-class system with parameters $(\lambda_2, \mu_2, \alpha_2, K_1I_1 + K_2, N_2)$, since I_1 is equal to the probability that there is no available Class-1 project in the system, and thus K_1I_1 represents the expected number of Type-1 members working on Class-2 projects. Then, the respective idle probability, rejection probability and abandonment probability for Class-2 projects can be approximated by

$$\tilde{I}_2 = \left\{1 + \sum_{n=1}^{N_2}\prod_{j=1}^{n}\frac{\lambda_2}{(K_1I_1 + K_2)\mu_2 + j\alpha_2)}\right\}^{-1} \quad (14)$$

$$\tilde{R}_2 = \prod_{j=1}^{N_2}\frac{\lambda_2}{(K_1I_1 + K_2)\mu_2 + j\alpha_2)} \cdot \tilde{I}_2 \quad (15)$$

$$\tilde{A}_2 = 1 - \frac{(K_1I_1 + K_2)\mu_2(1 - \tilde{I}_2)}{\lambda_2(1 - \tilde{R}_2)}. \quad (16)$$

We performed a comprehensive numerical study to evaluate the accuracy of this above scheme for approximating the three key measures given in (14)–(16). For our base case, we set $\lambda_1 = \lambda_2 = 5$, $\mu_1 = \mu_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, $K_1 = N_1 = 5$, and $K_2 = N_2 = 5$, and we obtain $\tilde{I}_2 = .347$ and $I_2 = .337$, $\tilde{R}_2 = .039$ and $R_2 = .045$, and $\tilde{A}_2 = .148$ and $A_2 = .155$.

We next illustrate how the different model parameters can affect the approximation errors of the three probability measures as defined by $I_2^{err} = \tilde{I}_2 - I_2$, $R_2^{err} = \tilde{R}_2 - R_2$, and $A_2^{err} = \tilde{A}_2 - A_2$. In particular, we varied the values of each of the model parameters as given in the base case within the following ranges: (i) $\lambda_1 \in [1, 10]$; (ii) $\mu_1 \in [0.1, 5]$; (iii) $\alpha_1 \in [0.1, 5]$; (iv) $K_1 \in [1, 15]$ (with $N_1 = K_1$); and (v) $K_2 \in [1, 9]$ (with $N_2 = K_2$). (We also examined the impact of the admission thresholds, N_1 and N_2, on the approximation errors but do not see any significant impact, and so we omit those results in our discussions below.)

Figs. 4–6 show the three approximation errors, I_2^{err}, R_2^{err} and A_2^{err}, as each model parameter varies between the range given above. From our numerical results, we have the following two main observations:

- The approximation consistently over-estimates the system performance for Class-2 projects, and gives a higher idle probability ($I_2^{err} > 0$), and smaller rejection and abandonment probabilities ($R_2^{err} < 0$ and $A_2^{err} < 0$) than the true values.
- The approximation errors are generally very small (around 0.01) except when K_1I_1 is relatively large when compared to K_2.

We can explain these two observations as follows. The approximation is based on the performance of a single-class system with $(K_1I_1 + K_2)$ members handling Class-2 projects. While the K_2 Type-2 members are always available, the K_1 Type-1 members is only available with probability I_1. In other words, the approximation simply ignores the supply variability of Type-1 members, and thus results in over-estimating the system performance. Furthermore, the impact of the underlying supply variability is especially significant when K_1I_1 is relatively high when compared to K_2.

Accordingly, we can approximate the profit function (see (1)) as

$$\tilde{\pi}(K_1, K_2, N_1, N_2) = \left\{\lambda_1(1 - R_1)[(1 - A_1)r_1 + A_1(d_1 - g_1)] - c_1K_1\right\} + \left\{\lambda_2(1 - \tilde{R}_2)[(1 - \tilde{A}_2)r_2 + \tilde{A}_2(d_2 - g_2)] - c_2K_2\right\}. \quad (17)$$

for any fixed crew size (K_1, K_2) and admission policy (N_1, N_2). We can use the approximate profit function (17) to search for the optimal crew size and admission policy. Table 12 summarizes the result of our numerical experiments. For this set of numerical results, we set $\lambda_1 = \lambda_2 = 5$, $\mu_1 = \mu_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, with different values of c_1, c_2, r_1 and r_2. Also, we assume that $d_i = 0.1r_i$ and $g_i = r_i$ for $i = 1, 2$ in each case.

Table 12 shows the optimal crew size and admission policy based on the approximate profit function given in (17) with the corresponding (approximate) optimal profit. In each case, we also show the true optimal crew sizes and admission policy with the corresponding optimal profit. This was done by solving the steady-state probabilities from (10) for each set of fixed values (K_1, K_2, N_1, N_2) from which we can compute the exact profit and then by conducting an exhaustive numerical search for the optimal (K_1, K_2, N_1, N_2).

We can observe from Table 12 that when the value of c_2 is much smaller than c_1, the approximation gives near-optimal results with the optimal profit being very close to the exact optimal value. As the value of c_2 is close to c_1, the approximate profit

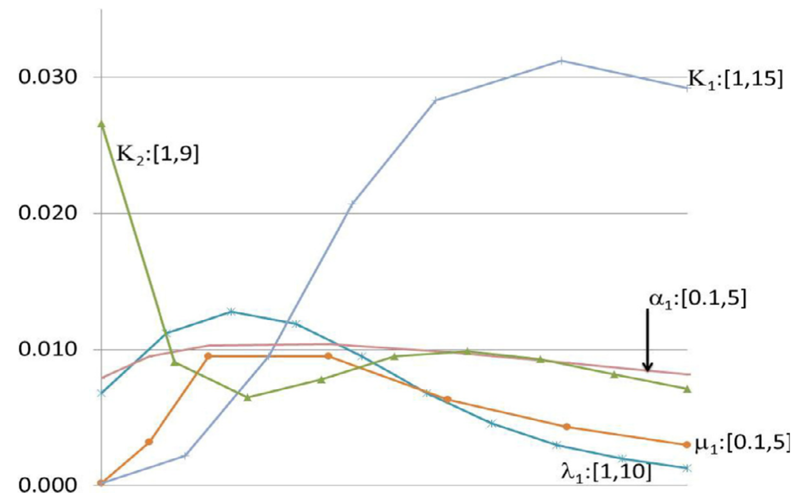

Fig. 4. Impact of model parameters on I_2^{err}.

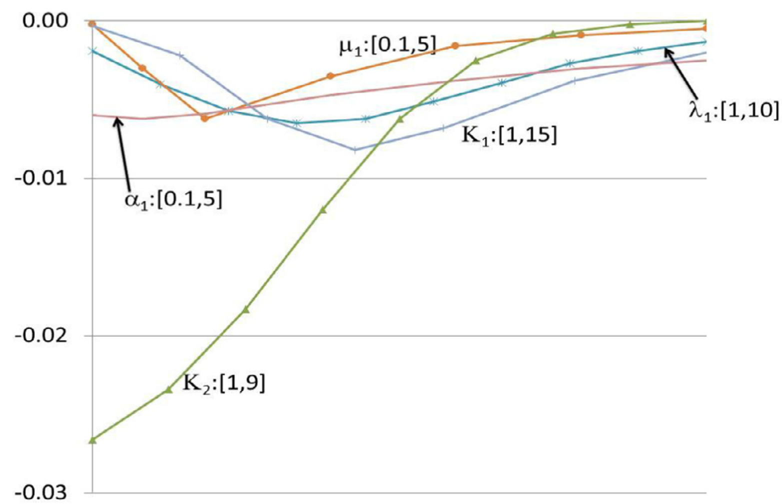

Fig. 5. Impact of model parameters on R_2^{err}.

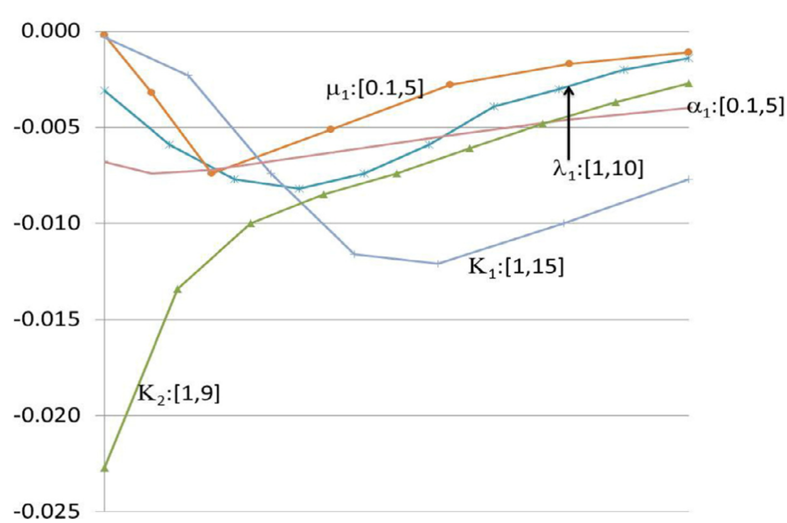

Fig. 6. Impact of model parameters on A_2^{err}.

Table 12
Optimal crew size and admission policy.

				Based on approximation			Exact solution		
c_1	c_2	r_1	r_2	(K_1^*, K_2^*)	(N_1^*, N_2^*)	$\tilde{\pi}^*$	(K_1^*, K_2^*)	(N_1^*, N_2^*)	π^*
5	1	10	1	(5,0)	(4,1)	4.86	(5,0)	(4,1)	4.63
			3	(5,5)	(4,6)	9.67	(5,5)	(4,6)	9.41
			5	(6,6)	(4,8)	17.23	(5,7)	(4,9)	16.97
			7	(6,7)	(5,10)	25.31	(5,8)	(4,10)	25.06
			10	(5,9)	(4,10)	37.94	(5,9)	(4,10)	37.68
5	2	10	2	(6,0)	(5,2)	5.68	(6,0)	(5,2)	5.11
			4	(6,3)	(5,4)	9.21	(6,3)	(4,4)	8.46
			6	(6,4)	(4,5)	15.70	(6,5)	(4,7)	14.98
			8	(6,5)	(4,7)	23.09	(6,6)	(5,8)	22.31
			10	(6,6)	(4,9)	30.87	(6,6)	(4,8)	30.02
5	3	10	3	(6,0)	(4,2)	6.75	(6,0)	(4,2)	5.91
			5	(7,1)	(5,3)	10.01	(7,2)	(5,4)	8.44
			7	(7,3)	(5,5)	15.68	(7,3)	(5,5)	14.07
			10	(7,4)	(5,6)	26.05	(6,5)	(4,7)	24.36
5	4	10	4	(7,0)	(5,2)	8.25	(7,0)	(5,2)	6.79
			6	(8,0)	(5,3)	12.04	(8,0)	(5,3)	9.66
			8	(10,0)	(8,5)	17.22	(8,2)	(5,5)	14.38
			10	(9,1)	(6,5)	23.48	(8,3)	(5,6)	20.62

shows a larger error as compared to the exact profit, it becomes beneficial to have mostly Type-1 members in these cases. As such, $K_1 I_1$ is large when compared to K_2, which results in larger approximation errors as discussed earlier. However, the optimal crew size and admission policy using the approximation are still very close to the true optimal values. Thus, our numerical results suggest that the approximation method can still be used to provide near-optimal results even for such scenarios.

7. Conclusion

When dealing with uncertain customer arrivals, a contractor has incentive to accept multiple projects to keep his crew busy by stringing along his customers. By doing so, the contractor faces the risk of customer abandonment and customer complaint. To quantify this tradeoff, we have presented a queuing model to examine the optimal admission policy in terms of the maximum number of projects that a contractor should accept at any point in time.

Our steady state analysis enables us to obtain closed form expressions for various system performance measures, from which we can analyze the optimal admission policy and the optimal crew size. Our model results can be useful for small project contractors to deal with the underlying market uncertainty. For instance, a contract can apply our model to determine the optimal admission policy as a tactical decision to tackle short-term seasonal demand fluctuations. Furthermore, the contractor can utilize the model results to address the strategic decisions of determining the optimal crew size or improving the worker productivity to deal with long-term demand changes. Finally, our model can also be applied to select the appropriate pricing strategy, in addition to optimal admission policy and crew size management, to cope with the underlying market uncertainty.

Ultimately, our analysis indicates that the contractor needs to carefully select his operating and pricing decisions in order to strike an optimal balance between keeping crew members busy and keeping customers happy. Furthermore, we have extended our model to allow for different types of projects with different arrival, service, and abandonment characteristics.

Our paper serves as an initial attempt to examine a new research topic arising from project management. There are other research questions that deserve further examination. For example, our model assumes that customers will abandon their projects according to some constant abandonment rate. In practice, customers are more likely to abandon their projects when the waiting time exceeds the completion time quoted by the contractor. It would be interesting to extend our model to analyze this situation.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ejor.2016.02.052.

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