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ABSTRACT

Recent work on dual Reggeized resonance models for vector currents is discussed. The properties vector current amplitudes in such models are expected to possess are first described in some detail; i.e., (i) factorization, (ii) divergence conditions, and (iii) good large-q^2 behavior. Presently existing models fall into two classes: factorizable models that emphasize (i) and (ii) at the expense of (iii), and phenomenological models that emphasize (ii) at the expense of (i) and (ii). These models are discussed and a new phenomenological model is proposed that incorporates exponentially falling form factors, a property we believe dual resonance models should possess. We find the partial successes described here a source of optimism for an eventual complete solution of the problem.

I. INTRODUCTION

The recently developed Reggeized dual resonance model (DRM) for the strong interactions promises to be a very useful theoretical tool, if not also a good phenomenological model, since it exhibits a large number of properties that physical scattering amplitudes are believed to possess. Clearly it would be desirable to extend this model to include the electromagnetic and weak interactions of the hadrons (see Sec. II). Here we discuss recent attempts to construct a DRM for the vector currents.

In any attempt to construct a model for vector currents three important types of physical properties should be kept in mind (see Sec. III): (i) Bootstrap consistency conditions (factorization)--the spectrum of resonances occurring as poles in energy variables and in current "masses" (q^2) should be consistent with the spectrum of the purely hadronic amplitudes. (ii) Divergence conditions implied by current conservation and, if desired, current algebra should be satisfied. (iii) The large q^2 behavior should be "good," i.e., as suggested by experiment and theoretical considerations. In a complete bootstrap theory, we expect that condition (i) will completely determine the currents and thus the divergence conditions (ii) and the large-q^2 behavior (iii). However, here we put aside the question of uniqueness and investigate only the existence of currents with acceptable properties (i) - (iii).

Presently existing models emphasize one or more of the above properties at the expense of the others. One class of models attempts to satisfy factorization (i) and the divergence conditions (ii) but has bad large-q^2 behavior (iii) (see Sec. IV). If the infinite set of universally coupled vector mesons are included as poles in q^2, amplitudes satisfying current algebra and factorizing on the M highest trajectories can be constructed for a form factor falling like $(q^2)^{-M}$. However, it is found that complete factorization cannot be obtained if the current couples only to the universal vector mesons.

The other class of present models attempts to obtain good large-q^2 behavior (iii) and sometimes (ii), but has bad factorization (i) properties (see Sec. V). It is possible to generalize previously proposed models in order to obtain amplitudes with form factors that fall faster than any power, which we believe should be the case in the DRM. These amplitudes have all the good large-q^2 behaviors discussed in Sec. III.

Although none of the present models satisfies all the expected properties, we find the partial successes outlined above reason for optimism about obtaining a full solution to the problem. In the concluding section we suggest what we believe may be fruitful directions for future work on obtaining such a solution.

II. DUAL RESONANCE MODELS

The prominent hadronic resonances generally have rather narrow widths, appear to lie on fairly linear Regge trajectories, and account for most of the observed scattering cross sections. These empirical facts and theoretical considerations on composite particles led Mandelstam to propose a dynamical model for the strong interactions in which scattering amplitudes are dominated by zero-width resonances lying on linearly rising Regge trajectories.[1,2] Such a model is dual in the sense that any amplitude can be expressed, using unsubtracted dispersion relations (USDR), equivalently as a sum over resonance poles in any given channel or in its crossed channels. The requirement of consistency between these equivalent expressions is hoped to determine the resonance masses and couplings, neglecting unitarity corrections.

The dominance of the hadronic amplitudes by narrow resonances leads one to expect that the singularities in q^2 in current amplitudes are also dominated by narrow resonances.[3] Conversely, the rigorous validity of resonance domination in q^2 requires resonance domination in energy variables, since otherwise dispersion relations in q^2 have contributions from cuts whose discontinuities are not determined by resonance amplitudes. Since, furthermore, electromagnetic form factors are experimentally observed, and theoretically expected,[1] to decrease rapidly for large negative q^2 and presumably satisfy USDR, it is natural to extend the DRM to amplitudes involving the electromagnetic and weak currents by assuming that they can be expressed as a sum over resonance poles in q^2 as well as in energy variables.[4] In such a bootstrap theory of currents,[5] the couplings of the resonances to the currents are presumably determined by crossing symmetry and consistency with the hadronic amplitudes.

Work on the hadronic problem received considerable impetus by Veneziano's proposal of a simple function as a prototype dual amplitude.[6] Veneziano's model and its generalizations[7] assume a particular type of duality called planar duality. The N-body scattering amplitude is decomposed into a sum of terms, one for each permutation of the external momenta (p_i). Each term has resonance poles in subenergies corresponding to adjacent momenta [e.g., the term for the permutation $p_1, p_2, \cdots, p_N$ has poles in $s_{ij} = (p_i + p_{i+1} + \cdots + p_j)^2$] and also Regge behavior in these subenergies; it is therefore dual in the sense described above. We shall assume that the current amplitudes also have such a dual decomposition. At present there is no fundamental reason for assuming planar duality; the only justification we offer is simplicity.[8]

The simplest present hadronic model for the meson bootstrap, and the one we shall demand our currents be consistent with here, takes for each term in the dual decomposition the product of an "orbital factor" $B(p_1, p_2, \cdots, p_N)$ (N-point beta function[9]), and an "internal symmetry factor,"[10,11] $\frac{1}{2}\,\mathrm{Tr}(\tau_1\tau_2\cdots\tau_N)$. Mandelstam has proposed a model which has in addition a "spin factor."[11] This model has a better particle spectrum and includes the baryons as well as the mesons. Perhaps the most serious difficulty with these and other models for N-body amplitudes is that amplitudes involving pions do not vanish as the pion momenta go to zero. This means that we cannot obtain physically reasonable partially conserved axial-vector currents consistent with these hadronic models. We therefore restrict our considerations to vector currents in this paper. We thus make the tacit assumption that the zero-width limit can be assumed independently of the $SU(2) \otimes SU(2)$ symmetric limit; there seems to be no reason why this should not be possible.

All existing hadronic models are rather conjectural at the moment, of course, and it may be possible that only models other than the simple one considered here will admit consistent vector currents. Furthermore at present it is by no means clear to what extent the hadronic and current amplitudes are uniquely bootstrapped in the zero-width approximation. Supplementary assumptions like requiring a minimal number of states may be necessary in order to specify a unique solution. In fact one should keep in mind the possibility that the existence of physically acceptable current amplitudes might be a necessary condition for determining the hadronic amplitudes, although such a situation is contrary to the bootstrap philosophy discussed above.

III. PROPERTIES OF VECTOR CURRENT AMPLITUDES

In this section we discuss the basic properties that vector current amplitudes in dual resonance models should have. Some of these properties follow directly from the zero-width approximation, duality, and the conserved vector current hypothesis (CVC);[12] for example, the divergence conditions on the full amplitude are shown to give conditions on each separate term in the dual decomposition. Other properties are suggested by experiment and field theory models; for example, the relationship of the compositeness of the hadrons to the absence of certain fixed poles and the asymptotic behavior of form factors, the behavior of electroproduction structure functions, etc. We stress that it should be kept in mind that most of the field theory results can only be regarded as suggestive since many are derived by considering only a subset of Feynman diagrams and treat crossed channels in an unsymmetric manner. For simplicity we discuss the amplitudes for N spinless hadrons and one current $[V^{\mu}(q)]$ or two currents $[M^{\mu\nu}(q_1,q_2)]$.

(i) Bootstrap Consistency Conditions (Factorization)

Strong restrictions are imposed on the current amplitudes by the requirement that the spectrum of resonances occurring as poles in q^2 and in energy variables be the same as the spectrum of the purely hadronic amplitudes. These consistency conditions, shown diagrammatically in Figs. 1 and 2, include generalized vector-meson dominance [Figs. 1(a) and 2(a)] and factorization in the various several particle channels [Figs. 1(b), 2(b), and 2(c)]. According to the bootstrap philosophy,[5,12] the current amplitudes are believed to be completely determined by these conditions. Quadratic factorization [Fig. 2(c)] is expected to play the crucial role, since the single-current amplitudes must be restricted so as to yield acceptable two-current amplitudes.[13] We emphasize that due to quadratic factorization and USDR, the two-current amplitudes are completely determined by the single-current amplitudes; this fact will be heavily exploited in Sec. IV.

(ii) Divergence Conditions and High-Energy Behavior

We first consider the implications[12] of current conservation for the single-current amplitudes and two-current amplitudes. We then discuss the consequences of current algebra and the possibility of non-Regge terms in the asymptotic behavior.

Each isospin invariant amplitude of $V^{\mu}(q)$ has a dual decomposition as described in Sec. II and is divergenceless. Each term in the decomposition has poles in a different set of variables, and therefore there is no possibility of cancellation between them in the divergence. Since duality rules out terms without singularities in the full set of variables, each term must itself be divergenceless[14] (see Fig. 3),

$$q_{\mu}\, V_{i,P}{}^{\mu}(q) \;=\; 0 \quad . \tag{3.1}$$

Here P specifies the permutation P of the hadron momenta and i specifies that the current is to the left of $p_{P(i)}$. From now on we consider only the hadron ordering $p_1,\cdots,p_N$ and drop the subscript P.

Current conservation has particularly interesting consequences for $q_\mu \to 0$. In this limit the dominant contributions to the amplitude come from the soft pole terms or external line insertions (ELI)--see Fig. 4. First, we remark that in our generalized vector-meson dominance model nonzero ELI are most naturally obtained through the existence of at least one vector meson which couples universally to the hadrons. If a single vector meson dominates, it must couple universally to provide nonzero ELI. If the hadronic spectrum does not include such universally coupled mesons, physically reasonable consistent vector currents will be difficult to obtain. Second, we note that V_i^μ has only two ELI, i.e., those corresponding to p_{i-1} and p_i at $(q + p_{i-1})^2 = m_{i-1}^2$ and $(q + p_i)^2 = m_i^2$. Since for $q_\mu \to 0$ these are the only possible contributions to (3.1), the residues of these poles must be equal and opposite,

$$V_i^\mu(q) \approx \left[\frac{q^\mu + 2p_{i-1}^\mu}{(q + p_{i-1})^2 - m_{i-1}^2} - \frac{q^\mu + 2p_i^\mu}{(q + p_i)^2 - m_i^2}\right] A_{\text{hadronic}} \,. \tag{3.2}$$

From now on the V_i^μ will always be understood to have their ELI poles normalized as in (3.2).

Let us discuss further the structure of V^μ as $q^\mu \to 0$ and its internal symmetry properties before restricting ourselves to the case of no exotic resonances. It is particularly convenient to represent the isospin state of each hadron, p_i, as a direct product of isospin one-half spinors--"quarks" or "antiquarks," i.e. lower indices $\alpha_i, \alpha_i', \cdots, \alpha_i^{(k)}$ and upper indices $\beta_i, \beta_i', \cdots, \beta_i^{(\ell)}$ (one may require some symmetry and tracelessness conditions but we may ignore this inessential complication). The number of upper and lower indices of an amplitude can always be made equal by using the raising and lowering matrix $C_\alpha^{\ \beta}$. Since $\delta_\alpha^{\ \beta}$ is the only invariant tensor in SU(2), an amplitude can always be expanded as a sum of terms, each consisting of a product of δ's and an isospin invariant amplitude.[11] Each such term has a natural diagrammatic representation--see Fig. 5(a) for an example of a purely hadronic amplitude. Each term in a dual decomposition has a similar expansion. For these it is convenient to draw the lines around the periphery of the diagram--compare Figs. 5(a) and 5(b).

Now consider for simplicity an isovector current (currents with other isospins can be easily treated in a similar manner) with spinor indices α and β where the isospin one part is obtained by using the projection operator $\sum_{\alpha,\beta} (\tau_a)^\alpha_{\ \beta}$. The ELI's for V^μ are then given by

$$V_a^{\ \mu} \, {}^{\beta_1 \cdots \beta_N{}^{(\ell)}}_{\alpha_1 \cdots \alpha_N{}^{(k)}} \approx \frac{q_\mu + 2p_i^{\ \mu}}{(q+p_i)^2 - m_i^{\ 2}} \left\{ \sum_m (\tau_a)^{\bar{\alpha}_i{}^{(m)}}_{\ \alpha_i{}^{(m)}} \, A^{\beta_1 \cdots \beta_N{}^{(\ell)}}_{\alpha_1 \cdots \bar{\alpha}_i{}^{(m)} \cdots \alpha_N{}^{(k)}} \right.$$

$$\left. - \sum_n A^{\beta_1 \cdots \bar{\beta}_i{}^{(n)} \cdots \beta_N{}^{(\ell)}}_{\alpha_1 \cdots \alpha_N{}^{(k)}} \, (\tau_a)^{\beta_i{}^{(n)}}_{\ \bar{\beta}_i{}^{(n)}} \right\} . \qquad (3.3)$$

One can easily verify that the vanishing of the divergence for $q_\mu \to 0$ is assured by the isospin invariance of A:

$$\sum_{i,m} (\tau_a)_{\alpha_i^{(m)}}^{\bar{\alpha}_i^{(m)}} A_{\alpha_1 \cdots \bar{\alpha}_i^{(m)} \cdots \alpha_N^{(k)}}^{\beta_1 \cdots \beta_N^{(\ell)}}$$

$$- \sum_{i,n} A_{\alpha_1 \cdots \alpha_N^{(k)}}^{\beta_1 \cdots \bar{\beta}_i^{(n)} \cdots \beta_N^{(\ell)}} (\tau_a)_{\bar{\beta}_i^{(n)}}^{\beta_i^{(n)}} = 0 \ .$$

Now consider a particular isospin invariant amplitude. For definiteness suppose it corresponds to $\delta_{\bullet}^{\bullet} \cdots \delta_{\alpha}^{\beta_k} \delta_{\alpha_j}^{\beta} \cdots \delta_{\bullet}^{\bullet}$, where only the δ's involving currents have been explicitly shown and α_j and β_k are any indices for p_j and p_k, respectively. Each term in the dual decomposition of this isospin invariant amplitude of course satisfies (3.1). This condition and the requirement that the full ELI contribution be given by (3.3) can easily be shown to imply that the amplitude has the form

$$\delta_{\bullet}^{\bullet} \cdots \delta_{\alpha}^{\beta_k} \delta_{\alpha_j}^{\beta} \cdots \delta_{\bullet}^{\bullet} \left[\sum_{i=j+1}^{k} V_i^{\mu}(q) \right] \ . \qquad (3.4)$$

In terms of diagrams, this means that in V_i^{μ} the current couples (by τ_a) to each "quark line" passing between p_{i-1} and p_i--see Fig. 6. We therefore see that current conservation, duality, and isospin invariance require the amplitude to have a very particular form as $q_{\mu} \to 0$ with the current in a sense probing the quark-like isospin structure of the amplitude. It should be emphasized that (3.4) only specifies the ELI structure; an arbitrary contribution containing no soft poles can always be added.

The isospin analysis for the two-current amplitudes proceeds in a very similar manner but there is an important new feature which has to do with the ELI and the terms in the dual decomposition with adjacent currents. Let us denote by $M_{ij}^{\mu\nu}(q_1,q_2)$ the term in the dual decomposition corresponding to the permutation $p_1,\cdots, p_{i-1}, q_1, p_i,\cdots,p_{j-1}, q_2, p_j,\cdots,p_N$ (or similarly for $i > j$). For adjacent currents, $i = j$, we let M_{ii} denote the term with q_1 to the left of q_2 and M'_{ii} denote the term for q_1 to the right of q_2. The terms with nonadjacent currents have two ELI's for each current and thus may be taken to be individually divergenceless, at least for $q_{i\mu} \to 0$. However, the adjacent current terms have only <u>one</u> ELI each: both $M_{ii}^{\mu\nu}(q_1,q_2)$ and $M'^{\mu\nu}_{ii}(q_1,q_2)$ are needed to supply the usual two ELI on p_{i-1} and p_i. The existence of only one ELI means that these amplitudes cannot be divergenceless as $q_{i\mu} \to 0$; specifically, we find

$$q_{1\mu}\, M^{\mu\nu}_{ii,ab}{}^{\beta_1\cdots\beta_N}_{\alpha_1\cdots\alpha_N}{}^{(\ell)}_{(k)}(q_1,q_2) \xrightarrow[q_{1\mu}\to 0]{} \sum_{\bar{\alpha}_x} (\tau_a)_{\alpha_x}{}^{\bar{\alpha}_x}$$

$$\times \quad V_{i;b}{}^{\nu}{}^{\beta_1\cdots\beta_N}_{\alpha_1\cdots\bar{\alpha}_x\cdots\alpha_N}{}^{(\ell)}_{(k)}(q_1+q_2)$$

$$- \sum_{\bar{\beta}_y} V_{i;b}{}^{\nu}{}^{\beta_1\cdots\bar{\beta}_y\cdots\beta_N}_{\alpha_1\cdots\alpha_N}{}^{(\ell)}_{(k)}(q_1+q_2)(\tau_a)_{\bar{\beta}_y}{}^{\beta_y} , \qquad (3.5)$$

where the sums are over all quark lines between p_{i-1} and p_i.

The important result that $M_{ii}^{\mu\nu}$ has a fixed singularity in addition to the usual Regge poles in the two-current channel (t channel) follows from Eq. (3.5), current conservation, and quadratic factorization. To see this, consider any channel s_i [see Fig. 2(c)]. Current conservation and quadratic factorization imply that $q_{1\mu}M^{\mu\nu}$ has no poles in this channel and so it must behave like a polynomial in s_i. Since the right-hand side of (3.5) is nonvanishing, we conclude that the constant term is nonvanishing at least for $q_1^2 = 0$ and $q_2^2 = t$ in those amplitudes not multiplied by q_1^ν. This implies that $M_{ii}^{\mu\nu}$ also has fixed power behavior as $s_i \to \infty$ and thus some fixed singularities in the t channel.[15] This result is stronger than that which obtains without duality, since it applies to M_{ii} occurring in the dual decomposition of isospin symmetric amplitudes (e.g., for physical photons) as well as isospin antisymmetric amplitudes (where the usual current algebra fixed pole occurs). In particular it implies the existence of wrong-signature fixed poles in the isospin symmetric amplitudes at least at the special point $q_1^2 = 0$ and $q_2^2 = t$.[16]

For the remainder of this paper we assume the absence of exotic $(I > 1)$ resonances. Then, by a trivial generalization of the results of Chan and Paton,[10] we must have the dual decompositions,

$$V_a^\mu(q) = \sum_{i,P} \frac{1}{2}\, \mathrm{Tr}[\tau_{P(1)} \cdots \tau_{P(i-1)} (\tfrac{1}{2}\, \tau_a) \tau_{P(i)} \cdots \tau_{P(N)}] V_{i,P}^\mu(q) \tag{3.6}$$

and

$$M_{ab}{}^{\mu\nu}(q_1,q_2)$$

$$= \sum_{i\neq j,P} \frac{1}{2}\,\mathrm{Tr}[\tau_{P(1)}\cdots\tau_{P(i-1)}(\tfrac{1}{2}\,\tau_a)\tau_{P(i)}\cdots\tau_{P(j-1)}(\tfrac{1}{2}\,\tau_b)\tau_{P(j)}\cdots\tau_{P(N)}] \cdot M^{\mu\nu}_{ij,P}(q_1,q_2)$$

$$+ \sum_{i} \frac{1}{2}\,\mathrm{Tr}[\tau_{P(1)}\cdots\tau_{P(i-1)}(\tfrac{1}{2}\,\tau_a)(\tfrac{1}{2}\,\tau_b)\tau_{P(i)}\cdots\tau_{P(N)}]M^{\mu\nu}_{ii,P}(q_1,q_2)$$

$$+ \sum_{i} \frac{1}{2}\,\mathrm{Tr}[\tau_{P(1)}\cdots\tau_{P(i-1)}(\tfrac{1}{2}\,\tau_b)(\tfrac{1}{2}\,\tau_a)\tau_{P(i)}\cdots\tau_{P(N)}]M^{\nu\mu}_{ii,P}(q_2,q_1). \tag{3.7}$$

We note that the absence of exotic resonances and Bose statistics for the currents have been used to replace $M'^{\mu\nu}_{ii}(q_1,q_2)$ by $M_{ii}{}^{\nu\mu}(q_2,q_1)$. The divergence conditions (3.5) on the two-current amplitudes now become simply (see Fig. 7),

$$q_{1\mu}M^{\mu\nu}_{ij,P}(q_1,q_2) = 0 \qquad (i \neq j) \tag{3.8}$$

and

$$q_{1\mu}M^{\mu\nu}_{ii,P}(q_1,q_2) = V_{i,P}{}^{\nu}(q_1+q_2) \;,$$

$$M^{\nu\mu}_{ii,P}(q_2,q_1)q_{1\mu} = -V_{i,P}{}^{\nu}(q_1+q_2) \;, \tag{3.9}$$

and similarly for q_2. These hold exactly for $q_{1\mu} \to 0$ and to within terms which vanish as $q_{1\mu} \to 0$ for all $q_{1\mu}$ with $q_1{}^2 = 0$ and $q_2{}^2 = t$.

If we further assume the Gell-Mann current algebra, (3.8) and (3.9) hold for all q_1 and q_2. Of course, according to the bootstrap philosophy, the bootstrap conditions are believed to determine completely the current amplitudes and therefore the current algebra. Since we are unable to fully implement these conditions here, we shall often assume CVC and current algebra as additional requirements.

Several comments on the divergence conditions for the two-current amplitudes are in order. Firstly, there is no necessary contradiction between current algebra and duality, i.e., (3.8) and (3.9) do not require that any invariant amplitude not satisfy USDR.[17]

Secondly, in order to dispel possible confusion, we should remark on the relationship of our program to the theorems proving that physically acceptable solutions of current algebra without states of spacelike momenta do not exist.[18] Of course, there is the possibility that the solution to the bootstrap conditions does not correspond to any current algebra;[19] there is then obviously no contradiction with the theorems. Even if the currents do satisfy current algebra, it has recently been shown that the theorems can be circumvented with a particle spectrum similar to that considered here either through the presence of Schwinger terms in the time-time commutators or through the existence of ghost states of negative norm.[20] Both of these possibilities are possible in our model, in fact it is well known that the N-point beta function has ghosts.[21]

Finally, it should be noted that the presence of the fixed poles required by (3.9) implies that certain sums must converge nonuniformly: (a) Since the purely hadronic vector-meson amplitudes do not (and cannot) have fixed power behavior as $s_i \to \infty$, the sum over vector mesons [Fig. 2(a)] must converge nonuniformly in s_i so that the limit $s_i \to \infty$ cannot be taken inside. (b) The sum rule for the fixed pole residue is of the form, assuming current algebra,

$$\int ds_i \; \text{Im}\, M(s_i, t; q_1^2; q_2^2) = \sum_n R_n(t, q_1^2, q_2^2) = F(t) \quad . \tag{3.10}$$

Since R_n is proportional to the product of form factors, we expect it to decrease rapidly as $q_i^2 \to \infty$. Thus the sum (3.10) must converge nonuniformly in q_i^2. These nonuniformities are exhibited by the functions discussed in Sec. V.

We have seen that the divergence conditions (3.4) require the existence of fixed singularities in the t-channel angular momentum plane of the two-current amplitude. However, we do not expect fixed singularities in any other channel or in any other amplitude, if the DRM does indeed correspond to infinitely composite particles as one suspects. This expectation is based on the results of certain field theory and potential theory model calculations.[22,23] For example, Rubenstein et al.[22] show that [see Fig. 8(a)], if particle X is a composite of two particles, the highest fixed pole possible in the s channel is absent. If X is a composite of more particles, lower fixed poles are also absent. Similarly, we might expect fixed poles in the s channel of the two-current amplitude to be absent, if the hadron is composite [see Fig. 8(b)]. Thus there are two suggested ways in which the compositeness of the hadrons manifests itself in the nonstrong interactions: (a) the absence of fixed poles except in the two-current channel and (b) the rapid decrease of form factors (see Ref. 1 and the following subsection). The functions discussed in Secs. IV and V illustrate the close connection between these two features: the faster the form factors fall, the lower the extra fixed poles[24]--in the limit of exponential form factors (Sec. V) only the t-channel fixed pole remains.

(iii) Large-q^2 Behavior

Here we discuss some features of the behavior of current amplitudes for large q^2 which are suggested by theory (usually field theory) and experiment. From the explicit examples given in Sec. V, it appears that the DRM is capable of incorporating all of them.

Mandelstam has suggested that in a theory with infinitely rising Regge trajectories form factors should fall faster than any power as $q^2 \to \infty$.[1] Further support for "exponential" form factors comes from Harte's bootstrap model for infinitely composite particles.[25] He finds that form factors must behave like $e^{-(-q^2)^\gamma}$, $0 < \gamma \leq \frac{1}{2}$, to within powers of q^2, in order to satisfy his bootstrap equations. Field theory models for composite particles also give rapidly decreasing form factors.[26] We thus expect that in the DRM form factors should fall faster than any power. The results of Secs. IV and V give some support to this belief.

The field theory models also make an interesting prediction about the asymptotic spin dependence of form factors.[26] They predict that the form factor falls faster the larger the spins J_1 and J_2 of the particles involved,

$$F^{(i)}_{J_1 J_2}(q^2) \sim \left(\frac{1}{q^2}\right)^{J_1+J_2-i} F(q^2) \quad , \tag{3.11}$$

where i labels the various amplitudes [see Eq. (5.15) for a precise definition]. In these calculations, $F(q^2)$ has an asymptotic power behavior that depends on the specific model. Roughly speaking, this behavior can be understood as a manifestation of the behavior of the bound-state wave function at the origin.

We now turn to the two-current amplitudes. The Bjorken limit[27] gives an interesting connection between the large q^2 behavior and the current commutation relations, if such exist. Defining $q = \frac{1}{2}(q_1 - q_2)$ and $Q = (q_1 + q_2)$ and taking the limit $|q_0| \to \infty$ with $\vec{q}$, Q^μ, and the hadronic momenta fixed, we have

$$M_{ab}^{\mu\nu} \xrightarrow[|q_0|\to\infty]{} -\frac{i}{q_0} \int d^3x\, e^{i\vec{q}\cdot\vec{x}} \langle\alpha|[V_a{}^\mu(0,\tfrac{1}{2}\vec{x}), V_b{}^\nu(0,-\tfrac{1}{2}\vec{x})]|\beta\rangle$$

$$-\frac{1}{q_0{}^2} \int d^3x\, e^{i\vec{q}\cdot\vec{x}} \langle\alpha|[\dot{V}_a{}^\mu(0,\tfrac{1}{2}\vec{x}), V_b{}^\nu(0,-\tfrac{1}{2}\vec{x})]|\beta\rangle$$

$$+ \cdots + \text{polynomial in } q_0 \;, \tag{3.12}$$

where α and β represent hadronic states. We note that in this limit that $s_i \to \infty$ also whereas $t = Q^2$ is fixed. Equation (3.12) is very useful for determining what current commutation relations amplitudes correspond to. To avoid possible confusion about the application of (3.12), we note that, since $M^{\mu\nu}$ has Regge behavior $(s_i)^{\alpha(t)}$, in general one can only expect[28] to be able to carry the expansion (3.12) to as many terms as correspond to fixed poles above $\alpha(t)$. If an attempt is made to carry it further, the remainder term will diverge. Any given term can, of course, be obtained by choosing t sufficiently negative, since $\alpha(t)$ is infinitely falling in our model, (or, in general, by subtracting out the divergence).

Recent experiments on deeply inelastic electron scattering have generated much interest in the behavior of the two-current amplitude with two hadrons in the limit $q^2 \to -\infty$ with $\rho \equiv -\frac{s-u}{2q^2} = -\frac{2m\nu}{q^2}$ and $t\ (=0)$ fixed. Experiment and theory[29,30] suggest that typical invariant amplitudes behave like

$$M_i \xrightarrow[\substack{q^2 \to \infty \\ \rho, t \text{ fixed}}]{} (-q^2)^{k_i} f_i(\rho,t) \quad , \tag{3.13}$$

where k_i is a small negative integer. In Regge behaved models one also expects[30]

$$\operatorname{Im} f_i(\rho,t) \xrightarrow[\rho \to \infty]{} \rho^{\alpha(t)-n_i} \quad . \tag{3.14}$$

It should be emphasized that so far experiment only suggests the behaviors (3.13) and (3.14) for the diffractive (Pomeranchon) contribution with $\alpha_P(0) = 1$.[4] It could well be, as conjectured by Harari,[4] that the contribution of the other (dual) trajectories vanishes very rapidly as $q^2 \to -\infty$ and only the Pomeranchon has the weak q^2 dependence (3.13). We remark that the amplitudes discussed in Sec. V have the above behaviors for <u>all</u> trajectories just as do the field theory models.[30]

The behavior of $f_i(\rho,t)$ at threshold seems to be related to the asymptotic behavior of the elastic form factors. Drell and Yan[31] have pointed out that, in their field theory model, if the elastic form factor has the asymptotic power behavior

$$F(q^2) \underset{q^2 \to \infty}{\sim} (q^2)^{\gamma} , \tag{3.15a}$$

one has the threshold behavior

$$\text{Im } f(\rho,0) \underset{\rho \to 1}{\sim} (\rho - 1)^{-2\gamma-1} \tag{3.15b}$$

for a specific amplitude (νW_2). The existence of relationships of this form is probably quite model independent.

This concludes our discussion of the properties we should like vector current amplitudes to satisfy. In the next two sections we discuss dual resonance models which attempt to satisfy them. Although there does not yet exist a model having all the above properties, there is no indication that any of them are inconsistent with the DRM.

IV. FACTORIZABLE MODELS

In this section we discuss models which attempt to satisfy (i) the factorization conditions, in particular, consistency with the simple N-point beta function hadronic model described in Sec. II, and (ii) the divergence conditions. The discussion is a summary of the work of Brower and Weis.[32,33]

As noted above, a physically acceptable vector-meson dominated conserved current suggests the existence of at least one universally coupled vector meson. Fortunately, such vector mesons are in fact present in the hadronic spectrum;[32,34] there is one at each mass $m_\ell^2 = m^2 + 1 + \ell$ $(\ell = 0,1,2,\cdots)$.[35] Since the low-lying trajectories in the DRM have a large degeneracy,[34] these vector mesons are only a small fraction of the total, but they play a particularly vital role in models for currents.

Applying vector-meson dominance for the universally coupled mesons, we give below one- and two-current amplitudes that (i) obey current algebra, (ii) factorize on the M highest trajectories, and (iii) have form factors that fall like $(q^2)^{-M}$. On the other hand, if only leading trajectory factorization is required, the current algebra condition can be satisfied for arbitrary form factors--see Appendix B of Ref. 33. We feel that these results give a good indication of the power of factorization in determining the structure of currents and suggest that in a full solution to the problem form factors will fall exponentially.

However, the limit $M \to \infty$ of these amplitudes does not lead to a full solution. Indeed, the requirements of complete factorization and USDR in q^2 for the single current amplitudes do imply exponential form factors, but the two-current amplitudes constructed from the single-current amplitudes using complete quadratic factorization are found to possess unphysical singularities which violate linear factorization. Therefore full factorization cannot be obtained if only the universally coupled vector mesons are included; approximate solutions are the most that can be obtained with this restriction. However, a completely factorizable solution may be obtainable if some or all of the other vector mesons are included. The major difficulty with this lies in the tremendous number of existing parameters, f_n, which are apparently quite arbitrary if only single-current amplitudes are considered, but are in fact severely constrained in a nonobvious manner by the connection of these amplitudes to the two-current amplitudes through quadratic factorization.

Before delving into the details of this model, we make a general comment on the method used to obtain conserved currents. Consider the amplitude coupling a current to N spinless particles of lowest mass ("scalars"). Current conservation (3.1) is equivalent to the statement that the divergence of the current does not couple to N scalars, i.e., it is either a "spurious state"[34] or identically zero. The problem of finding conserved currents is thus closely related to the problem of finding spurious states. For example, in our model this amplitude is given by (see Fig. 9),

$$V^{\mu}(q) = F(q^2)\langle 0|(\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu})|p\rangle \tag{4.1}$$

with

$$|p\rangle = V(p_1)\, D(R,s_1)\, V(p_2)\cdots V(p_{N-1})|0\rangle \ ,$$

where we have used the operator notation of Fubini, Gordon, and Veneziano.[36] The form factor $F(q^2)$ gives poles at $m^2 + 1 + \ell$ and the residues are proportional to amplitudes for universally coupled vector mesons. The divergence of V^{μ} is proportional to

$$\langle 0|(\sqrt{2}\, q\cdot a_{(1)} + q^2) = \langle 0|S(q) \equiv \langle 0_s|$$

which is the first spurious state generated by the spurious state operator,[37] $S(q)$, and thus by definition does not couple to the N-scalar state $|p\rangle$, i.e., $\langle 0_s|p\rangle = 0$.

In order to exhibit the absence of spurious intermediate states, $|\lambda_s\rangle$, in (4.1) we should replace $V(p)$ by $\hat{V}(p)$[38] which has no couplings to spurious states

$$\langle\lambda_s|\hat{V}(p) = \hat{V}(p)|\lambda_s\rangle = 0 \ . \tag{4.2}$$

This has no effect in the all scalar amplitude (4.1) but is necessary to assure that the current has a conserved coupling to an arbitrary excited state $|\lambda\rangle$,

$$q_{\mu}\left[F(q^2)\langle 0|(\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu})\ \hat{V}(p)|\lambda\rangle\right] = 0 \ . \tag{4.3}$$

One observes from (4.2) and (4.3) that there is a one-to-one correspondence between current conservation and the elimination of spurious intermediate states. Roughly speaking this is because the divergence has its coupling proportional to a spurious state and thus can only couple to other spurious states.[39] If such states are eliminated, so is the divergence.

In the partially factorizable current-algebra parameterization, we use a modified vertex, $\hat{V}_M$, obtained by terminating the expansion for $\hat{V}$ after M terms,

$$\hat{V}_M(p) = \sum_{\ell=0}^{M} \frac{(-1)^\ell}{\binom{-m^2-1}{\ell}} \binom{S^+(-k) - m^2}{\ell} V(p) \binom{S(k+p) - m^2}{\ell}, \tag{4.4}$$

and the specific form factor,

$$F_M(q^2) = \prod_{\ell=0}^{M-1} \left(1 - \frac{q^2}{m^2+1+\ell}\right)^{-1} = \sum_{\ell=0}^{\infty} f_\ell [1 - q^2/(m^2+1+\ell)]^{-1}$$

$$\underset{q^2\to\infty}{\sim} (q^2)^{-M} . \tag{4.5}$$

The amplitude for a single current, $N-1$ scalars, and one excited state is then[41]

$$V_\lambda{}^\mu(q) = F_M(q^2)\langle 0|(\sqrt{2}\, a_{(1)}{}^\mu + q^\mu)\, \hat{V}_M(p_1)\, D(R,s_1)\, V(p_2)\cdots V(p_{N-1})|\lambda\rangle. \tag{4.6}$$

It is conserved for states λ lying on trajectories displaced less than M units below the leading one and also factorizes for such intermediate states but not for lower trajectories. For future use we note that the coupling of the current to a scalar and an excited state on a trajectory k units below the leading one in general behaves like

$$F_M(q^2)(q^2)^k \underset{q^2\to\infty}{\sim} (q^2)^{-M+k} \quad . \qquad (4.7)$$

This behavior is clearly quite different from the field theory suggestion (3.11) in that the asymptotic behavior depends on the trajectory a particle lies on and not its spin. In general it appears very difficult to obtain behavior like (3.11) when form factors are introduced in the multiplicative way (4.6). Such simple multiplicative form factors are also responsible for bad large-q^2 behavior of the two-current amplitudes.

We now discuss the two-current amplitudes. Amplitudes for two nonadjacent universal vector mesons are automatically conserved[32] and thus amplitudes satisfying (3.8) are constructed by just multiplying by the form factors as in (4.1). Thus the nonadjacent terms need not be considered further. Amplitudes for adjacent currents satisfying (3.9) are constructed by a generalization of the work of Brower and Halpern.[40] We write

$$M^{\mu\nu}(q_1,q_2) = M_H^{\ \mu\nu}(q_1,q_2) + M_C^{\ \mu\nu}(q_1,q_2) + M_{FP}^{\ \ \mu\nu}(q_1,q_2) \quad , \qquad (4.8)$$

where $M_H^{\ \mu\nu}$ and $M_C^{\ \mu\nu}$ are purely Regge behaved, $M_H^{\ \mu\nu}$ contains all the vector-meson poles and $M_C^{\ \mu\nu}$ cancels its unwanted Regge behaved divergence. The exact current algebra divergence comes from $M_{FP}^{\ \ \mu\nu}$ which has fixed singularities in J_t.

The "hadronic amplitude", $M_H^{\mu\nu}$, is constructed using $\hat{V}_M$,

$$M_H^{\mu\nu}(q_1,q_2) = -F_M(q_2^2)\langle 0|(\sqrt{2}\, a_{(1)}^{\nu} + q_2^{\nu})V(p_1)\, D(R,s_1)\cdots$$

$$\cdots D(R,s_{N-1})\, \hat{V}_M(p_N)(\sqrt{2}\, a_{(1)}^{\mu+} + q_1^{\mu})|0\rangle F_M(q_1^2)$$

$$+ \frac{(m^2+1)\cdots(m^2+M)}{M!}\langle u^M \mathcal{V}_{(1)}^{\nu}\, \bar{\mathcal{V}}_{(1)}^{\mu} + 2g^{\mu\nu}(u^M + Mu^{M+1})\rangle_{N+2} , \qquad (4.9)$$

where

$$\mathcal{V}_{(1)}^{\nu} = q_2^{\nu} + 2p_1^{\nu} + 2p_2^{\nu}u_1 + \cdots + 2p_N^{\nu}(u_1\cdots u_{N-1}) ,$$

the u_i are the usual integration variables[9] $[u = (u_1u_2\cdots u_{N-1})]$, and the bracketts $(\langle\ \rangle_R)$ represent an integral, $\int_R du\, I_R$, over the usual integrand for the R-point function.[9] This term factorizes (without spurious states) on trajectories displaced by less than M units below the leading one, since $\hat{V}_M$ assures this for the first term and the second term contributes only to lower trajectories due to the factor u^M.

The other two terms are given by

$$M_C^{\mu\nu}(q_1,q_2)$$

$$= \sum_{\ell=0}^{M-1} \frac{f_\ell(m^2+1+\ell)}{\alpha_t - 1 - \ell} \langle[\mathcal{V}_{(1)}^{\nu} + (q_2^{\nu} + 2q_1^{\nu})u][\bar{\mathcal{V}}_{(1)}^{\mu} + (q_1^{\nu} + 2q_2^{\nu})u]$$

$$\times (1-u)^{\ell}\rangle_{N+2} \qquad (4.10)$$

and

$$M_{FP}{}^{\mu\nu}(q_1,q_2) = \sum_{\ell=0}^{M-1} \frac{f_\ell(m^2+1+\ell)}{\alpha_t - 1 - \ell}$$

$$\times \langle [\mathcal{V}_{(1)}{}^{\nu} + (q_2{}^{\nu} + 2q_1{}^{\nu})u][\overline{\mathcal{V}}_{(1)}{}^{\mu} + (q_1{}^{\nu} + 2q_2{}^{\nu})u](1-u)^{\alpha_t - 1}\rangle_{N+2}$$

$$+ \; F(t) \; g^{\mu\nu}\langle 1\rangle_{N+1} \; , \qquad (4.11)$$

where the f_ℓ are determined by (4.5). The divergences have the properties described above, since

$$q_{1\mu} \; M_H{}^{\mu\nu}(q_1,q_2) = -q_{1\mu} \; M_C{}^{\mu\nu}(q_1,q_2)$$

$$= \frac{(m^2+1)\cdots(m^2+M)}{(M-1)!} \langle u^M \mathcal{V}_{(1)}{}^{\nu} + (q_2{}^{\nu} + 2q_1{}^{\nu})u^{M+1}\rangle_{N+2} \; ,$$

and

$$q_{1\mu} \; M_{FP}{}^{\mu\nu}(q_1,q_2) = F(t)\langle \mathcal{V}_{(1)}{}^{\nu}(q_1+q_2)\rangle_{N+1} = V^{\nu}(q_1+q_2).$$

The sum $M_C{}^{\mu\nu} + M_{FP}{}^{\mu\nu}$ has poles in s_i determined by

$$\sum_{\ell=0}^{M-1} \frac{f_\ell(m^2+1+\ell)}{\alpha_t - 1 - \ell}[1 - (1-u)^{\alpha_t - 1 - \ell}](1-u)^{\ell} \; ,$$

which due to (4.5) is proportional to u^M and so contributes only to trajectories displaced by at least M units. Therefore, since linear factorization is always trivially satisfied, the two-current amplitude factorizes on all trajectories displaced less than M units below the leading one as asserted.

We have seen the $M_C^{\mu\nu} + M_{FP}^{\mu\nu}$ contributes only to trajectories displaced by at least M units--this contribution is nonfactorizable. Furthermore, since this piece has no poles in q^2, it corresponds to subtractions in q^2 dispersion relations contrary to our requirements. This fact along with (4.7) means that the current algebra sum rule (3.10) is satisfied <u>uniformly</u> in q^2 and is saturated for large q^2 by the low-lying nonfactorized poles. This unphysical feature [see Sec. III (ii)] gives a hint of the source of the failure of this parameterization as $M \to \infty$. In fact in this limit $M_C^{\mu\nu} + M_{FP}^{\mu\nu}$ would have no poles in s_i. Since it is Regge behaved for $s_i \to -\infty$ and nonzero, it must have nonpower behavior for $s_i \to +\infty$.

Let us point out several other features of the amplitudes (4.9) to (4.11). First, the only fixed power behavior in addition to that associated with the current algebra fixed pole is t^{-M} as $t \to \infty$ and comes from $M_{FP}^{\mu\nu}$. The faster the form factor falls the lower this power behavior.[24] Secondly, these amplitudes, particularly the nonadjacent current terms, have very bad behavior for deeply inelastic electron scattering as one can readily verify (see Ref. 46 below). As noted above, this can be traced to the simple multiplicative nature of the form factors.

We believe that more general parameterizations with the M highest trajectories factorizing can be constructed with only the condition that $F(q^2)$ decrease at least as rapidly as $(q^2)^{-M}$. This connection between the asymptotic behavior of form factors and factorization is very suggestive--but only suggestive, due to the nonexistence proof mentioned above which we now discuss.

A likely candidate for the completely factorized single-current amplitude is[41]

$$V_\lambda{}^\mu(q) = F(q^2)\langle 0|(\sqrt{2}\, a_{(1)}{}^\mu + q^\mu)\, \hat{V}(p_1)\, D(R,s_1)\cdots\hat{V}(p_{N-1})|\lambda\rangle , \tag{4.12}$$

since $\hat{V}$ eliminates spurious states and makes $V_\lambda{}^\mu$ exactly conserved. Indeed, assuming unsubtracted dispersion relations in q^2 and requiring the absence of spurious states $|\lambda_s\rangle$ on-mass-shell $(p_\lambda{}^2 = m^2 + J)$ [or, alternatively, current conservation on-mass-shell $(p_\lambda{}^2 = m^2 + J)$] we find (4.12) is the required amplitude provided $F(q^2)$ falls faster than any power, i.e.,

$$\sum_{\ell=0}^{\infty} f_\ell (m^2 + 1 + \ell)^n = 0 , \qquad n = 1,2,3,\cdots . \tag{4.13}$$

Some examples of form factors satisfying (4.13) are given in Appendix A.

The two current amplitudes are determined from (4.12) through quadratic factorization and USDR. It should not be surprising that the result is [compare with (4.9)],

$$M^{\mu\nu}(q_1,q_2) = F(q_1{}^2)\, \hat{B}^{\mu\nu}(q_1,q_2)\, F(q_2{}^2) , \tag{4.14}$$

where

$$\hat{B}^{\mu\nu}(q_1,q_2) = -\langle 0|\left(\sqrt{2}\, a_{(1)}{}^\nu + q_2{}^\nu\right)\hat{V}(p_1)D\cdots\hat{V}(p_N)\left(\sqrt{2}\, a_{(1)}{}^{\mu+} + q_1{}^\mu\right)|0\rangle .$$

The structure of this amplitude is most easily studied using its integral representation which is readily obtained using the explicit form for $\hat{V}$.[38] We find

$$\hat{B}^{\mu\nu}(q_1,q_2) = -\langle \hat{\mathcal{B}}^{\mu\nu} \rangle_{N+2} \quad , \tag{4.15}$$

where

$$\hat{\mathcal{B}}^{\mu\nu} = \left[\left(\bar{\mathcal{V}}_{(1)}{}^{\nu} + \frac{q_2{}^{\nu}}{m^2 + 1 - q_2{}^2} u' \frac{\partial}{\partial u'} \right) \left(\mathcal{V}_{(1)}{}^{\mu} + \frac{q_1{}^{\mu}}{m^2 + 1 - q_1{}^2} u' \frac{\partial}{\partial u'} \right) + 2u\, g^{\mu\nu} \right] {}_2F_1(m^2 + 1 - q_1{}^2, m^2 + 1 - q_2{}^2; m^2 + 1; u')\Big|_{u'=u} \quad .$$

We examine the singularities in the two-current (t) channel. They arise from divergences of the integrand as $u \to 1$ [i.e., $I_{N+2} \propto (1 - u)^{-\alpha_t - 1}$] where the hypergeometric function has the behavior

$$\begin{aligned} &{}_2F_1(m^2 + 1 - q_1{}^2, m^2 + 1 - q_2{}^2; m^2 + 1; u) \\ &= \frac{\Gamma(m^2 + 1)\, \Gamma(q_1{}^2 + q_2{}^2 - m^2 - 1)}{\Gamma(q_1{}^2)\, \Gamma(q_2{}^2)} \\ &\times {}_2F_1(m^2 + 1 - q_1{}^2, m^2 + 1 - q_2{}^2; m^2 + 2 - q_1{}^2 - q_2{}^2; 1 - u) \\ &+ (1 - u)^{q_1{}^2 + q_2{}^2 - m^2 - 1} \frac{\Gamma(m^2 + 1)\, \Gamma(m^2 + 1 - q_1{}^2 - q_2{}^2)}{\Gamma(m^2 + 1 - q_1{}^2)\, \Gamma(m^2 + 1 - q_2{}^2)} \\ &\times {}_2F_1(q_1{}^2, q_2{}^2; q_1{}^2 + q_2{}^2 - m^2; 1 - u) \quad . \end{aligned} \tag{4.16}$$

The first term yields the usual poles on the trajectory α_t and its daughters. The second term, however, gives poles at

$\alpha_t - q_1^2 - q_2^2 + m^2 + 1 = 2q_1 \cdot q_2 + 1 = 0,1,2,\cdots$. Such singularities are clearly unphysical since their positions depend on the current "masses" q_i^2. The presence of these anomalous singularities in place of the desired fixed pole can be understood, if we notice that our amplitude (4.14) has vanishing divergence, $q_{1\mu} \hat{B}^{\mu\nu} \equiv 0$. As we argued in Sec. III, the absence of an unphysical $J = 1$ intermediate state at $t = q_2^2$ implies a nonvanishing divergence $q_{1\mu} M^{\mu\nu} \to V^\nu$ for $q_{1\mu} \to 0$ which, when combined with quadratic factorization, implies a fixed pole. Our anomalous singularity violates the conditions of this theorem by providing just such an unphysical state.[42]

The origin of the vanishing divergence of our $M^{\mu\nu}$ can be seen clearly in (4.12). While, if the invariant amplitudes are evaluated on-mass-shell at $s_i = m^2 + J$, the infinite series for $\hat{V}$ terminates and the basic equation, $q_\mu V_\lambda{}^\mu(q) \propto (p_\lambda{}^2 - m_\lambda{}^2)$, holds, it is clear that (4.12) as it stands represents a certain off-shell continuation which is divergenceless everywhere. Since in our case the two-current amplitude can be rewritten in terms of this off-shell continuation,

$$M^{\mu\nu} = \sum_{\lambda\lambda'} V_\lambda{}^\nu \langle\lambda|D(R,s_i)|\lambda'\rangle V_{\lambda'}{}^\mu$$

it is obvious that $M^{\mu\nu}$ has vanishing divergence. We note that this off-shell continuation is never needed in our derivation of $M^{\mu\nu}$, since it obeys USDR in s_i, but unhappily it provides an equivalent formulation. This appears to be the origin of the difficulty with the universally coupled vector meson approximation.

V. PHENOMENOLOGICAL MODELS

The form of dual resonance-dominated amplitudes for currents is certainly extremely nonunique if factorization and consistency with the hadronic amplitudes are not required. Nevertheless, it may be useful to temporarily set aside these requirements and study the general structure of dual resonance dominated functions having good large-q^2 behavior and, if possible, satisfying the requirements of current conservation and current algebra. Perhaps the most important outcome of such a study could be an improved understanding of the role of high mass vector mesons which could then help solve the factorization problem, but such functions are also interesting and useful from a strictly phenomenological point of view.

A number of such phenomenological functions have previously been proposed by various authors.[43-48] We feel that their chief virtue is better large-q^2 behavior than the functions discussed in the preceding section, since in most cases[43-46,48] current conservation and current algebra have been enforced in a very ad hoc manner if at all and their factorization properties are very bad. These functions will be discussed further at the end of the section.

All the above functions have power behaved form factors whereas we have already remarked several times that one expects exponentially decreasing form factors in models with linear trajectories. Here we propose a new class of functions which have such exponential form factors. It is very intriguing that these functions also exhibit <u>all</u> the properties suggested by field theory discussed in Sec. III (iii).

We therefore believe that they are particularly interesting functions although no attempt is made to satisfy the divergence conditions.

In order to motivate our proposal for current amplitudes we recall the basic features of the N-point beta function[9]

$$B(p_1,\cdots,p_N) = \int_0^1 \int_0^1 \cdots \int_0^1 \frac{du_1 du_2 \cdots du_{N-3}}{J(u_i)} \prod_{ij} u_{ij}^{-\alpha_{ij}-1} , \qquad (5.1)$$

where $\alpha_{ij} = (p_i + p_{i+1} + \cdots + p_j)^2 - m^2$ and J is an appropriate Jacobian factor depending on the choice of the $N - 3$ independent u_i from the full set of $\frac{N(N-1)}{2}$ dependent u_{ij}. The u_{ij} are constrained so that, if $u_{ij} \to 1$, the $u_{i'j'}$ for at least one overlapping trajectory must vanish. Thus the behavior for $\alpha_{ij} \to -\infty$, which is determined by the behavior for $u_{ij} \approx 1$, depends upon the $\alpha_{i'j'}$ for some overlapping trajectory,

$$B \underset{\alpha_{ij}\to -\infty}{\sim} (\alpha_{ij})^{\alpha_{i'j'}} ,$$

i.e., Regge behavior.

These functions can be modified to yield functions appropriate for currents by introducing two ficticious "lepton" lines for each current (see Fig. 10). There are then a number of ficticious trajectories corresponding to one lepton line and several hadron lines (γ, γ', δ, etc. in Fig. 10) that have no physical meaning. The ficticious trajectories were taken to be -1 in Refs. 44 and 47 and the expression (5.1) itself was used. As discussed above we expect terms in the asymptotic behavior of a given variable corresponding to the various overlapping "trajectories," thus, if the ficticious trajectories are set equal to constants, we obtain fixed powers as well as Regge powers in the energy variables, e.g., α_{12}^{γ}, α_{23}^{δ}, etc. as well as $\alpha_{12}^{\alpha_{23}}$, $\alpha_{23}^{\alpha_{12}}$ for the amplitude of Fig. 10. Further we see that the form factors are power behaved, since as $q^2 \to -\infty$ we have $(\alpha_{q^2})^{\gamma}$, $(\alpha_{q^2})^{\gamma'}$, etc. We note that the power behavior of the form factors is correlated with the presence of fixed powers in the subenergies just as is suggested by field theory [see Sec. III (ii)]; in fact one can verify that the connection is between precisely the same form factors and subenergies as is the case in field theory (i.e., fixed poles are absent in channels consisting of a current and a hadron with an exponential form factor--see, e.g., Fig. 8).

In general the factors in (5.1) corresponding to ficticious trajectories, $u_{ij}^{-\alpha_{ij}-1}$, can be replaced by an arbitrary function, $g(u_{ij})$. The asymptotic behavior of form factors and high energy behavior are determined by the behavior of $g(u_{ij})$ as $u_{ij} \to 0$. Exponential form factors obtain $g(u_{ij})$ vanishes faster than any power, for example,

$$g(u_{ij}) \sim \exp\left[-\frac{1}{(u_{ij})^P}\right], \quad P > 0.$$

To obtain form factors that satisfy USDR we must further restrict P to $0 < P < 1$ (see Appendix A). This behavior will also guarantee the absence of the undesired fixed poles. We can rigorously prove the existence of only the desired Regge powers for subenergies approaching infinity in their left-half plane $(\mathrm{Re}\ \alpha_{ij} < 0)$. The behavior in the right-half plane (Re $\alpha_{ij} > 0$) is much more difficult to determine as it involves the analytic continuation of (5.1) through contour deformations; we conjecture that the same Regge behavior obtains at least for $P < 1$.[49]

In the case of amplitudes involving several currents there is another type of variable u_{ij} where i and j correspond to the lepton lines of two different currents.[50] If the currents are nonadjacent we make the same replacement as above. If they are adjacent we use $h(u_{ij}) \underset{u_{ij}\to 0}{\sim} u_{ij}^{-k-1}$ where k is an integer and i and j correspond to adjacent "lepton" lines of the two different currents. These replacements thus give amplitudes with exponential form factors and fixed poles only in the two-current (t) channel.

We now investigate the properties of such amplitudes in more detail by studying some simple examples. Actually, we shall use a somewhat different prescription from the above in order to obtain simpler functions. For each current, we take all u_{ij} in the set S corresponding to one of its "lepton" lines and any nonzero number of adjacent hadron momenta or any nonzero number of adjacent hadron momenta and a single lepton line of another (necessarily nonadjacent) current and insert a factor

$$\exp[-1/(\prod_S u_{ij})^P] , \qquad 0 < P < 1 \tag{5.2}$$

in (5.1). For variables corresponding to adjacent "lepton" lines of adjacent currents we simply set $\alpha_{ij} \to k$, an integer. One can readily convince himself that this gives a behavior of the integrand for

$u_{ij} \to 0$ similar to the above yielding the same general properties for the amplitudes.

Let us study as an example a typical invariant amplitude for two adjacent currents and two hadrons,

$$M = \int_0^1 du\, u^{-\alpha_s - 1}(1-u)^{-\alpha_t - 1} \int_0^1 \int_0^1 du_1 du_2\, u_1^{-\alpha_1 - 1}(1-u_1)^{-\gamma-1}$$

$$\times\, u_2^{-\alpha_2 - 1}(1-u_2)^{-\gamma-1}(1-uu_1)^{-\delta+\gamma+\alpha_t}(1-uu_2)^{-\delta+\gamma+\alpha_t}$$

$$\times\, (1-uu_1u_2)^{-k+2\delta-\alpha_t} \exp\left[-\left(\frac{1-uu_1u_2}{1-u_1}\right)^P\right] \exp\left[-\left(\frac{1-uu_1u_2}{1-u_2}\right)^P\right],$$

$$0 < P < 1\,, \qquad (5.3)$$

which reduces to the amplitude (5.1) shown in Fig. 11 for $P = 0$. In (5.3), $\alpha_i = q_i^2 - m^2 - 1$, and γ, δ, k are arbitrary negative parameters. The elastic form factor obtained from the residue of the pole at $\alpha_s = 0$ is clearly exponential (see also Appendix A),[51]

$$F(q^2) = \int_0^1 du\, u^{-\alpha-1}(1-u)^{-\gamma-1} \exp - \frac{1}{(1-u)^P}$$

$$\underset{q^2 \to -\infty}{\sim} \sqrt{\frac{2\pi}{P(P+1)}} \left(-\frac{q^2}{P}\right)^{\frac{\gamma - \frac{1}{2}P}{P+1}} \exp\left[-(P+1)\left(-\frac{q^2}{P}\right)^{\frac{P}{P+1}}\right] . \qquad (5.4)$$

One may readily verify that the only fixed power high energy behavior is $(\alpha_s)^{k-n}$ $(n = 0,1,2,\cdots)$, at least for $\alpha_s, \alpha_t \to -\infty$, as discussed above. This is most conveniently done using the "Veneziano transform,"[52] a Mellin-like transform particularly suited to functions of the form (5.1) and (5.3). We write

$$A(\alpha_s,x) = \frac{1}{2\pi i}\int_{-\epsilon-i\infty}^{-\epsilon+i\infty} d\sigma\ \tilde{A}(\sigma,x)\ B(-\sigma,-\alpha_s)\ , \qquad (0 < \epsilon < 1)$$

$$\tilde{A}(\sigma,x) = \frac{1}{2\pi i}\int_{-\eta-i\infty}^{-\eta+i\infty} d\alpha_s\ A(\alpha_s,x)\ B(\sigma+1,\ \alpha_s+1), \qquad (0 < \eta < 1)$$

where x represents all the other variables in A. If A has the form

$$A(\alpha_s,x) = \int_0^1 du\ u^{-\alpha_s-1}\,\mathcal{A}(u,x)\ ,$$

then[45]

$$\tilde{A}(\sigma,x) = \int_0^1 du(1-u)^\sigma\,\mathcal{A}(u,x)\ . \tag{5.5}$$

In our case, $\tilde{A}$ will generally be given by a sum of poles

$$\tilde{A}(\sigma,x) = \sum_i \frac{r_i(x)}{\sigma - \sigma_i(x)}$$

and thus

$$A(\alpha_s,x) = \sum_i r_i(x)\ B[-\sigma_i(x),\ -\alpha_s]$$

$$\underset{\alpha_s\to-\infty}{\sim}\ \Gamma[-\sigma_0(x)]\ r_0(x)\ (-\alpha_s)^{\sigma_0(x)}\ , \tag{5.6}$$

where σ_0 is the pole furthest to the right. The advantage of this method is that it is rather easy to find the poles in (5.5).

We thus find for (5.3) the Regge powers $(-\alpha_s)^{\alpha_t}$ and $(-\alpha_t)^{\alpha_s}$ as $\alpha_s, \alpha_t \to -\infty$ and in addition the leading fixed power behavior

$$M \underset{\alpha_s \to -\infty}{\sim} (-\alpha_s)^k \, \Gamma(-k) \int_0^1 \int_0^1 dx_1 dx_2 x_1^{-\gamma-1} (1 - x_1)^{-\delta+k-1}$$

$$\times \; x_2^{-\gamma-1} (1 - x_1)^{-\delta+k-1} \, (1 - x_1 x_2)^{-\alpha_t + 2\delta - k}$$

$$\times \; \exp\left\{ - \left[\frac{1 - x_1 x_2}{x_2(1 - x_1)} \right]^P \right\} \exp\left\{ - \left[\frac{1 - x_1 x_2}{x_1(1 - x_2)} \right]^P \right\}$$

$$\equiv (-\alpha_s)^k \int_0^1 \int_0^1 dx_1 dx_2 \, R_k(x_1, x_2; t) \equiv (-\alpha_s)^k R_k(t) \quad , \qquad (5.7)$$

corresponding to a fixed pole in the two-current channel. The residue of this fixed pole is not equal to the elastic form factor (5.4) as is the case in current aglebra although it is independent of q_1^2 and q_2^2 and exponentially behaved as $\alpha_t \to -\infty$. The absence of poles in q_i^2 is a manifestation of the fact that the single current amplitudes obtained by taking the residue of a pole in α_i or α_s of (5.3) are consistent with the prescription (5.2) and indeed have no fixed poles. Since it appears that (5.3) can be written entirely in terms of its poles in α_i, this means that the sum over vector mesons converges nonuniformly as discussed in Sec. III.[53] Also, if there is indeed power behavior for $\alpha_s \to +\infty$, the sum rule for the fixed pole (5.7) converges nonuniformly in q^2 as discussed in Sec. III.[53]

We now discuss the behavior of (5.3) for deeply inelastic electron scattering: $q_1^{\,2} = q_2^{\,2} = q^2 \to -\infty$, $\rho = -\frac{s-u}{2q^2}$ fixed and t fixed. Since $\alpha_s = q^2(1-\rho) - \frac{t}{2} \sim q^2(1-\rho)$ becomes infinite in this limit, formally this corresponds to a Regge-like limit[46] in the leftmost link of a "multiperipheral" diagram like Fig. 11 when the "leptons" of the second current are taken as the incoming lines. The corresponding "momentum transfer" trajectory is k so we expect a behavior $(q^2)^k$ as given in (3.13). Indeed the asymptotic limit is easily calculated by changing variables to those appropriate to this "multiperipheral" configuration,

$$M = \int_0^1 \int_0^1 \int_0^1 dv_1 dv_2 dv_3 \; v_1^{-k-1}(1-v_1)^{-\alpha_1-1} \; v_2^{-\delta-1}(1-v_2)^{-\gamma-1}$$

$$\times \; v_3^{-\gamma-1}(1-v_3)^{-\alpha_t-1}(1-v_1v_2)^{-\alpha_s+\alpha_1+\gamma}(1-v_2v_3)^{-\delta+\gamma+\alpha_t}$$

$$\times \; (1-v_1v_2v_3)^{-\alpha_2+\alpha_s+\delta-\gamma} \exp\left[-\left(\frac{1-v_1v_2}{1-v_2}\right)^P\right] \exp\left[-\left(\frac{1}{v_2v_3}\right)^P\right]$$

$$\underset{\substack{q^2\to-\infty \\ \rho \text{ fixed}}}{\sim} (-q^2)^k \int_0^1 \int_0^1 dx_1 dx_2 \left[\frac{(1-x_1x_2)}{(1-x_1)(1-x_2)} - \rho\right]^k R_k(x_1,x_2;\, t) \,, \tag{5.8}$$

where a further change of variables has been made to obtain the final expression. We note that the Bjorken limit (3.12) corresponds to $\rho \to 0$ and that in this limit there is near identity of (5.8) to the fixed pole in t (5.7). In general we see that the assignment of the power k to the channel containing one lepton from each of two adjacent currents is the mechanism which produces both the desired fixed poles and the good electroproduction limit (5.8); there is thus a strong correlation between these two behaviors in this model.

As noted above our prescription yields amplitudes for nonadjacent currents with no fixed poles. They also vanish exponentially in the $q^2 \to -\infty$ electroproduction limit. Thus in our model the only nontrivial scaling contribution comes from the adjacent current terms. We see that both the adjacent and nonadjacent terms exhibit a close connection between asymptotic power behavior and the electroproduction limit.[54]

Let us examine the behavior of (5.8) in more detail in the case $k = -1$. This choice would be appropriate for the amplitude M_1 which contains the current algebra fixed pole and gives the dominant contribution in electroproduction experiments $(W_2 = \frac{m^2}{\pi} \operatorname{Im} M_1)$.[55] The scaling function, i.e., the coefficient of $(-q^2)^{-1}$, becomes, after some changes of variables,

$$f(\rho,t) = \frac{1}{2}\int_1^\infty \frac{dw}{\rho - w}\int_{-(w-1)}^{(w-1)} dv \left(\frac{w+v-1}{2}\right)^{-\gamma-1}\left(\frac{w-v-1}{2}\right)^{-\gamma-1}$$

$$\times \left(\frac{w+v+1}{2}\right)^{\gamma-\delta+\alpha_t}\left(\frac{w-v+1}{2}\right)^{\gamma-\delta+\alpha_t} w^{-\alpha_t+2\delta+1}$$

$$\exp\left[-\left(\frac{2w}{w-v-1}\right)^P \exp -\left(\frac{2w}{w+v-1}\right)^P\right] \quad . \qquad (5.9)$$

This expression exhibits clearly the physical cut in ρ for $1 \le \rho \le \infty$. This cut is, of course, only an asymptotic approximation since M itself has only poles and no cuts. For large ρ we obtain the conjectured behavior (3.14), since then large w dominates in (5.9) and we obtain

$$\mathrm{Im}\, f(\rho,t) \underset{\rho\to\infty}{\sim} \rho^{\alpha_t} \cdot \frac{\pi}{2} \int_{-1}^{1} dz \left(\frac{1 - z^2}{4}\right)^{\alpha_t - \delta - 1}$$

$$\times \exp\left[-\left(\frac{2}{1+z}\right)^P\right] \exp\left[-\left(\frac{2}{1-z}\right)^P\right], \qquad (5.10)$$

$$\mathrm{Re}\, f(\rho,t) \underset{\rho\to\infty}{\sim} (-\rho)^{-1} R_{-1}(t) .$$

The threshold behavior of $f(\rho,t)$ is[51]

$$\mathrm{Im}\, f(\rho,t) \underset{\rho\to 1}{\sim} \left[\tfrac{1}{2}(\rho - 1)\right]^{-2\gamma-1} \int_{-1}^{1} dz (1 - z^2)^{-\gamma-1}$$

$$\times \exp\left[-\left(\frac{2}{(\rho - 1)(1 + z)}\right)^P - \left(\frac{2}{(\rho - 1)(1 - z)}\right)^P\right]$$

$$\sim \sqrt{\frac{\pi}{P(P + 1)}} \left[\tfrac{1}{2}(\rho - 1)\right]^{-2\gamma-1+\frac{P}{2}} \exp\left[-2\left(\frac{2}{\rho - 1}\right)^P\right]. \qquad (5.11)$$

Comparing (5.11) and (5.4) for $P = 0$ we see that the relationship of Drell and Yan[31] (3.15) is exactly satisfied. For exponential form factors the two behaviors are related through the parameter P:

$$F(q^2) \underset{q^2 \to -\infty}{\sim} e^{-a(-q^2)^{\frac{P}{P+1}}} ,$$

$$\mathrm{Im}\, f(\rho,t) \underset{\rho \to 1}{\sim} e^{-b\left(\frac{1}{\rho-1}\right)^P} . \tag{5.12}$$

Finally we discuss the spin dependence of the asymptotic behavior of form factors [see (3.11)]. Form factors for one scalar particle and one particle with arbitrary spin can be extracted from (5.3) by taking the residues at poles $\alpha_s = N$. However, we would like to discuss the general case of form factors for two resonances of arbitrary spin. These form factors can be obtained by considering, for example, the amplitude for one current and four scalars (Fig. 12),

$$M = \int_0^1 du\, u^{-\delta-1}(1-u)^{-\alpha-1} \int_0^1 \int_0^1 du_1 du_2\, u_1^{-\alpha_{12}-1}(1-u_1)^{-\gamma-1}$$

$$\times\, u_2^{-\alpha_{34}-1}(1-u_2)^{-\gamma-1}\,(1-uu_1)^{-\alpha_{24}+\gamma+\alpha}\,(1-uu_2)^{-\alpha_{13}+\gamma+\alpha}$$

$$\times\,(1-uu_1u_2)^{-\alpha_{23}+\alpha_{13}+\alpha_{24}-\alpha}\,\exp\left\{-\left[\frac{(1-uu_1u_2)^2}{u(1-uu_1)(1-uu_2)}\right]^P\right\}. \tag{5.13}$$

When (5.13) is factorized at poles $\alpha_{12} = N_1$, $\alpha_{34} = N_2$ the overlapping energy variables correspond to

$$\alpha_{23} \propto \epsilon_1 \cdot q, \qquad \epsilon_1 \cdot \epsilon_2, \qquad q \cdot \epsilon_2,$$

$$\alpha_{24} \propto \epsilon_1 \cdot q,$$

$$\alpha_{13} \propto q \cdot \epsilon_2,$$

where ϵ_i are the polarization vectors of the resonances. Thus a particle of spin J_1 requires J_1 factors of ϵ_1 in the residue and thus J_1 powers of α_{23} or α_{24}. Further one finds that there are extra powers of the integration variables associated with these α_{ij}, thus (5.13) always contains the combinations

$$\alpha_{23} u u_1 u_2 ; \quad \alpha_{24} u u_1 ; \quad \alpha_{13} u u_2 \; . \tag{5.14}$$

Note that the powers of u_1 and u_2 in (5.14) assure that a particles of spin J_i only occurs if the appropriate α is greater than J_i. The factors of u are very interesting because they cause the corresponding form factors to fall more rapidly for $|q^2| \to \infty$. Thus one easily sees that if the form factor is expanded as (see Amati, et al., Ref. 26)

$$F_{\mu_1 \cdots \mu_{J_1} ; \nu_1 \cdots \nu_{J_2}} = \sum_{i=1}^{\min(J_1, J_2)} T^{(i)}_{\mu_1 \cdots \mu_{J_1} ; \nu_1 \cdots \nu_{J_2}} F^{(i)}_{J_1 J_2}(q^2) \; , \tag{5.15}$$

where

$$T^{(i)}_{\mu_1 \cdots \mu_{J_1} ; \nu_1 \cdots \nu_{J_2}} = g_{\mu_1 \nu_1} g_{\mu_2 \nu_2} \cdots g_{\mu_i \nu_i} q_{\mu_{i+1}} \cdots q_{\mu_{J_1}} q_{\nu_{i+1}} \cdots q_{\nu_{J_2}} ,$$

then

$$F^{(i)}_{J_1 J_2}(q^2) = \int_0^1 du \, u^{-\alpha-1} (1 - u)^{-\delta-1+(J_1+J_2-i)} e^{-\frac{1}{(1-u)^P}} f(u) \; . \tag{5.16}$$

For $P = 0$ and power behaved form factors, (5.16) yields

$$F^{(i)}_{J_1J_2}(q^2) \sim (q^2)^{-(J_1+J_2-i)} F(q^2) \tag{5.17}$$

which is precisely the result of Amati et al.,[26] (3.11) simplified to scalar currents. For exponential form factors the spin dependence of (5.17) is weakened by a fraction $\left(\frac{1}{P+1}\right)$ as one sees from (5.4).

The preceding discussion has shown that functions of the form proposed above (those of Refs. 44-47 are special cases of these) have the large-q^2 behavior expected on the basis of other theoretical considerations.[56] We have not attempted to satisfy the divergence conditions for conserved currents or current algebra. In Refs. 43, 46, and 48 this was attempted, but the methods do not seem to give much promise of leading to a complete solution for general current amplitudes. We note however that the "hybrid" amplitudes given in Ref. 46 do satisfy CVC and current algebra and have all the good large-q^2 behaviors discussed here for a power-behaved form factor $F(q^2) = [1 - q^2/(m^2 + 1)]^{-1}$. Freedman[47] has combined the divergence identity techniques used in Sec. IV with amplitudes of the form proposed in Refs. 44 and 45 to obtain an elegant model for current algebra with one vector current and one scalar current. In Appendix B we present a generalization of this model to exponential form factors. However, only the scalar current has nontrivial structure like that discussed in this section; the vector current has trivial multiplicative form factors like those considered in Sec. IV and therefore bad large-q^2 behavior. Therefore, as yet the vector nature of the current has not been successfully combined with a current structure sufficiently complicated to give good large-q^2 behavior.

We have also not attempted to satisfy the factorization conditions (Figs. 1 and 2) for these amplitudes. It can easily be seen that the contribution of the leading trajectories in models of the type suggested here is factorizable (i.e., nondegenerate) for the same reason as for the N-point beta function (5.1).[57] However, as discussed by Freedman[47] for the case of power behaved form factors, the lower trajectories will have a much greater degeneracy than (5.1). This in itself would not be a fatal flaw because there exists the possibility of modifying the hadronic amplitudes, since these are not yet firmly established. However, the spectrum of such current amplitudes is internally inconsistent: it is different in different channels.[47]

Even leaving aside the problem of factorization, we believe further development of models of the form proposed here would be very useful. It could give important suggestions on the role of high mass vector mesons in giving good large-q^2 behavior and satisfying the divergence conditions. Furthermore, functions of this form could be useful for phenomenological applications.

VI. CONCLUSION

Although there are a very large number of physical properties that dual resonance amplitudes for vector currents should satisfy, we feel that the partial successes discussed above give a good deal of hope for the discovery of a full solution to the problem. The two different partial approaches to the problem discussed in Secs. IV and V both point up the important role that must be played by the high mass vector mesons. The vast number of vector mesons existing in the DRM is at once a source of optimism, since the great freedom it allows may be a crucial factor in being able to obtain a solution, and a source of difficulty, due to the great complexity it introduces. Clearly some guide to selecting the appropriate current is needed. We mention two approaches that may yield this guide.

First, one could attempt to formulate the problem in a more algebraic manner.[33] The fundamental object in a zero-width model for currents is the vertex for a current and two arbitrary resonances. The current algebra divergence conditions have a natural algebraic expression in terms of this vertex, but, at the present, the conditions that duality imposes are not well-understood. Generally, one would like to be able to see directly how the singularities in dual channels (e.g., s_i and t) are related. This would help circumvent difficulties like those encountered in Ref. 33 (see Sec. IV) where a solution of factorization in one channel (s_i) was found to give unfactorizable singularities in the dual channel (t). We anticipate that a deeper understanding of duality will allow a concise vertex formulation of the conditions on currents and thus give insight into their vector-meson structure.

A second approach is to temporarily ignore the factorization constraints and explore in more detail dual models having good large-q^2 behavior such as those discussed in Sec. V. If a model with these properties could be found which also satisfied the divergence conditions and factorizes on just the leading trajectories, much could probably be learned about the role of the high mass vector mesons.

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APPENDIX A. EXPONENTIAL FORM FACTORS

In this appendix we give some simple examples of narrow-resonance-dominated form factors which satisfy dispersion relations and decrease faster than any power for $|q^2| \to \infty$, e.g., "exponential" form factors.

Form factors with these properties can easily be constructed from elementary functions. For example, consider the class of functions

$$F_n(q^2) = G_n(q^2)/G_n(0) \quad , \qquad (n > 2 \text{ and even})$$

where

$$G_n(q^2) = \frac{1}{\prod_{k=0}^{\frac{1}{2}n-1} \cos \frac{\pi}{2}(\alpha\ e^{2\pi i k})^{\frac{1}{n}}} \tag{A.1}$$

and

$$\alpha = q^2 - m^2 - 1 \ .$$

One can easily verify that the only singularities of $F_n(q^2)$ are simple poles at positive integral values of α [the apparent cuts are absent since the product is invariant under $\alpha \to \alpha e^{2\pi i}$], that it decreases exponentially

$$F_n(q^2) \underset{q^2 \to \infty}{\sim} \exp[-a(-q^2)^{\frac{1}{n}}] \ ,$$

and that it satisfies an USDR. We note if we were to take $n = 2$,

$$G_2(q^2) = \frac{1}{\cos \frac{\pi}{2} \alpha^{\frac{1}{2}}} \ ,$$

$F_2(q^2)$ would not satisfy USDR since it does not decrease for $\mathrm{Re}\, q^2 \to +\infty$, $\mathrm{Im}\, q^2$ fixed.

It is very convenient to represent form factors by an integral representation of the form considered in Sec. V,

$$F(q^2) = \int_0^1 du\, u^{-\alpha-1} f(u) . \tag{A.2}$$

The poles in q^2 are then determined by the behavior of $f(u)$ near $u = 0$ whereas the asymptotic behavior for q^2 in the left-half plane is determined by the behavior of $f(u)$ near $u = 1$. We remark that (A.2) can easily be cast in the form of a Laplace transform

$$F(q^2) = \int_0^\infty dz\, e^{\alpha z}\, \tilde{F}(z) \tag{A.3}$$

where $\tilde{F}(z) = f(e^{-z})$. This expression may be useful for studying the conditions on $f(u)$ necessary to assure a given behavior of $F(q^2)$, since the known properties of Laplace transforms can be used.

A simple example of an exponential form factor in the form (A.2) is

$$F(q^2) = \int_0^1 du\, u^{-\alpha-1}\, e^{-\frac{1}{(1-u)^P}} . \tag{A.4}$$

One finds using standard techniques

$$F(q^2) \underset{|q^2|\to\infty}{\sim} \left(\frac{2\pi}{P(P+1)}\right)^{\frac{1}{2}}$$

$$\times \left\{ \frac{e^{-i\pi\alpha}\left(\frac{\alpha e^{-i\pi}}{P}\right)^{-\frac{(P+2)}{2(P+1)}} \exp\left[-(P+1)\left(\frac{\alpha e^{-i\pi}}{P}\right)^{\frac{P}{P+1}}\right]}{e^{-i\pi\alpha} - e^{i\pi\alpha}} - \frac{e^{i\pi\alpha}\left(\frac{\alpha e^{i\pi}}{P}\right)^{-\frac{(P+2)}{2(P+1)}} \exp\left[-(P+1)\left(\frac{\alpha e^{i\pi}}{P}\right)^{\frac{P}{P+1}}\right]}{e^{-i\pi\alpha} - e^{i\pi\alpha}} \right\} .$$

Therefore, for $0 < P < 1$, $F(q^2)$ is exponentially falling in the whole complex q^2 plane and satisfies USDR.

Exponential form factors satisfy an infinite set of superconvergence relations. If

$$F(q^2) = \sum_{\ell=0}^{\infty} \frac{f_\ell}{1 - \frac{q^2}{m^2 + 1 + \ell}}$$

then

$$\sum_{\ell=0}^{\infty} f_\ell (m^2 + 1 + \ell)^N = 0 , \quad N = 1,2,3,\cdots . \tag{A.5}$$

These relations are easily proved by noting that $(q^2)^N F(q^2)$ also satisfies an unsubtracted dispersion relation. The great wealth of such form factors is illustrated by the theorem of Atkinson and Halpern:[58] Given one exponential form factor, e.g., a solution to (A.5), an infinite number of other solutions to (A.5) can be constructed.

APPENDIX B. MODEL FOR ONE VECTOR AND ONE SCALAR CURRENTS

Freedmans's model for one vector current and one scalar current satisfying current algebra[47] is generalized to exponential (or arbitrary) form factors. In this model the vector current (q_2) is introduced using the divergence identity techniques discussed in Sec. IV and the scalar current (q_1) is introduced using the techniques of Sec. V. Thus only the scalar current has good large-q^2 behavior.

Freedman's model was constructed from amplitudes of the form shown in Fig. 13(a) and (b) with $\gamma_i = -1$. Changing variables to those corresponding to the multiperipheral configuration of Fig. 13(c), letting k be arbitrary, and using the current algebra identity of Ref. 29, we obtain

$$\begin{aligned} 0 &= \int_0^1 dv_1 \cdots dv_N \frac{d}{dv_1} [I_{N+3}\, v_1(1 - v_1)] \\ &= \int_0^1 dv_1 \cdots dv_N [-k(1 - v_1) + q_{2\nu} \mathcal{V}_{(1)}{}^{\nu} v_1] I_{N+3} , \end{aligned} \tag{B.1}$$

where $\mathcal{V}_{(1)}{}^{\nu}$ is defined following Eq. (4.9) with the change of variables to the v_i. Letting $k \to 0$ picks out the residue of the lowest pole in the first term of (B.1). This $N + 2$ point function is precisely the amplitude for a single scalar current [Fig. 13(b)]. Therefore, changing back to the variables of Fig. 13(a) we obtain

$$q_{2\nu} \cdot \int_0^1 du_1 \cdots du_N \; \mathcal{V}_{(1)}{}^{\nu} \; I_{N+3} \;=\; -\int_0^1 du_2 \cdots du_N \; I_{N+2} \;, \qquad (B.2)$$

or

$$q_{2\nu} \, M^{\nu}(q_1, q_2) \;\equiv\; -\,\phi(q_1 + q_2) \;,$$

where the integrands now have the choice of trajectories shown in Fig. 13(a) and (b). Equation (B.2) is the analogue of (3.9) for the current commutation relation

$$[V_a{}^0(\vec{x},t),\; \phi_b(\vec{y},t)] \;=\; i\epsilon_{abc}\, \phi_c(\vec{x},t)\; \delta^3(\vec{x} - \vec{y}) \;.$$

We note that the identity (B.1) did not depend the variables $v_2, \ldots, v_N$ and thus any function of these variables can be inserted without spoiling the result. In particular exponential form factors and the absence of all fixed poles beside those in the two-current channel can be achieved following the prescription of Sec. V and inserting the factor

$$\exp\left[-\left(\frac{1}{v_2 \cdots v_N}\right)^P\right] \;.$$

This gives in M^{ν} the factor

$$\exp\left[-\left(\frac{1 - u_1 \cdots u_N}{1 - u_N}\right)^P\right] \qquad (B.3)$$

and in ϕ the factor

$$\exp\left[-\left(\frac{1}{1 - u_N}\right)^P\right] \;.$$

As it stands (B.2) has a trivial constant form factor for the vector current. Arbitrary form factors can be introduced through the simple expedient of multiplying M^ν by

$$\Delta_{\nu'}{}^{\nu}(q_2) = \sum_{\ell=0}^{\infty} f_\ell \left(1 - \frac{q_2^{\,2}}{m^2 + 1 + \ell}\right)^{-1} \left(g_{\nu'}{}^{\nu} - \frac{q_2^{\,\nu}\, q_{2\nu'}}{m^2 + 1 + \ell}\right) . \tag{B.4}$$

since

$$q_\nu\, \Delta_{\nu'}{}^{\nu}(q^2) = q_{\nu'}$$

the current algebra identity (B.2) still holds. Note that the vector-meson amplitudes implied by this generalization will have the same fixed poles as the vector current amplitudes, a manifestation of the simple multiplicative nature of the form factors (B.4).

FOOTNOTES AND REFERENCES

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13. This is nicely illustrated by the explicit calculations of Sec. IV.

14. In the case of more general internal symmetry factors than those given in (3.6) below, we may need to assume also the absence of fixed power behavior to obtain this conclusion.

15. See Ref. 12 for a more general form of this argument. The above argument should be thought of as applying separately to each invariant amplitude.

16. For $N = 2$, this can also be shown with just the assumption of current conservation, unitarity (factorization), and the validity of dispersion relations for signatured amplitudes.

17. However, we note that, in the parameterizations for the two-current amplitude satisfying current algebra given in Sec. IV,

the invariant amplitude multiplying $g^{\mu\nu}$ has constant behavior as $s_i \to \infty$, i.e., it is nondual. This leads to a $J = 0$ fixed pole in the physical forward Compton scattering amplitude with a residue 1/2 of the Born term (5/9 with SU(3) quark isospin structure). The experimental evidence for such a duality violating fixed pole has been recently discussed by M. Damashek and F. J. Gilman, SLAC Report (1969).

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34. K. Bardakci and S. Mandelstam, Phys. Rev. 184, 1640 (1969); S. Fubini and G. Veneziano, Nuovo Cimento 64A, 811 (1969).

35. For $m^2 = -1$ only the meson at $m_0^2 = 0$ exists.

36. S. Fubini, D. Gordon, and G. Veneziano, Phys. Lett. 29B, 679 (1969). See also Y. Nambu, University of Chicago preprint (1969) and L. Susskind, Yeshiva University preprints (1969).

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41. Note we consider the case where the excited state is adjacent to the current, since we wish to compare this to the two-current amplitude with adjacent currentsusing quadratic factorization. If the excited state is not adjacent to the current, the current is automatically conserved and spurious intermediate states are automatically absent.

42. The anomalous singularity imitates the fixed pole at $q_1 \cdot q_2 = 0$, since it is then at an integer. Further we observe from (4.14) that, like a fixed pole, it does not contribute to the residues at the vector meson poles.

43. M. Bander, Nucl. Phys. B13, 587 (1969).

44. H. Sugawara, Tokyo University of Education Report, 1969 (unpublished); I. Ohba, Prog. Theor. Phys. 42, 432 (1969).

45. M. Ademollo and E. Del Giudice, Nuovo Cimento 63A, 639 (1969). We discuss here only their results to lowest order in the weak coupling.

46. R. C. Brower, A. Rabl, and J. H. Weis, Nuovo Cimento 65A, 654 (1970). We discuss here chiefly the material in Secs. III and IV.

47. D. Z. Freedman, Phys. Rev. (to be published).

48. P. V. Landshoff and J. C. Polkinghorne, Cambridge preprint DAMTP 70/5, 1970.

49. Thus all statements about asymptotic behavior made below, unless stated otherwise, implicitly mean for the left-half plane only.

50. These variables can be asssumed not to overlap the other lepton line of either current, since if they did they would be equivalent by momentum conservation to purely hadronic variables or variables with only one lepton line.

51. For $P = 0$ the asymptotic form given below has the correct dependence on the asymptotic variable but not the correct coefficient.

52. G. Veneziano, in Proceedings of First Coral Gables Conference on Fundamental Interactions at High Energy, Jan. 1969 (to be published). See also Ref. 45.

53. These statements are certainly true for $P = 0$.

54. This should not be too surprising since the electroproduction limit followed by $\rho \to \infty$ means $|s| \gg |q^2| \to \infty$ which for sufficiently smooth functions might be expected to be the same as the Regge limit, $|s| \to \infty$ with $|q^2|$ fixed and large. Recall the fixed pole residues are q^2 independent.

55. We should also replace α_t by $\to \alpha_t - 2$ since this amplitude corresponds to two units of helicity flip.

56. Bander (Ref. 43) and Landshoff and Polkinghorne (Ref. 48) have also discussed some aspects of the large-q^2 behavior of their proposed functions. In general the correspondence to the field theory results is not as close as that obtained here.

57. On the other hand, as pointed out in Ref. 46 the leading trajectory in Bander's model is infinitely degenerate.

58. D. Atkinson and M. B. Halpern, Phys. Rev. 166, 1724 (1968).

FIGURE CAPTIONS

Fig. 1. Constraints on the single-current amplitude. (a) Vector-meson dominance. (b) Factorization. The amplitude must be expressible as a sum over the poles shown by the heavy lines and these poles must correspond to states in the hadronic spectrum.

Fig. 2. Constraints on the two-current amplitude. (a) Vector-meson dominance. (b) Linear factorization. (c) Quadratic factorization. The amplitude must be expressible as a sum over the poles shown by the heavy lines and these poles must correspond to states in the hadronic spectrum.

Fig. 3. Divergence condition for single current amplitude.

Fig. 4. An external line insertion (ELI) for the particle x.

Fig. 5. (a) Diagram for hadronic isospin factor $\delta_{\alpha_1}^{\beta_3} \delta_{\alpha_3}^{\beta_4} \delta_{\alpha_3'}^{\beta_6} \times \delta_{\alpha_6}^{\beta_1} \delta_{\alpha_6'}^{\beta_2}$. Each line represents a δ. (b) Modified diagram. Each cusp represents a sum, e.g., $\sum_{x_1} \delta_{\alpha_b'}^{x_1} \delta_{x_1}^{\beta_2}$.

Fig. 6. Diagram for coupling of isovector current in $V_1^{\mu}(q)$.

Fig. 7. Divergence conditions for two-current amplitudes. (a) Nonadjacent currents. (b) Adjacent currents.

Fig. 8. Field theory models illustrating the relationship between compositeness and the absence of fixed poles.

Fig. 9. Factorized single current amplitude $(k_i^2 = s_i)$.

Fig. 10. Construction for current amplitudes.

Fig. 11. Choice of variables in 6-point beta function corresponding to Eq. (5.3) for $P = 0$.

Fig. 12. Single-current amplitude with four hadrons [Eq. (5.13)].

Fig. 13. Vector-scalar current algebra amplitudes. (a) Two-current amplitude, $M^{\nu}(q_1,q_2)$. (b) Single-scalar current amplitude, $\phi(q_1 + q_2)$. (c) Choice of variables for Eq. (B.1).

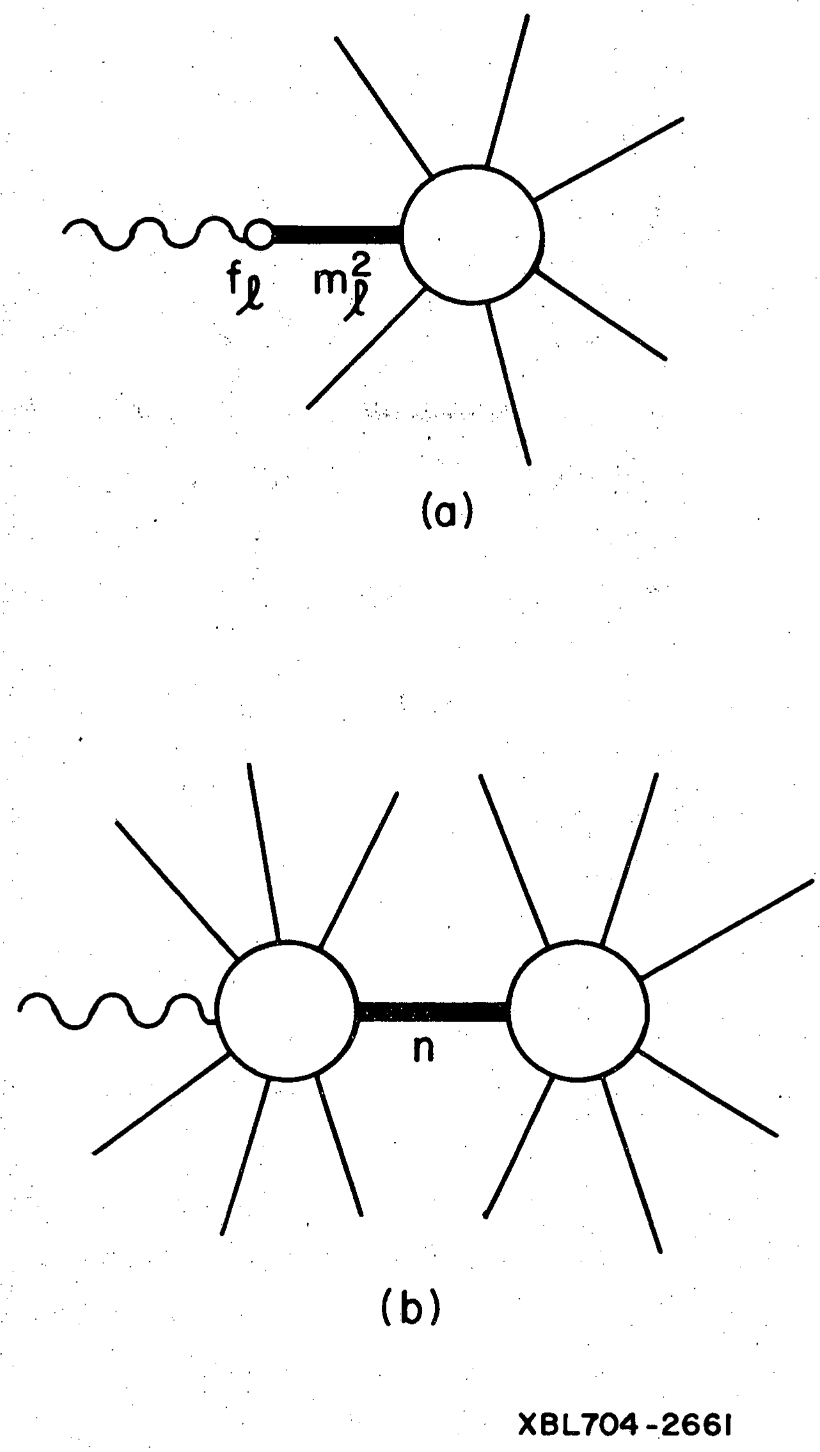
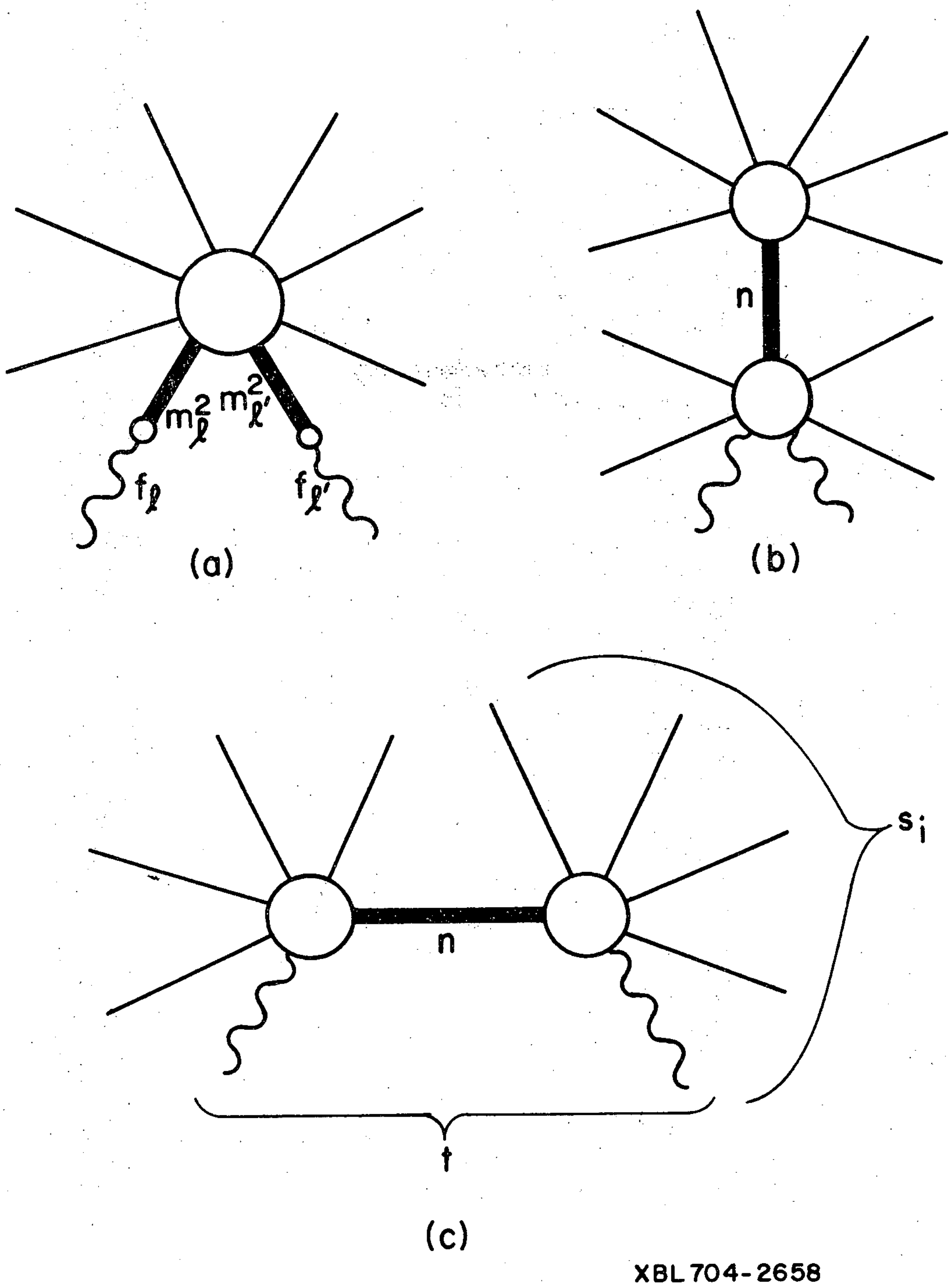


Fig. 1



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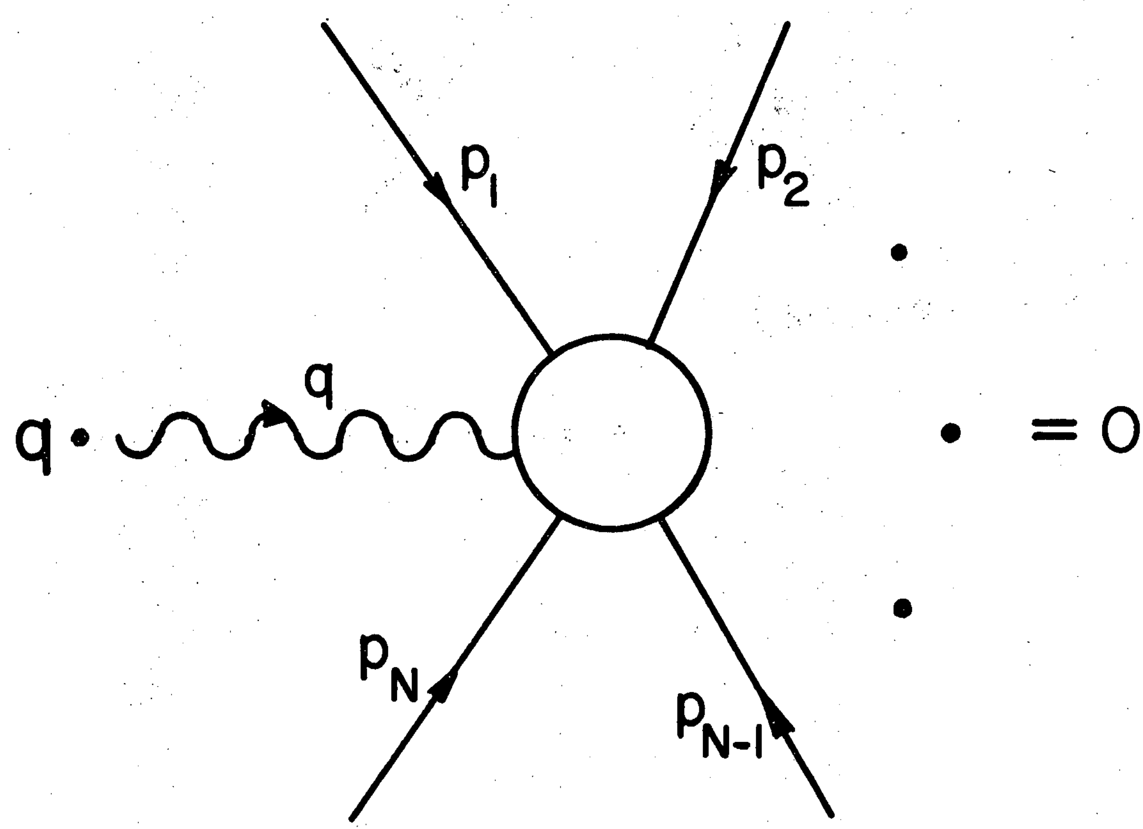

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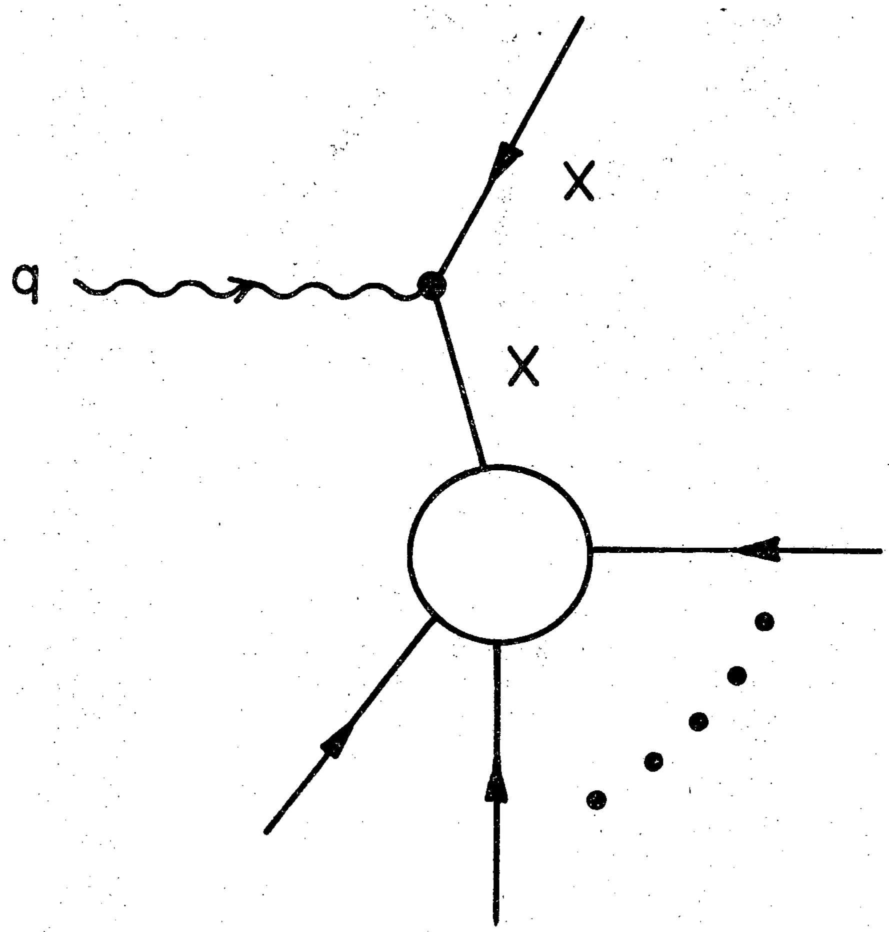

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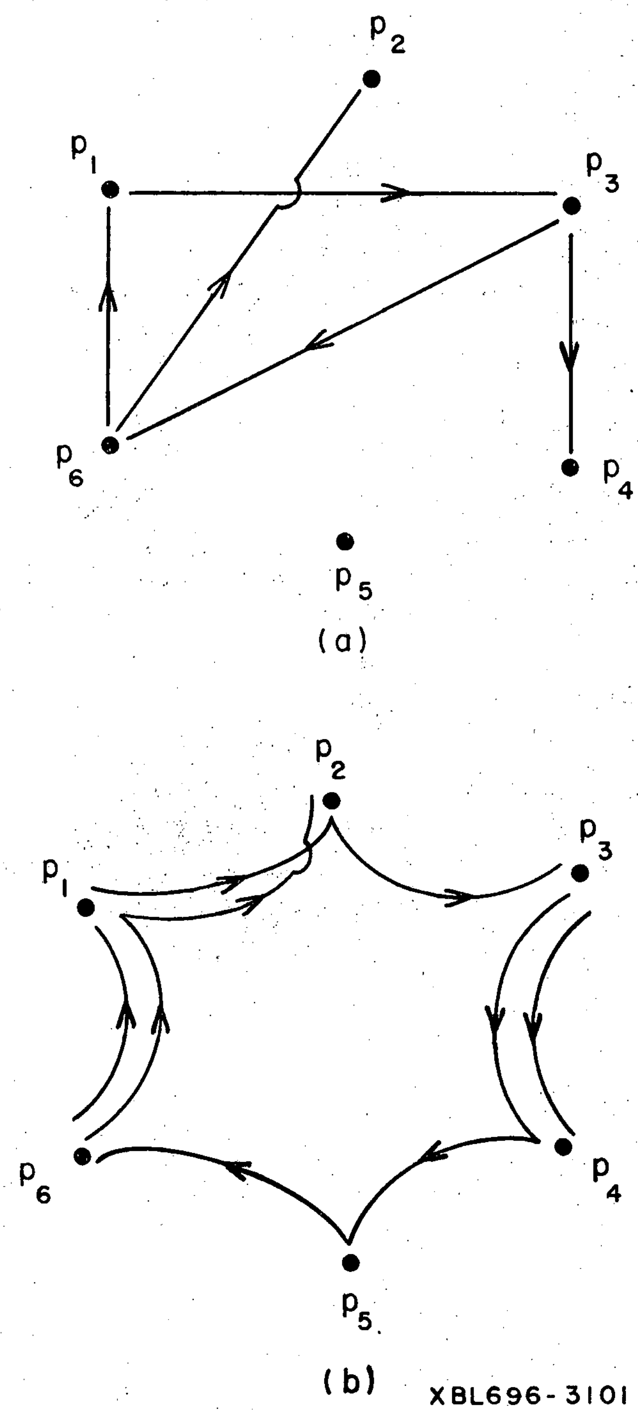

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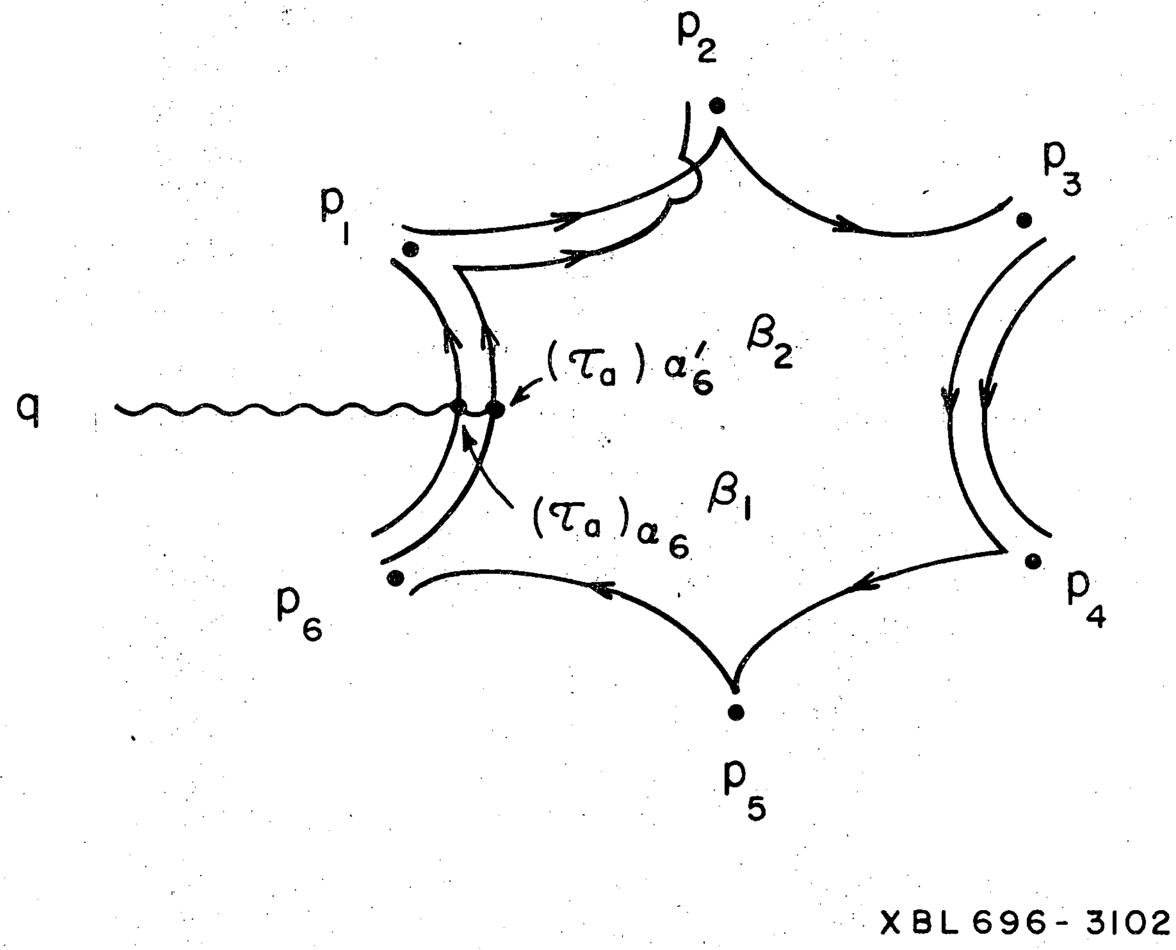

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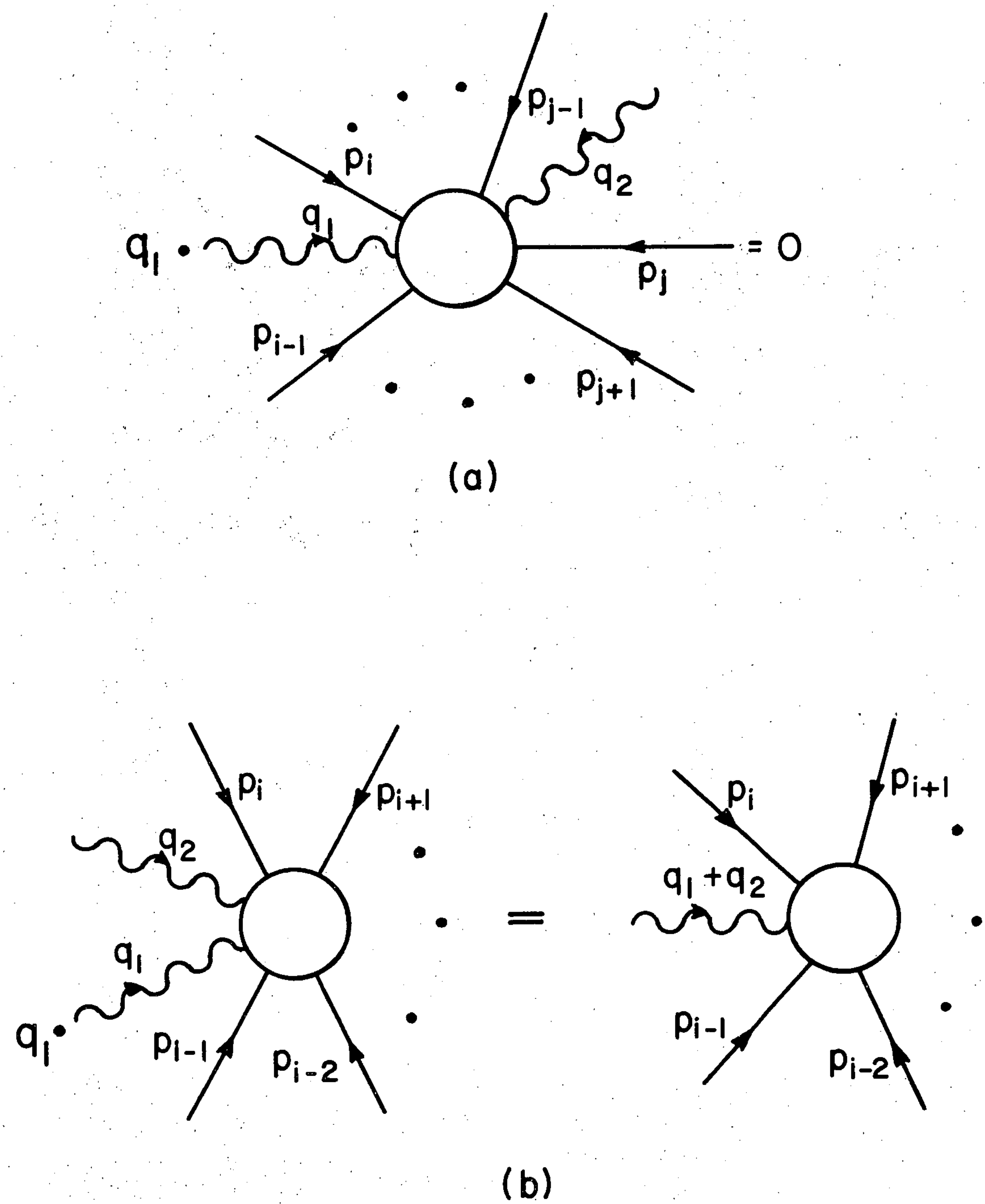

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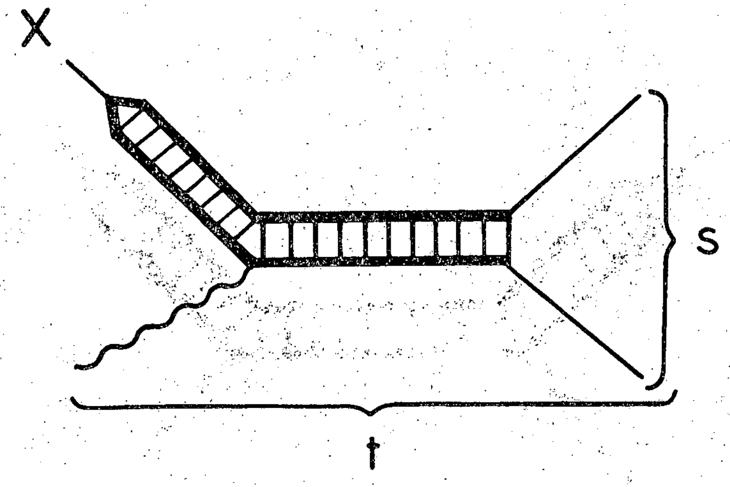
Fig. 7



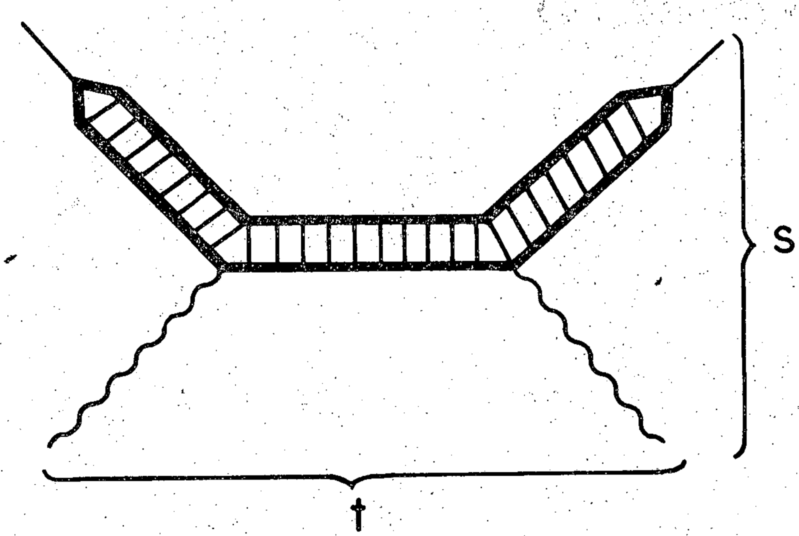


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Fig. 8

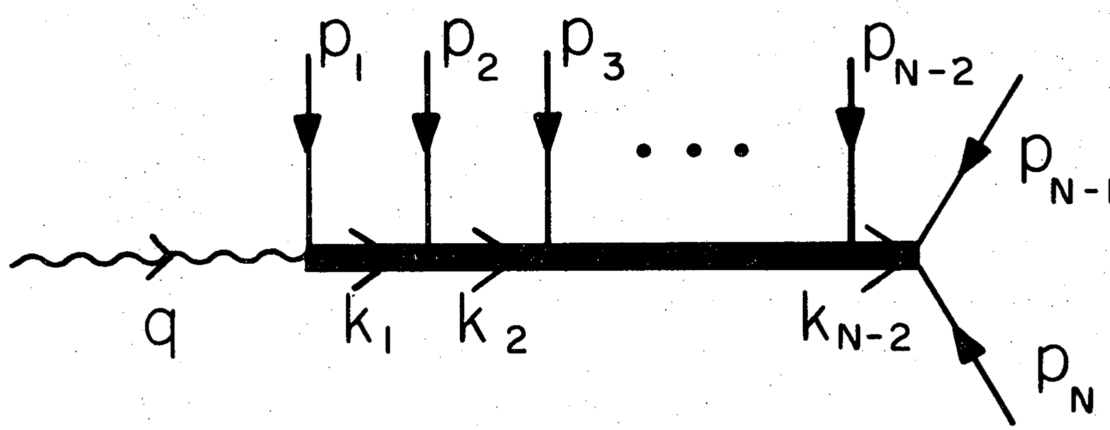

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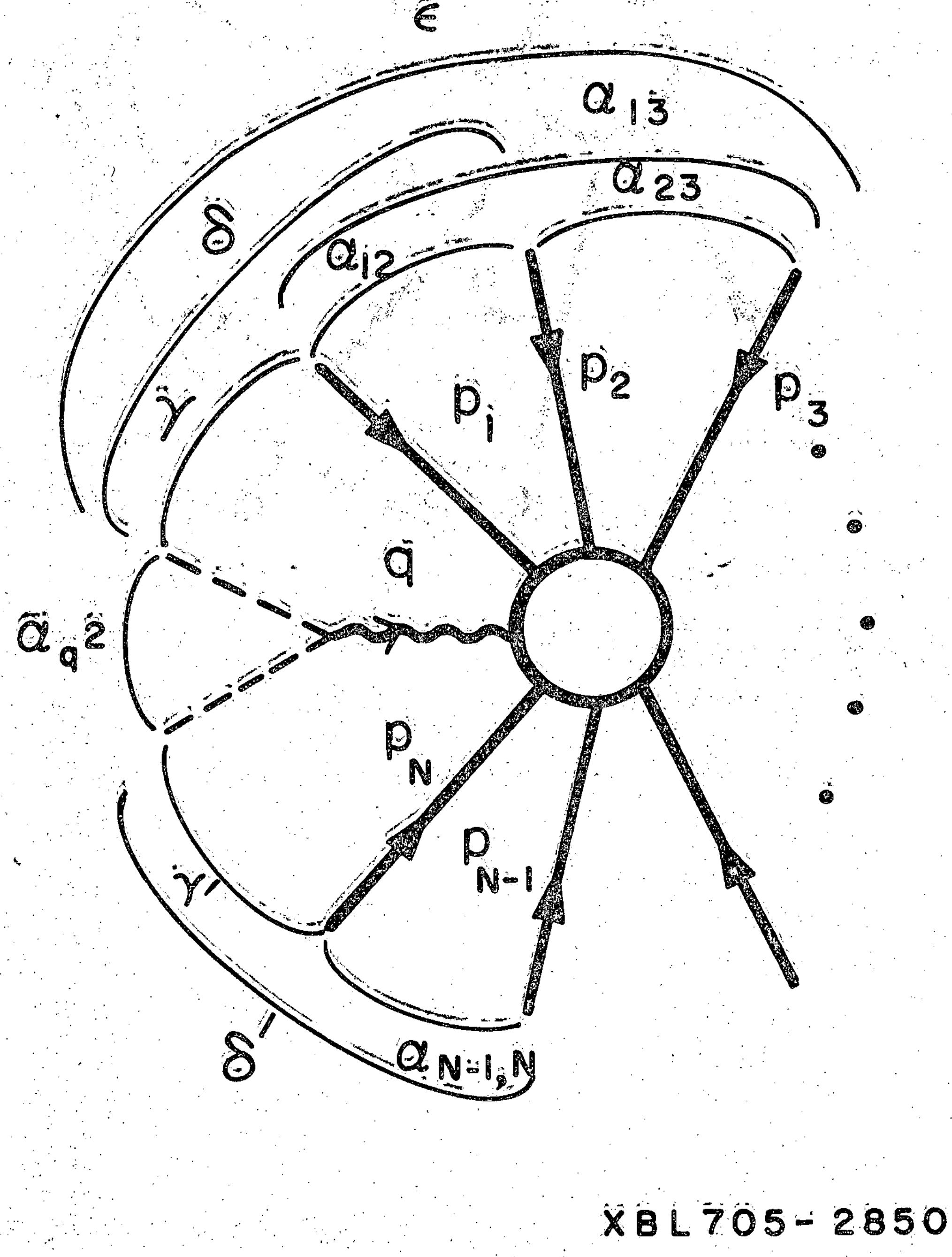
Fig. 9



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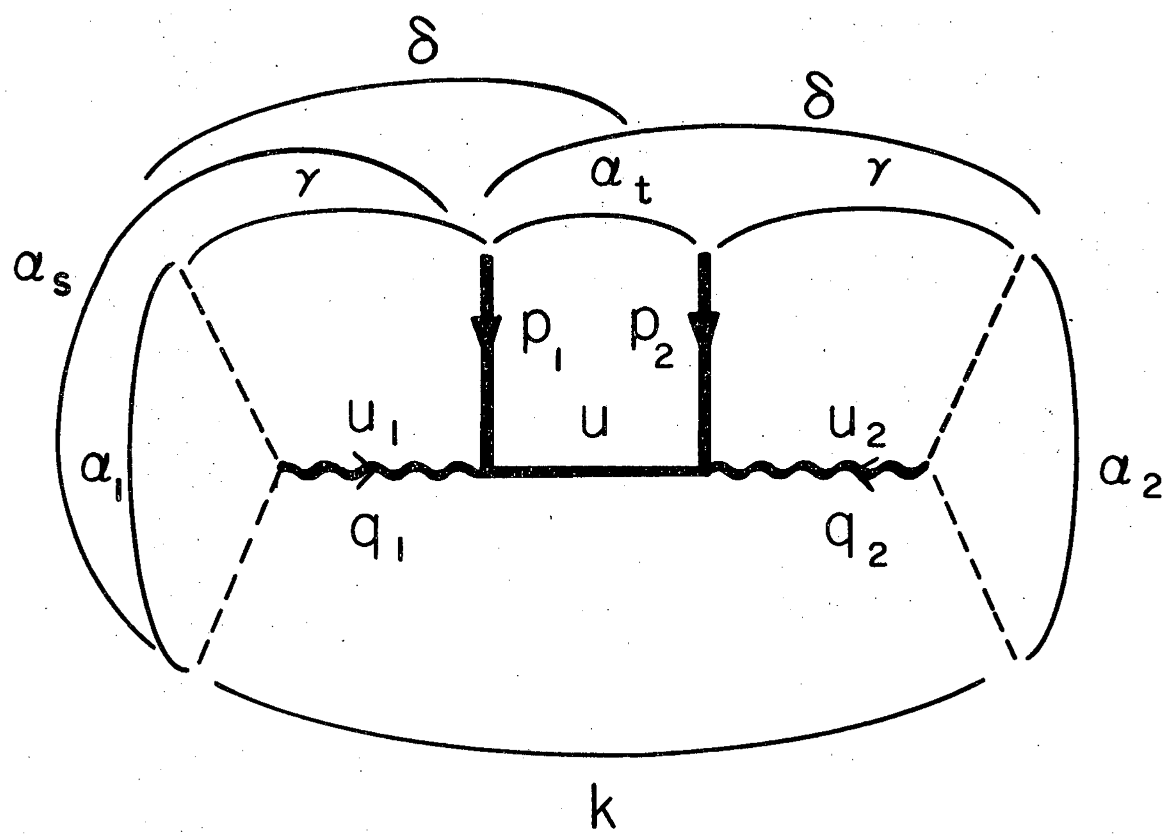
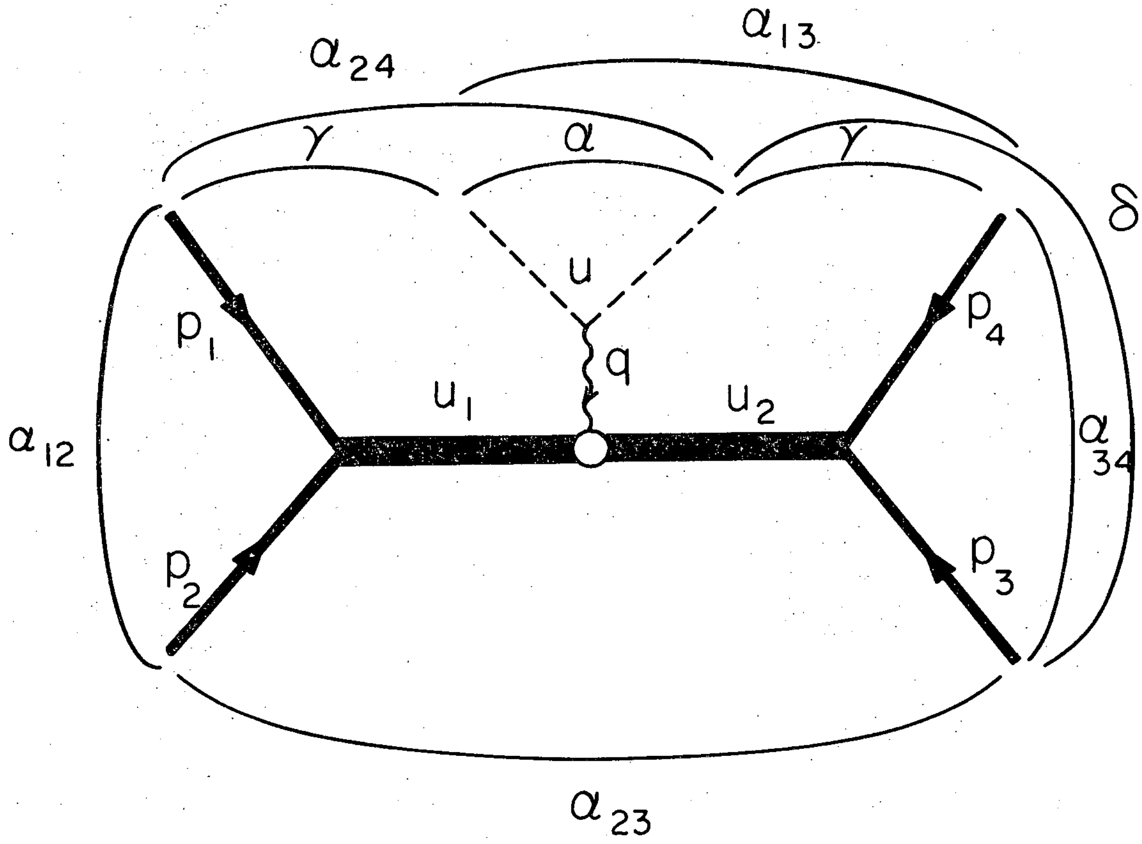

δ
δ
γ
α_t
γ
α_s
p_1
p_2
u_1
u
u_2
α_1
α_2
q_1
q_2
k
XBL705-2851

Fig. 11



α_{24}
α_{13}
γ
α
γ
δ
u
p_1
q
p_4
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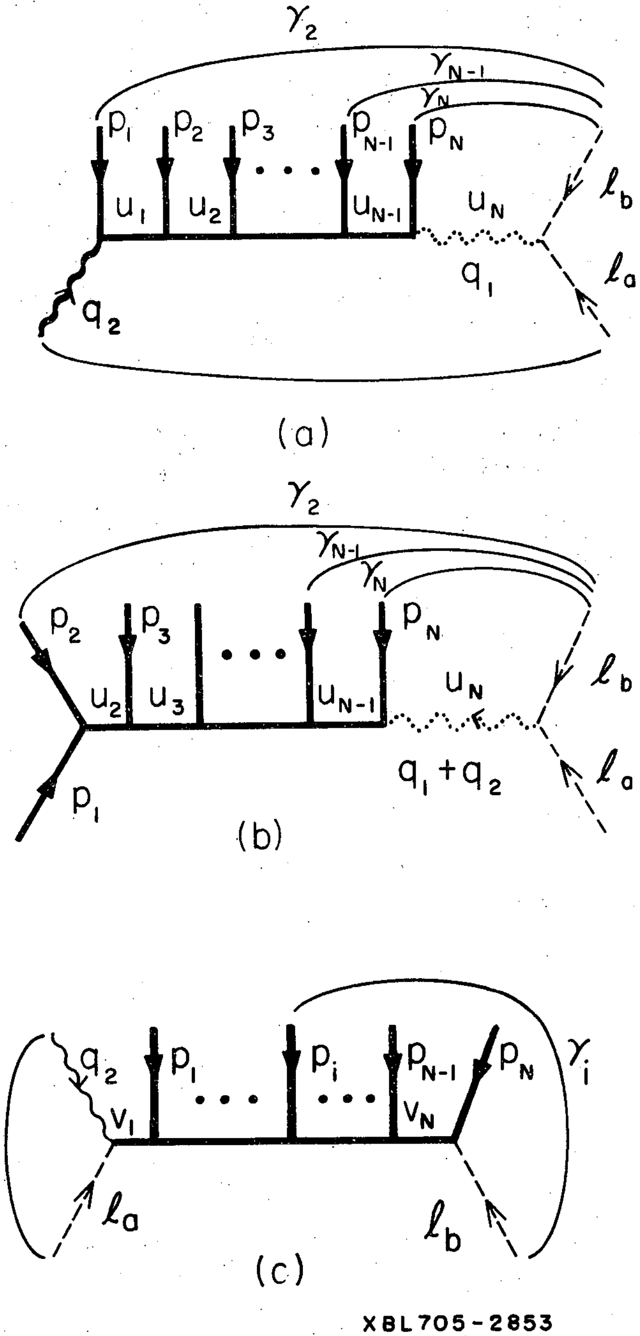
p_2
p_3
α_{23}
XBL705-2852

Fig. 12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p3
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ℓa
q1
q2
(a)
γ2
γN-1
γN
p2
p3
pN
u2
u3
uN-1
uN
ℓb
ℓa
p1
q1 + q2
(b)
q2
p1
pi
pN-1
pN
γi
k
v1
vN
ℓa
ℓb
(c)
XBL705-2853

Fig. 13

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