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to be delivered by R. T. Van de Walle at the CERN Conference. On Page 4, Equation (2a) now reads:

$$G_V(P,E_\mu)dP\, d\, E_\mu = \frac{2P}{E}\left\{ f_V^{\,2}\left[4\, E_\mu\, (W - E_\mu) - (W^2 - P^2 - m_L^{\,2})^2\right] + 4\, f_V\, g_V\, (W - E_\mu)\,\frac{m_L^{\,2}}{M_K} + \frac{m_L^{\,2}}{M_K^{\,2}}\, g_V^{\,2}\, (W^2 - P^2 - m_L^{\,2})\right\} dP\, d\, E_\mu \; .$$

This equation should read:

$$G_V(P,E_\mu)dP\, d\, E_\mu = \frac{2P}{E}\left\{ f_V^{\,2}\left[4\, E_\mu\, (W - E_\mu) - (W^2 - P^2 - m_L^{\,2})\right] + 4\, f_V\, g_V\, (W - E_\mu)\,\frac{m_L^{\,2}}{M_K} + \frac{m_L^{\,2}}{M_K^{\,2}}\, g_V^{\,2}\, (W^2 - P^2 - m_L^{\,2})\right\} dP\, d\, E_\mu$$

On Page 3, the last line of the third paragraph now reads:

216 events, we have obtained from a maximum-likelihood analysis $\lambda = 0.4 \pm 0.4$.

This sentence should read:

216 events, we have obtained from a maximum-likelihood analysis $\lambda = 0.04 \pm 0.04$.

Page 6, last line now reads: that $f_V^e/f_V^\mu =$

equation should read: that $f_V^\mu/f_V^e =$

EXPERIMENTAL STUDY OF THE THREE-BODY LEPTONIC DECAY MODES OF THE K^+ MESON*

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I. INTRODUCTION

We report in this paper the results of an experimental study of the three-body leptonic decay modes (K^+_{L3}) of the K^+ mesons ($K^+_{e3} \rightarrow e^+ + \pi^0 + \nu$, $K^+_{\mu3} \rightarrow \mu^+ + \pi^0 + \nu$).

It is obvious from the conservation laws that all momenta and angles in a K^+_{L3} decay at rest are determined by the specification of any two independent variables. Pais and Treiman have pointed out that the pion momentum and the angle between pion and neutrino directions are convenient variables and have obtained the following expressions for the distribution functions in these variables (assuming pure couplings):[1]

Vector coupling

$$F_V(P,\theta)dP\,d\cos\theta = \frac{P^2}{E}\,\frac{\left(W^2 - P^2 - m_L{}^2\right)^2}{(W + P\cos\theta)^4}\left\{P^2 \sin^2\theta\, f_V{}^2 + \frac{m_L{}^2}{M_K{}^2}\left[M_K f_V + (W + P\cos\theta)g_V\right]^2\right\} dP\,d\cos\theta \tag{1a}$$

Scalar coupling

$$F_S(P,\theta)dP\,d\cos\theta = \frac{P^2}{E}\,\frac{\left(W^2 - P^2 - m_L{}^2\right)^2}{(W + P\cos\theta)^2}\; f_S{}^2\; dP\,d\cos\theta \tag{1b}$$

Tensor coupling

$$F_T(P,\theta)dP\,d\cos\theta = \frac{P^4}{M_K{}^2 E}\,\frac{\left(W^2 - P^2 - m_L{}^2\right)^2}{(W + P\cos\theta)^4}\; f_T{}^2\left[(W\cos\theta + P)^2 + m_L{}^2\sin^2\theta\right] dP\,d\cos\theta \tag{1c}$$

Here P is the pion momentum and E is the total pion energy; θ is the angle between the pion and the neutrino; M_K is the K^+ mass; $W = M_K - E$; m_π and m_L are the pion and lepton masses respectively, and f_V, g_V, f_S, and f_T are functions ("form factors") of $q^2 = M_K{}^2 + m_\pi{}^2 - 2M_K E$, the square of the invariant four-momentum transfer. We assume time-reversal invariance and take f_V, g_V, f_S, and f_T to be real.

In the present investigation we have attempted to study the following questions:

1. What couplings are responsible for K^+_{e3} and $K^+_{\mu 3}$ decay?
2. Are the muon and electron couplings identical; i.e., are the form factors the same?
3. What can be said about the q^2 dependence of the form factors?

The experiment was run by exposing a xenon bubble chamber to a separated K^+ beam of such momentum that the K^+ mesons were stopped near the center of the chamber. Details of the experiment, identification of the various modes, sample selection, etc. have been published elsewhere.[2-5]

II. K^+_{e3} DECAYS

A. Nature of the Coupling

We have already shown that the vector coupling seems to be the only one that agrees well with our K^+_{e3} data.[3] This conclusion has one qualification: to rule out the scalar hypothesis, we have assumed a "gentle" q^2 dependence of the form factor f_V^e (we use superscripts, e, μ, to separate the form factors in K^+_{e3} and $K^+_{\mu 3}$ decay in those analyses where we are not specifically assuming the universality of the muon and electron couplings).

B. The q^2 Dependence of f_V^e

We have represented the q^2 dependence of f_V^e by the first two terms in a series expansion $f_V^e(q^2) = A(1 + \lambda q^2/m_\pi^2)$. By comparison with data from 216 events, we have obtained from a maximum-likelihood analysis $\lambda = 0.4 \pm 0.4$.

III. $K^+_{\mu 3}$ DECAYS

A. Nature of the Coupling

To introduce the corrections due to chamber geometry, we can conveniently rewrite the various distribution functions in terms of the variables P and E_μ, where E_μ is the muon total energy:

Vector coupling

$$G_V(P,E_\mu)dP\,d\,E_\mu = \frac{2P}{E}\left\{f_V^2\left[4\,E_\mu\,(W-E_\mu)-(W^2-P^2-m_L^2)^2\right]\right.$$

$$\left.+4\,f_V\,g_V\,(W-E_\mu)\frac{m_L^2}{M_K}+\frac{m_L^2}{M_K^2}\,g_V^2\,(W^2-P^2-m_L^2)\right\}dP\,d\,E_\mu \quad (2a)$$

Scalar coupling

$$G_S(P,E_\mu)dP\,d\,E_\mu = \frac{2P}{E}\,f_S^2\,(W^2-P^2-m_L^2)dP\,d\,E_\mu \quad (2b)$$

Tensor coupling

$$G_T(P,E_\mu)dP\,d\,E_\mu = \frac{2P}{M_K^2E}\,f_T^2\left[P^2(W^2-P^2-m_L^2)-4\,P^2\,(W-E_\mu)^2\right.$$

$$\left.+(W^2+P^2+m_L^2-2\,W\,E_\mu)^2\right]dP\,d\,E_\mu\,. \quad (2c)$$

We have calculated expected pion-energy distributions by integrating Eqs. (2) over E_μ, taking due account of chamber geometry, with the following further assumptions:

1. All form factors are taken to be constant.

2. The ratio g_V^μ/f_V^μ is chosen, on the assumption of the identity of muon and electron coupling strengths, from the measured ratio of $K_{\mu3}^+$ and K_{e3}^+ decay rates,[2] 0.96 ±0.16. Because the rates are quadratically related to the f_V and g_V, this ratio is satisfied by two values of g_V/f_V, namely 0.5 ±0.4 and −4.8 ±0.4.

In Fig. 1 we show the experimental pion momentum spectrum and compare it with the scalar, tensor, and both vector distributions calculated as described above. If one makes a χ^2 test (see Table I) of the experimental data with these various theoretical distributions based on constant form factors, one finds that the most likely coupling is vector (with $g_V/f_V = +0.58$), although tensor is not ruled out. The scalar coupling, as well as the vector coupling with $g_V/f_V = -4.85$, does not fit the data at all.

Table I. Comparison of experimental $K^+_{\mu 3}$ π^0 momentum distribution with various couplings having constant form factors

Coupling	χ^2 Probability (%)
vector, $g_V/f_V = +0.58$	45
vector, $g_V/f_V = -4.85$	< 0.1
scalar	< 0.1
tensor	9

B. Energy Dependence of f_V^μ and g_V^μ

Our data are insufficient to permit significant conclusions on the energy dependence of f_V^μ and g_V^μ. We have, however, compared our results with the predictions of the conserved current theory:[6]

$$\frac{g_V}{f_V} = -\frac{M_K W}{W^2 - P^2} = -\frac{M_K W}{q^2} .$$

In this theory the ratio g_V/f_V has a very strong energy dependence due to the pole at $q = 0$, which causes a marked peak in the pion spectrum at high energy. There is less than a 2% chance that the observed $K^+_{\mu 3}$ spectra are correctly described by this theory. Moreover, if μ-e universality is assumed, the ratio of $K^+_{\mu 3}$ and K^+_{e3} decay rates is predicted to be

$$\frac{R(K^+_{\mu 3})}{R(K^+_{e3})} = 0.38.$$

This is to be compared with the observed ratio:[2] 0.96 ±0.15.

We conclude that our experiment is in substantial disagreement with this prediction. On the other hand, more sophisticated models[7] based on assumptions about partially conserved currents lead to much less violent energy dependences which are compatible with our data.

IV. IDENTITY OF THE MUON AND ELECTRON COUPLING STRENGTHS

Since we have shown above that the experimental evidence favors vector coupling for both K^+_{e3} and $K^+_{\mu 3}$ decay, we are now ready to test the actual identity of the coupling by comparing f^μ_V and f^e_V (remembering that g^e_V cannot be observed, and hence no comparison of g^μ_V and g^e_V is possible). Again we assume the constancy of the f_V's, noting that this assumption is compatible with our measurement of λ discussed in Section II. We then note that:

(a) the K^+_{e3} rate determines f^e_V

(b) the $K^+_{\mu 3}$ rate determines a relation between f^μ_V and g^μ_V

(c) the study of the $K^+_{\mu 3}$ decay (P, E_μ) distribution provides another relation between f^μ_V and g^μ_V.

Thus from (b) and (c), f^μ_V can be determined and compared to f^e_V obtained from (a). The result is that $f^e_V/f^\mu_V = 1.09 \pm 0.15$, in good agreement with the expectations.

V. FURTHER DISCUSSION

We have already noted in Sec. III that if universality of the muon and the electron is assumed, the measured K^+_{e3} and $K^+_{\mu 3}$ rates lead to two values of g_V/f_V, namely +0.53 and -4.83. On the other hand, the study of the (P,E_μ) distribution in $K^+_{\mu 3}$ decay leads to

$$\frac{g_V}{f_V} = +0.3 \pm 0.5,$$

thus ruling out the large negative g_V/f_V possibility. This result, already clear from Fig. 1, disagrees with the published results of Dobbs et al.[8] Various theoretical models predict values of g_V/f_V from -0.2 to -0.9.[7] These are in reasonable agreement with the conclusion from the $K^+_{\mu 3}$ (P,E_μ) distribution and a little lower (i.e., more negative) than the result from the ratio of $K^+_{\mu 3}$ and K^+_{e3} rates. However, in view of the quoted statistical errors, as well as conceivable systematic biases in determining the $K^+_{\mu 3}$ rate, we interpret our data as being fully compatible with the predictions of these models.

To make the line of reasoning more apparent, we have assumed throughout our discussion that f_V and g_V are constant, which is in agreement with our data. A more complete (but more complicated) analysis, allowing additional parameters to describe the energy dependence of f_V and g_V, has been published elsewhere.[4,5] The conclusions drawn in this case are substantially the same as those of the present paper, except that one can no longer rigorously exclude the possibility of scalar and tensor couplings for $K^+_{\mu 3}$ decay.

FOOTNOTES AND REFERENCES

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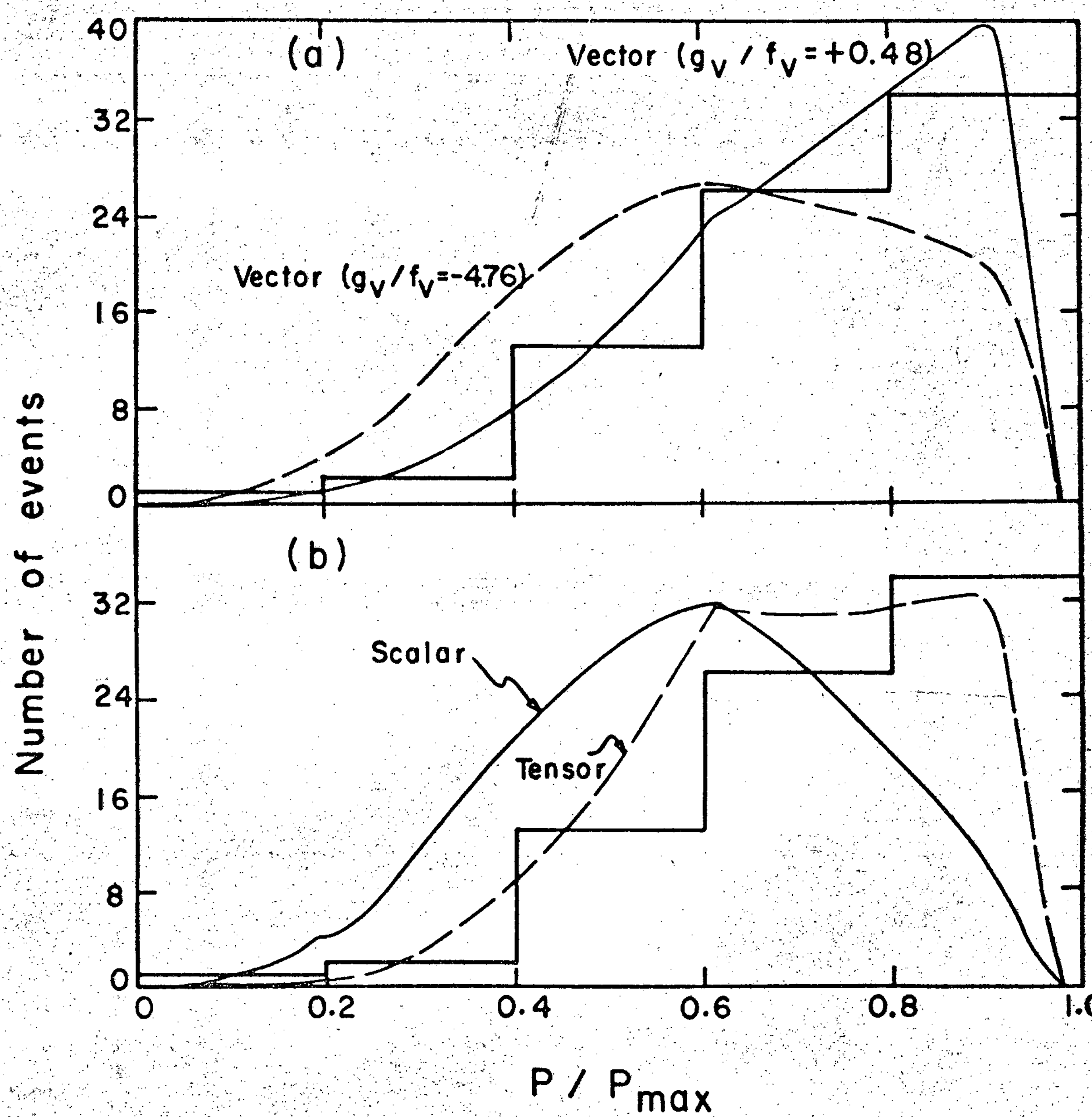
FIGURE LEGENDS

Fig. 1. Expected $K^+_{\mu 3}$ π^0 momentum distributions for (a) vector coupling with $g_V/f_V = +0.53$ (solid curve) and $g_V/f_V = -4.82$ (dashed curve), and (b) scalar (solid curve) and tensor (dashed curve) couplings, with constant form factors. The histogram represents experimental data (76 events). Irregularities in the curves result from modifications folded into the theoretical distributions to correct for criteria used in the selection of data.



(a)
Vector $(g_V / f_V = +0.48)$
Vector $(g_V/f_V=-4.76)$
(b)
Scalar
Tensor
Number of events
40
32
24
16
8
0
0
0.2
0.4
0.6
0.8
1.0
P / P_{max}

MU-27075