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It has become customary to interpret nuclear scattering experiments in terms of the optical model in which one introduces a general single-particle operator (optical potential) for the incident projectile and attempts to determine its properties from experiment. Although this procedure has yielded many useful results it has a number of drawbacks, particularly for very high energies. On the other hand there are a number of simplifications which obtain at very high energies which permit a more satisfactory treatment to be given.

To be specific, we consider the scattering of a neutral spinless particle by a spherical symmetric object. The scattering amplitude $f(\theta)$ in the usual notation is

$$k\, f(\theta) = \frac{1}{2i} \sum_{\ell=0}^{\infty} (2\ell + 1)(\eta_\ell - 1) P_\ell(\cos\ \theta) .$$

The amplitude η_ℓ of the ℓ th outgoing wave is related to the phase shift δ_ℓ by the equation $\eta_\ell = \exp(2i\,\delta_\ell)$. We propose to treat the scattering in terms of these coefficients rather than with a potential model. In doing so we shall make the quasi-classical approximation in which η is a continuous function of ℓ. In addition we emphasize the role of strong absorption at high energies by writing the scattering amplitude as a sum,

$$f(\theta) = f_0(\theta) + f_1(\theta) .$$

*Work performed under the auspices of the U.S. Atomic Energy Commission.

The amplitude $f_0(\theta)$ corresponds to the complete absorption of L partial waves: $\eta_\ell = 0$ for $\ell \leq L$ and $\eta_\ell = 1$ otherwise. Thus $f(\theta)$ is the amplitude for the "black-sphere" model,[1,2] for which exact solutions exist.

We now generalize the simple diffraction theory by writing

$$\eta(\ell) = \left|\eta(\ell)\right| e^{i\alpha(\ell)}$$

and by assuming that (a) the opacity function, $1 - \left|\eta\right|^2$, decreases monotonically with ℓ from an essentially constant value β for small ℓ to zero for large ℓ, (b) this transition occurs mainly within an interval of width 2Δ centered about a large value of the angular momentum L, (c) the phase function α is continuous and vanishes for sufficiently large ℓ. By expressing these assumptions in terms of certain definite analytic functions for $1 - \left|\eta\right|^2$ and α, one can find closed expressions for $f(\theta)$, $\sigma^{(r)}$, and $\sigma^{(t)}$. We have chosen various functional forms to represent the transition region, assuming constant phase, but find the results to be independent of the details in this region.

The significant feature of this result is that a scattering formalism of sufficient generality for high-energy collisions has been obtained which eliminates the need for any lengthy calculations. Closed-form expressions are thus available to discuss a large number of measurements in terms of a few physically significant parameters. Assuming constant phase, these parameters are: L, the number of partial waves strongly absorbed; 2Δ, the range over which the opacity function decreases from β to 0; β, the opacity for small ℓ; and α, the phase.

As an example of this method we consider neutron total and reaction cross sections in the energy range from 0.3 to 4.5 Bev for C, Cu, and Pb. The measurements at the highest energy are reported in the preceding letter.[3] The following reasonable assumptions are made about the dependence of L and Δ on k and A (atomic weight):

$$L \propto k A^{1/3}, \qquad \Delta \propto k .$$

Good agreement can be obtained only if the phase is close to zero. In other words the scattering amplitude is practically pure imaginary. The results are plotted in Fig. 1, and the values deduced for β are:

	Energy (Bev)		
	0.3	1.0	4.5
Pb	0.97	1.00	0.94
Cu	0.94	0.99	0.92
C	0.80	0.93	0.90

The dependence of β on k and A can be interpreted in terms of a classical picture of exponential absorption with distance, the absorption coefficient being related to the observed nucleon-nucleon total cross sections. The energy variations of the cross sections are best understood in terms of the basic formulae for the partial reaction and total cross sections:

$$\sigma_\ell^{(r)} = (1 - |\eta_\ell|^2)(2\ell + 1)\pi \lambda\!\!\!^{-2} ,$$

$$\sigma_\ell^{(t)} = (1 - \mathrm{Re}\ \eta_\ell)\, 2(2\ell + 1)\pi \lambda\!\!\!^{-2} .$$

For small real η, $\sigma_\ell^{(r)}$ is less sensitive to changes in η than $\sigma_\ell^{(t)}$, since the former is a quadratic function of η whereas the latter is linear. In addition, the reaction cross section does not depend on the phase of η, whereas the total cross section does. The observed large energy variation in the total cross section can only be obtained by choosing the phase for η close to zero.

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3. Atkinson, Hess, Perez-Mendez, and Wallace, Phys. Rev. Letters [preceding letter]. The writers would like to thank these authors for early communication of their results.

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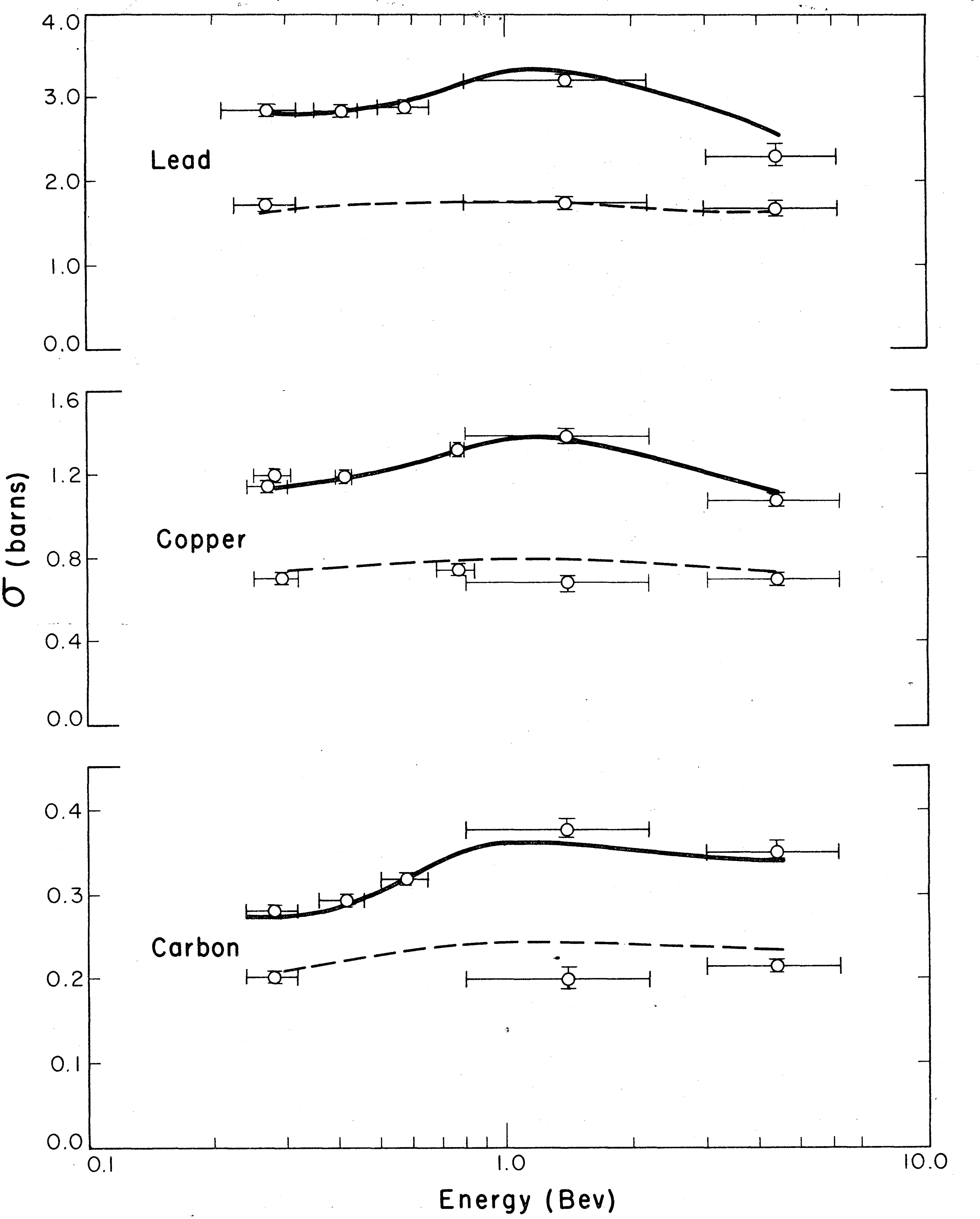
Fig. 1. Neutron total and reaction cross sections. The solid curves are the theoretical total cross sections and the dash curves are the theoretical reaction cross sections. The circles are the experimental measurements. The following values were used in the analysis: $L = (1.26 \times 10^{-13}\ \mathrm{cm})kA^{1/3}$ and $\Delta = (0.672 \times 10^{-13}\ \mathrm{cm})k$.



4.0
3.0
2.0
1.0
0.0
Lead
1.6
1.2
0.8
0.4
0.0
Copper
0.4
0.3
0.2
0.1
0.0
Carbon
σ (barns)
0.1
1.0
10.0
Energy (Bev)