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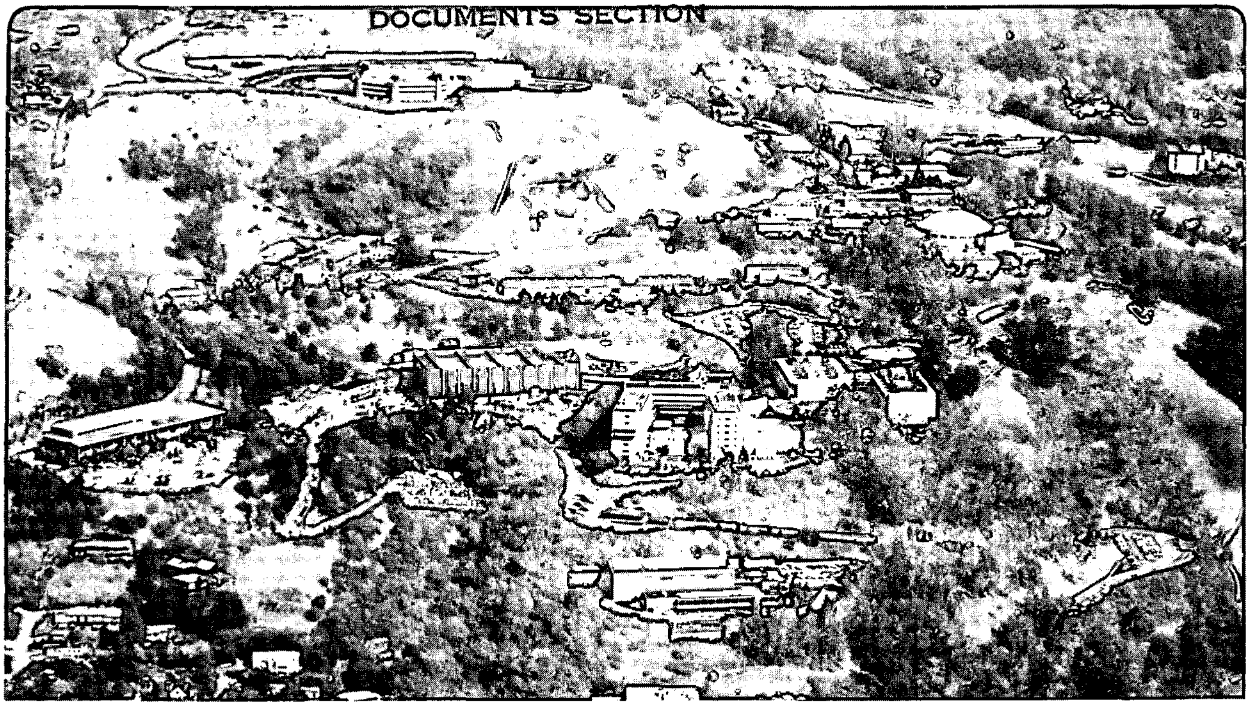

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LBID-172

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1. Introduction

This report describes the modifications made in the magnetostatic program Trim to incorporate a technique for handling the boundary conditions in the case of a magnet with quadrant symmetry. The program Trim solves the equation for the z-component A of the magnetic vector potential for a two-dimensional geometry in the x,y-plane. The symmetry assumed for the boundary condition described here is that A is zero along the y-axis and that the derivative of A normal to the x-axis is zero. The technique consists of applying a boundary condition to a circular boundary closely surrounding the region of interest, thus eliminating the necessity of a large air region surrounding the essential features of the magnet. The program Trim is described in the report UCRL-18439 titled "Trim: A Magnetostatic Computer Program for the CDC 6000" by John S. Colonias. The boundary condition is discussed in the internal report ESCAR-28 titled "On a Boundary Condition Applicable to Magnetostatic Relaxation Computations" by L. Jackson Laslett. The version of the program described here is essentially the same as described in the report by Colonias except that in order to keep the program within the bounds of small core memory the limit mentioned in the definition of PRBCON(3) has been reduced from 4000 to 3500. A flag has been added to the input parameters for the program Mesh to enable the user to choose the boundary condition described herein. The modifications to the programs Mesh and Field are described in the following pages, and the appendix contains an example of an input deck for calculating the field of a magnet with quadrant symmetry.

2. Program Mesh

Input for the program Mesh is prepared in the same way as for the standard version of Trim. The program assumes the first input region to be the universe, and it assumes that all succeeding input regions lie within the universe. Due to the symmetry of the problem all the input regions lie within the first quadrant. The universe need contain only those features essential to the problem without any surrounding air space. The program constructs two additional regions surrounding the universe which give an outer boundary of two concentric circles, and these regions are given numbers consecutive to the numbered input regions. In the printed output from Mesh the data specifying these two additional regions are listed before the data for the input regions.

The program constructs the two additional regions in the following way. A value JMAX = max(KMAX,LMAX)+3 is calculated from the input values KMAX and LMAX. In mesh coordinates the outer region is then a square of dimension JMAX on a side, and the inner region is a square of dimension JMAX-1 on a side. The x and y coordinates of the points on the circular portions of the two regions are calculated as follows. If there are M points on the circular boundary, excluding the points on the axes, then the angular distribution of the points is given by

$$\theta(k) = \pi(2k-1)/4M \; ; \; k = 1,2,\ldots,M.$$

Because of the storage limit of 3500 mentioned in the introduction, the maximum allowable value of JMAX is 109. The number of points along the outer boundary is N = 2JMAX-3, and the number of points on the inner arc is N-2, not counting points on the axes. The radius R of the circular boundaries is then calculated as follows. Let

$$ZMAX = \max\sqrt{x^2+y^2}$$

for points of the universe. Then the radius of the inner boundary is

$$R = ZMAX+2H$$

and the radius of the outer boundary is

$$R = ZMAX+3H$$

where H satisfies the relation

$$H = \pi(ZMAX+3H)/2N.$$

The option to use the circular boundary feature is taken if PRBCON(41) is set to 1.0, in which case the boundary value flags in PRBCON(5)-PRBCON(8) need not be set. If PRBCON(41) is not specified the program behaves in the manner of the standard version of Trim. The parameter PRBCON(41) is transfered to the program Field as TVAR(43). In figure 1 is shown one of the dipole magnet designs that was proposed for use in the Experimental Superconducting Accelerating Ring. Figure 2 is the mesh for this magnet with the circular boundary, and figure 3 is the mesh from the standard version of Trim.

3. Program Field

If the option to use the circular boundary feature is chosen, then the boundary condition is applied in the following way. With N points on the outer circular arc and N-2 points on the inner arc, not counting points on the axes, one regards the potential function in this region to be expressible in the form

$$A = \sum_{m=1}^{N-2} b_m [(R-H)/r]^{2m-1} \cos(2m-1)\theta$$

where R is the radius of the outer arc, H is the distance between the inner and outer arcs, and (r,θ) are polar coordinates. In particular A can be written

$$A_i^I = \sum_{m=1}^{N-2} b_m \cos(2m-1)\theta_i^I$$

at the points $i=1,2,\ldots,N-2$ of the inner arc. The coefficients b_m are then expressible by

$$b_m = \sum_{i=1}^{N-2} (M^{-1})_{m,i} A_i^I$$

where M^{-1} is the inverse of the matrix M for which the elements are

$$(M)_{p,q} = \cos(2q-1)\theta_p^I.$$

The extension of the potential function to the outer arc then leads to the values

$$A_k^O = \sum_{i=1}^{n-2} E_{k,i} A_i^I$$

where the elements of E are given by

$$E_{k,i} = \sum_{m=1}^{n-2} \cos(2m-1)\theta_k^O \, [(R-h)/R]^{2m-1} (M^{-1})_{m,i}$$

where $i=1,2,\ldots,N-2$ and $k=1,2,\ldots,N+1$, and $k=1$ corresponds to the point on the x-axis. The points θ^I on the inner arc have been regularly spaced in the sense that

$$\theta_p^I = \pi(2p-1)/[4(N-2)]$$

where $p=1,2,\ldots,N-2$, and therefore the inverse of M is readily expressible in explicit form as

$$\begin{aligned}(M^{-1})_{m,i} &= [2/(N-2)]\cos(2m-1)\theta_i^I \\ &= [2/(N-2)]\cos(2m-1)(2p-1)\pi/[4(N-2)]\end{aligned}$$

and E has the elements

$$E_{k,i} = \frac{2}{N-2} \sum_{m=1}^{N-2} [(R-H)/R]^{2m-1} \cos(2m-1)\theta_k^O \cos(2m-1)\theta_i^I.$$

In the version of Trim decribed here the azimuthal distribution of the points on the outer boundary is $\theta_1^O = 0$ and

$$\theta_k^O = \pi(2k-3)/4N$$

for $k=2,3,\ldots,N+1$.

Values of A_k^O as obtained above are introduced as values of the potential function on the outer boundary followng the completion of any full relaxation pass through the remainder of the mesh, and such values on the outer boundary then are employed in the subsequent relaxaion revision of interior potentials. It is to be noted that the factor

$$[(R-H)/R]^{2m-1} \simeq 1-(2m-1)\frac{H}{R}$$

appears in the equations above, and this agrees through first order in H/R with the factor

$$[\frac{a}{h}-m]/[\frac{a}{h}+(m-1)]$$

proposed in the report ESCAR-28.

4. Program Trip

The plotting program Trip has not been changed, but if one wishes to plot the entire grid, including the circular regions constructed by the program, then the values XMAX and YMAX used in the input to Trip must be specified accordingly.

5. Examples

To test the method the potential A was calculated for a window-frame current distribution with no iron as illustrated in figure 4. The results of the Trim run were compared with analytical results. In table 1 the two sets of values are compared for some of the calculated points. In figures 5 and 6 are shown the mesh plot and the equipotential plot for the window-frame calculation.

The program was run for variable permiability iron for the mesh configurations of figures 2 and 3. In figure 7 is shown the equipotential plot corresponding to the mesh of figure 2. This plot shows there is probably no advantage to using the circular boundary for this configuration because the potential outside the iron has fallen off to about two percent of its maximum value. The potential values for the two runs agreed to about four figures in the air region inside the iron. It took about 1.25 times as long to do the calculation for the circular boundary as for the mesh shown in figure 3, and since the number of mesh points are about the same for the two runs this gives a good idea of the relative speed of computation for the two methods. The possible gain in speed of convergence by over-relaxing the updated values on the outer boundary was investigated by running this variable permiability case for several values of a relaxation parameter. Thus if A is the potential on the outer boundary and n is the cycle number, then we used the relation

$$A^{n+1} = \phi A^{new}+(1-\phi)A^{n}$$

for ϕ=1.00,1.30,1.60,1.90,1.94 and found no change in the number of cycles needed for convergence. The value of ϕ was therfore left at unity.

Table 1

Comparison of Numerical and Analytic Results for Window-Frame Current Distribution

X	Y	A From Program	A analytic
10.0	.0	249.7	250.2
20.0	.0	493.1	493.9
30.0	.0	730.6	731.6
5.0	5.0	125.9	126.1
20.0	20.0	511.8	513.2
25.0	25.0	631.5	633.2
30.0	30.0	727.3	729.1

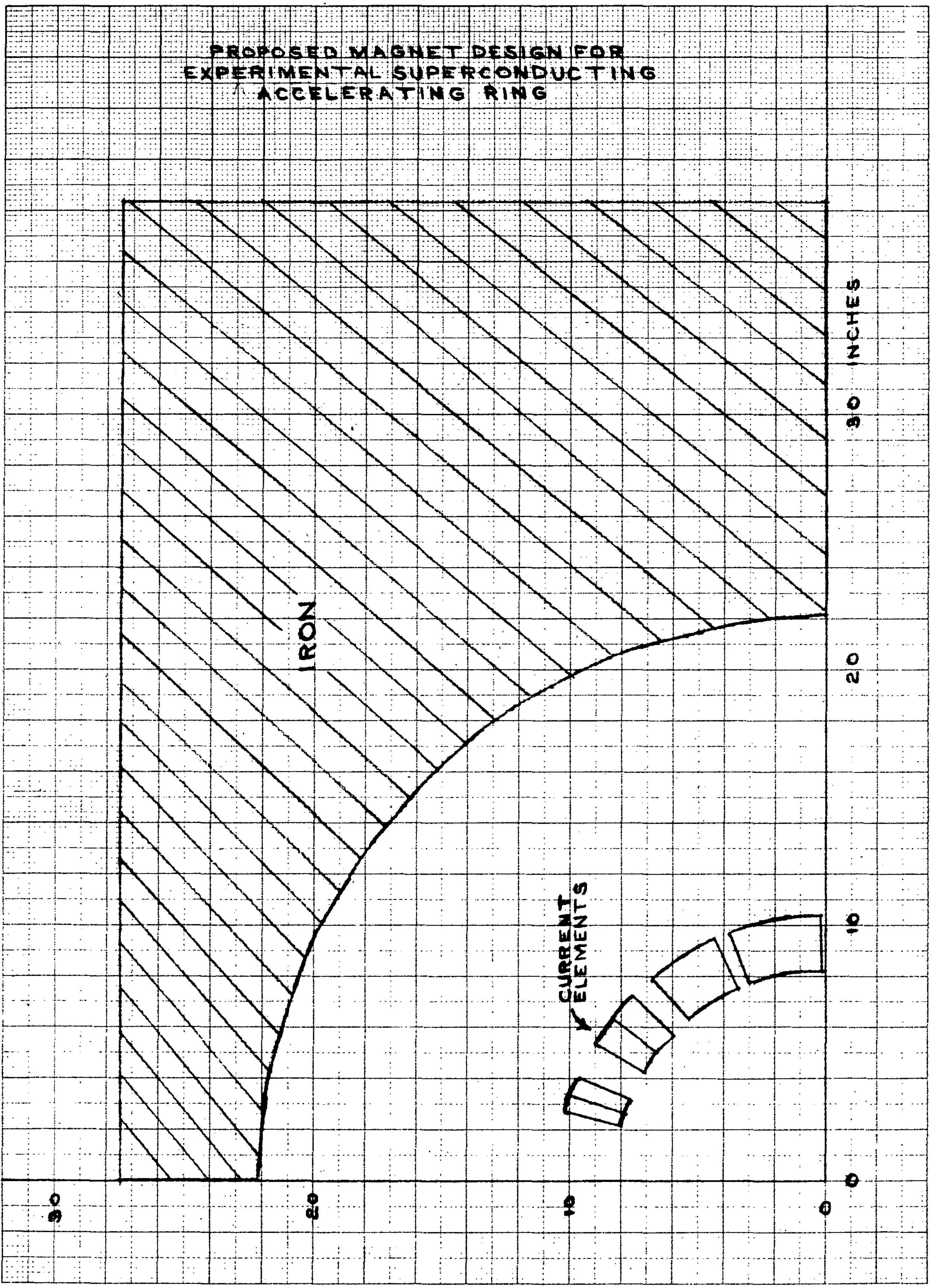
6. Appendix

The following appendix contains a set of control cards and data input cards for calculating the magnet field for the magnet illustrated in figure 1 using the circular boundary condition.

```
TRVOB,4,850,170000.XXXXXX,BRADY
FLOOR(3)
GETTAPE,OLD1=TRIM/QUADSYM/MESH,21416.
GETTAPE,OLD2=TRIM/QUADSYM/FIELD,21416.
GETTAPE,OLD3=TRIM/QUADSYM/TRIP,21416.
UPDATE(P=OLD1,C=C1,F,C=LIST)
RETURN(CLD1)
RUN76(SC,I=C1,B=MSH,O=LIST,NL75000)
RETURN(C1)
LINK(F=MSH,X)
UPDATE(P=OLD2,C=C2,F,O=LIST)
RETURN(CLD2)
RUN76(SC,I=C2,B=FIELD,O=LIST,NL50000)
RETURN(C2)
REWIND(TAPE35)
LINK,F=FIELD,X.
UPDATE(P=OLD3,C=C3,F,O=LIST)
RETURN(CLD3)
RUN76(SC,I=C3,B=TRP,O=LIST)
REWIND(TAPE35)
RETURN(C3)
LINK(F=TRP,X)
EXIT.
DUMP,170000,L=LIST.
GRUMP,L=LIST.
CXIT.
FIN.
TIM.
DOB.
COPY,OUTPUT/RB,LIST.
COPY,DAYFILE/RB,LIST.
DISPOSE,LIST=MF,R=[FLOOR 3].
END OF RECORD
END OF RECORD
 DIPOLE MAGNET
 *1+8. +44. +53. 0.  *41+1.    S
    1    1   0.          0.          2            $  UNIVERSE  $
    1   13   0.          8.27
    1   16   0.         10.439
    1   33   0.         22.1056
    1   53   0.         38.35
   38   53  22.1056     38.35
   44   53  27.43       38.35
   44    1  27.43        0.
   38    1  22.1056      0.
   17    1  10.439       0.
   14    1   8.27        0.
    1    1   0.          0.          C
    2    1  220000.      0.          1
    2   13     .14229    8.26878
    2   16     .17961   10.43745
    3   16     .91860   10.39850
    4   16    1.65296   10.30730
    5   16    2.37901   10.16430
    6   16    3.09312    9.97022
    7   15    3.79167    9.72604
    7   14    3.52906    9.05242
    6   14    3.26646    8.37880
    6   13    3.00385    7.70518
    5   13    2.31005    7.94082
    4   13    1.59811    8.11412
    3   13     .87363    8.22373     C
```

```
  3     2    0.           0.          1          $  IRON REGION  $
 38     1    22.10560      .00000
 38     2    22.09690      .61998
 38     3    22.07082     1.23947
 38     4    22.02738     1.85799
 38     5    21.96660     2.47504
 37     5    21.88855     3.09015
 37     6    21.79327     3.70283
 37     7    21.68085     4.31259
 37     8    21.55137     4.91896
 37     9    21.40493     5.52146
 36    10    21.24165     6.11962
 36    11    21.06167     6.71296
 36    12    20.86511     7.30102
 35    12    20.65213     7.88333
 35    13    20.42291     8.45945
 35    14    20.17762     9.02891
 35    15    19.91646     9.59126
 34    16    19.63962    10.14607
 34    17    19.34734    10.69290
 33    17    19.03983    11.23131
 33    18    18.71735    11.76089
 33    19    18.38013    12.28121
 32    20    18.02846    12.79188
 31    20    17.66261    13.29248
 31    21    17.28285    13.78262
 30    22    16.88950    14.26191
 29    22    16.48287    14.72999
 29    23    16.06326    15.18648
 28    24    15.63102    15.63102
 27    24    15.18648    16.06326
 26    25    14.72999    16.48287
 25    25    14.26191    16.88950
 24    26    13.78262    17.28285
 24    27    13.29247    17.66261
 23    27    12.79188    18.02846
 22    28    12.28121    18.38013
 21    28    11.76089    18.71735
 20    29    11.23131    19.03983
 19    29    10.69290    19.34734
 18    29    10.14607    19.63962
 17    29     9.59126    19.91646
 16    30     9.02891    20.17762
 15    30     8.45945    20.42291
 14    31     7.88333    20.65213
 13    31     7.30102    20.86511
 12    32     6.71296    21.06167
 11    31     6.11961    21.24165
 10    32     5.52146    21.40493
  9    32     4.91896    21.55137
  8    33     4.31259    21.68085
  7    32     3.70283    21.79327
  6    33     3.09015    21.88855
  5    33     2.47504    21.96660
  4    33     1.85799    22.02738
  3    33     1.23947    22.07082
  2    33      .61998    22.09690
  1    33      .00000    22.10560
  1    53    0.          38.35
 44    53   27.43        38.35
 44     1   27.43         0.          C
  4     1   168000.       0.          1
```

```
     7   12    3.44553    7.51806
     7   13    3.74675    8.17532
     8   14    4.04798    8.83258
     8   15    4.34920    9.48985
     9   14    4.99584    9.16593
    10   14    5.61850    8.79802
    11   13    6.21419    8.38788
    12   13    6.78006    7.93748
    12   12    6.31048    7.38773
    11   11    5.84089    6.83798
    10   11    5.37130    6.28824
     9   11    4.76823    6.75699
     8   12    4.12447    7.16810      C
     5    1   70000.      0.           1
    11    9    5.98236    5.71002
    12   10    6.50536    6.20921
    12   11    7.02837    6.70841
    13   11    7.55137    7.20760
    14   11    7.94566    6.77047
    15   10    8.31468    6.31180
    12    9    6.58707    5.00035      C
     6    1   46000.      0.           1
    12    9    6.58707    5.00035
    15   10    8.31468    6.31180
    15    9    8.65726    5.83306
    16    9    8.97230    5.33578
    13    7    7.10805    4.22712
    12    8    6.85846    4.62108      C
     7    1   36000.      0.           1
    14    6    7.64355    3.15739
    17    7    9.64824    3.98549
    17    6    9.88492    3.35574
    14    5    7.83105    2.65849      C
     8    1   24000.      0.           1
    14    5    7.83105    2.65849
    17    6    9.88492    3.35574
    17    5   10.08055    2.71205
    14    4    7.98603    2.14855      C
  END OF RECORD
  END OF RECORD
   0
   1
   S
   *24*56. *26*56.
   S
   0
  END OF RECORD
  END OF RECORD
   0. 52. 0. 52. 74.2857 1. 0. 1.
  0. 52. 0. 52. 74.2857 0. 50. 2.
   S
```



PROPOSED MAGNET DESIGN FOR EXPERIMENTAL SUPERCONDUCTING ACCELERATING RING

FIG. 1

DIPOLE MAGNET

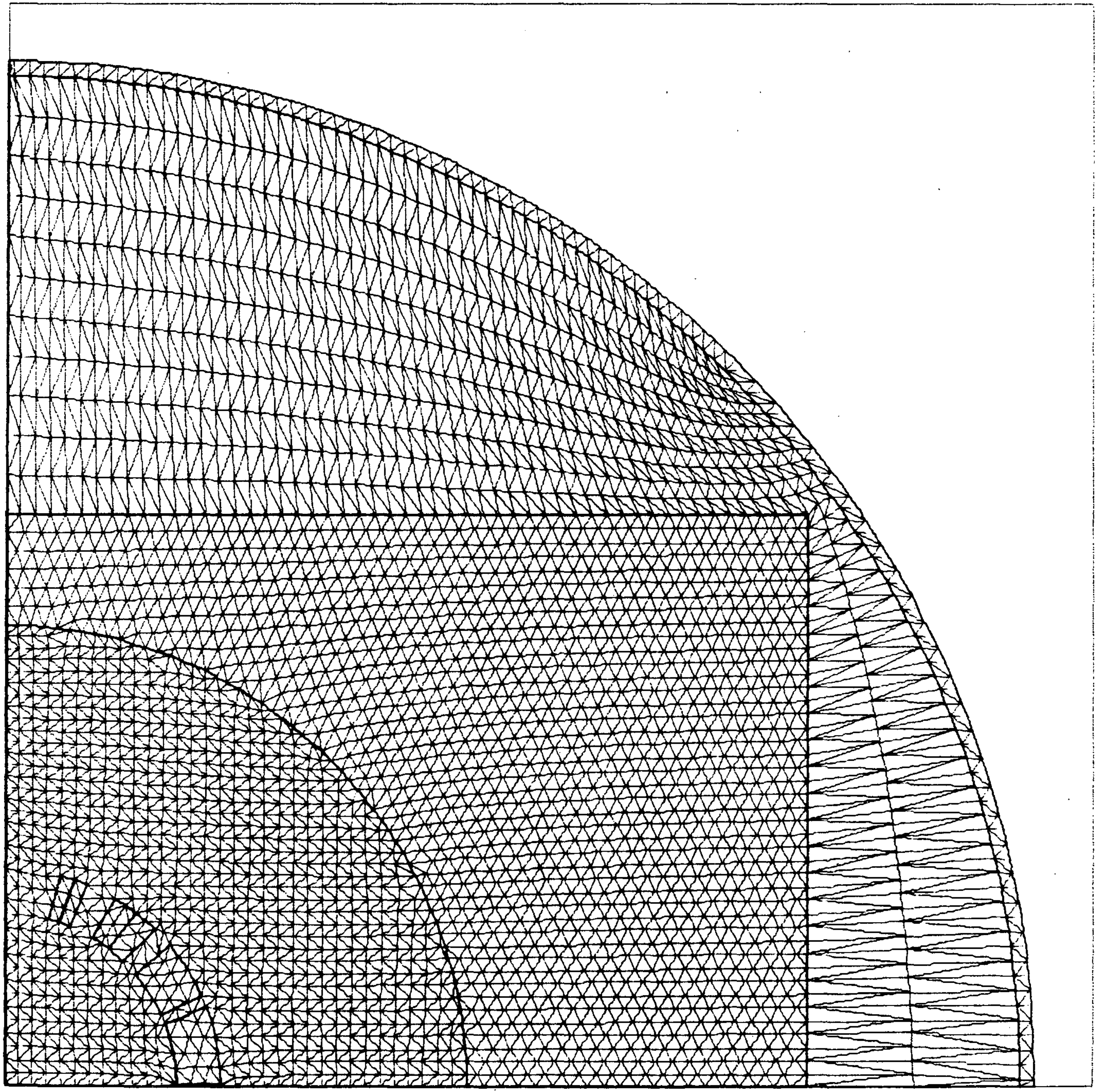


FIG. 2

DIPOLE MAGNET

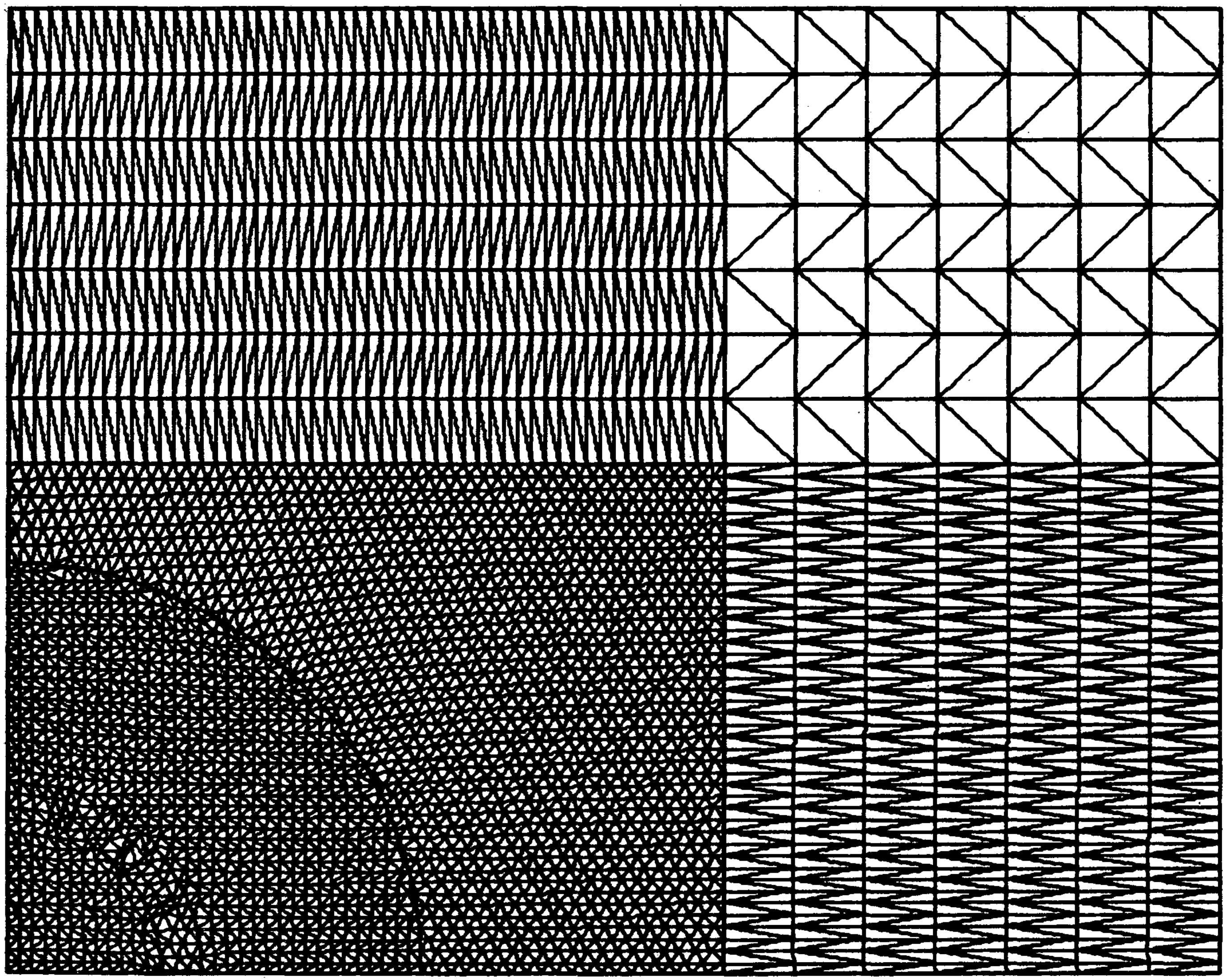


FIG.3

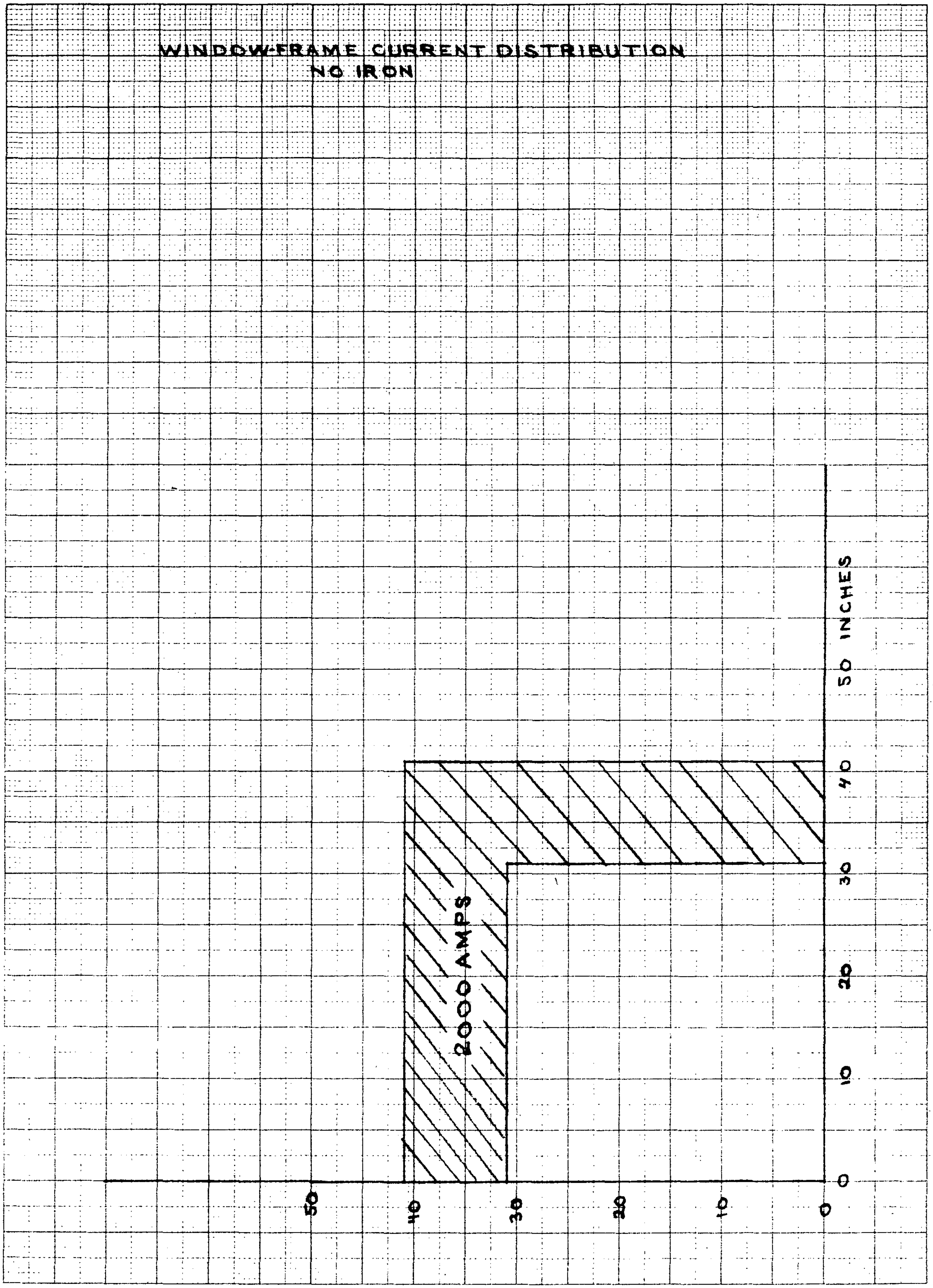

WINDOW-FRAME CURRENT DISTRIBUTION
NO IRON
2000 AMPS
50 INCHES
0
10
20
30
40
50

FIG. 4

WINDOW-FRAME. 2000 AMPS. NO IRON.

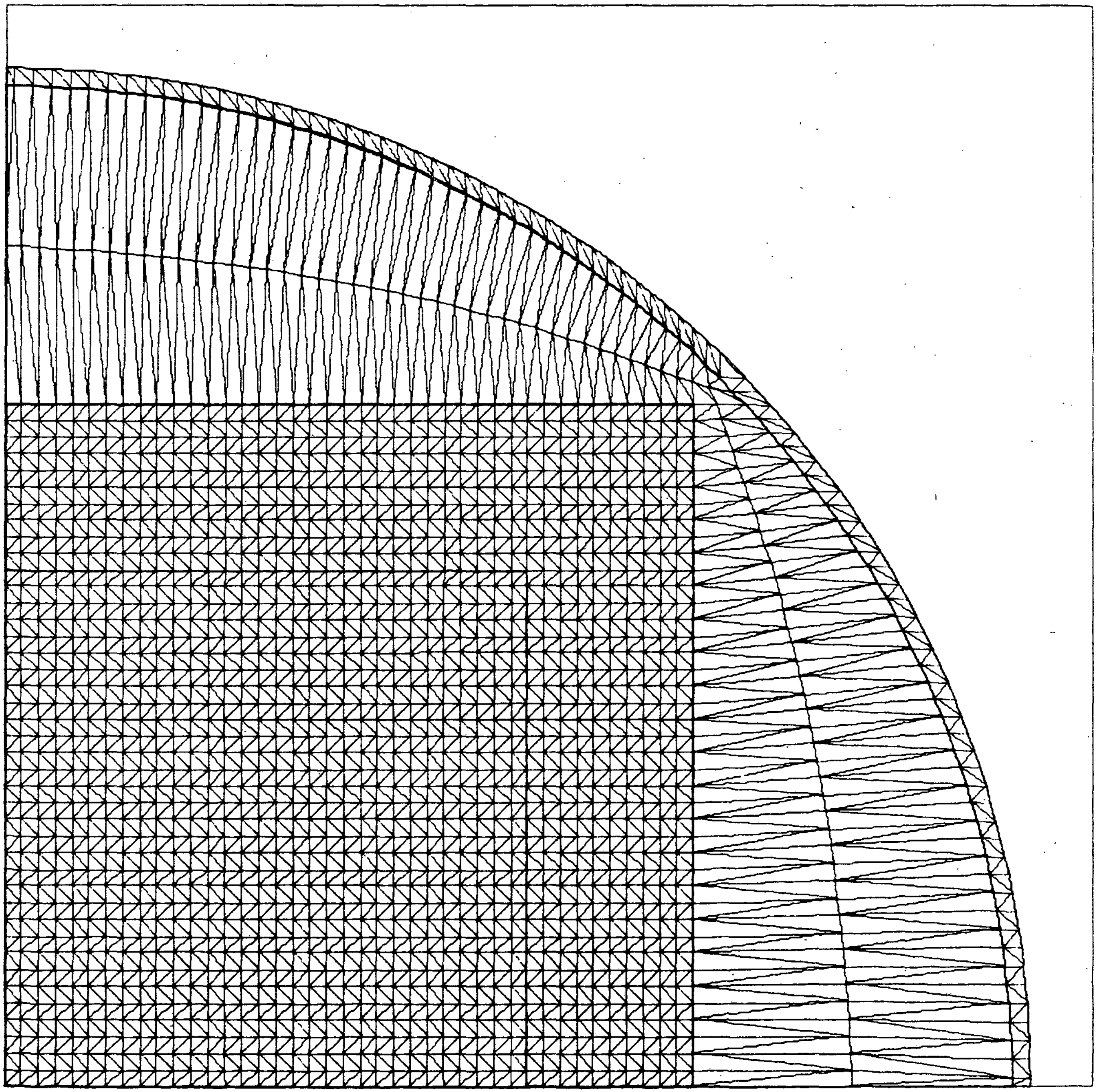
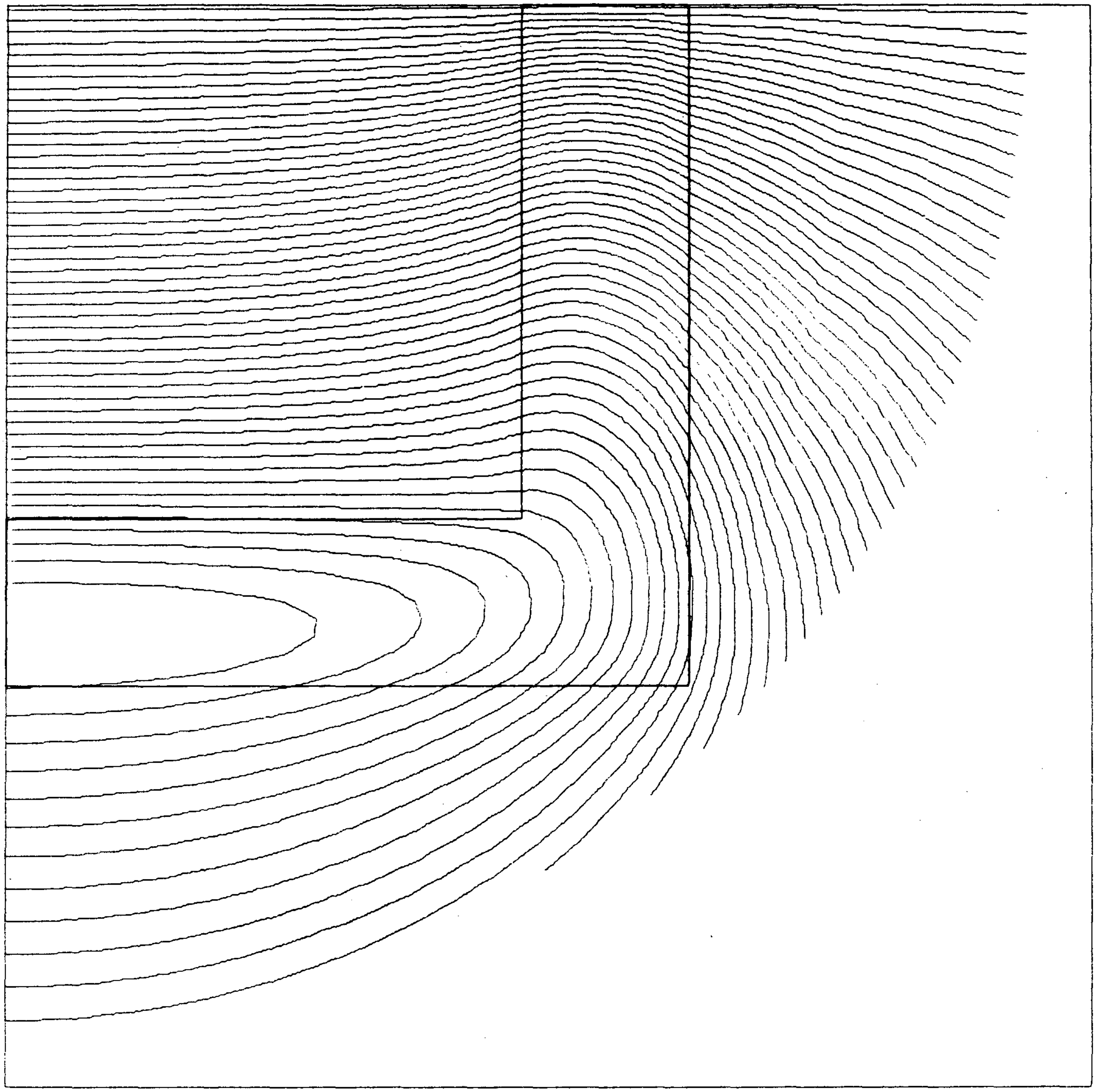


FIG. 5



```
WINDOW-FRAME. 2000 AMPS. NO IRON.
YMIN=        0.000   CYCLE  344    DUMP  2
YMAX=       65.000   MODE AIR
XMIN=        0.000   AMIN= 7.9533E+00
XMAX=       65.000   AMAX= 8.3565E+02
 11 OCT 79   LINEAR  MODEL
```

FIG. 6

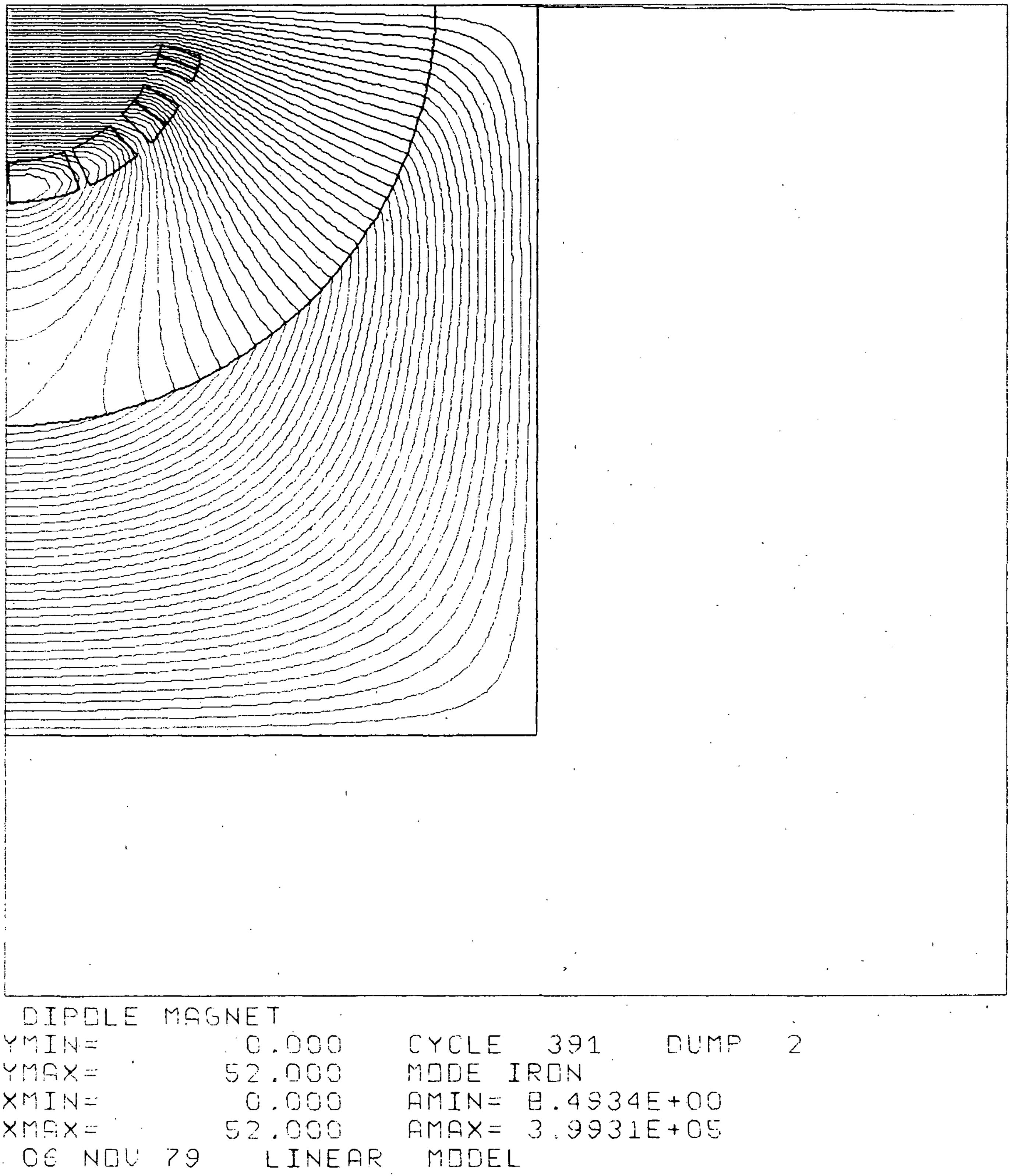

DIPOLE MAGNET
YMIN= 0.000 CYCLE 391 DUMP 2
YMAX= 52.000 MODE IRON
XMIN= 0.000 AMIN= 8.4934E+00
XMAX= 52.000 AMAX= 3.9931E+05
06 NOV 79 LINEAR MODEL

FIG. 7

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BERKELEY, CALIFORNIA 94720