Title

Exploiting Heterogeneous Channel Coherence Intervals for Blind Interference Alignment

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Abstract

We explore 5 network communication problems where the possibility of interference alignment, and consequently the total number of degrees of freedom (DoF) with channel uncertainty at the transmitters, are unknown. These problems share the common property that in each case the best known outer bounds are essentially robust to channel uncertainty and represent the outcome with interference alignment, but the best inner bounds — in some cases conjectured to be optimal — predict a total collapse of DoF, thus indicating the infeasibility of interference alignment under channel uncertainty at transmitters. Our main contribution is to introduce the idea of blind interference alignment. Specifically, we show that even with no knowledge of channel coefficient values at the transmitters, the knowledge of the channels' coherence structure can be exploited to achieve interference alignment. In each case, we show that under certain heterogeneous block fading models, i.e., when certain users experience smaller coherence time/bandwidth than others, the transmitters are able to align interference without the knowledge of channel coefficient values.

This work was presented in part at IEEE GLOBECOM 2010 [1].

I. INTRODUCTION

There is much recent research activity aimed at characterizing the degrees of freedom of wireless networks. The topic is of interest due to the novel insights, in particular those related to interference alignment, that have emerged out of this perspective. Interference alignment refers to the design of signals in such a way that they cast overlapping shadows at the receivers where they constitute interference while remaining separable from the interference at the receivers where they are desired. While the capacity benefits of interference alignment schemes have been shown to be substantial, the caveat behind most of these results has been the assumption of perfect, and sometimes global, channel state information at the transmitters (CSIT). Indeed, in the absence of channel knowledge, especially with i.i.d. isotropic fading models, it is well known that the degrees of freedom of many networks collapse entirely to what is achievable simply by orthogonal time-division among users [2], [3], [4], [5]. However, if the fading is not i.i.d. isotropic, the possibility of interference alignment without the knowledge of channel realizations at the transmitters, becomes very intriguing and very little is known about the network degrees of freedom. A common trend is discernible in the diverging inner and outer bounds for various networks under such scenarios [6], [7], [8]. While the best known inner bounds predict a total collapse of degrees of freedom, the best known outer bounds remain fairly robust to channel uncertainty. A few representative examples extracted from prior work, that illustrate this trend, are listed below.

1. **MISO BC with no CSIT for one User:** Consider the multiple input single output (MISO) broadcast channel (BC) with 2 antennas at the transmitter and 2 receivers (users), each equipped with a single antenna. The channel state of one user is known perfectly to the transmitter while the other users' channel states are unknown. Assuming that the channel states are generic and fixed throughout the duration of communication, the best known outer bound on the total DoF, obtained in [7], is equal to $\frac{3}{2}$. However, the best known inner bound on the total DoF if User 2's channel state is drawn from some continuous distribution, is only 1 - which is trivially achieved by orthogonal time-division between users. The inner bound is conjectured to be optimal.

2. **MISO BC with no CSIT for both Users:** For the same 2 user MISO BC setting, if the channel states of both users are unknown to the transmitter, and held fixed, then the best known outer bound on the total DoF, also found in [7] is $\frac{4}{3}$. If the channel states are time-varying, the best known outerbound on the total DoF, derived in [6] is still equal to $\frac{4}{3}$. In both cases the best inner bound is only 1. In both cases, the inner bound is conjectured to be tight.

3. **X Channel:** The best known outer bound on the total DoF of the X channel with 2 transmitters and 2

receivers, whether the channel is time-varing or held constant, follows from the arguments as presented in [9] and is equal to $\frac{4}{3}$, same as with perfect channel knowledge. The best known inner bound in this case is also only 1.

4. **MIMO Interference Channel:** A similar open problem is pointed out in the context of the 2 user time-varying MIMO interference channel studied in [8]. Consider the setting with $1, 3$ antennas at transmitters $1, 2$ and $2, 4$ antennas at receivers $1, 2$ respectively. If User 1 achieves 1 DoF (his maximum possible DoF), then what is the maximum DoF simultaneously achievable by User 2? The outer bound for the DoF achieved by User 2 found in [8] is $\frac{3}{2}$ but the best known achievable DoF for User 2 is only equal to 1, which is achieved by simple zero forcing at the receivers.

5. **Interference Network:** The best known outer bound on the total DoF of the interference channel with K transmitters and K receivers, whether the channel is time-varing or held constant, follows from the arguments as presented in [10] and is equal to $\frac{K}{2}$, same as with perfect channel knowledge. The best known inner bound with channel uncertainty in this case is also only 1.

Note that in all these cases the channel coefficients are assumed to be generic, i.e., drawn from a continuous distribution, and their values are assumed known to the receivers perfectly. The DoF inner and outer bounds mentioned above are meant in the "almost surely" sense, i.e., for almost all values of channel coefficients. If the channel uncertainty at the transmitters is not spread over a continuum, especially if the channel coefficient values are drawn from a set of finite cardinality, the results can be quite different from the pessimistic conjectures that favor the inner bounds [11].

This paper is motivated by the need to find *robust* alignment schemes with the understanding that an interference alignment scheme is robust if the alignment is achieved even with the transmitters remaining oblivious of the values of channel coefficients which may be drawn from a continuum. The main contribution of the paper is to introduce the first blind interference alignment scheme, i.e., an interference alignment scheme that does not need knowledge of channel realizations at the transmitters. The scheme is enabled by the following new insight.

New Insight — *Even if the transmitters have no knowledge of channel coefficient values, they can still align interference under heterogeneous block fading models based on the knowledge of the distinct coherence-intervals in time/frequency dimensions associated with different users.*

That channel coherence intervals play a significant role determining the capacity of a network, even under the assumption of no CSIT and perfect CSIR, is a surprising observation. For example, the single user point to point wireless channel, under the assumption of perfect CSIR and no CSIT, has the same capacity under the block fading model (with i.i.d. variations across blocks) *regardless of the value of channel coherence interval*. However, surprisingly, for networks, not only is the coherence interval important, but so much so that it determines the degrees of freedom of the network and even the outer bounds considered unachievable in the benchmark scenarios outlined above, are achievable. Moreover, the coding scheme proposed in this paper to achieve this alignment with no CSIT has several desirable features as highlighted below.

- No knowledge of the values of channel coefficients is required at the transmitters. This is in contrast to the infinite precision channel knowledge needed at transmitters to achieve the DoF outer bounds, for almost all interference alignment schemes proposed so far.
- The encoding is a simple linear beamforming scheme. This is in contrast to sophisticated lattice alignment schemes used in, e.g., [12], [13].
- The size of supersymbols and the number of independently encoded streams over which beamforming takes place is small. This is in contrast to the long symbol extensions needed for the alignment scheme in [10].
- Alignment is achieved over very few channel coherence times. This is in contrast to the very long waiting times required for pairing complementary matrices in the ergodic interference alignment scheme of [14].
- No knowledge of *any* channel coefficient values is needed even at the receivers in order to cancel the interference. However, after cancelling interference, in order to perform coherent detection of desired signals (and for the DoF metric to be meaningful) the receivers do need to know their own channel states. Note that the receivers do not need to learn the channel coefficient values associated with other receivers[1]

II. Channel Model - Heterogeneous Block Fading

We will assume a block fading model in both time and frequency, where the channel states are constant for a time duration T_c, known as the channel coherence time, and over a frequency interval B_c known as a coherence bandwidth, and then switch to a different generic (i.e. drawn from a continuous distribution but not necessarily independent) value. Unless explicitly mentioned otherwise, the channel realizations are assumed to be known perfectly to the receivers and not known to the transmitters. Since we are

[1]This is in contrast with the more recently developed idea of retrospective interference alignment [15], [16] where each receiver also needs to learn the channel coefficient values associated with other receivers.

interested in interference alignment, it is important to recall that the enabling premise of interference alignment is the *relativity of alignment*, i.e., receivers should be statistically distinguishable as seen from the transmitters. Otherwise, if the receivers are statistically equivalent it is easy to show that the DoF do collapse to unity (assuming single antenna receivers) as all messages can be decoded by the same receiver [2], [3]. In this paper we will assume this relativity of alignment comes from the heterogeneous nature of the block fading process, so that different users experience different coherence time and coherence bandwidth intervals.

With a few exceptions towards the end (the interference channels) that will be pointed out explicitly, we will focus on communication scenarios with two receivers where one receiver experiences a fading process with a smaller coherence time while the other receiver experiences a smaller coherence bandwidth. To highlight this distinction between the two users, we will identify User 1, the user with the smaller coherence time as the *time-selective user* and User 2, the user with the smaller coherence bandwidth, as the *frequency-selective user*. Since coherence time is related to the mobility and coherence bandwidth is related to multipath delay spread, evidently this setting implies that the time-selective user is more mobile and in a less cluttered environment so that he experiences less multipath (e.g., only line of sight propagation), whereas the frequency-selective user is less mobile and in a more cluttered environment, i.e., experiences more multipath.

Since the heterogeneous block fading channel model is essential to this work, an explicit illustration is provided in Fig. 1 as an example. The top of the figure shows the time-frequency plane. The coherence times ($T_c^{[1]}, T_c^{[2]}$) and coherence bandwidths ($B_c^{[1]}, B_c^{[2]}$) of the two users are indicated along the time/frequency axes so that the time-selective user (User 1) has a smaller coherence time, and the frequency-selective user (User 2) has a smaller coherence bandwidth. Within the time-frequency plane, the time-frequency tiles over which communication takes place are indicated as the numbered squares. The numbered tiles collectively define a supersymbol for the purpose of interference alignment schemes. For this example let us focus on the 8 numbered time-frequency tiles, corresponding to 8 channel uses that constitute a supersymbol. The bottom of the figure shows the resulting supersymbol structure which is the most crucial element of the interference alignment schemes proposed in this work. Since time-frequency tiles $1, 4, 7$ occur within the same coherence interval for User 1 in both time and frequency, the user's channel state (indicated in the figure as $h^{[1]}(1)$) remains identical over these three channel uses. Note, however, that User 2's channel state changes each time (indicated in the figure as $h^{[2]}(1), h^{[2]}(2), h^{[2]}(3)$) over these three channel uses, because time-frequency tiles $1, 4, 7$ are separated by more than the coherence

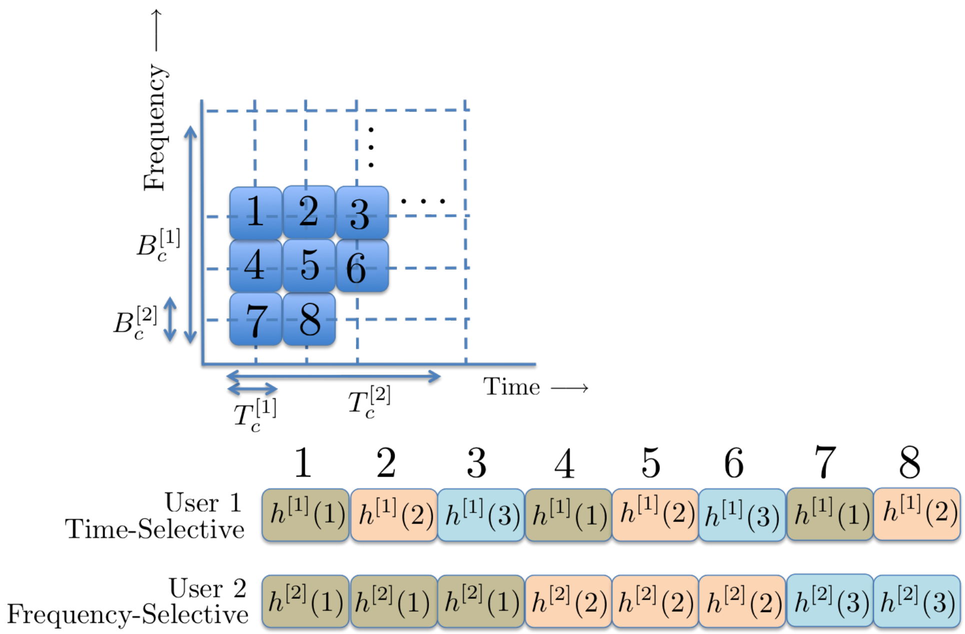

Fig. 1. Heterogeneous Block Fading Model and the Resulting Supersymbol Structure

bandwidth for User 2. Similarly, over time-frequency tiles $1, 2, 3$, User 2's channel remains unchanged but User 1's channel state changes each time. The remainder of the supersymbol structure can be similarly verified to follow from the heterogeneous block fading model shown at the top of the figure.

We assume throughout that the heterogeneous block fading model is known perfectly to the transmitters, i.e., the transmitters know the values of the users' coherence times and coherence bandwidths. The transmitter also knows the continuous distributions according to which the channel coefficient values are drawn. However, most importantly, the specific realizations of channel coefficients are *not known* to the transmitter(s).

Our goal in the next few sections is to consider each of the representative problems highlighted in the introduction and to show, in each case, that the outer bound is achievable with only the statistical knowledge of the heterogeneous block fading model and no knowledge of instantaneous channel realizations.

We expect the reader to be very familiar with the Shannon-theoretic definitions of achievable rates, capacity, degrees of freedom, etc., which are used in this work only in the standard sense. Therefore, we will proceed directly to the technical content.

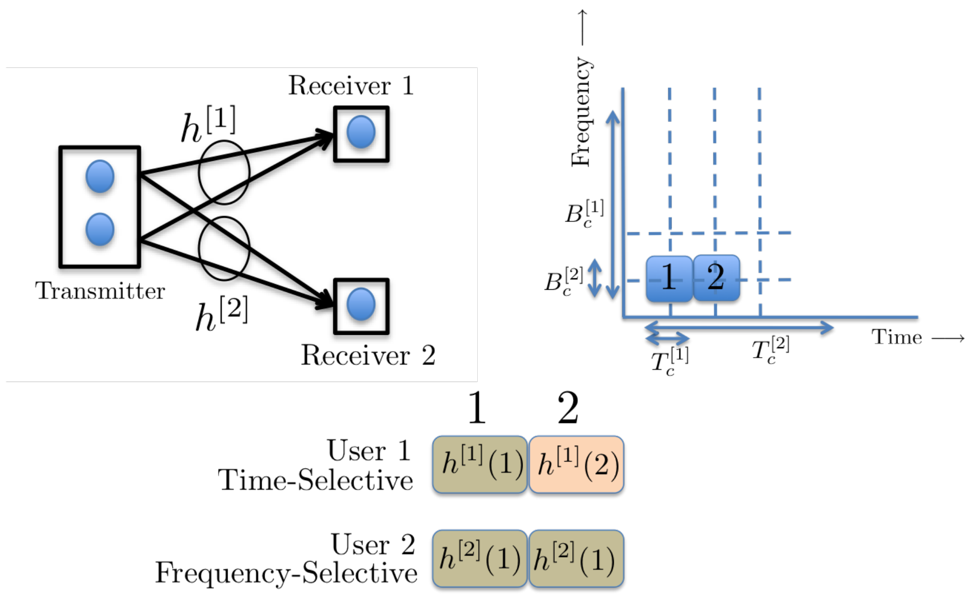

Fig. 2. 2 User MISO BC and the Channel Coherence Structure over a Supersymbol

III. MISO BC with no CSIT for One User

Consider the MISO BC, as shown in Fig. 2, with two antennas at the transmitter and two users with single antenna each, under a heterogeneous block fading model. The channel realization of User 1, $h^{[1]}(n) = [h_1^{[1]}(n) \quad h_2^{[1]}(n)]$, is known perfectly to the transmitter while the channel realization of User 2, defined similarly, *is unknown to the transmitter*. Perfect CSIR is assumed at both receivers. The main result for this model is stated in the following theorem.

Theorem 1: The 2 user MISO BC, under the heterogeneous block fading model, with full CSIT for one user and no CSIT for the other user, has $\frac{3}{2}$ DoF, almost surely.

Proof: That $\frac{3}{2}$ is the DoF outer bound found for this channel follows from the corresponding outer bound established for the finite state compound network setting in [7]. Note that an outer bound for the finite state compound network setting is automatically an outer bound for our setting because in our case the channel uncertainty for User 2's channel is spread over a continuum (i.e., a set of infinite cardinality) instead of a set of finite cardinality.

Next we describe the interference alignment scheme that achieves $\frac{3}{2}$ DoF. We define a supersymbol as comprised of two symbols in the manner shown in Fig. 2. Thus, User 1's channel state changes over the supersymbol while User 2's channel state maintains a fixed value over a supersymbol.

The received signals of the two users are expressed as follows.

$$\begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2) \end{bmatrix} = \begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) & 0 & 0 \\ 0 & 0 & h_1^{[1]}(2) & h_2^{[1]}(2) \end{bmatrix} \begin{bmatrix} x_1(1) \\ x_2(1) \\ x_1(2) \\ x_2(2) \end{bmatrix} + \begin{bmatrix} z^{[1]}(1) \\ z^{[1]}(2) \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} y^{[2]}(1) \\ y^{[2]}(2) \end{bmatrix} = \begin{bmatrix} h_1^{[2]}(1) & h_2^{[2]}(1) & 0 & 0 \\ 0 & 0 & h_1^{[2]}(1) & h_2^{[2]}(1) \end{bmatrix} \begin{bmatrix} x_1(1) \\ x_2(1) \\ x_1(2) \\ x_2(2) \end{bmatrix} + \begin{bmatrix} z^{[2]}(1) \\ z^{[2]}(2) \end{bmatrix} \quad (2)$$

The symbol y is used for received signals, h for channel coefficients, x for input signals and z for additive white Gaussian noise (AWGN). The superscript within square parentheses indicates the user index, the subscript is the antenna index and the index within the round parentheses is the time index within the supersymbol. In compact notation we write equivalently:

$$\mathbf{Y}^{[1]} = \mathbf{H}^{[1]}\mathbf{X} + \mathbf{Z}^{[1]} \quad (3)$$

$$\mathbf{Y}^{[2]} = \mathbf{H}^{[2]}\mathbf{X} + \mathbf{Z}^{[2]} \quad (4)$$

Our goal is to send 2 DoF to User 1 and 1 DoF to User 2, for a total of 3 DoF. Since this is accomplished over two channel uses (time-frequency tiles), the normalized total DoF value is $\frac{3}{2}$. The achievable scheme is based on simple linear beamforming. The transmitted signal is constructed as:

$$\mathbf{X} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \begin{bmatrix} h_2^{[1]}(1) \\ -h_1^{[1]}(1) \\ 0 \\ 0 \end{bmatrix} u^{[2]} \quad (5)$$

Here $u_1^{[1]}, u_2^{[1]}$ are the independently encoded scalar Gaussian codewords for User 1, each carrying one DoF, while $u^{[2]}$ is the independently encoded scalar Gaussian codeword for User 2, also carrying one DoF. With this coding scheme, the received signals at User 1 becomes:

$$\begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2) \end{bmatrix} = \begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) \\ h_1^{[1]}(2) & h_2^{[1]}(2) \end{bmatrix} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \begin{bmatrix} z^{[1]}(1) \\ z^{[1]}(2) \end{bmatrix} \quad (6)$$

Thus User 1 sees no interference from User 2, and accesses a full rank 2×2 MIMO channel through which he is able to achieve 2 DoF. Now consider the received signal of User 2.

$$\begin{bmatrix} y^{[2]}(1) \\ y^{[2]}(2) \end{bmatrix} = \begin{bmatrix} h_1^{[2]}(1) & h_2^{[2]}(1) \\ h_1^{[2]}(1) & h_2^{[2]}(1) \end{bmatrix} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \end{bmatrix} u^{[2]} + \begin{bmatrix} z^{[2]}(1) \\ z^{[2]}(2) \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} (h_1^{[2]}(1)u_1^{[1]} + h_2^{[2]}(1)u_2^{[1]}) + \begin{bmatrix} \alpha \\ 0 \end{bmatrix} u^{[2]} + \begin{bmatrix} z^{[2]}(1) \\ z^{[2]}(2) \end{bmatrix} \tag{8}$$

where $\alpha = h_1^{[2]}(1)h_2^{[1]}(1) - h_2^{[2]}(1)h_1^{[1]}(1) \neq 0$, almost surely. Thus, the two streams carrying User 1's signal align into one dimension at User 2's receiver, leaving the remaining dimension to achieve 1 DoF for his desired signal. In this case, a simple projection of the received signal along the vector $[1 \quad -1]$ provides the interference free signal needed to achieve the desired 1 DoF.

$$y^{[2]} = y^{[2]}(1) - y^{[2]}(2) = \alpha u^{[2]} + z^{[2]}(1) - z^{[2]}(2) \tag{9}$$

Thus, User 1 achieves 2 DoF and User 2 achieves 1 DoF as desired. Note that the transmitter does not know User 2's channel coefficients at all. Moreover, even for User 1, whose channel is known to the transmitter perfectly, note that only the knowledge of the channel coefficient values over the first coherence interval is used. In other words, the transmitter does not need to know even User 1's channel coefficients over the second coherence interval.

IV. MISO BC WITH NO CSIT FOR BOTH USERS

Consider the same 2 user MISO BC channel as in the previous section, with one exception — now we assume that the channel realizations of *both* users are unknown to the transmitter.

The main result for this model is stated in the following theorem.

Theorem 2: The 2 user MISO BC, under the heterogeneous block fading model and with no CSIT, has $\frac{4}{3}$ DoF, almost surely.

Proof: Once again the $\frac{4}{3}$ DoF outer bound follows directly from the outer bound found for the corresponding scenario in the finite state compound network setting in [7]. A constructive achievability proof is presented next.

In this case we define the supersymbol as comprised of three symbols. As shown in Fig. 3 the 3 symbols are chosen such that the channel state of User 1 changes after the first symbol and returns to the original value after the second symbol, while the channel state of User 2 is fixed for the first 2 symbols

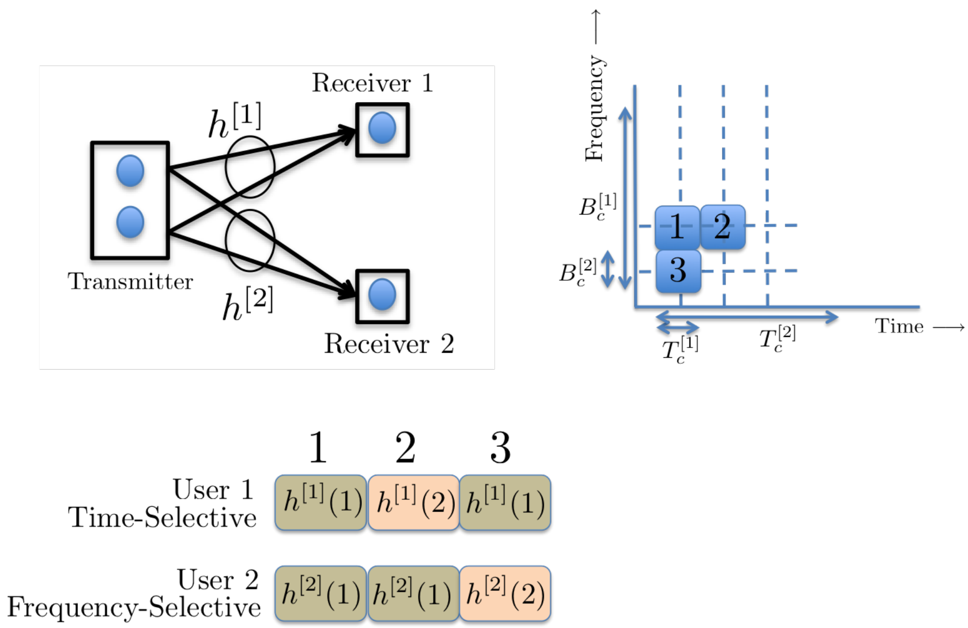

Fig. 3. Supersymbol Structure for Theorem 2

and changes in the last symbol. The received signals over one supersymbol defined in this manner, are expressed as follows.

$$\begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2) \\ y^{[1]}(3) \end{bmatrix} = \begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) & 0 & 0 & 0 & 0 \\ 0 & 0 & h_1^{[1]}(2) & h_2^{[1]}(2) & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1^{[1]}(1) & h_2^{[1]}(1) \end{bmatrix} \begin{bmatrix} x_1(1) \\ x_2(1) \\ x_1(2) \\ x_2(2) \\ x_1(3) \\ x_2(3) \end{bmatrix} + \begin{bmatrix} z^{[1]}(1) \\ z^{[1]}(2) \\ z^{[1]}(3) \end{bmatrix}$$

$$\begin{bmatrix} y^{[2]}(1) \\ y^{[2]}(2) \\ y^{[2]}(3) \end{bmatrix} = \begin{bmatrix} h_1^{[2]}(1) & h_2^{[2]}(1) & 0 & 0 & 0 & 0 \\ 0 & 0 & h_1^{[2]}(1) & h_2^{[2]}(1) & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1^{[2]}(2) & h_2^{[2]}(2) \end{bmatrix} \begin{bmatrix} x_1(1) \\ x_2(1) \\ x_1(2) \\ x_2(2) \\ x_1(3) \\ x_2(3) \end{bmatrix} + \begin{bmatrix} z^{[2]}(1) \\ z^{[2]}(2) \\ z^{[2]}(3) \end{bmatrix}$$

Our goal is to achieve two DoF for each user over this supersymbol consisting of 3 symbols, to achieve an overall normalized DoF equal to $\frac{4}{3}$, consistent with the outer bounds in [6], [7] for similar settings.

To accomplish this objective, we construct the input vector $\mathbf{X}$ as follows.

$$\mathbf{X} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix} \tag{10}$$

where $u_1^{[k]}, u_2^{[k]}$ are the independently encoded scalar Gaussian codeword symbols for User $k = 1, 2$ respectively, each carrying one DoF. Note that the beamforming vectors do not depend on the values of channel coefficients. With this scheme the signal at receiver 1 becomes:

$$\begin{aligned} \begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2) \\ y^{[1]}(3) \end{bmatrix} &= \underbrace{\begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) \\ h_1^{[1]}(2) & h_2^{[1]}(2) \\ 0 & 0 \end{bmatrix}}_{\text{rank}=2} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \underbrace{\begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) \\ 0 & 0 \\ h_1^{[1]}(1) & h_2^{[1]}(1) \end{bmatrix}}_{\text{rank}=1} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix} + \begin{bmatrix} z^{[1]}(1) \\ z^{[1]}(2) \\ z^{[1]}(3) \end{bmatrix} \\ &= \begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) \\ h_1^{[1]}(2) & h_2^{[1]}(2) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} (h_1^{[1]}(1)u_1^{[2]} + h_2^{[1]}(1)u_2^{[2]}) + \begin{bmatrix} z^{[1]}(1) \\ z^{[1]}(2) \\ z^{[1]}(3) \end{bmatrix} \end{aligned}$$

Thus in the 3 dimensional received signal space of Receiver 1, the interference from User 2's signal aligns within 1 dimension, while the desired signals, carrying 2 DoF, occupy two linearly independent dimensions. It remains to check that the two desired signal dimensions do not overlap with the one interference dimension. This is easily verified as the determinant of the matrix

$$\begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) & 1 \\ h_1^{[1]}(2) & h_2^{[1]}(2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

is not equal to zero almost surely. In other words, User 1 is able to achieve 2 DoF. By symmetry the same arguments can be used to show that User 2 achieves 2 DoF as well, so that the desired $\frac{4}{3}$ (normalized) DoF are achieved.

V. The X Channel with no CSIT

The X channel that we consider in this section, consists of two transmitters and two receivers, each equipped with a single antenna and 4 independent messages, one for each transmitter-receiver pair. Note that if we separate the two antennas at the transmitter of the MISO BC considered in the previous section to form two separate transmitters, i.e., if we do not allow joint processing of signals or common knowledge of messages at the two transmit antennas of the MISO BC, then we obtain the X channel. The remaining assumptions — no CSIT, perfect CSIR — and the heterogeneous block fading model, are the same as the MISO BC in the previous section. The channel input output relationships are also the same as the previous section. The DoF result for the X channel is stated in the following theorem.

Theorem 3: The X channel, under the heterogeneous block fading model and with no CSIT, has $\frac{4}{3}$ DoF, almost surely.

Proof: Note that $\frac{4}{3}$ is the DoF outer bound found even with perfect CSIT. Therefore it is also an outer bound with no CSIT.

The proof of Theorem 3 follows trivially from the proof of Theorem 2. Note that no cooperation between the two transmit antennas is needed for the achievable scheme of the 2 user MISO BC with no CSIT described in the previous section. Thus, the same achievable scheme can be applied directly for the 2 user X channel as well. In both cases $\frac{4}{3}$ DoF are achieved. Note that this insight is consistent with the result from the finite state compound setting found in [11], where also it is found that with enough channel uncertainty, the MISO BC devolves into the X channel as the DoF benefits of joint processing across transmit antennas are lost.

VI. Generalization - the $M \times 2$ X Channel with no CSIT

Consider an $M \times 2$ X channel, i.e., an X channel with M transmitters and 2 receivers as shown in Fig. 4, where each transmitter has 2 independent messages, one for each receiver. The remaining assumptions are the same as before, i.e., Receiver 1 experiences time-selective fading, while Receiver 2 experiences frequency-selective fading, and there is no knowledge of channel realizations at the transmitters. The DoF for this channel are presented in the following theorem.

Theorem 4: The $M \times 2$ X channel, under the heterogeneous block fading model and with no CSIT, has $\frac{2M}{M+1}$ DoF, almost surely.

Proof: The outer bound is straightforward since it is the same outer bound as when perfect channel knowledge is available to the transmitters [17].

The achievability argument generalizes the proofs presented in the previous sections and also illuminates the essential principle behind these blind interference alignment schemes. On the right side of Fig. 4 is illustrated the supersymbol structure in the time-frequency plane. As shown in the figure, the transmissions

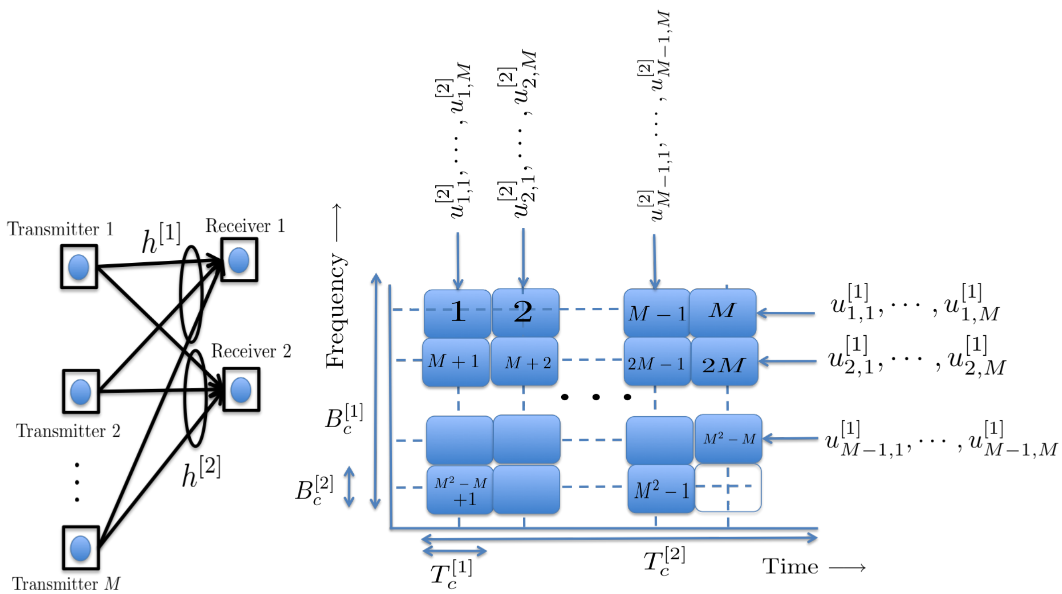

Fig. 4. $M \times 2$ X Channel and the Corresponding Interference Alignment Scheme

occur over an $M \times M$ grid in the time-frequency plane, with the exception of one last tile on the bottom right corner that is not a part of this super-symbol. Thus, the total number of channel uses (tiles in the time-frequency plane) is $M^2 - 1$. Over these $M^2 - 1$ time-frequency dimensions, each transmitter sends a total of $2(M-1)$ independent information symbols, half of which are intended for Receiver 1 and the other half for Receiver 2. Thus, the goal is to convey a total of $2M(M-1)$ independent information symbols, each carrying 1 DoF, over $M^2 - 1$ time-frequency dimensions, so that a normalized DoF value of $\frac{2M(M-1)}{M^2-1} = \frac{2M}{M+1}$ is achieved per channel use.

To understand how this is accomplished, let us first consider the symbols intended for Receiver 1. Consider the first row of the time-frequency space, consisting of tiles number $1, 2, \cdots, M$. Of the $M-1$ symbols that each transmitter wants to send to Receiver 1, only the first symbol is transmitted by each transmitter, repeatedly, over each of these M channel uses. Thus, M symbols intended for Receiver 1, one from each transmitter, are transmitted over these M dimensions, through simple repetition. Since Receiver 1's channel changes every time in a generic manner over these M channel uses, Receiver 1 observes a different linear combination of these M desired symbols each time. Thus, over these M channel uses, Receiver 1 obtains M linearly independent combinations of the desired M symbols. At the same time, the unintended receiver, Receiver 2, only sees the same linear combination of undesired symbols repeated M times. This is because the channel experienced by Receiver 2 does not change over time-frequency tiles $1, 2, \cdots, M$, because all of these channel uses are located within the same coherence interval for Receiver 2. Thus, these M symbols intended for Receiver 1, occupy an M dimensional space at Receiver 1, but only a 1 dimensional space at Receiver 2.

Similarly, consider the second row of the time-frequency tiles grid that is a part of the super-symbol. Over these M tiles numbered $M+1, M+2, \cdots, 2M$, each transmitter once again repeatedly transmits its second symbol intended for Receiver 1. Receiver 1 sees M linearly independent combinations of these M symbols, while the unintended receiver, Receiver 2, only sees the same linear combination of unintended symbols repeated M times. Once again, these M symbols intended for Receiver 1 occupy an M dimensional space at the desired receiver, and only a 1 dimensional space at the undesired receiver.

Similarly, over each of the first $M-1$ rows, each transmitter transmits one of the $M-1$ symbols intended for Receiver 1, repeated a total of M times (since each of these rows consists of M channel uses). As a result, a total of $M(M-1)$ desired symbols for Receiver 1 occupy a total of $M(M-1)$ dimensions at the desired receiver, while they occupy only $M-1$ dimensions at the undesired receiver.

Now let us consider the symbols intended for Receiver 2. Consider the first column of the time-frequency grid consisting of tiles numbered $1, M+1, 2M+1, \cdots, (M-1)M+1$. Over these M channel uses, Receiver 1 sees the same channel every time, and Receiver 2 sees a different generic channel each time. Of the $M-1$ symbols intended for Receiver 2 that originate at each transmitter, the transmitter sends only the first symbol repeated a total of M times across these channel uses. Thus, Receiver 2 sees a different linear combination of these M symbols (one from each transmitter) each time, while Receiver 1 sees the same linear combination repeated M times. As a result, these M symbols that are intended for Receiver 2 span a M dimensional space at Receiver 2 and only a 1 dimensional space at Receiver 1. Following the same reasoning for each of the first $M-1$ columns of the time-frequency grid, we find that each transmitter sends a total of $M-1$ symbols intended for Receiver 2, one over each column, such that the symbols from different transmitters are seen in different linear combinations at Receiver 2 each time, but appear in the same linear combination at Receiver 1 each time. Thus, these $M(M-1)$ symbols intended for Receiver 2 occupy $M(M-1)$ dimensions at Receiver 2 and only $M-1$ dimensions at Receiver 1.

While we described the transmission of symbols intended for Receiver 1 and 2 separately, what is effectively transmitted over each time/frequency tile is the superposition of the symbols intended for Receiver 1 and for Receiver 2, as determined by the pattern described above. Mathematically, Transmitter k, $k = 1, 2, \cdots, M$ wants to send symbols $u_{i,k}^{[1]}$, $i = 1, 2, \cdots, M-1$, to Receiver 1 and symbols $u_{j,k}^{[2]}$,

$j = 1, 2, \cdots, M-1$, to Receiver 2. What Transmitter k actually sends over the time-frequency tile corresponding to the i^{th} row and j^{th} column of the supersymbol grid, is the superposition $u^{[1]}_{i,k} + u^{[2]}_{j,k}$.

Since desired signals occupy a total of $M(M-1)$ dimensions at each receiver and interference occupies $M-1$ dimensions and the total size of the signal space is M^2-1, the generic nature of channel realizations guarantees the linear independence of desired signal space from the interference space, and all desired signals can be resolved from the interference. Thus, a normalized value of $\frac{2M}{M+1}$ DoF is achieved. ■

To conclude this section, we note that $\frac{2M}{M+1}$ is also the DoF outer bound for the MISO Broadcast Channel obtained by allowing all M transmitters in the $M \times 2$ X channel to fully cooperate as antennas located on the same device. This is because the finite state compound MISO BC with M antennas at the transmitter and a single antenna at each of the two receivers, has only $2M/(M+1)$ DoF when the number of states exceeds M. In our case, the number of possible states is drawn from a continuum, i.e., has infinite cardinality, and therefore the compound MISO BC outer bound applies. Thus, we automatically have the following DoF result.

Theorem 5: The 2 user MISO Broadcast Channel with M antennas at the transmitter and 1 antenna at each receiver, under the heterogeneous block fading channel model and with no CSIT, has $\frac{2M}{M+1}$ DoF, almost surely.

VII. The 2 User MIMO Interference Channel

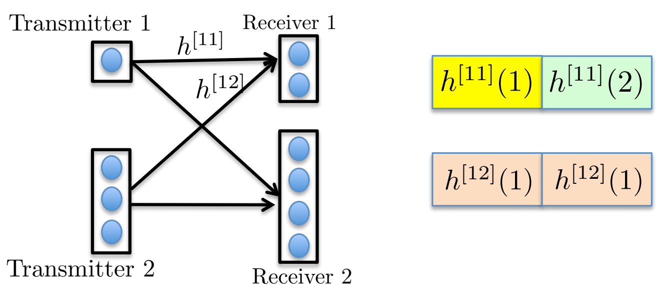

Fig. 5. MIMO Interference Channel and the Supersymbol Structure

This example is based on the open problem in [8]. We have a two user MIMO interference channel with 1 and 3 antennas at the transmitters and 2 and 4 antennas at their corresponding receivers, respectively. As in all preceding sections we assume no CSIT, perfect CSIR, and a heterogeneous block fading model. However, there is one important distinction in the fading model for this case. Instead of identifying

Receiver 1 as time-selective and Receiver 2 as frequency-selective, we identify Transmitter 1 as time-selective and Transmitter 2 as frequency-selective, as seen by Receiver 1. For the problem we are interested in, it does not matter what correlation model is seen by Receiver 2. Since the key to this problem is the possibility of interference alignment only at Receiver 1 — Receiver 2 has enough antennas to separate all signal and interference — it is not surprising that only the fading block structure at Receiver 1 is of significance.

The question posed in [8] is the following — what is the maximum DoF achievable by User 2 simultaneously as User 1 achieves his maximum (one) DoF ? The best outer bound found in [8] is $\frac{3}{2}$ but the best inner bound is only able to achieve 1 DoF for User 2.

The DoF result for this channel is presented in the following theorem.

Theorem 6: For the 2 user MIMO interference channel defined in this section, users 1 and 2 can simultaneously achieve 1 and $\frac{3}{2}$ DoF, respectively, almost surely.

Note that this result also matches the outer bound found in [8]. The key to this achievability, as suggested in [8] is interference alignment at Receiver 1. A constructive proof follows next.

Proof: Our goal is for User 1 to achieve 2 DoF and for User 2 to achieve 3 DoF over this two symbol extension, which corresponds to normalized values of 1 and $\frac{3}{2}$, respectively. Interference alignment is needed to accomplish this objective and like all previous examples we rely on linear beamforming techniques.

For this example, we define a supersymbol as comprised of two dimensions, which leads to the structure shown in Fig. 5. Within a supersymbol, the signal at Receiver 1 is expressed as:

$$\begin{bmatrix} y_1^{[1]}(1) \\ y_2^{[1]}(1) \\ y_1^{[1]}(2) \\ y_2^{[1]}(2) \end{bmatrix} = \begin{bmatrix} h_{11}^{[12]}(1) & h_{12}^{[12]}(1) & h_{13}^{[12]}(1) & 0 & 0 & 0 \\ h_{21}^{[12]}(1) & h_{22}^{[12]}(1) & h_{23}^{[12]}(1) & 0 & 0 & 0 \\ 0 & 0 & 0 & h_{11}^{[12]}(1) & h_{12}^{[12]}(1) & h_{13}^{[12]}(1) \\ 0 & 0 & 0 & h_{21}^{[12]}(1) & h_{22}^{[12]}(1) & h_{23}^{[12]}(1) \end{bmatrix} \begin{bmatrix} x_1^{[2]}(1) \\ x_2^{[2]}(1) \\ x_3^{[2]}(1) \\ x_1^{[2]}(2) \\ x_2^{[2]}(2) \\ x_3^{[2]}(2) \end{bmatrix}$$

$$+ \begin{bmatrix} h_{11}^{[11]}(1) & 0 \\ h_{21}^{[11]}(1) & 0 \\ 0 & h_{11}^{[11]}(2) \\ 0 & h_{21}^{[11]}(2) \end{bmatrix} \begin{bmatrix} x^{[1]}(1) \\ x^{[1]}(2) \end{bmatrix} + \begin{bmatrix} z_1^{[1]}(1) \\ z_2^{[1]}(1) \\ z_1^{[1]}(2) \\ z_2^{[1]}(2) \end{bmatrix} \tag{12}$$

Equivalently, in compact notation

$$\mathbf{Y}^{[1]} \quad = \quad \mathbf{H}^{[12]}\mathbf{X}^{[2]} + \mathbf{H}^{[11]}\mathbf{X}^{[1]} + \mathbf{Z}^{[1]}. \tag{13}$$

The key to the alignment is to design User 2's signal as follows.

$$\mathbf{X}^{[2]} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \\ u_3^{[2]} \end{bmatrix} \tag{14}$$

where $u_i^{[1]}$ is the i^{th} independently coded scalar stream sent from Transmitter 2. Each stream carries 1 DoF. With this coding scheme, the signal at Receiver 1 becomes

$$\mathbf{Y}^{[1]} \quad = \quad \underbrace{\begin{bmatrix} h_{11}^{[12]}(1) & h_{12}^{[12]}(1) & h_{13}^{[12]}(1) \\ h_{21}^{[12]}(1) & h_{22}^{[12]}(1) & h_{23}^{[12]}(1) \\ h_{11}^{[12]}(1) & h_{12}^{[12]}(1) & h_{13}^{[12]}(1) \\ h_{21}^{[12]}(1) & h_{22}^{[12]}(1) & h_{23}^{[12]}(1) \end{bmatrix}}_{\text{rank}=2} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \\ u_3^{[2]} \end{bmatrix} + \begin{bmatrix} h_{11}^{[11]}(1) & 0 \\ h_{21}^{[11]}(1) & 0 \\ 0 & h_{11}^{[11]}(2) \\ 0 & h_{21}^{[11]}(2) \end{bmatrix} \begin{bmatrix} x^{[1]}(1) \\ x^{[1]}(2) \end{bmatrix} + \begin{bmatrix} z_1^{[1]}(1) \\ z_2^{[1]}(1) \\ z_1^{[1]}(2) \\ z_2^{[1]}(2) \end{bmatrix}$$

The interference alignment is manifested in the rank deficiency of the 4×3 effective channel matrix between Transmitter 2 and Receiver 1. Note that the first and third rows are identical, as are the second and fourth rows. Thus this matrix has rank only 2. Equivalently, the interference space seen by Receiver 1 is spanned by the first two columns of this matrix. In a 4 dimensional received space at Receiver 1, since interference spans only two dimensions, the remaining 2 dimensions are available to achieve its desired 2 DoF. However, we must ensure that the desired signals arrive along linearly independent directions from the interference. In other words, the following matrix must be full rank.

$$\begin{bmatrix} h_{11}^{[11]}(1) & 0 & h_{11}^{[12]} & h_{12}^{[12]} \\ h_{21}^{[11]}(1) & 0 & h_{21}^{[12]} & h_{22}^{[12]} \\ 0 & h_{11}^{[11]}(2) & h_{11}^{[12]} & h_{12}^{[12]} \\ 0 & h_{21}^{[11]}(2) & h_{21}^{[12]} & h_{22}^{[12]} \end{bmatrix} \tag{15}$$

Here the first two columns span the desired signal space while the last two columns span the interference space. It is easy to compute the determinant of this matrix, which is seen to be non-zero as long as $(h_{11}^{[11]}(1), h_{21}^{[11]}(1)) \neq (h_{11}^{[11]}(2), h_{21}^{[11]}(2))$, i.e. the channel $h^{[11]}$ changes from one coherence block to another. Since this is true almost surely (by definition of the block fading model), User 1 is able to achieve his maximum 2 DoF over 2 symbols. The achievability of User 2's three DoF is straightforward, because with 4 receive antennas, Receiver 2 is able to invert the channel from both transmitters simultaneously, which allows it to separate the two users' signals, regardless of the coherence times.

VIII. The K User Interference Channel

The final interference alignment problem that we consider in this paper is the K user interference channel, comprised of K transmitters, K receivers, each equipped with a single antenna, and K independent messages, one from each transmitter to its corresponding receiver. As usual we assume no CSIT, perfect CSIR and heterogeneous block fading. Specifically, we assume a rather specialized form of channel coherence structure. All the direct channels (carrying desired signals) are time-selective and all the cross-channels (carrying interference) are frequency-selective. Admittedly this correlation structure is less natural than other cases studied above. Nevertheless it serves to show that interference alignment can be achieved without the knowledge of channel coefficient values in this setting as well.

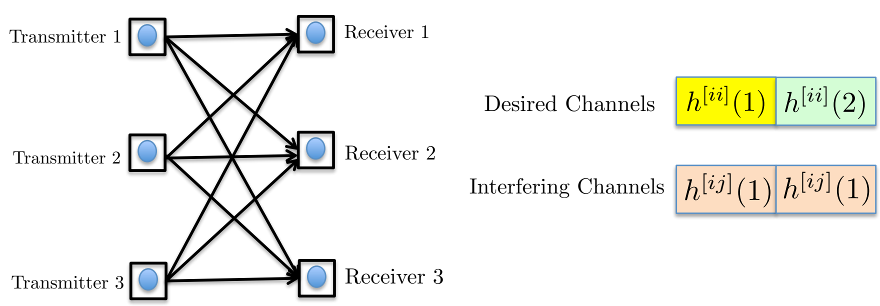

Fig. 6. 3 User Interference Channel and the Supersymbol Structure

A 3 user example is shown in Fig. 6. The DoF result for this setting is stated in the following theorem.

Theorem 7: For the K user interference channel defined in this section, a total of $\frac{K}{2}$ DoF are achievable, almost surely.

Note that $\frac{K}{2}$ is the DoF outer bound even with perfect CSIT [18]. Therefore it is also an outer bound with no CSIT. A constructive achievability proof is presented next.

Proof: For simplicity of exposition we present the proof for the $K = 3$ user interference channel. The extension to $K > 3$ is straightforward.

We define a supersymbol as comprised of two symbols in the manner shown in Fig. 6, i.e., the desired channels change values but the interfering channels are held constant within a supersymbol. The signal received by Receiver 1 over one supersymbol defined in this manner, is expressed as follows.

$$\begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2) \end{bmatrix} = \begin{bmatrix} h^{[11]}(1) & 0 \\ 0 & h^{[11]}(2) \end{bmatrix} \begin{bmatrix} x^{[1]}(1) \\ x^{[1]}(2) \end{bmatrix} + \begin{bmatrix} h^{[12]}(1) & 0 \\ 0 & h^{[12]}(1) \end{bmatrix} \begin{bmatrix} x^{[2]}(1) \\ x^{[2]}(2) \end{bmatrix}$$
$$+ \begin{bmatrix} h^{[13]}(1) & 0 \\ 0 & h^{[13]}(1) \end{bmatrix} \begin{bmatrix} x^{[3]}(1) \\ x^{[3]}(2) \end{bmatrix} + \begin{bmatrix} z^{[1]}(1) \\ z^{[1]}(2) \end{bmatrix}$$

Each user $k = 1, 2, 3$, sends one scalar coded stream $u^{[k]}$ carrying one DoF along the beamforming vector $[1 \quad 1]^T$. The received signal can then be written as:

$$\begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2) \end{bmatrix} = \begin{bmatrix} h^{[11]}(1) \\ h^{[11]}(2) \end{bmatrix} u^{[1]} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} (h^{[12]}(1)u^{[2]} + h^{[13]}(1)u^{[3]}) + \begin{bmatrix} z^{[1]}(1) \\ z^{[1]}(2) \end{bmatrix}$$

Thus, in the two dimensional received signal space of Receiver 1, all the interference aligns along the vector $[1 \quad 1]^T$ and the desired signal, which is received in a linearly independent direction almost surely, can be separated from interference to yield one DoF. By symmetry, the same is true for each receiver and a total of K DoF are achieved over 2 symbols. In other words, the K user interference channel has $K/2$ DoF almost surely.

IX. Conclusion

The main contribution of this work is the idea that heterogeneous coherence intervals in time/frequency associated with different users can be exploited to achieve interference alignment, even when the transmitters have no information about the precise values taken by the channel coefficients, which may be drawn from a continuum of values. Under a heterogeneous block fading model, what is surprising is not only that interference alignment is achieved without CSIT, but also that the alignment schemes are quite simple. The main insight is that when one receiver's channel remains constant to all transmitters, and another receiver's channel changes, then by each transmitter simply repeating its symbol multiple times, the receiver with changing channels is able to see a different linear combination of transmitted symbols each time, whereas the receiver with fixed channels sees the same linear combination repeated every time. This leads to the idea that *transmitters should repeat their symbols over those time-frequency dimensions where the undesired receivers' channels remain fixed and the desired receivers' channels change.* This idea guarantees that desired symbols can be resolved from the linearly independent combinations, while the interfering symbols are aligned into one dimension.

One limitation of this scheme appears to be the need to identify for each user, a dimension along which his channel changes and the other users' channels remain fixed. Time and frequency naturally provide two such dimensions where the heterogeneous nature of channel coherence time and channel coherence bandwidth create opportunities for interference alignment. However, if more than 2 users are to be accommodated, it is not clear how such dimensions can be naturally associated with each user. It is for this reason that in this paper we are limited to two user communication scenarios.

The ideas presented in this work have inspired several more recent works that we briefly describe here. In [19], Wang et. al. show that channel coherence structures that enable interference alignment for any number of users, e.g., the $M \times N$ X channel setting, can be created artificially through staggered switching of reconfigurable antennas which allows the X channel with M transmitters and N receivers to achieve, with no CSIT, a total of DoF = $\frac{MN}{M+N-1}$ which is also the DoF outer bound even with perfect and global channel knowledge. Maddah-Ali and Tse in [15] have shown that interference alignment can be achieved based on only outdated CSIT. The key observation there, as also pointed out originally in this work, is that the interference space can be minimized by only allowing undesired symbols to be heard in the same linear combination each time, while the desired symbols are heard with a different linear combination over different channel uses. By repeating the same linear combination of interference symbols, those symbols are aligned along the all-ones vector at each undesired receiver, hence achieving interference alignment.

While the present work focuses on some limited examples to convey the fundamental idea, a promising direction for future would be to generalize these results to arbitrary channel fading autocorrelation models. In particular, the validity of the Lapidoth-Shamai-Wigger conjecture [6] for the MISO BC is an important benchmark for further progress in this direction. Note that in this work our channel model differs from [6] only in the assumption that channel values within a coherence interval are identical, which makes the differential entropy rate of the channel sequence equal to $-\infty$. Any reasonable departure from the assumption of precisely identical channel coefficients within a coherence block will remove this limitation. However, what remains to be established is if such a departure would necessarily lead to a collapse of all DoF in the absence of CSIT.

Finally, we underscore the generality of the concepts presented here with two observations that show that these blind interference alignment schemes also have applications in seemingly unrelated settings.

1) *Ergodic Interference Alignment without Uniform Phase Assumption:* Ergodic Interference Alignment refers to the idea introduced by Nazer et al. in [14] by which interference alignment is achieved

in a K user interference channel with i.i.d. channel coefficients, uniform phase distribution on each channel coefficient, and global channel knowledge, by a simple repetition of information symbols over two carefully chosen complementary channel states. The complementary channels satisfy the property that the direct channel coefficients $h^{[ii]}$ assume the same values in both channel states, but the cross-channel coefficients $h^{[jk]}, j \neq k$ are the negatives of each other in the two states. Adding the received symbols at the receivers has the effect of adding the $K \times K$ channel matrices for the two complementary states, thereby cancelling all interference while the desired signals add up in phase. A critical assumption of this scheme is the need for symmetric phase distributions which guarantees that for almost every channel state there exists a complementary state that occurs equally frequently so that the two can be paired evenly. However, the blind interference alignment scheme described in this paper for the K user interference channel provides an alternative ergodic interference alignment scheme that does not require any assumptions on the phase distributions. In this scheme, we define two channel states to be complementary if the cross-channel realizations are identical and direct-channel realizations are distinct. Again we use a repetition code over these complementary states, but instead of adding symbols at the receiver, we subtract one received symbol from another. This has the effect of canceling interference while desired signals are not cancelled, thus again achieving $K/2$ DoF, but without the need for symmetric phase distributions. The blind interference alignment schemes of [19] can similarly be directly adopted to achieve ergodic interference alignment for X networks without the need for uniform phase distributions.

2) *Half Rate Feasibility for Multiple Unicasts in Network Coding:* Consider a network coding problem with K sources that wish to send independent information, each to its corresponding destination, from a total of K destination nodes. If the min-cut for each source-destination pair can be normalized to 1, then the half-rate feasibility problem asks whether it is possible for each source-destination pair to simultaneously achieve a rate 0.5. The problem is very challenging even for 3 users and a rather cumbersome set of conditions is obtained for half-rate feasibility for 3 users in [20]. Interestingly, the blind interference alignment scheme for the K user interference channel presents an alternative, rather simple condition for half-rate feasibility for the general K user setting. Simply, half-rate is feasible if the direct channels $m^{[kk]}(\xi)$ (in the notation of [20]), which are polynomials of network coding variables ξ, are not functions of the set of cross channels $m^{[ij]}(\xi)$, i.e., given the cross-channel realizations (i.e., a given choice of network coding coefficients ξ), there exists more than one realization for each direct channel. If this condition is satisfied, the new ergodic

alignment scheme described above, which is inspired by the blind interference alignment scheme proposed in this work, can be directly applied to simultaneously achieve rate-half for every user.

Beyond these two examples, we expect that there may be a variety of settings which give rise to opportunities for blind interference alignment schemes to be applicable. The search for such settings is an interesting direction for future work.

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