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Stochastic Optimal Load Shedding with Heterogeneous Load Zones

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***Abstract*—Reliability and resilience in power distribution networks are vitally important to ensure seamless electricity delivery to end users. In this paper, we provide an optimization framework for stochastic optimal load shedding in power distribution networks that consists of heterogeneous load zones. Considering uncertain estimates of power consumption and energy shortfall, we formulate the system planning task as a chance-constrained integer quadratic program. By leveraging binary decomposition and McCormick relaxations, we develop two efficient algorithms yielding minimum total load shedding cost while respecting fairness among different end users. Simulation results corroborate the merits of our proposed framework and algorithms, which outperform the off-the-shelf solver BARON.**

I. Introduction

By turning off service or cutting back the supply voltage, load shedding (a.k.a. rolling blackout) is the last but indispensable resort when power generation along with transmission & distribution systems cannot meet the demand. Load shedding regularly occurs in many underdeveloped regions, which suffer from insufficient generation capacity or aging transmission infrastructure [1], [2]. For developed countries, load shedding can happen in exceptional situations due to rare natural disasters [3], as well as economic forces at the expense of system reliability (e.g., the California electricity crisis of 2000–2001).

Under various risks of possible load surge, power degeneration/failure, and multi-hazard threats, proactive load shedding must be meticulously planned in order to prevent uncontrolled service disruptions and equipment damage. In [4], dynamic and static models of load shedding are developed in power systems with distributed generation. Operation and security constraints such as the power flow equations are incorporated in the resulting optimization problem. A load shedding flowchart is proposed for an islanded microgrid based on frequency and rate of change of frequency [5]. Impacts of island load shedding and restoration strategies on reliability of microgrids are studied via the sequential Monte Carlo simulations [6]. Considering the participation of smart buildings, Xu et al develop a distributed load shedding to alleviate the rate of frequency drop [7]. Their solution is based on the Lagrangian relaxation and a time-division multiple access wireless network. A mid-term stochastic optimization program, which incorporates a unit commitment based market clearing, is proposed to deal with outage scheduling performed by transmission system operators [8]. These existing works mainly focus on the infrastructures of either transmission systems or a single microgrid. In addition, the inherent integrality of decision variables in load shedding is not considered. Finally, rotational load shedding via bilinear integer programming is developed for heterogeneous load zones controlled by a utility [9]. McCormick relaxations along with a heuristic procedure of feasible solution recovery are proposed therein without theoretical guarantee of feasibility.

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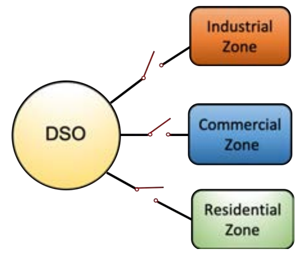

Fig. 1. A distribution network consisting of industrial, commercial, and residential load zones is managed by a distribution system operator (DSO).

Inspired by prior work in the literature, we formulate a novel stochastic load shedding problem for distribution power grids with heterogeneous end users. Our contribution is twofold. First, we take the uncertainty of parameters (i.e., power consumption and required energy shortfall) into account for the decision making. We propose a chance-constrained pure integer programming framework, whose objective is to minimize the total load shedding cost while promote fairness among multiple microgrids (Section II). The resulting formulation is more practical. Second, by leveraging the binary decomposition, McCormick relaxations, and the bounded structure of all integer variables, we develop two novel algorithms that are much faster than BARON – a state-of-the-art solver for global optimization of nonconvex optimization problems (Sections IV and V).

II. Problem Formulation

Consider a power distribution network supporting a total of N electrical loads, as shown in Fig. 1. Those loads can be classified into different groups according to the consumption patterns of electricity end users; e.g., industrial, commercial, and residential zones. We assume that each load zone can be shed independently without affecting the rest of the network. Whenever needed, the distribution system operator (DSO) conduct load shedding program which can be formulated as an optimization problem whose details are elaborated as follows.

1) Decision variables: We have two sets of optimization variables: i) k_n: total number of outages for load n in the

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planning horizon (a few days or weeks); and ii) d_n: duration of each outage for load n that is a multiple of the unit time period (e.g., 15 or 30 minutes). Note that d_n can also be modeled as a continuous variable. However, it is often restricted to be integer for the planning problem under study.

2) Cost functions: We model the load shedding cost of zone $n \in \mathcal{N} := \{1, 2, \ldots, N\}$ as:

$$C_n(d_n, k_n) = c_{n,1} d_n k_n + c_{n,2} d_n + c_{n,3} k_n,\ \forall n \in \mathcal{N}. \quad (1)$$

The cost is proportional to the total duration of load shedding $d_n k_n$ in the planning horizon. The coefficients $c_{n,1}, c_{n,2}$, and $c_{n,3}$ are pre-determined based on the preference of each zone. For example, residential zones often have large $c_{n,2}$ and relatively small $c_{n,3}$ due to the fact that they prefer multiple bursts of shorter duration of power outages [9].

3) Fairness regularizers: Besides minimizing the total shedding cost, the DSO may strive for the shedding fairness, which essentially guarantees that the load shedding costs or damages of different zones do not vary dramatically. We propose to promote fairness by using soft constraints in this work. Let C^{ave} and $C^{\text{ave}}_{g_i}$ denote the average cost for all zones and g_i-th group of zones, respectively. Then, fairness can be boosted by adding certain regularizers with appropriate weighting coefficients. Here, we focus on the ℓ_1-norm regularizers of inter-group faireness $\Phi^{\text{inter}}(C_n) := |C_n - C^{\text{ave}}|$ and intra-group fairness $\Phi^{\text{intra}}_{g_i}(C_n) := |C_n - C^{\text{ave}}_{g_i}|$. Note that we can also consider the ℓ_2-norm quadratic penalty or generally a convex function for the regularizers.

4) Load shedding constraint: Let P^{ave}_n be the average power consumption for zone n and E^{ave}_{sf} denote the average energy shortfall, which is the required amount of energy that being shed across the planning period. In order to ensure that the demand after load shedding can be met with the generation capacity, we have the coupling constraint:

$$E^{\text{ave}}_{sf} - \sum_{n=1}^{N} P^{\text{ave}}_n d_n k_n \leq 0. \quad (2)$$

However, the actual power consumption of each zone and the required energy shortfall have inherent uncertainties due to the forecasting errors. We leverage the following chance constraint to make the planning decisions robust to those uncertainties:

$$\Pr\left(E_{sf} - \sum_{n=1}^{N} p_n d_n k_n \leq 0\right) \geq \eta, \quad (3)$$

where E_{sf} and p_n are the random total energy shortfall and power consumption of zone n, respectively. The chance constraint ensures that the post-shedding demand can be balanced with a high probability $\eta \in [0, 1]$.

To this end, the DSO needs to solve the following optimal load shedding problem:

$$\min_{\{d_n, k_n\}} \quad \sum_{n=1}^{N} C_n(d_n, k_n) + \sum_{n=1}^{N} \alpha_n |C_n(d_n, k_n) - C^{\text{ave}}| + \sum_{i=1}^{N_g} \sum_{n \in \mathcal{N}_{g_i}} \beta_n |C_n(d_n, k_n) - C^{\text{ave}}_{g_i}| \quad (4a)$$

$$\text{s.t.} \quad \Pr\left(E_{sf} - \sum_{n=1}^{N} p_n d_n k_n \leq 0\right) \geq \eta \quad (4b)$$

$$\underline{d}_n \leq d_n \leq \overline{d}_n, \quad n \in \mathcal{N} \quad (4c)$$

$$\underline{k}_n \leq k_n \leq \overline{k}_n, \quad n \in \mathcal{N} \quad (4d)$$

$$d_n \in \mathbb{Z}_+,\ k_n \in \mathbb{Z}_+,\ n \in \mathcal{N}, \quad (4e)$$

where α and β are the weighting parameters for the inter/intra-group fairness regularizers. Lower and upper limits of d_n, k_n are posed as the constraints (4c) and (4c). The resulting problem is a chance-constrained pure integer quadratic program, which is generally NP-hard to solve.

III. Model Variations

In this section, we discuss variants of problem (4) that are dependent on probability distributions of uncertain parameters as well as lower limits of the decision variables.

A. Chance Constraint Reformulations

Let $\mathbf{a} := [E_{sf}, p_1, p_2, \ldots, p_N]^\top = \bar{\mathbf{a}} + \boldsymbol{\nu}$ collect the random input parameters of problem (4). Note that vector $\bar{\mathbf{a}} := [E^{\text{ave}}_{sf}, P^{\text{ave}}_1, P^{\text{ave}}_2, \ldots, P^{\text{ave}}_N]^\top$ is the mean of $\mathbf{a}$ while $\boldsymbol{\nu}$ quantifies the estimation or forecasting errors. Suppose additive errors are Gaussian distributed. In this case, we can convert the chance constraint into a second-order cone (SOC) constraint via the following lemma.

Lemma 1 (Section 4.4 in [10])**.** *Let* $\mathbf{a} \sim \mathrm{N}(\bar{\mathbf{a}}, \boldsymbol{\Sigma})$ *be a Gaussian random vector. Given* $\eta \geq 0.5$*, the chance constraint* $\Pr(\mathbf{a}^\top \mathbf{x} \leq 0) \geq \eta$ *is equivalent to the second-order cone constraint*

$$\bar{\mathbf{a}}^\top \mathbf{x} + \phi^{-1}(\eta) \|\boldsymbol{\Sigma}^{\frac{1}{2}} \mathbf{x}\|_2 \leq 0, \quad (5)$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$ *is the cumulative distribution function of a standard Gaussian random variable.*

Let $\mathbf{x} = [1, -d_1 k_1, \ldots, -d_N k_N]^\top$. Thus, the chance constraint (4b) can be equivalently rewritten as the SOCP (5), where $\boldsymbol{\Sigma} = \mathbb{E}(\boldsymbol{\nu}\boldsymbol{\nu}^\top)$ is the covariance matrix of $\mathbf{a}$.

If any entries of $\mathbf{a}$ are non-Gaussian or have unknown distributions, equivalent closed-form reformulation may be intractable. In this case, we leverage the sample average approximation (SAA) method that replaces (4b) with the following deterministic constraint $\frac{1}{S} \sum_{s=1}^{S} \mathbb{1}(\mathbf{a}_s^\top \mathbf{x}) \leq 1 - \eta$, where $\{\mathbf{a}_s\}_{s=1}^{S}$ are independent identically distributed (i.i.d.) samples of $\mathbf{a}$; the indicator function $\mathbb{1}(x) = 1$ if $x > 0$ and 0 otherwise. Let ϑ^* and Ω^* denote the optimal value and optimal solution set of the original problem (4), while ϑ_S and Ω_S the counterparts of the SAA problem. Furthermore, let $\mathcal{Y}$ be the feasible set of problem (4). Under mild conditions, the approximate problem "converges almost surely" to the original one as guaranteed by the following theorem.

Theorem 1 (Asymptotic optimality)**.** *Assume there is an optimal solution* $\mathbf{y}^*$ *of problem* (4) *such that for any* $\epsilon > 0$ *there is* $\mathbf{y} \in \mathcal{Y}$ *with* $\|\mathbf{y}^* - \mathbf{y}\| \leq \epsilon$ *and* $\Pr(\mathbf{a}^\top \mathbf{y} \leq 0) > \eta$*. Then,* $\vartheta_S \to \vartheta^*$ *and* $\sup_{\mathbf{y}_S \in \Omega_S} \mathrm{dist}(\mathbf{y}_S, \Omega^*) \to 0$ *with probability* 1 *as* $S \to \infty$*.*

The proof is based on Proposition 2.2 in [11]. We omit details here due to space limit. The sampling-based scenario approximation provides an alternative approach to approximate the chance constraint by using a set of deterministic linear constraints $\mathbf{a}_s^\top \mathbf{x} \leq 0, s = 1, 2, \ldots, S$. The finite sample performance is guaranteed with enough i.i.d. samples $\{\mathbf{a}_s\}$ [12], [13].

B. Zero Lower Limits for Decision Variables

If the lower limits $\underline{d}_n = \underline{k}_n = 0$, then we inherently have $d_n = 0$ if $k_n = 0$, and vice versa. By using auxiliary binary variables $\{z_n\}$ and the big-M technique, such conditional constraints can be cast as

$$z_n \leq d_n \leq M_1 z_n,\ z_n \in \{0,1\},\ \forall n \in \mathcal{N} \tag{6}$$

$$z_n \leq k_n \leq M_2 z_n,\ z_n \in \{0,1\},\ \forall n \in \mathcal{N}, \tag{7}$$

where $M_1 = \max_{n\in\mathcal{N}}\{\overline{d}_n\}$ and $M_2 = \max_{n\in\mathcal{N}}\{\overline{k}_n\}$.

IV. Proposed Approaches

We propose two approaches to solve problem (4), namely the binary decomposition plus McCormick relaxation (BDMR) and the matrix-based binary program (MBP) reformulation.

A. Binary Decomposition & McCormick Relaxation (BDMR)

McCormick envelopes are the commonly used convex relaxations for bilinear nonlinear programs [14]. A load shedding problem with a fairness constraint is solved by the McCormick relaxation, where cost reductions are obtained over two industry practices [9]. However, the relaxation is generally not exact. An extra procedure should be developed to recover a feasible solution, which is often ad-hoc for complicated feasible sets.

In this paper, instead of directly applying the McCormick relaxations, we develop a joint scheme of binary decomposition plus McCormick relaxations. As shown in the following, the proposed relaxation is exact and can efficiently solve the original problem (4).

To introduce the least number of new binary variables, and consider the fact that $\overline{d}_n \leq \overline{k}_n$ for all $n \in \mathcal{N}$, we apply the binary decomposition (a.k.a. binary expansion) [15] [16] to $\{d_n\}_{n\in\mathcal{N}}$ that yields $d_n = \sum_{r=0}^{\lfloor \log_2 \overline{d}_n \rfloor} 2^r b_{n,r}$, where the new binary variables $b_{n,r} \in \{0,1\}$ for $r = 0, 1, \ldots, \lfloor \log_2 \overline{d}_n \rfloor$. For simplicity of notation, define $R_n := \lfloor \log_2 \overline{d}_n \rfloor$ and $\mathcal{R}_n := \{1, 2, \ldots, R_n\}$. Hence, the bilinear terms $\{d_n k_n\}$ can be rewritten as $d_n k_n = \sum_{r=0}^{R_n} 2^r b_{n,r} k_n = \sum_{r=0}^{R_n} 2^r z_{n,r}$, $\forall n \in \mathcal{N}$, where $z_{n,r} = b_{n,r} k_n, \forall n \in \mathcal{N}$. By using the McCormick relaxation, we have

$$\begin{cases} z_{n,r} \geq b_{n,r}\underline{k}_n, & \forall n \in \mathcal{N}, \forall r \in \mathcal{R}_n \\ z_{n,r} \geq k_n + (b_{n,r}-1)\overline{k}_n, & \forall n \in \mathcal{N}, \forall r \in \mathcal{R}_n \\ z_{n,r} \leq k_n + (b_{n,r}-1)\underline{k}_n, & \forall n \in \mathcal{N}, \forall r \in \mathcal{R}_n \\ z_{n,r} \leq b_{n,r}\overline{k}_n, & \forall n \in \mathcal{N}, \forall r \in \mathcal{R}_n \end{cases} \tag{8}$$

To this end, each load shedding cost function can be rewritten as $C_n(b_{n,r}, z_{n,r}, k_n) = \sum_{r=0}^{R_n} (c_{n,1} 2^r z_{n,r} + c_{n,2} 2^r b_{n,r}) + c_{n,3} k_n$. Similarly, the load shedding coupling constraint (2) becomes $E_{sf}^{\text{ave}} - \sum_{n=1}^{N} \sum_{r=0}^{R_n} P_n^{\text{ave}} 2^r z_{n,r} \leq 0$. The corresponding chance constraint takes the form of $\overline{\mathbf{a}}^\top \mathbf{x} + \phi^{-1}(\eta) \|\mathbf{\Sigma}^{\frac{1}{2}} \mathbf{x}\|_2 \leq 0$, where $\mathbf{x} = \left[1, -\sum_{r=0}^{R_1} 2^r z_{1,r}, \ldots, -\sum_{r=0}^{R_n} 2^r z_{n,r}\right]^\top$.

The complete problem formulation by using the proposed BDRM is given as follows:

$$\min_{\substack{\mathbf{x},\\ \{k_n, b_{n,r}, z_{n,r}\}}} \sum_{n=1}^{N} C_n + \sum_{n=1}^{N} \alpha_n |C_n - C^{\text{ave}}| + \sum_{i=1}^{N_g} \sum_{n\in\mathcal{N}_{g_i}} \beta_n |C_n - C_{g_i}^{\text{ave}}| \tag{9a}$$

$$\text{s.t.} \quad \overline{\mathbf{a}}^\top \mathbf{x} + \phi^{-1}(\eta) \|\mathbf{\Sigma}^{\frac{1}{2}} \mathbf{x}\|_2 \leq 0 \tag{9b}$$

$$\mathbf{x} = [1, -\sum_r 2^r z_{1,r}, \ldots, -\sum_r 2^r z_{n,r}]^\top \tag{9c}$$

$$\underline{k}_n \leq k_n \leq \overline{k}_n,\ \forall n \in \mathcal{N} \tag{9d}$$

$$\underline{d}_n \leq \sum_r 2^r b_{n,r} \leq \overline{d}_n,\ \forall n \in \mathcal{N} \tag{9e}$$

$$k_n \leq M_2 \sum_r b_{n,r},\ \forall n \in \mathcal{N} \tag{9f}$$

$$b_{n,r} \leq k_n,\ b_{n,r} \in \mathbb{B},\ \forall n \in \mathcal{N},\ \forall r \in \mathcal{R}_n \tag{9g}$$

$$z_{n,r} \in \mathbb{Z}_+,\ k_n \in \mathbb{Z}_+,\ \forall n \in \mathcal{N},\ \forall r \in \mathcal{R}_n \tag{9h}$$

$$\text{all constraints in (8).} \tag{9i}$$

Note that the constraints (9f) and (9g) ensure that the problem formulation is also applicable for the case of zero lower limits of d_n and/or k_n, as discussed in Section III-B.

Lemma 2. *The McCormick relaxation is exact when at least one of the variables of the bilinear terms is binary.*

By leveraging this well-known result for the McCormick relaxation (see e.g., [15], [16]), we can show the equivalence of problems (9) and (4) as given in the following proposition.

Proposition 1. *The binary decomposition plus McCormick relaxation (BDMR) reformulation* (9) *has the same optimal solution to the original load shedding problem* (4). *Hence, these two problems are equivalent.*

B. Matrix-based Binary Programming (MBP)

In addition to the BDMR approach, we propose a matrix-based reformulation by using the fact that both d_n and k_n are bounded integers. Performance of the two proposed approaches will be demonstrated in the ensuing section.

Define $\overline{\mathbf{d}}_n := [\underline{d}_n, \underline{d}_n + 1, \ldots, \overline{d}_n]^\top$, and $\overline{\mathbf{k}}_n := [\underline{k}_n, \underline{k}_n + 1, \ldots, \overline{k}_n]^\top$. Furthermore, let $\mathbf{H}_n := \overline{\mathbf{d}}_n \overline{\mathbf{k}}_n^\top$, $\mathbf{D}_n := \overline{\mathbf{d}}_n \mathbf{1}^\top$, and $\mathbf{K}_n := \mathbf{1}\overline{\mathbf{k}}_n^\top$, where $\mathbf{1}$ is the all-ones vector with an appropriate dimension. Note that matrix $\mathbf{H}_n$ contains all possible values of the bilinear terms $\{d_n k_n\}_{n\in\mathcal{N}}$. Matrices $\mathbf{D}_n$, $\mathbf{K}_n$ have all the values of d_n, k_n respectively. The key idea is to define matrix variables that consist of all possible values of the integer decision variables. In this way, the decision variables can be cast as binary matrices of which all the entries are zero except one. The value "1" entries of all matrix variables indicate the optimal values of $\{d_n, k_n\}$.

Specifically, define $\mathbf{Q}_n = c_{n,1}\mathbf{H}_n + c_{n,2}\mathbf{D}_n + c_{n,3}\mathbf{K}_n$ and a binary matrix variable $\mathbf{X}_n \in \mathbb{B}^{(\bar{k}_n - \underline{k}_n)\times(\bar{d}_n - \underline{d}_n)}$, where at most one of all its elements is 1. Thus, it can be seen that the load shedding costs become $C_n = \mathrm{Tr}(\mathbf{Q}_n^\top \mathbf{X}_n)$, $\forall n \in \mathcal{N}$. The shedding constraint is given as $E_{sf}^{\text{ave}} - \sum_{n=1}^N P_n^{\text{ave}} \mathrm{Tr}(\mathbf{H}_n^\top \mathbf{X}_n) \leq 0$ while the corresponding chance constraint is $\bar{\mathbf{a}}^\top \mathbf{x} + \phi^{-1}(\eta)\|\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{x}\|_2 \leq 0$, where $\mathbf{x} = [1, -\mathrm{Tr}(\mathbf{H}_1^\top \mathbf{X}_1), \ldots, -\mathrm{Tr}(\mathbf{H}_N^\top \mathbf{X}_N)]^\top$.

To this end, we can equivalently rewrite problem (4) as the a matrix-based binary SOCP:

$$\min_{\mathbf{x}, \{\mathbf{X}_i\}_{i=1}^N} \quad \sum_{n=1}^N \mathrm{Tr}(\mathbf{Q}_n^\top \mathbf{X}_n) + \sum_{n=1}^N \alpha_n \left|\mathrm{Tr}(\mathbf{Q}_n^\top \mathbf{X}_n) - C^{\text{ave}}\right| + \sum_{i=1}^{N_g} \sum_{n \in \mathcal{N}_{g_i}} \beta_n \left|\mathrm{Tr}(\mathbf{Q}_n^\top \mathbf{X}_n) - C_{g_i}^{\text{ave}}\right| \tag{10a}$$

$$\text{s.t.} \quad \bar{\mathbf{a}}^\top \mathbf{x} + \phi^{-1}(\eta)\|\mathbf{\Sigma}^{\frac{1}{2}}\mathbf{x}\|_2 \leq 0 \tag{10b}$$

$$\mathbf{x} = [1, -\mathrm{Tr}(\mathbf{H}_1^\top \mathbf{X}_1), \ldots, -\mathrm{Tr}(\mathbf{H}_N^\top \mathbf{X}_N)]^\top \tag{10c}$$

$$\sum_{i,j} \mathbf{X}_{n,ij} \leq 1, \ \mathbf{X}_{n,ij} \in \{0, 1\}, \ n \in \mathcal{N}. \tag{10d}$$

Remark 1. *First, both formulations* (9) *and* (10) *are equivalent to the original load shedding problem* (4)*. Second, the BDMR approach has auxiliary binary variables* $\{b_{n,r}, z_{n,r}\}$*, which is also applicable when* $\{d_n\}_{n\in\mathcal{N}}$ *are modeled as continuous variables. In this case, binary decomposition should be applied to* $\{k_n\}_{n\in\mathcal{N}}$*. The matrix-based binary program approach introduces the sparse matrix variables* $\{\mathbf{X}_n\}_{n\in\mathcal{N}}$*. The box constraints are implicitly enforced by the coefficient matrices* $\{\mathbf{H}_n, \mathbf{D}_n, \mathbf{K}_n\}_{n\in\mathcal{N}}$*. The BDMR has a total of* $N + 2\sum_{n=1}^N \lfloor \log_2 \bar{d}_n \rfloor$ *binary variables while the matrix-based approach has* $\sum_{n=1}^N \left[(\bar{k}_n - \underline{k}_n) \times (\bar{d}_n - \underline{d}_n)\right]$ *binary variables. Finally, both formulations are binary linear or SOCP programs, depending whether uncertainties of load shedding constraints are considered.*

V. Numerical Results

In this section, we show the comparisons and effectiveness of the proposed approaches vis-à-vis BARON, which is the state-of-the-art solver for global optimization of nonconvex optimization problems [17]. We first study the problem without fairness regularizers and uncertainties. Then, the discussion of the regularizers and the robustness of the optimal solution associated with the chance constraint will be provided. All simulations are conducted under an iOS system with 3.6 GHz Intel Core i7 and 32 GB memory. The modeling languages CVX [18] and Yalmip [19], along with two solvers: Gurobi 8.1 [20] and BARON 17.8.9 are used. Throughout all the simulations, we have the parameters: $E_{sf}^{\text{ave}} = 10^6$, $500 \leq P_n^{\text{ave}} \leq 1000$, and $\alpha_n = \beta_n$ for all $n \in \mathcal{N}$. Finally, in all case studies, the three different approaches always yield the same (sub)optimal solutions with negligible relative gaps between the lower and upper bounds of the objective.

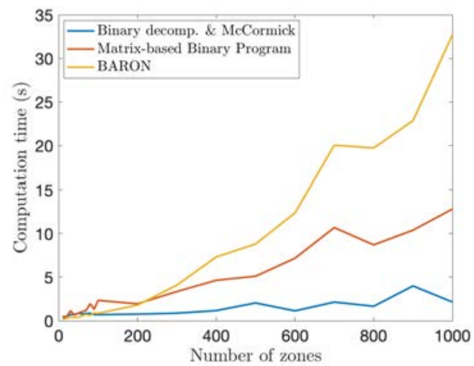

Fig. 2. The solver computation time of BDMR, BMP using Gurobi 8.1, and the original problem (4) directly solved by BARON. Fairness regularizers and uncertainties of the load shedding constraints are not included.

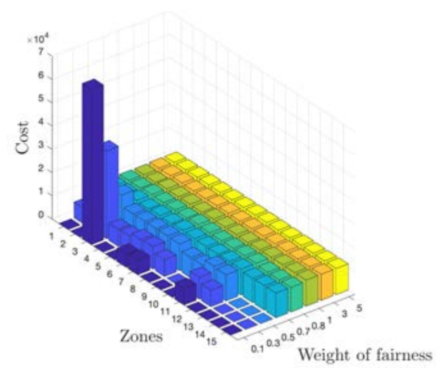

Fig. 3. Individual load shedding costs vs the fairness weighting parameters.

A. Results without Fairness Regularizers

First, we test the proposed approaches for problem (4) without fairness regularizers and uncertainties in the coefficient vector $\mathbf{a}$. As shown in Fig. 2, the problem can be solved within 35 seconds even for the large-scale instance $N = 1,000$. The running times of the two methods are in the same range when $N < 100$. Interestingly, as the problem size grows, the BDMR has the best performance compared with the matrix-based binary linear program and the BARON solver.

B. Results with Fairness Regularizers

Fig. 3 shows the individual cost of each zone with respect to the increasing weight of fairness. It can be seen that shedding costs vary dramatically across different zones with a very small weight; e.g., $\alpha = \beta = 0.1$ or 0.3. The fairness is enforced when the weights exceed a certain threshold. A case study of $N = 30$ zones (7 industrial, 12 residential, and 11 commercial zones) is presented in Fig. 4 to further illustrate the effectiveness and the scalability performance of the algorithms. The weights are fixed at $\alpha = \beta = 2$.

Table I compares the computation time of different approaches. Both BDMR and MBP significantly outperform BARON. For $N = 30$, BDMR and BARON are terminated for running 625 minutes with relative gaps 1.34% and 10.97%, respectively. The MBP formulation is better than the other two alternatives whose relative gaps are still large, even with more

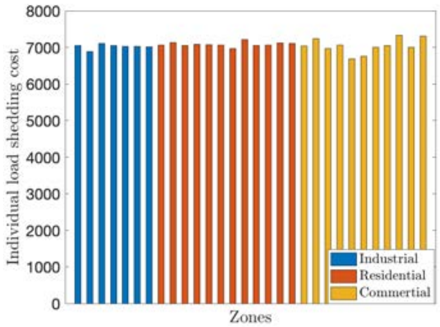

Fig. 4. Optimal load shedding cost of each zone with $N = 30$ and $\alpha = \beta = 2$. Uncertainties of the load shedding constraint is not included.

TABLE I
COMPUTATION TIME (MINUTE) OF DIFFERENT APPROACHES FOR $N = 15$, $\alpha = \beta = 5$ AND $N = 30$, $\alpha = \beta = 2$.

	BDMR	MBP	BARON
$N = 15$	5.2	26.5	114.9
$N = 30$	625	544	625

running time. Interestingly, adding the fairness regularizers makes the problem much harder to solve.

C. Result with the Chance Constraint

We consider again the scenario of $N = 15$ zones and assume that the random elements of $\mathbf{a}$ are correlated within each group, but independent across different groups. Therefore, the covariance $\mathbf{\Sigma}$ is a block diagonal positive definite matrix. We generate $5,000$ i.i.d. samples of normal vectors $\mathbf{a} = \bar{\mathbf{a}} + \mathbf{\Sigma}^{\frac{1}{2}}\mathbf{v}$, where $\mathbf{v} \sim \mathrm{N}(\mathbf{0}, \mathbf{I})$ is a standard normal vector. The chance constraint formulation is compared with a nonrobust benchmark that fails to take uncertainties into account. For the latter, optimal decisions are essentially made by solving the problem with only the static constraint (2). Note that the shedding requirement is met whenever $\mathbf{a}^\top\mathbf{x}$ is nonpositive. Fig. 5 shows the robustness of the optimal solutions obtained by solving the chance-constrained problem. It can be seen that the shedding constraint is always satisfied with the binary SOCP, while about 34% of the time the constraint is violated for the nonrobust benchmark.

VI. CONCLUSION

This paper studies the stochastic optimal load shedding problem for networked microgrids. Taking into account the uncertainty of actual power consumption and energy shortfall, the system operator aims to obtain the optimal load shedding decisions with least total shedding cost while keep inter-/intra-group fairness among a large amount of end users. We formulate such a task as an integer quadratic program, which is generally NP-hard to deal with. Capitalizing on the binary decomposition and McCormick relaxations, we develop two efficient algorithms whose runtime is 60%–95% shorter than the state-of-the-art solver BARON.

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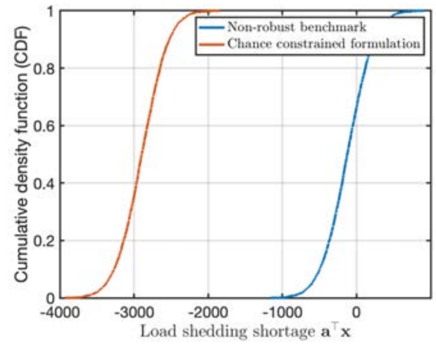

Fig. 5. Load shedding constraint violation (deemed as violation if $\mathbf{a}^\top\mathbf{x} > 0$): chance constraint (4b) vs static constraint (2). A total of $5,000$ Monte Carlo simulations are conducted for $N = 15$ with nodes $\alpha = \beta = 5$ and $\eta = 0.95$.

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