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Managing City Evacuations

by

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Committee in charge:

Professor Carlos F. Daganzo
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Managing City Evacuations

Abstract

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Doctor of Philosophy in Engineering

University of California, Berkeley

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The city evacuation problem is analyzed physically at the freeway and network levels. On a freeway, a macroscopic approach is used to identify the critical bottlenecks that determine the system's evacuation capacity. Knowledge of these bottlenecks leads to the development of an input control strategy that maximizes exit flows at all times, effectively minimizing total evacuation time. The optimality results are true for the complete system and for "population nests". The strategy, called innermost first out (InFO), has many other benefits: it is decentralized, adaptive and robust. Additionally, since the strategy gives priority to upstream, most-at-risk residents, InFO is likely to be socially acceptable. Finally, relaxed versions of the strategy exist, giving flexibility to freeway evacuation management.

At the network level, a tree-shaped topology allows for similar results to be obtained. Specifically, a tree-based innermost first out (T-InFO) strategy is developed, combining InFO with an intuitive routing scheme. It is shown that if a reasonable driver adaptive behavior can be assumed for the local access streets, then T-InFO maximizes exit flows and minimizes evacuation time for population nests and therefore the complete system. Similar to InFO, T-InFO has the following benefits when implemented in a tree-shaped network: decentralization, adaptiveness, robustness, and social optimality. Due to these reasons, the strategies proposed in this dissertation have the potential to greatly improve current traffic management practices in emergency evacuations.

To Berkeley, CA,

for her demonstration of life and spirit as a world citizen.

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Acknowledgments

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Chapter 1

Introduction

1.1 Motivation

Emergency evacuations have seldom been a focus of transportation planning and research. This lack of focus came at a great cost. In August 2005, Hurricane Katrina made landfall in southeastern Louisiana, causing a storm surge that topped and broke the levee system in New Orleans. While many people successfully evacuated ahead of the massive flooding that took place in New Orleans, numerous lives were lost as the government failed to provide evacuation assistance to special needs residents.[1]

Even when people have the means to self-evacuate, an evacuation can turn into a disaster itself if unmanaged. In September 2005, Hurricane Rita threatened a direct strike at Houston. The threat spurred the egress of over two million residents and caused heavy congestion on all freeway exit routes.[2] The heavy congestion that emerged and indications that alternate routes led to shorter evacuation times[3] suggest that authorities failed both in planning and management. Certainly, some people could have waited at home a little longer instead of embarking so early on their evacuation journey, only to immediately find themselves stuck in congested traffic. Also, more information could have been provided to drivers to guide them onto some of the less-utilized surface streets.

In addition to these recent experiences, predictions by climate scientists which point to an increasing trend of extreme weather patterns underscore the need for good evacuation planning.[4] The work is vital for populous coastal areas where available escape routes can easily be overwhelmed by people's evacuation demand. Therefore, this dissertation is an attempt to fulfill an important area of research that was once neglected by

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[1]According to the Reports of Missing and Deceased published by Louisiana Department of Health and Hospitals in August 2006, 1,464 lives were lost in Louisiana due to Katrina. The major cause of death was drowning, and people over age 75 made up 49% of the death toll (Brunkard et al., 2008).

[2]Survey results from Rice University indicate that two million people evacuated from Harris County alone (Stein et al., 2009). The total number of evacuees from the entire Houston metropolitan area is estimated at 2.5 million people (Mack, 2005). The severity of the congestion is supported by evacuees' stories reported in numerous news articles — it is not difficult to find that people spent over 12 hours on the road driving to destinations that are usually reachable in four hours (Houston Chronicle, 2005; Sallee, 2005).

[3]The shorter evacuation times are reported in the news: Sallee (2005); Swartz (2005).

[4]Climate change projections are cited on the U.S. Environmental Protection Agency website as of September 8, 2009.

transportation professionals. Specifically, optimal strategies that rely only on readily available data and use realistic traffic controls are developed. They are designed to be as general as possible, i.e., they apply to any type of evacuation where the physics or capacity of the available network can be readily estimated (e.g., nuclear power plant meltdown, wild fire, etc.). Finally, even though this dissertation focuses on improving general traffic mainly used by self-reliant individuals, special needs residents can also benefit from the results since most of them would be evacuated in transit vehicles that travel alongside other modes. In the future, ideas from this dissertation could be extended to include preferences for those evacuated using transit.

The rest of this chapter summarizes past evacuation experiences and reviews transportation models that are relevant for city evacuations. Chapter 2 introduces an analytical framework for modeling evacuations in a one-dimensional framework (i.e., a freeway), and presents a general strategy for traffic management in this context. Chapter 3 extends these freeway results to a two-dimensional framework, further proposing management techniques for networks. Finally, the last chapter highlights the significance of the results, and discusses practical matters and policy implications.

1.2   Past Evacuation Experiences

People may behave in unexpected ways when they evacuate in a panic. The understanding of these actions can help improve the overall evacuation planning process and guide the development of good management strategies. This section reviews evacuees' behaviors that have been observed in the past. Those that are transportation-specific are further discussed.

One of the most well-known behaviors is the tendency for families to reunite and evacuate together (Aguirre and Swisher, 1977; Perry et al., 1981). Many have observed this behavior in real emergencies. For example, disaster management officer Karen Geerlings in Queensland, Australia, reported this family gathering activity during the tsunami scare that took place in Cairns on April 2, 2007. At the start of the evacuation, it was observed that many people traveled first to schools to pick up their children before heading to higher grounds (personal communication with Geerlings, 2007).

Also, past evacuations have demonstrated that people rely heavily on the interstate system for their egress, leaving under-utilized some local streets which could serve as alternate routes. This preference for the freeway was observed during the Hurricane Floyd evacuation in South Carolina in 1999 (Dow and Cutter, 2002). Among the evacuees who had a roadmap in their possession, half of them did not use their maps and chose to simply stay on the congested freeways through their journey. Meanwhile, traffic on smaller highways and local roads was reportedly much lighter. This phenomenon was also prevalent in the Rita evacuation in Houston. Anecdotal evidence indicates that many evacuees arrived at their destinations quicker when they evacuated through the rural streets than through the freeways (Sallee, 2005; Swartz, 2005). That alternate egress routes have been under-utilized should motivate officials to develop better ways to communicate information to travelers in an emergency so that they can make the most use of existing infrastructure and arrive at safety in a shorter time.

Other behavioral observations have been made. Surveys conducted on evacuees from Hurricanes Floyd, Rita and Ike[5] reveal that many people prefer to flee with more cars than necessary (Dow and Cutter, 2002; Stein et al., 2009). In Rita, for example, it is estimated that the average vehicle occupancy was two evacuees per vehicle.[6] Dow and Cutter (2002) speculates that this could be due to people's high valuation for their cars, their desire to carry with them much personal belongings, differences in household members' job locations, or an attempt to increase transportation flexibility in preparation for their return.

Two other habits of evacuees have been found to hinder the evacuation process. Firstly, there is a significant number of people who prefer to evacuate at once. In the case of a hurricane, most residents leave two days prior to the storm's landfall (Dow and Cutter, 2002; Stein et al., 2009). This means that the transportation network and all supporting resources become strained just shortly before the disaster strikes, as evacuation demand suddenly far exceeds system capacity. Secondly, many residents evacuate from non-evacuation zones. Stein et al. (2009) argues that this "shadow evacuation" problem worsened the traffic congestion that "real evacuees" faced during the Hurricane Rita evacuation, since nearly half of the Rita evacuees were residents from non-evacuation zones. The problem remained during Ike as shadow evacuees outnumbered real evacuees by more than a factor of two. It was fortunate that only a quarter of the population evacuated during the Ike event and took fewer cars with them, so congestion did not become a big issue.

In summary, people have behaved in counter-productive ways in past evacuations. Future strategies should therefore be developed to account for the traffic problems that result from these behaviors. This can be done in two ways. Directly, strategies can help resolve evacuation traffic problems; indirectly, strategies can be designed to be robust to changes during an evacuation.

1.3   Literature Review

Transportation models that are applicable to an evacuation are reviewed here. It will become apparent that most existing models either have unrealistic data demands, or are non-adaptive to emergency events. Hence, they fall short of what is needed in evacuations. The models are classified into two types: flow-based optimization and simulation.

Conventional mathematical programming has been used widely to solve the evacuation problem in a flow-based traffic assignment framework. Though they may consider different evacuation scenarios such as shelter locations (Sherali et al., 1991), household trip-chain sequencing (Murray-Tuite and Mahmassani, 2004), and stochastic routing (Shen et al., 2009), all these models require estimated demand data as inputs. However, as shown in Section 1.2, demand is very unpredictable in evacuations. Hence, such models cannot provide traffic management solutions in real-time.

Simulation models were first developed in the 70s for the emergency planning of communities near nuclear power plants in the U.S. Two pioneer computer simulation

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[5]Hurricane Ike struck the Houston-Galveston area in September 2008.

[6]This figure is obtained by dividing 1.25 million vehicle-trips made in the Rita evacuation (Benson et al., 2005) into 2.5 million person-trips made (Mack, 2005).

models developed for evacuations are micro-simulator NETSIM (Peat, Marwick, Mitchell & Co., 1974) and macro-simulator NETVACl (Sheffi et al., 1982). These models used adhoc numerical schemes of questionable realism, which have since been superseded by models using more systematic methods: cellular automata for microscopic simulation (Nagel and Schreckenberg, 1992) and the cell transmission model (CTM) for macroscopic simulation (Daganzo, 1994, 1995). Besides being scenario specific, computer simulators are very time-consuming to operate and require large amounts of data inputs and parameter calibration. Thus, they cannot adapt to changing conditions and are not useful for quick emergency implementation.

Two notable simulation-type works of relevance for evacuations are the physical analysis of the morning commute problem by Daganzo and Lin (1993) and the solution for the single destination traffic assignment problem developed by Ziliaskopoulos (2000). As applications of the CTM, both works are not as data-intensive as the other computer simulations. However, the Daganzo and Lin result is only for the very limiting case of a homogeneous freeway. Although solving more general network problems, the Ziliaskopoulos model requires estimated demand data as inputs.

The only known model developed for more general networks that do not require demand estimation is Lovell and Daganzo (2000). The work proposes an access control algorithm that optimizes many-to-one traffic by keeping congestion outside of the system. Unfortunately, the strategy requires centralization, which may not be possible in an evacuation. Also, its solution applies only to networks without route choice.

Thus, realistic evacuation strategies are needed for general network problems. They should derive solutions using minimal, readily available data, and should be adaptive to changing traffic conditions. The analytical framework and strategies developed in this dissertation will meet these criteria. The results will further be generalizable to different types of emergencies and to other kinds of traffic problems.

Chapter 2

Evacuations of Freeways

As noted in the previous chapter, people have relied heavily on freeways for their evacuations in recent major disasters. Hence, this chapter examines a single freeway as a first step. Chapter 3 will extend these freeway results to the network level.

2.1 Freeway Definitions and Assumptions

Figure 2.1 is an illustration of the freeway analyzed here. Let $P \doteq$ total population to be evacuated from the freeway, and $I \doteq$ total number of links and on-ramps on the freeway. Note that off-ramps are ignored since traffic flows leaving the system at intermediate locations are assumed negligible. As shown, links and ramps are numbered from 1 to I in the upstream direction. An exit separating danger from safety is defined at location $i = 0$, just downstream of link 1. The distribution of evacuees beyond this location is outside the scope of this dissertation: it is assumed that there is always sufficient capacity downstream of the exit to absorb all evacuated traffic; hence, queues from congested downstream off-ramps do not spill back and block the exit.

Figure 2.2 depicts the triangular flow-density relation postulated for the freeway (Newell, 1993). Various traffic states of interest are labeled on the figure for reference in later sections. The time-invariant parameters that characterize link i, $i \in [1, I]$, just downstream of ramp i are as follows: $l_i \doteq$ link length, $c_i \doteq$ capacity flow, $\kappa_i \doteq$ jam density,
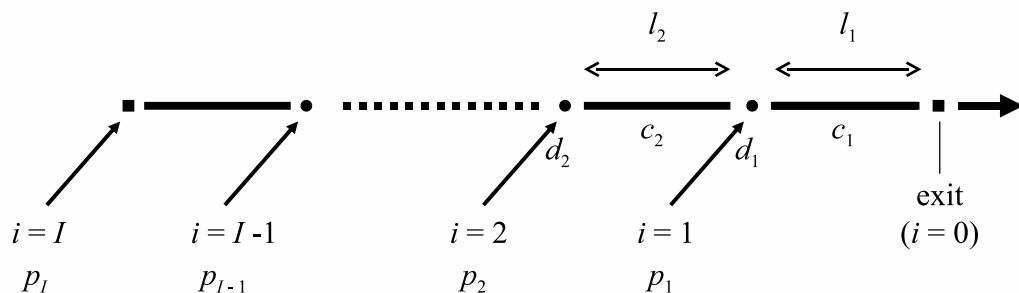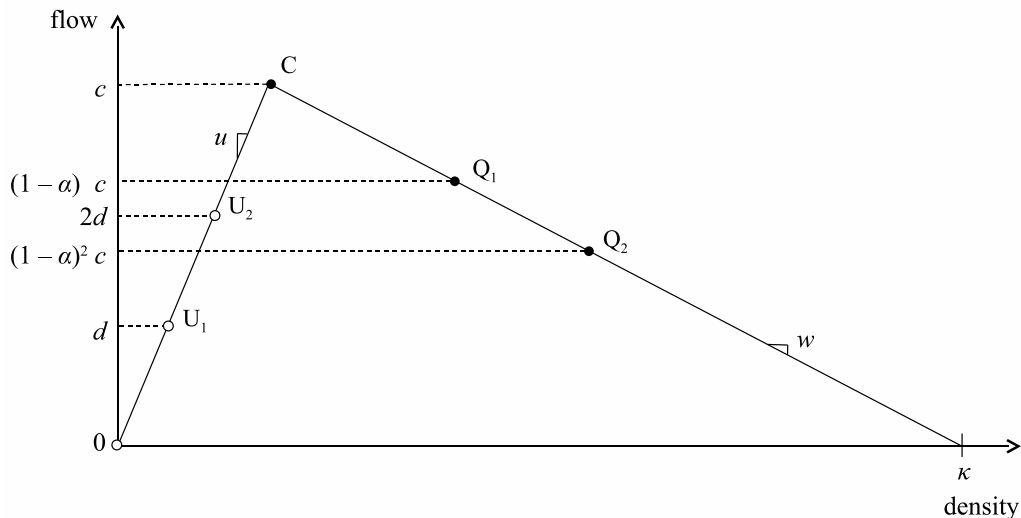

Figure 2.1: Freeway Illustration

Figure 2.2: Homogeneous Freeway Fundamental Diagram ($0 < \alpha < 1$)

$u_i \doteq$ free-flow travel speed, and $w_i \doteq$ backward wave speed. Note that the exit capacity is c_1. The parameters for ramp i are: $d_i \doteq$ discharge capacity of ramp i, and $p_i \doteq$ population to be evacuated from ramp i (so $\sum_{i=1}^{I} p_i = P$).

Two well-established traffic flow theories are assumed for analysis. The kinematic wave theory (KWT) is used to depict the evolution of aggregate traffic flows over time and space (Lighthill and Whitham, 1955; Richards, 1956). Secondly the theory underlying the cell transmission model (CTM) is assumed for predicting conditions at freeway-ramp merges (Daganzo, 1993). Empirical evidence has solidified these theories as good approximations for traffic behaviors in the real world (Windover and Cassidy, 2001; Brockfeld et al., 2003; Cassidy and Ahn, 2005). As in the CTM, α_i is assumed to define the merge proportion from ramp i, i.e., the fraction of vehicles on link i coming from ramp i when both link $i+1$ and ramp i have queues.

Finally, two other behavioral assumptions are made for this freeway analysis:

- Queuing persists at every on-ramp until the total population of that ramp is evacuated.

- The mainline freeway is empty at the start of the evacuation.

The above can be justified by observations made on recent major evacuations. For example, it was noted during the approaches of Hurricanes Gustav and Ike which struck the U.S. Gulf Coast in September 2008 that people saturated all freeway entrances throughout the duration of an evacuation. Secondly, personal interviews with emergency officials in New Orleans shortly after the Gustav evacuation also revealed that authorities prefer starting an evacuation in the early morning period (personal communication with Sneed et al., 2008). At that time the freeway can be assumed to be nearly empty.

2.2 Special Case: Homogeneous Freeway

2.2.1 Time-Space Evolution of Queues

A freeway is homogeneous if its physical parameters are constant for all i: $\{l_i,$ $c_i,\ \kappa_i,\ u_i,\ w_i,\ d_i,\ \alpha_i,\ p_i\} \equiv \{l,\ c,\ \kappa,\ u,\ w,\ d,\ \alpha,\ p\}$. The following discusses analytical and simulated results previously derived in Daganzo and Lin (1993) for a single corridor morning commute problem which is identical to the present evacuation problem. Original work derived in this dissertation is also presented for comparison.

In Daganzo and Lin (1993), the evolution of queues resulting from morning traffic heading to a single downstream destination is analyzed for a homogeneous freeway. (In an evacuation, the destination would be the exit to safety as shown in Figure 2.1.) Specifically, the case of a freeway with 12 on-ramps and $\alpha = 1$ is presented in the paper. Formulas are given for computing the time to reach an equilibrium, where traffic states on the freeway remain stable for a long duration, and for computing the equilibrium link flows. From these results, it is shown that the freeway discharges traffic at capacity throughout much of the morning rush without management. The result does not depend on the ramp discharge capacity; hence, it is concluded that ramp metering cannot help reduce people's delay in the system.

Daganzo and Lin (1993) also shows simulated results for the case with $\alpha = 1/4$ using the cell transmission model. This is displayed in Figure 2.3. In the figure, a darker shade indicates a denser traffic state. Note how the downstream end of the freeway becomes saturated by capacity flow shortly after time 0 and remains this way until approximately time 550. A very similar picture is displayed in Figure 2.4. It displays a generic solution of the problem for very large I (and $\alpha < 1$), and is derived as original work in this dissertation using KWT and CTM. Lines in the figure illustrate interfaces separating different traffic states on the freeway: under-capacity state U (dotted), queued state Q (solid shading), capacity state C (criss-crossed), and empty state 0 (blank).

During the initial transient, t_0, all ramps discharge the maximum possible flow d ($< c$). The freeway remains in this under-capacity state (U) until it reaches saturation. From then on, congestion (Q) develops everywhere, and the freeway discharges at capacity (C) for most of the remaining time, T^{mid}. Eventually, the front of the queue recedes upstream as downstream ramps start to empty. In the final transient, t_f, sub-capacity flows return as the back of the queue propagates forward to meet the front of the queue, and the few unfinished ramps once again discharge at the maximum rate d. [1] The evacuation is over when the last residents from ramp I^* cross the exit.

The labels along the vertical dotted line in Figure 2.4 indicate the events at the ramps for the corresponding freeway traffic states. At any time t: ramps in states C and 0 are empty, ramps in Q are discharging flows $\leq d$, and those in U are discharging at the ramp capacity. The reader can refer to Appendix A for more details on the evacuation process.

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[1] The number of ramps in the final transient cannot be greater than c/d since otherwise, some section of the freeway would be congested.
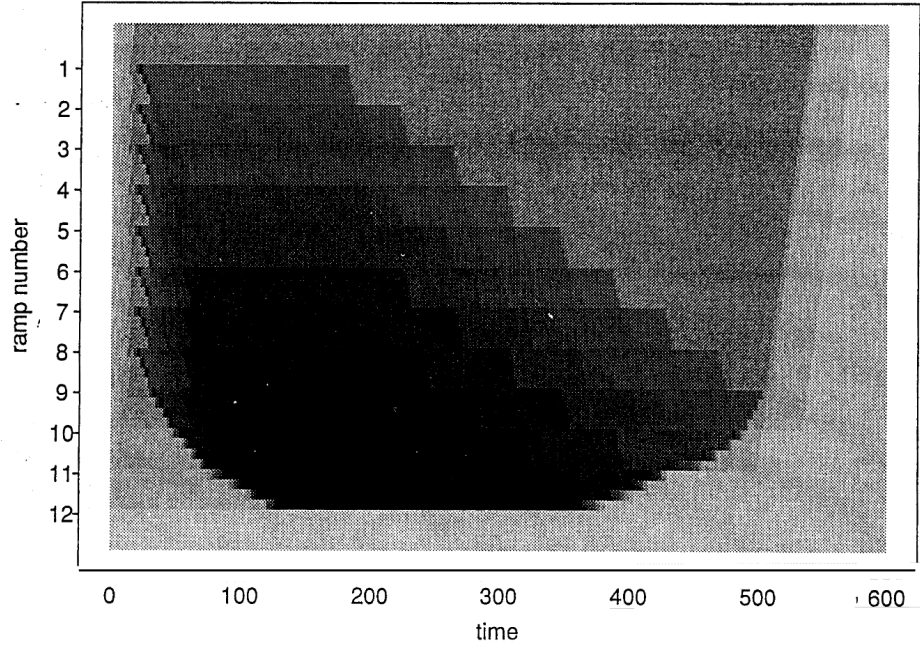
Figure 2.3: Simulated CTM Output: $I = 12$ and $\alpha = 1/4$ (Daganzo and Lin, 1993)
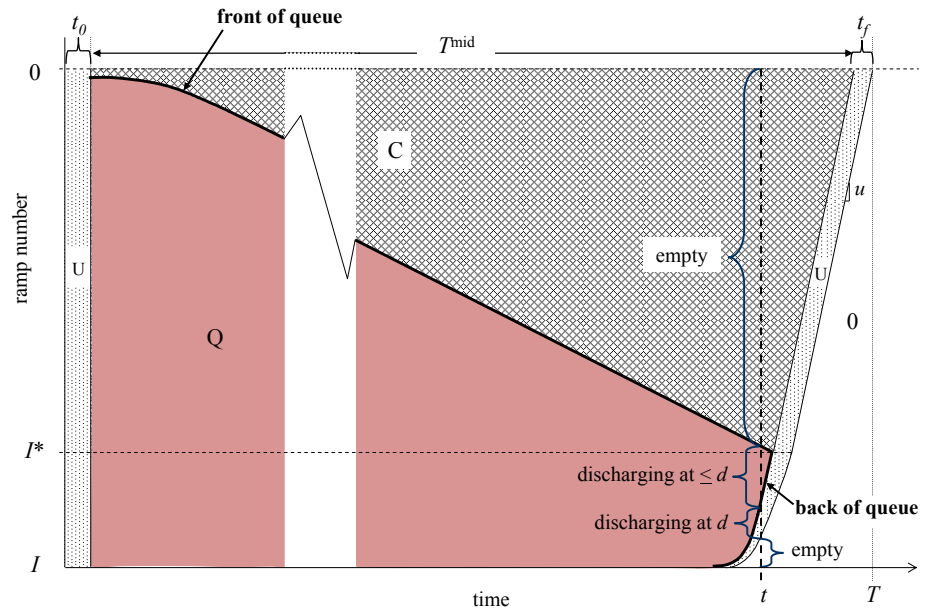

Figure 2.4: Analytical Result: $I \to \infty$ and $\alpha < 1$

2.2.2 Evacuation Time Analysis

A lower bound for evacuation time is P/c. It is established by imagining that residents were pre-staged immediately upstream of the exit. This value is significant in two ways. If the actual evacuation time of a freeway with capacity c and population P equals this lower bound, then 1) the freeway evacuated in the minimum time possible, and 2) P/c is the best lower bound.

Obviously, P/c cannot realistically be achieved since it does not account for the time to overcome distance. However, note from Figures 2.3 and 2.4 that the majority of a homogeneous freeway's evacuation time is attributable to the discharge of residents from the freeway when it is saturated. Free flow travel time which factors into the computation of the initial and final transients seems to matter much less. So, perhaps the actual evacuation time is not too different from the lower bound P/c. The rest of this section proves that this is indeed the case for a populous freeway.

From Section 2.2.1, total evacuation time, T, can be computed as the sum of t_0, T^{mid}, and t_f. Obviously, $T^{\mathrm{mid}} \leq P/c$ since the total number of evacuees discharged in the middle period ($T^{\mathrm{mid}} \cdot c$) cannot exceed the total population (P). Thus, an upper bound for T is $t_0 + P/c + t_f$, and together with the lower bound, T is expressed as follows:

$$\frac{P}{c} \quad \leq \quad T \quad \leq \quad t_0 + \frac{P}{c} + t_f. \tag{2.1}$$

As shown in Appendix A, t_0 is analytically approximated to be $(c/d) \cdot (l/u)$. Also, t_f can be shown to be bounded from above by p/d. Now, a freeway is "populous" if it satisfies two conditions: (i) $P/c \gg (c/d)(l/u)$, and (ii) $P/c \gg p/d$. The following key result is derived for the homogeneous freeway:

Theorem 2.1. *The evacuation time of a populous, homogeneous freeway is approximately equal to the lower bound, i.e.,*

$$T \approx P/c \qquad as \quad I \to \infty. \tag{2.2}$$

Proof. Note that neither the approximation to t_0 nor the upper bound of t_f depends on I. Therefore, as $I \to \infty$, $pI = P \to \infty$, and the transient terms in the upper bound of (2.1) become negligible compared with the middle term. $\square$

This result is true for any homogeneous freeway, i.e., for all parameter values of l, c, κ, u, w, d, α, and p. Additionally, this is true when populations at the ramps are different, i.e, when $p_i \neq p$ for some i. In this case, the qualitative solution remains largely the same as in Figures 2.3 and 2.4: a single queue evolves and persists on the freeway as ramps take no time to use up any available capacity on the freeway made available by a just-emptied downstream ramp. The front of the queue will still mark the spot separating downstream finished ramps from upstream unfinished ones. The only difference is the time-space paths of the front and the back of the queue, which depend on the different p_i.

Even with the different p_i, the total evacuation time will continue to consist of three distinct phases: (i) an initial transient of subcapacity flows, (ii) a prolonged capacity flow period, and (iii) a final transient of subcapacity flows. For as long as the population at

each ramp is sufficiently large such that the time required by a ramp to discharge evacuees is greater than the time for the freeway to be saturated [2], i.e., $p_i/d > (c/d) \cdot (l/u), \forall\ i$, the initial transient is still approximately $(c/d) \cdot (l/u)$. The final transient now becomes bounded from above by $\max_i p_i/d$, or the time it takes to clear the most populous ramp.

Corollary 2.2. *Even when the ramp populations are not the same, the evacuation time of the populous freeway is approximately equal to the lower bound.*

Proof. Replace t_0 and t_f by $(c/d) \cdot (l/u)$ and $\max_i p_i/d$ in the upper bound of (2.1) to see that:

$$\frac{P}{c} \quad \leq \quad T \quad \leq \quad \frac{c \cdot l}{d \cdot u} + \frac{P}{c} + \max_i \frac{p_i}{d}. \tag{2.3}$$

Again, the first and third terms of the right-hand-side of (2.3) do not depend on I, but P increases with I. Hence, (2.2) still holds. $\qquad\square$

Note the simplicity of (2.2). It has important implications. The lower bound closely approximates the actual evacuation time of a populous freeway. This means that when travel time is negligible (so initial and final transients take no time), P/c gives the best lower bound. It also means that traffic on an uncontrolled, homogeneous freeway self-manages and evacuates in a time that is close to the minimum possible. This is in line with the insight derived in Daganzo and Lin (1993) — that ramp metering cannot help to reduce the total time people spend in the system.

Therefore, this section shows that traffic on a homogeneous (or nearly homogeneous) freeway needs no management if the sole objective is to achieve minimum evacuation time. However, ramps seem to generally finish in the upstream direction. This can be observed from Figures 2.3 and 2.4. This implies that upstream residents who are likely to be most-at-risk are evacuated last. Perhaps a more socially acceptable approach is to allocate priority to these residents first. Thus, in the end, traffic control can help in managing the evacuations of homogeneous freeways: by allowing residents facing greater risk to exit first.

2.3 General Freeway Analysis

Not surprisingly, the evacuation of an inhomogeneous freeway (i.e., l_i, c_i, κ_i, u_i, w_i, d_i, α_i, and p_i vary over i) is more complex. The variability in the freeway and ramp characteristics potentially means a time-space solution that is more difficult to derive. In some cases, it is possible for a section of the freeway to never reach saturation. Separate queues may also form. Moreover, the distinct time phases found in Section 2.2 are not necessarily well-defined. Due to the above, a time-space solution is not derived here. Rather, different evacuation objectives are analyzed for strategy development starting here in this chapter.

Besides minimizing total evacuation time (the objective analyzed in the previous section), officials may want to evacuate as many people as possible within a short amount of time. This alternative objective is important especially when a disaster approaches too

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[2]On a realistic freeway, saturation only takes a few minutes.

quickly for everyone to be evacuated. While most existing research has so far only considered one objective at a time, this dissertation focuses on both objectives at once.

Definition. A strategy is a **comprehensive strategy** for a system if it minimizes the system's evacuation time and maximizes the number of residents that can be evacuated in a given time.

Note that a strategy that maximizes the number of evacuees by a time t before the end of the evacuation does not always minimize the total evacuation time. (Consider a strategy that stops evacuating people after t, so the total evacuation time is infinite. Certainly, an alternative strategy exists that can evacuate the rest of the population in finite time.) However, one that maximizes the number of evacuees at all times does:

Theorem 2.3. *If a strategy maximizes the number of residents that can evacuate from a system at all times, it minimizes the system's evacuation time.*

Proof. Given a strategy X that maximizes the number of evacuated residents at all times. If there exists a strategy Y which can evacuate everyone from the system by time $T^Y < T^X$, then it would have evacuated more people by T^Y. But this would contradict the given condition. □

In what follows, bounds are established for the two evacuation objectives. They serve as benchmarks in the sense that a strategy whose performance matches the lower bound on evacuation time, for example, is optimal. It minimizes evacuation time. Furthermore, having this strategy also proves that the lower bound is tight, i.e., it is the best lower bound. The same logic applies for a strategy whose performance matches the upper bound; i.e., the strategy maximizes the number of evacuees, and the upper bound is tight.

2.4 Benchmarks for Evacuation Objectives

Let the set of ramps from i to I, inclusively, be known as nest i. Also let the total population in nest i be $P_i = \sum_{j=i}^{I} p_j$, $\forall\ i$. Note that $P_i \leq P_{i-1}, \forall i$. Two bounds are developed here: a lower bound for evacuation time and an upper bound for the number that can evacuate by a given time.

A lower bound on evacuation time is clearly $\max_i P_i/c_i$. A simple upper bound on the number of evacuees is now defined for some time t. Partition the freeway into two segments at some arbitrary link location x ($x > 1$), such that a downstream segment extends from the exit to location x and an upstream segment extends from x to ramp I. Then an upper bound on the number of evacuated residents from the downstream segment is simply: $\sum_{i=1}^{x-1} p_i$. From the upstream segment, since all evacuees would have to pass freeway link x, a better upper bound at time t is $\min[\ P_x\ ,\ c_x t\ ]$. The upper bound for the freeway is therefore: $\sum_{i=1}^{x-1} p_i + \min[\ P_x\ ,\ c_x t\ ]$. Note that if $x = 1$, there is no downstream segment, and the upper bound for the freeway is simply $\min[\ P_1\ ,\ c_1 t\ ]$.

Now a new function is defined and used to improve the two bounds established above. Let $\tilde{c}_i$ be the most restrictive capacity on the freeway downstream of ramp i ($\tilde{c}_i = \min_{j \leq i} c_j$). Hereafter, $\tilde{c}_i$ is called the "downstream capacity" (d-capacity) for i, and the corresponding link location on the freeway is called the "downstream bottleneck"

(d-bottleneck) for i. If there are multiple locations on the freeway with capacity $\tilde{c}_i$, the most upstream link location downstream of i with capacity $\tilde{c}_i$ is defined as the d-bottleneck for i.

Figure 2.5 depicts the freeway capacity function, c_i, and the downstream capacity function, $\tilde{c}_i$, approximated as continuous curves. Note that $\tilde{c}_i \leq c_i$ and $\tilde{c}_i \leq \tilde{c}_{i-1}, \forall\ i$. Also, the d-bottlenecks are locations where the curves touch. Furthermore, the dotted line is horizontal wherever the curves are separated, and finally, the two curves meet at $i = 0$.

The lower and upper bounds are now improved using the function $\tilde{c}_i$. Let $T^L \doteq$ the improved lower bound on evacuation time of the freeway, and $N_x^U(t) \doteq$ the improved upper bound on the number evacuated by time t with respect to location x. The results are expressed as follows:

Lemma 2.4. *The improved lower bound for evacuation time is $T^L = \max_i P_i/\tilde{c}_i$.*

Proof. The nested population P_i must pass the (d-bottleneck) link with capacity $\tilde{c}_i$. Hence, $\max_i P_i/\tilde{c}_i$ is a lower bound. Note that $\max_i P_i/\tilde{c}_i$ is improved from (greater than) the previous $\max_i P_i/c_i$. $\qquad\square$

Lemma 2.5. *The improved upper bound for the number of evacuees is:*

$$N_x^U(t) = \begin{cases} \sum_{i=1}^{x-1} p_i + \min[\ P_x\ ,\ \tilde{c}_x t\ ] & \text{if } x > 1 \\ \min[\ P_x\ ,\ \tilde{c}_x t\ ] & \text{if } x = 1 \end{cases}$$

Proof. Residents from the upstream segment of the partitioned freeway with respect to x must pass through the (d-bottleneck) link with capacity $\tilde{c}_x$. Hence, an improved (decreased) upper bound for the upstream segment is $\tilde{c}_x t$. $\qquad\square$

In the next section, the lower bound T^L will be used to show that an inhomogeneous freeway, unlike the homogeneous case, may discharge inefficiently when unmanaged. However, the implementation of a simple priority scheme can help the freeway achieve optimal evacuation time.

2.5   Benefits of Management

The example shown in Figure 2.6 is used to illustrate that, unlike in the case of a homogeneous freeway, if an inhomogeneous freeway is unmanaged, its evacuation time can be sub-optimal. The assumptions used here include: $\alpha_1 = \alpha_2 = 1$; $l_1 = l_2 = l$; $c_1 = 2c_2 = 2c$; $p_1 = p_2 = p$; and $d_i > c_i$ for $i = 1, 2$. Also assumed is that the p_i/c_i are so large compared with l/u that the free flow speed can be assumed infinite: $u = \infty$ (thus free flow travel time is zero).

Under these conditions, the lower bound computed with Lemma 2.4 is $T^L = p/c$. Now if the freeway is uncontrolled, users from ramp 2 would have to wait until users from ramp 1 finish evacuating before they can advance past ramp 1. While users from ramp 1 can pass the exit at the maximum rate $2c$, the capacity restriction on link 2 imply that ramp 2 users can at most exit at rate c. Thus, the total evacuation time with no control is: $T^N = p/2c + p/c = 1.5(p/c)$. This exceeds the lower bound by 50%.
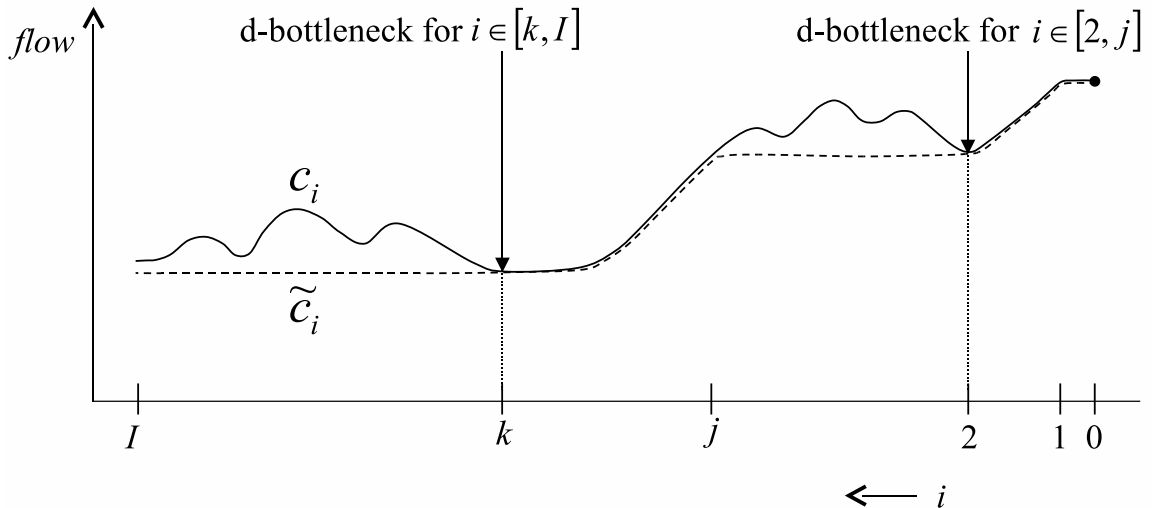
Figure 2.5: Capacity and d-Capacity Functions

Imagine now that a meter is placed on ramp 1, giving absolute priority to users coming from ramp 2 but allowing people from ramp 1 to use all the residual capacity of link 1. In this case, the users from ramp 2 will evacuate in time p/c; during this time, the users from ramp 1 will discharge at the residual rate $2c - c = c$. Since there are p users from ramp 1, they too finish discharging at time p/c. So the evacuation time with this form of control is: $T^C = p/c = T^L$. Hence, the control is optimal and reduces by 33% the evacuation time with no control. This happens because by giving priority to upstream residents, the freeway's capacity is better utilized.

An important insight is learned from this example: priority matters. When unmanaged, an inhomogeneous freeway may evacuate inefficiently as flows from less restrictive parts of the freeway block upstream flows from reaching the exit. A scheme giving priority to upstream residents can help reduce the total evacuation time of this simple freeway. So the question is whether this scheme can generalize and be applied to optimize the evacuation of any freeway. The answer is yes. The next section shows how.

2.6   The Innermost First Out (InFO) Control Strategy

This section introduces a strategy to manage evacuation traffic on a freeway. The strategy is easy to understand and uses minimal, real-time data. Note that these benefits make this strategy realistic for implementation in an emergency. Specifically, the latter benefit means that no predictive information is required — this is very important for an emergency during which officials would have no time to estimate demand data for model inputs.

Furthermore, the strategy is decentralized, allowing on-ramps to operate as independent units of the freeway. Decentralization in control is very important for emergency
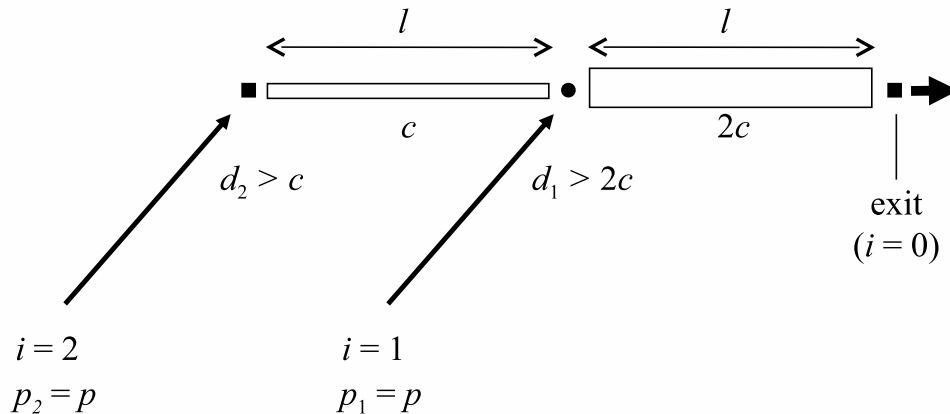
Figure 2.6: A Two-Link Inhomogeneous Freeway

management. The Katrina incident taught that central communication networks are prone to breakdowns in an emergency and hence, cannot be relied upon for evacuations.

2.6.1 Modeling Assumptions

Assumptions and definitions in addition to those in Section 2.1 are needed for the discussion that follows. First, free flow travel time is assumed negligible. It has already been shown for the homogeneous case that if the freeway is very populous, then initial and final transients which are driven by people's free flow travel time can be ignored. For an inhomogeneous freeway, free flow travel time can also be ignored without any loss of generality if it can similarly be assumed that the freeway is very populous. Additionally, it is assumed that $d_i \geq \tilde{c}_i, \forall\ i$.

2.6.2 Modeling Definitions

Only three new time-dependent variables need to be defined. For all ramp i and time t, let $q_i(t) \doteq$ traffic flow on link i at t, and $r_i(t) \doteq$ flow discharged by ramp i at t. Thus, the conservation equations are $q_i(t) = r_i(t) + q_{i+1}(t), \forall\ i \in [1, I-1]$, and $q_I(t) = r_I(t)$. Also, let $p_i(t) \doteq$ population remaining to be evacuated from ramp i at time t (so, $p_i = p_i(0)$).

Finally, the **"innermost first out" (InFO)** control strategy can be introduced. InFO instructs ramp i to discharge according to the following rules at time t:

$$r_i(t) = \begin{cases} 0 & \text{if } p_i(t) = 0, \ \forall\ i \\ \tilde{c}_i - q_{i+1}(t) & \text{if } p_i(t) > 0 \text{ and } i \in [1, I-1] \\ \tilde{c}_i & \text{if } p_i(t) > 0 \text{ and } i = I \end{cases} \tag{2.4}$$

Note that $q_{i+1}(t)$ $(= r_{i+1}(t) + q_{i+2}(t)$ by conservation) can be at most equal to $\tilde{c}_{i+1} \leq \tilde{c}_i$, so $r_i(t)$ is non-negative. Thus, InFO control gives absolute priority to upstream flows over downstream flows. Unfinished ramps are only allowed to emit flows onto the

freeway if there is residual d-capacity after upstream use. The following discussion proves that this strategy is optimal.

2.6.3   Optimality of InFO

Note from the definition above that InFO maintains d-capacity flow on the freeway at all times, for as long as there are remaining residents to be evacuated from upstream of the corresponding d-bottleneck. This property of InFO is proven in the following lemma:

Lemma 2.6. *Under InFO control, if $p_i(t) > 0$, then $q_i(t) = \tilde{c}_i$, and the d-bottleneck for i is saturated at time t.*

Proof. Re-arranging and substituting (2.4) into the conservation equations (for $p_i(t) > 0$) give the desired result. Since free flow travel time is negligible, the flow $q_i(t) = \tilde{c}_i$ on link i reaches the d-bottleneck for i at time t, saturating it at that time. □

Thus, no over-saturating flows would be released onto the freeway under InFO; all queues are held at the on-ramps.[3] Congestion would never arise on the freeway and evacuees would always travel there at free flow speed. Now, note that under InFO, at least one d-bottleneck is saturated while the evacuation is still in progress. (This is true since $p_i(t) > 0$ for some i if the evacuation is in progress; thus, by Lemma 2.6, $q_i(t) = \tilde{c}_i$.) Let the most downstream of these saturated d-bottlenecks at time t be denoted as $\delta(t)$.

Lemma 2.7. *Under InFO, $\delta(t)$ has two properties:*

1. *Ramps downstream of $\delta(t)$ have finished evacuating by t.*

2. *The d-bottleneck $\delta(t)$ has been saturated since $t = 0$.*

Proof. Note that if $\delta(t) = 1$, then there are no downstream ramps to consider. Assume that $\delta(t) > 1$. If the first property is not true, then $p_i(t) > 0$, for some $i < \delta(t)$. But Lemma 2.6 implies $q_i(t) = \tilde{c}_i$, or the d-bottleneck for i, which would be downstream of $\delta(t)$ would be saturated at t. This would contradict the definition of $\delta(t)$. Now, for the second property, identify a most downstream ramp j that is discharging a flow at t. Note that $j \geq \delta(t)$. Since $\delta(t)$ is saturated and j is the nearest ramp emitting a flow, the d-bottleneck for j must be $\delta(t)$. Obviously, $p_j(t) > 0$. Also $p_j(t') > 0$ for $t' \leq t$ since the function is non-increasing in t. By Lemma 2.6, $q_j(t') = \tilde{c}_j$ for $t' \leq t$, so the d-bottleneck for j or $\delta(t)$ is saturated for any $t' \leq t$. □

Theorem 2.8. *InFO maximizes the number of evacuees from the freeway at all times.*

Proof. Consider any time t and identify $\delta(t)$. Partition the freeway into two segments at $\delta(t)$. The first property of Lemma 2.7 implies that all residents downstream of $\delta(t)$, i.e., $\sum_{i=1}^{\delta(t)-1} p_i$, have been evacuated by t. The second property of the lemma suggests that the number to have evacuated from the upstream segment of the freeway is equal to $\tilde{c}_{\delta(t)} t$. The total number of residents that have evacuated from the freeway by t is thus: $\sum_{i=1}^{\delta(t)-1} p_i + \tilde{c}_{\delta(t)} t = \sum_{i=1}^{\delta(t)-1} p_i + \min[\ P_{\delta(t)}\ ,\ \tilde{c}_{\delta(t)} t\ ] = N_{\delta(t)}^U(t)$. Note that if $\delta(t) = 1$, then the total number is $\tilde{c}_{\delta(t)} t = \tilde{c}_1 t = \min[\ P_1\ ,\ \tilde{c}_1 t\ ] = N_1^U(t)$. □

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[3]To maintain flows below saturation, the freeway cannot store queues.

Hence, the upper bound can be met by selecting $\delta(t)$ as the location to partition the freeway. The last two corollaries follow directly from this theorem.

Corollary 2.9. *InFO minimizes total evacuation time.*

Proof. This is clearly true by Theorems 2.8 and 2.3. $\qquad\square$

Corollary 2.10. *InFO is a comprehensive strategy for the management of freeway evacuations.*

Proof. By Theorem 2.8 and Corollary 2.9, InFO satisfies the definition of a comprehensive strategy as presented in Section 2.3. $\qquad\square$

Hence, InFO is optimal as defined by the two most common evacuation objectives. It is also a useful strategy in that it relies only on readily available information for implementation: an operator at every on-ramp only needs to know the mainline's arriving traffic flows and the ramp's corresponding d-capacity. The former can be obtained using counters such as loop detectors on the road. Such devices are widely available. The latter can be determined using the system's capacity function, which is readily known.

Furthermore, InFO is adaptive since it uses only real-time information (instead of demand predictions). Also, since InFO can be implemented independently at every on-ramp, it is decentralized. Most importantly, the strategy is socially acceptable as it always gives priority to those most-at-risk. In summary, the InFO control strategy can be expected to be realistically implemented in emergency evacuations.

2.7 Extensions of InFO

2.7.1 Optimality for Nests

It turns out that InFO is comprehensive for every nest on the freeway. Let $N_x^{U(i)}(t) = \min[\ P_x\ ,\ \tilde{c}_x t\ ] + \sum_{j=i}^{x-1} p_j \doteq$ an upper bound on the number of residents that can be evacuated from nest i by time t with respect to location $x > i$. If $x = i$, then $N_x^{U(i)}(t) = \min[\ P_x\ ,\ \tilde{c}_x t\ ]$.

Theorem 2.11. *InFO maximizes the number evacuated from every nest at all times.*

Proof. Consider any nest i and time t before the end of the evacuation. Define $f(t)$ as the most downstream ramp in nest i that still has remaining evacuees at t. The nest is now partitioned into 1) an upstream section equal to nest $f(t)$ and 2) a downstream section from ramps i to $f(t) - 1$, inclusively. The number of residents evacuated from each section is now calculated. Obviously, $p_{f(t)}(t) > 0$, and since the function is non-increasing in t, $p_{f(t)}(t') > 0$, for $t' \leq t$. By Lemma 2.6, $q_{f(t)}(t') = \tilde{c}_{f(t)}$ for $t' \leq t$ under InFO, and the d-bottleneck for $f(t)$ has been saturated by nest $f(t)$ since the start of the evacuation. Thus, the number of evacuees at t from the upstream section is $\tilde{c}_{f(t)} t = \min[\ P_{f(t)}\ ,\ \tilde{c}_{f(t)} t\ ]$.

If $f(t) > i$, then ramps downstream of $f(t)$ in nest i have obviously finished evacuating, and the number of evacuees from downstream would be $\sum_{j=i}^{f(t)-1} p_j$. If $f(t) = i$, then there is no downstream section. The total number of evacuees from nest i, for $f(t) > i$,

is therefore $\tilde{c}_{f(t)}t + \sum_{j=i}^{f(t)-1} p_j = \min[\ P_{f(t)}\ ,\ \tilde{c}_{f(t)}t\ ] + \sum_{j=i}^{f(t)-1} p_j = N_{f(t)}^{U(i)}(t)$. The result for $f(t) = i$ is: $\tilde{c}_i t = \min[\ P_i\ ,\ \tilde{c}_i t\ ] = N_i^{U(i)}(t)$. $\qquad\square$

Once again, Theorem 2.3 and the definition of a comprehensive strategy, in addition to the above theorem, prove the following true:

Corollary 2.12. *InFO minimizes the evacuation time of every nest.* $\qquad\square$

Corollary 2.13. *InFO is comprehensive for every nest.* $\qquad\square$

2.7.2 Time-dependent Capacity

It turns out that InFO remains optimal even when capacity changes on the freeway, perhaps due to the occurrence of incidents. To demonstrate this, assume a time-dependent capacity function: $c_i(t)$. This function is now used in place of c_i. Then a time-dependent d-capacity function can be defined: $\tilde{c}_i(t) = \min_{j \leq i} c_j(t)$. In this case, InFO is defined by replacing $\tilde{c}_i(t)$ for $\tilde{c}_i$ in equation (2.4). This is feasible if the location and severity of each change in capacity can be quickly reported to all ramp controllers.

Theorem 2.14. *Even when capacity on the freeway changes with time, InFO continues to maximize the number of evacuees at all times.*

Proof. It suffices to prove this true for any nest on the freeway. Note that Lemma 2.6 is true even if $\tilde{c}_i(t)$ is used in place of $c_i(t)$. Now the upper bound $N_x^{U(i)}(t)$ is updated by using $\tilde{C}_x(t) = \int_0^t \tilde{c}_x(s)ds$ instead of $\tilde{c}_x t$. It follows that Theorem 2.11 holds when $\tilde{C}_{f(t)}(t)$ is used to replace $\tilde{c}_{f(t)}t$ in the proof. $\qquad\square$

Once again, the above implies that InFO continues to minimize evacuation time and is therefore comprehensive for every nest even when capacity is time-dependent. A caveat to the result is that time lags in communication are not taken into account. In reality, the transmission of the message that an incident has happened can take some time. During that time, over-saturating flow (based on the reduced d-capacity function) may have already been released onto the freeway. Congestion can develop on the freeway in this case.

2.7.3 Driver Adaptation

Even when drivers are allowed to an upstream ramp to access the freeway, perhaps because they know that InFO is being implemented and thus upstream ramps have higher priority, InFO continues to be optimal. Specifically, it is assumed that people would switch to an *upstream* ramp if doing so would allow them to reduce their individual evacuation time. Further assumed is that drivers have full knowledge of the system status, that ramp switches take no time, and that they occur at $t = 0$, right before the start of the evacuation. (Downstream switches are not considered, since such movements would imply alternate routes exist, invalidating the single freeway assumption.)

A **user-equilibrium under InFO (InFO-UE)** describes the resulting populations at the on-ramps at $t = 0$ after all the switches take place. Specifically, the population at ramp i under InFO-UE is known as p_i^A. (The superscript stands for "after adaptation".)

Also, let all ramps sharing the same d-bottleneck be grouped, and number these groups, k, in the upstream direction from 1 to K, where $K \doteq$ the number of d-bottlenecks on the freeway.

Define T_i^I as the time at which nest i finishes evacuating under InFO. Brackets are used for group variables, so $\{T\}_k^I \doteq$ the time at which group k finishes evacuating under InFO; $\{p\}_k$ and $\{p\}_k^A$ denote the original and adaptive population of group k, respectively; and $\{P\}_k$ and $\{P\}_k^A$ the original and adaptive population nested upstream of group k, respectively. Finally, $\{\tilde{c}\}_k \doteq$ the d-capacity for group k. Note that if ramp i is in group k, then $\tilde{c}_i = \{\tilde{c}\}_k$. Also, by construction, $\{\tilde{c}\}_k > \{\tilde{c}\}_{k+1}$.

Lemma 2.15. *Under InFO-UE, a ramp i of group k is used only if i is the most upstream ramp in group k.*

Proof (by contradiction). If a ramp that is not the most upstream in the group was used, it would finish after the most upstream ramp in the group. Therefore, it could not be in equilibrium. $\square$

Hence, only one ramp is used in a group under InFO-UE. Without loss of generality, each group is now assumed to have only one ramp. Only groups will be used from now on. Now let k^* index the most downstream group to finish the non-adaptive InFO evacuation at T_1^I. Note that $\{T\}_{k^*}^I = \{T\}_1^I = T_1^I$.

Lemma 2.16. *If k^* acts as a barrier such that residents downstream do not cross k^* during adaptation, then an equilibrium exists in which groups $k \geq k^*$ would finish evacuating concurrently, in time $\{T\}_{k^*}^I \leq \{T\}_1^I$. This equilibrium is: $\{p\}_k^A = \{T\}_{k^*}^I \cdot (\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1})$, for $k \geq k^*$.*

Proof. Note that $\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1} > 0$. Identify the group $k \geq k^*$, which is the first to finish evacuating among the groups $k^* \dots K$. Note that it finishes at $t = \{p\}_k^A / (\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1})$. But this equals $\{T\}_{k^*}^I$ when the equilibrium condition is substituted into this expression for $\{p\}_k^A$. $\square$

The next result shows that the equilibrium described in Lemma 2.16 only entails upstream motion.

Lemma 2.17. *The nested populations in the equilibrium of Lemma 2.16 satisfy: $\{P\}_k^A \geq \{P\}_k$, for $k \geq k^*$.*

Proof (by contradiction). Note, for $k \geq k^*$, $\{P\}_k^A = \sum_{j \geq k} \{p\}_j^A = \{T\}_{k^*}^I \{\tilde{c}\}_k = \{T\}_1^I \{\tilde{c}\}_k$, where the right-hand-side is the maximum number of vehicles that can discharge past d-bottleneck k in the non-adaptive evacuation time, $\{T\}_1^I$. Now, if the lemma is false, there would be a $k \geq k^*$ such that $\{P\}_k > \{P\}_k^A$. But this is impossible since it would mean that $\{P\}_k$ would exceed this maximum number of vehicles that can discharge past d-bottleneck k in the allotted time. $\square$

The final theorem shows that an equilibrium exists for the complete freeway system. In this equilibrium, the total evacuation time of the system remains optimal when InFO is implemented:

Theorem 2.18. *Under InFO, there is a user equilibrium with $\{P\}_k^A \geq \{P\}_k$, $\forall k$, that does not change the total evacuation time of the system.*

Proof. If $k^* = 1$, the result is obvious. Otherwise, divide the problem into an upstream part from k^* to K and a downstream part from 1 to $k^* - 1$. If the equilibrium in Lemma 2.16 exists, people downstream do not backtrack past k^*, and the final residents from the upstream part would finish evacuating at $t = \{T\}_{k^*}^I$. Note that by definition, this evacuation time is longer than any one group's evacuation time downstream. During this whole time, the d-bottleneck k^* is saturated by the evacuation of upstream residents. Thus, all d-capacities in the downstream part of the freeway are effectively reduced by $\{\tilde{c}\}_{k^*}$. However, note that the populations in the downstream part remain the same.

Now let the downstream part become a reduced freeway problem (denoted with a prime), with d-capacities $\{\tilde{c}\}_k' = \{\tilde{c}\}_k - \{\tilde{c}\}_{k^*}$. Note that this reduced problem is of the same type as the one in Lemmas 2.16 and 2.17, and therefore can be treated in the same way with the same conclusions: there will be a new $k'^* < k^*$ and the new discharge time for residents in k ($\in [k'^*, k^* - 1]$) will not exceed $\{T\}_1^I$. Furthermore, $\{P\}_k^A \geq \{P\}_k$, $\forall k \in [k'^*, k^* - 1]$ (thus, $\forall k \geq k'^*$ when combined with the previous result). So, again, if $k'^* = 1$, the problem is solved. Otherwise, repeat this step enough times until the last group to discharge is $k = 1$. $\qquad \square$

Thus, InFO continues to achieve minimum evacuation time even when drivers try to adapt to the strategy and backtrack upstream.

2.8   Strategy Insights

In this section, it is shown that even when InFO is made less restrictive, it remains a comprehensive strategy. This knowledge gives greater flexibility to managing freeway evacuations.

Let ramps sharing the same d-bottleneck be grouped and numbered from 1 to K as in Section 2.7.3. Note again that $\{\tilde{c}\}_k$ is the d-capacity for group $k \leq K$. Figure 2.7 depicts a d-capacity function for a freeway with six groups. Under InFO, upstream flows receive absolute priority over downstream flows. In the figure, the shaded area depicts the aggregate flow released from Group 6 at the start of the evacuation. Note how it reduces the d-capacities for all downstream groups by $\{\tilde{c}\}_6$.

Lemma 2.19. *Under InFO, only the most upstream unfinished ramp of a group is discharging at any given time. Ramps downstream in the same group are blocked.*

Proof. Given a ramp i that is discharging at some time t. Under InFO, $r_i(t) = \tilde{c}_i$ or $= \tilde{c}_i - q_{i+1}(t) \leq \tilde{c}_i$. So enough flow is sent from i to saturate the corresponding d-bottleneck. Hence, if there is a downstream ramp j in the same group, $q_{j+1}(t) = \tilde{c}_i = \tilde{c}_j$, so $r_j(t) = 0$. Ramps upstream of i that are in the same group are empty at t. This can be proven true as follows: if not, there exists an upstream ramp $i' > i$ that is emitting flow $= \tilde{c}_{i'} = \tilde{c}_i$ or $= \tilde{c}_{i'} - q_{i'+1}(t) = \tilde{c}_i - q_{i'+1}(t)$, just enough to saturate the d-bottleneck for i. So, i would not be able to emit any flow, contradicting the given condition. $\qquad \square$
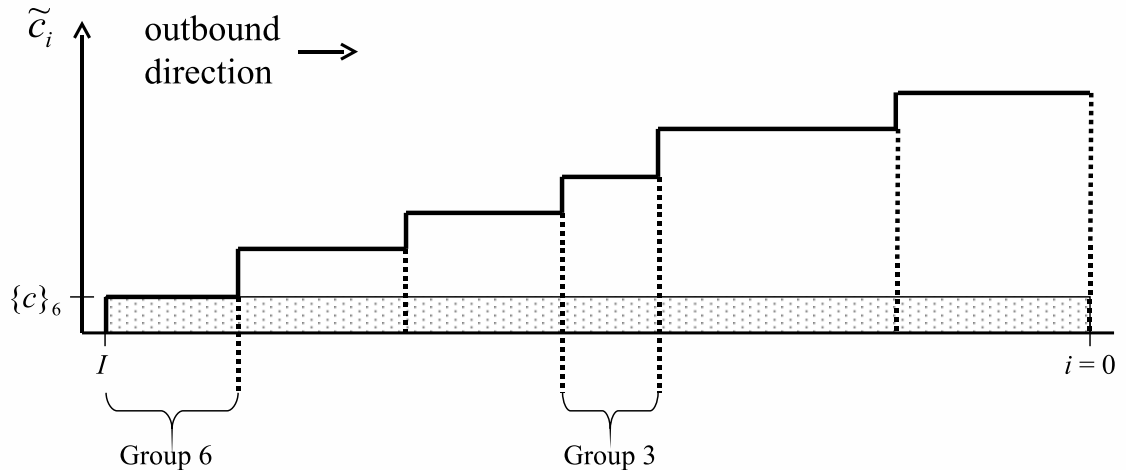
Figure 2.7: A d-Capacity Function

Note from observing Figure 2.7 that under InFO, the residual d-capacity at the d-bottleneck for group k is $\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1}$ at the start of the evacuation. Hence, group k would be instructed to discharge this much flow at the start of the evacuation to saturate the corresponding d-bottleneck. This happens until all ramps in the group are empty of residents. Let $\{\tau\}_k \doteq$ the evacuation time of group k. Note that this is the amount of time when the d-bottleneck for group k remains saturated.

Theorem 2.20. *The total evacuation time of the freeway is minimized as long as every group k emits a flow equal to $\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1}$ if $\{p\}_k > 0$.*

Proof. In the above, note that $\{\tilde{c}\}_{k+1} = 0$ if $k = K$. Note that $\{\tau\}_k$ stays the same as in InFO. Since the total evacuation time can be computed as $\max_k\{\tau\}_k$, or as the time when the last residents are evacuated, it remains unchanged from InFO. The evacuation time is therefore minimum. $\square$

The above implies that a strategy is comprehensive for the freeway as long as upstream *groups* have priority over downstream *groups*. Ramps of the same group can discharge in any order as long as their corresponding d-bottleneck is saturated whenever there are remaining residents in the group. This gives flexibility to traffic managers to switch around the order of the evacuation among ramps in the same group when necessary, to perhaps give priority to transit or special vehicles. However, note that this strategy is not necessarily comprehensive for a nest, unless if the nest is defined at a d-bottleneck location.

Chapter 3

Evacuations of Trees

This chapter extends the freeway results from Chapter 2 to the network level. As before, the focus is on the development of an evacuation management strategy that can be implemented using only readily available data and realistic controls. The management of an evacuation tree — a network comprising only of diverges and whose routes all lead to the same area of safety — is considered.

3.1 Network Management Difficulties

3.1.1 Sub-Optimality of Priority Control

A freeway is a special case network. It has been shown in Chapter 2 that a freeway requires priority control. Similarly, control is also needed to manage more general networks. Unfortunately, the InFO upstream priority scheme alone cannot guarantee system optimum in more general networks. The point is made here with a simple example shown in Figure 3.1.

In the figure, all residents are distributed in the upstream link. No one lives downstream of the diverge. It should be obvious that system optimum, defined here as the minimum evacuation time, is achieved when all routes finish evacuating at the same time. The proof of optimality can be easily argued: if routes do not finish at once, the exit capacity of a route that finishes early will be left unused for some time. Certainly, the evacuation time of the system could be reduced if some of this capacity can be used to help discharge the final residents.
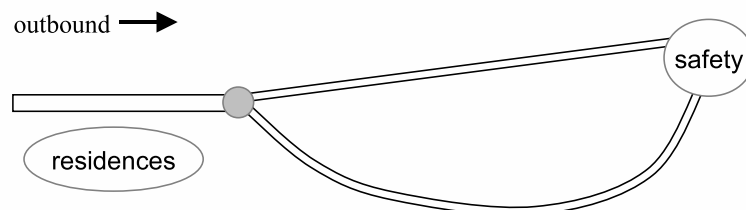

Figure 3.1: Insufficiency of Priority Control

Assume that InFO is implemented for the upstream link, where all residents live. Since this strategy does nothing to allocate traffic to routes, these upstream residents at the diverge will have to choose to evacuate through the upper or lower branch. The upper branch, as it is a more direct path to safety, may be more attractive. People may therefore be more inclined to take this route, leaving the lower branch under-utilized. Then a system optimum could not happen.

In summary, the above example shows that an upstream priority scheme is not always optimal for a network, due to people's unknown route choice behaviors. A network strategy, hence, requires a different kind of control. The next section shows that a traffic assignment strategy would not suffice either if demand is unknown.

3.1.2   Sub-Optimality of Traffic Assignment

Like InFO, a traffic assignment algorithm cannot be optimal if demand cannot be predicted. This is true because different demand patterns require different strategies. Figure 3.2 is an example used to demonstrate this point. In this example, the populations at the links (p_1, p_2, p_3) are unknown information. Known only is the capacity of each link.

If $p_2 = p_3$, a reasonable route split at the diverge is a 50-50 strategy since the downstream links have equal capacity (c). This would send a flow of $c/2$ into each downstream link and both would finish at the same time. However, imagine that the populations turn out to be $p_3 = 0$ and $p_1 = p_2 = p$. Then the 50-50 route split is not optimal: for optimality, residents discharging from the upstream link should all be diverted into the lower downstream branch. If all of p_1 is diverted to the lower branch, both downstream branches would finish evacuating at the same time, at p/c.[1] Thus, any amount of traffic diverted into the upper branch would delay the finishing time of the system.
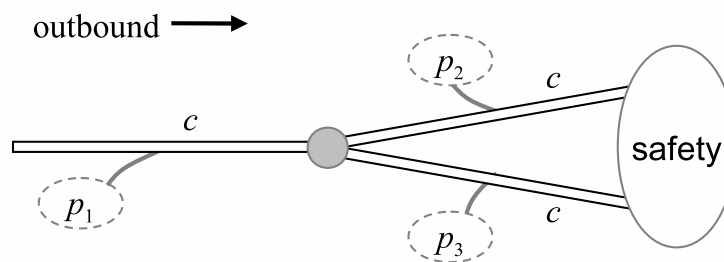
Figure 3.2: Routing without Advance Demand Information

Note that the optimality condition for the above example — that all traffic needs to be diverted into the lower branch — could only be determined after knowing the populations at the links. However, since only capacity information is given, the total number of people that use each link can never be known until after the evacuation is over. The best that anyone can do is to make predictions. But as demonstrated by past evacuation experiences (e.g., people evacuating with more cars than necessary), demand is very unpredictable in

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[1]Free flow travel time is assumed negligible.

an emergency. Even in the best of circumstances, demand predictions are never right. Therefore, the rest of this chapter presents a strategy at the network level that combines the upstream priority control with a traffic assignment scheme.

3.2 Evacuation Tree Definitions and Assumptions

3.2.1 Definitions

An evacuation tree is a network made up of (i) a set of outbound links and (ii) a set of nodes which separate successive links and define diverge locations. In such a network, no freeway merges are allowed (but ramp-to-freeway merges are); all routes lead to one general area of safety; and numerous on-ramps lie along each link where traffic enters. It is assumed here that emergency officials can readily select an evacuation tree from a general transportation network for management. This is reasonable especially for cases of mono-centric risk.

An example of an evacuation tree with five routes is displayed in Figure 3.3a. Since upstream residents are likely to face greater risk than downstream residents in an evacuation, a risk function that is non-increasing in the downstream direction is assumed for each route. Thus, every location on the tree is associated with some readily quantifiable risk value. Note that each wavy line in Figure 3.3a is a risk contour that passes through locations of the same risk.

The problem is now transformed so that a risk function can be explicitly defined at the tree level. For a node on a route in the original tree, add a new node to every other route at a location that has the same risk as this node. Using this procedure, the original problem from Figure 3.3a now looks like Figure 3.3b. Note from Figure 3.3b how the transformation has added a node to wherever the contour line crosses a route (if a node does not already exist). The white dots therefore indicate the new nodes. Observe that the transformed problem has the same number of routes as the original. Now, all routes have an equal number of nodes. The transformed network is now known as a "risk-indexed evacuation tree".

Definitions that are now given refer to the risk-indexed evacuation tree. Let $I \doteq$ the number of links on a route, and $S \doteq$ the total number of destination nodes at safety. The links on each route are now numbered in order in the upstream direction from 1 to I. Let the ith risk level of a route contain the ith link on the route along with its immediate downstream node. The destination nodes and the routes are now numbered from 1 to S, such that a route ending at destination node $s \leq S$ is known as "route s".

The links will be labeled as follows. A link (i, s) lies within risk level i and on route s. The link's immediate downstream node will have the same label. In addition, the threat node is labeled as $(I + 1, s)$, $\forall s$. Since a link or node may be used by multiple routes, it can have multiple labels. For example, link/node $(3, 1) \equiv (3, 2) \equiv (3, 3)$ in Figure 3.3b. Note that some on-ramps are shown on this particular link. Like the freeway analysis of Chapter 2, this network analysis assumes that outflows upstream of the destination nodes in an evacuation tree are negligible.

Now group all ramps that lie along the same link. The following link-level aggregate variables are defined. Let $q(i, s, t) \doteq$ the flow leaving link (i, s) at time t; $r(i, s, t) \doteq$ the
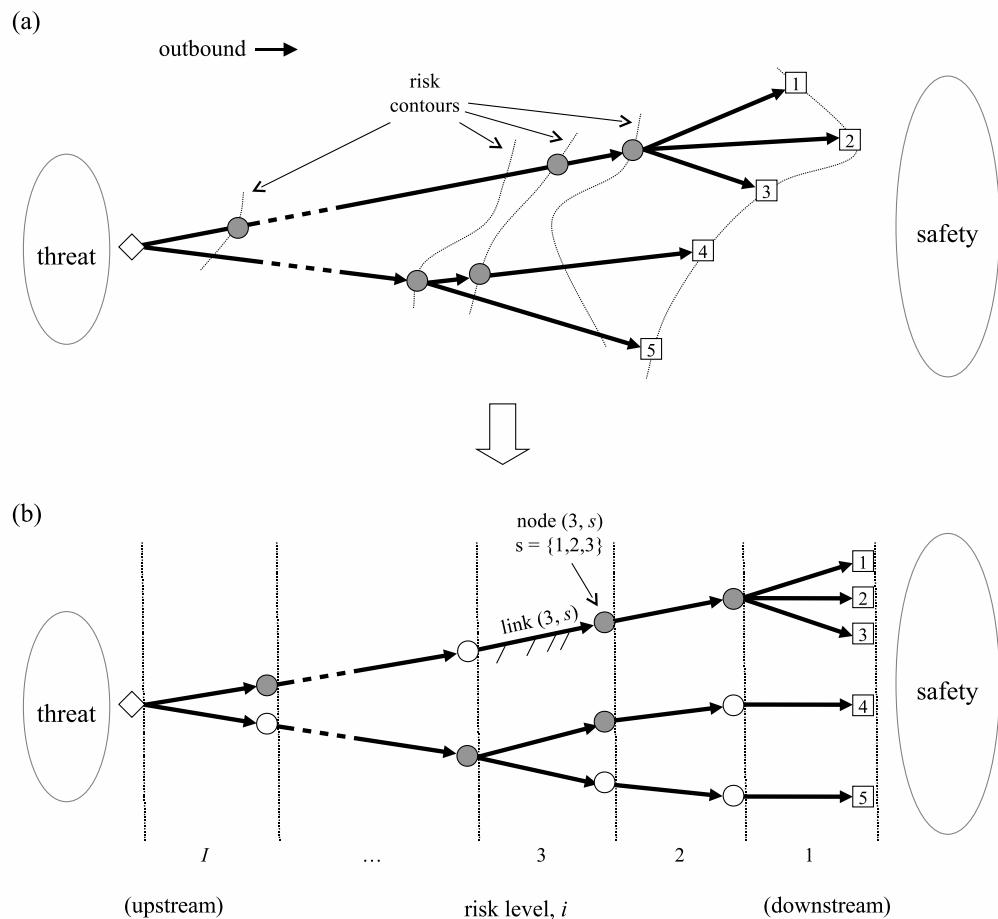
Figure 3.3: Transformation from (a) an evacuation tree to (b) a risk-indexed evacuation tree.

aggregate flow entering link (i, s) from its on-ramps at t; and $p(i, s, t) \doteq$ the number of remaining residents at the on-ramps at link (i, s) at t.

Note that in such a network, every node, except for the threat node, has one input link upstream. However, a node, if not a destination, can have one or more links emerging from it downstream. The set of all links emerging from node (i, s) is denoted $\mathcal{E}_{i,s}$, $\forall s$ and $i \in [2, I+1]$. Obviously, if node (i, s) is not a diverge, $|\mathcal{E}_{i,s}| = 1$ and link $(i-1, s)$ would be the only emerging link.

If $c(i, s)$ is the capacity of link (i, s), then let $\tilde{c}(i, s)$ be defined as follows:

$$\tilde{c}(i, s) = \begin{cases} c(i, s) , & \text{for } i = 1; \text{ and} \\ \min(c(i, s) , \ \sum_{e \in \mathcal{E}_{i,s}} \tilde{c}(e)) , & \text{for } i \in [2, I]. \end{cases} \tag{3.1}$$

Hence, these $\tilde{c}(i, s)$ are similar to the d-capacities defined for the freeway in Chapter 2. It will be shown later in this chapter that $\tilde{c}(i, s)$ is the maximum feasible flow from link (i, s). That is, this is the maximum flow that can travel from link (i, s) to safety without

congesting the network. Now, $\tilde{c}(i, s)$ is called the "t-capacity" for (i, s), where "t" stands for the "sub-**t**ree of (i, s)", or the tree that comprises link (i, s) and all downstream links and nodes reachable by flows passing link (i, s). In practice, $\tilde{c}(i, s)$ is found recursively in the upstream direction starting from the first risk level. The most upstream link(s) in the sub-tree corresponding to $\tilde{c}(i, s)$ is now called the "t-bottleneck" for (i, s).

Note that at a link, the total outflow from its downstream end must be equal to the total inflow at its upstream end, minus any (entry) flows in between. Assuming that travel time is negligible, the conservation of flows can be expressed as: $q(i, s, t) - r(i, s, t) = q(i+1, s, t) \cdot \beta(i, s)$, where $\beta(i, s) \doteq$ the proportion of flow from link $(i+1, s)$ that is routed onto link (i, s), $\forall i \in [1, I-1]$. Note that $0 \leq \beta(i, s) \leq 1$, and $\sum_{e \in \mathcal{E}_{i,s}} \beta(e) = 1$. Hence:

$$q(i, s, t) = \begin{cases} r(i, s, t) \quad + \quad \beta(i, s) \cdot q(i+1, s, t) \,, & \text{for } i \in [1, I-1]; \\ r(i, s, t) \,, & \text{for } i = I. \end{cases} \tag{3.2}$$

All links (and on-ramps) and nodes belonging to a risk level i are now classified into a set $\mathcal{L}_i$, called the risk-level set i. (Note that $|\mathcal{L}_i| \leq S$.) At any time t: $\langle q \rangle_i(t) = \sum_{l \in \mathcal{L}_i} q(l, t) \doteq$ the total flow leaving $\mathcal{L}_i$; $\langle r \rangle_i(t) = \sum_{l \in \mathcal{L}_i} r(l, t) \doteq$ the total entry flow from the on-ramps at $\mathcal{L}_i$; and $\langle p \rangle_i(t) = \sum_{l \in \mathcal{L}_i} p(l, t) \doteq$ the total remaining population at the on-ramps at $\mathcal{L}_i$. The risk-level capacity and t-capacity are, respectively: $\langle c \rangle_i = \sum_{l \in \mathcal{L}_i} c(l)$ and $\langle \tilde{c} \rangle_i = \sum_{l \in \mathcal{L}_i} \tilde{c}(l)$. In classic network flows literature, the aggregate link(s) at or downstream of risk level i whose total capacity corresponding to $\langle \tilde{c} \rangle_i$ is called the "minimum cut" for risk level i. [2]

Now the union of risk-level sets $i \ldots I$ is called "bounded risk set i". Finally, let $P_i(t) = \sum_{j=i}^{I} \langle p \rangle_j(t) = \sum_{j=i}^{I} \sum_{l \in \mathcal{L}_j} p(l, t)$ be the total remaining residents in bounded risk set i at time t.

3.2.2 Assumptions

Several assumptions are made here for the analysis of an evacuation tree. First, free flow travel time is assumed negligible. This is reasonable since the strategy proposed here allows people to always travel at the free flow speed once in the system, similar to the freeway strategy. In this case, the system's exit flows are more likely driven by the constraining capacity of internal bottleneck(s), rather than people's travel time.

Second, it is assumed that there are no queues in the system at the start. As mentioned in Chapter 2, this is justified by a conversation with emergency management officials in New Orleans in 2008 (Sneed et al., 2008). Section 3.6.1 shows that this assumption is not restrictive. Even when it is relaxed (so that there are pre-existing queues), similar performance results hold for the proposed strategy.

Third, it is assumed that people "laterally" switch ramps within the same risk-level set to minimize their system access time. They do this so well that:

$$p(i, s, t) = 0 \qquad \Leftrightarrow \qquad \langle p \rangle_i(t) = 0 \quad , \quad \forall s. \tag{3.3}$$

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[2] This can be seen by imagining that the total demand arising from risk level i comes from a source node, and all destination nodes are connected to an artificial sink node.

The above condition is reasonable only if a background of surface streets exists outside of the evacuation tree. The condition expresses people's desire to evacuate as quickly as possible by choosing the fastest way to enter the evacuation tree. This can be expected, especially if people know that management will try to keep the system uncongested (since this way, once they are in the system, they spend negligible time to reach safety). However, people would not want to increase the risk they face. So, they do not move into an upstream risk-level set.[3] Later, Section 3.6.2 will further analyze movements in the upstream direction, in addition to lateral movements.

The justifications for lateral adaptation give rise to another assumption. Since people try to evacuate as quickly as possible, whenever there are residents remaining in a risk-level set, each ramp will be holding a queue. This, along with (3.3), implies that all ramps within a risk-level set will finish evacuating at the same time. Finally, as there are many on-ramps on a link (so that the aggregate ramp discharge capacity is very large), the link's corresponding t-bottleneck will be readily saturated.

3.3 A Benchmark for the Evacuation

This section presents an upper bound for the number of residents that can be evacuated from every bounded risk set at a given time. The next section will use this bound to demonstrate that the proposed strategy is optimal.

First, note that if feasible, a flow $q(i, s, t)$ passing link (i, s) must travel from the upstream link of the sub-tree of (i, s) through the entire sub-tree without causing congestion. Since, as will be shown, $\tilde{c}(i, s)$ is an upper bound to the flow through link (i, s), this implies: $q(i, s, t) \leq \tilde{c}(i, s)$. Now if this is repeated for all parallel links within a risk-level set, then an upper bound for the set i is: $\langle \tilde{c} \rangle_i$. Note that this is also an upper bound for feasible flows from bounded risk set i.

An upper bound can now be established for the number of residents that can evacuate from a bounded risk set. Let $\mathcal{N}_i(t) \doteq$ the number of residents that can evacuate from bounded risk set i by time t, under a generic strategy. An upper bound can be found by dividing the set of links into two parts using a "vertical cut" at risk level j ($j \geq i$). The two parts are: an upstream part containing risk-level sets $j \ldots I$ and a downstream part contains risk-level sets $i \ldots j - 1$. Let $P_{i,j}^+$ and $P_{i,j}^-$ be the total original population in the upstream and downstream parts of the set, respectively. If $i = j$, then $P_{i,j}^- = 0$. An upper bound for the number of evacuees from bounded risk set i at time t, $\mathcal{N}_{i,j}^U(t)$, is:

$$\mathcal{N}_{i,j}^U(t) \quad = \quad \min(\ P_{i,j}^+\ ,\ \langle \tilde{c} \rangle_j \cdot t\ ) + P_{i,j}^- \quad \geq \quad \mathcal{N}_i(t). \tag{3.4}$$

The maximum number of people that can evacuate from each part of the set at any time is of course bounded from above by its total population. Therefore, $P_{i,j}^+$ and $P_{i,j}^-$ are the upper bounds to the upstream and downstream parts, respectively. The upstream part's bound is made tighter by introducing $\langle \tilde{c} \rangle_j \cdot t$.

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[3]Downstream risk-decreasing movements are not allowed, since otherwise, there would be alternate evacuation routes outside of the original problem.

3.4 The Tree-based Innermost First Out (T-InFO) Control

This section introduces a strategy for managing traffic in an evacuation tree. The strategy, called tree-based innermost first out (T-InFO), combines priority control and routing control.

The strategy is defined as follows. At any time t, T-InFO instructs all unfinished ramps at a link (i, s) to meter traffic according to (3.5) and routes vehicles at the diverge node $(i + 1, s)$ according to (3.6):

$$r(i, s, t) = \begin{cases} 0 , & \text{if } p(i, s, t) = 0 ; \\ \tilde{c}(i, s) - \beta(i, s) \cdot q(i + 1, s, t) , & \text{if } p(i, s, t) > 0 \text{ and } i \in [1, I - 1]; \\ \tilde{c}(i, s) , & \text{if } p(i, s, t) > 0 \text{ and } i = I . \end{cases} \quad (3.5)$$

$$\beta(i, s) = \frac{\tilde{c}(i, s)}{\sum_{e \in \mathcal{E}_{i+1,s}} \tilde{c}(e)} , \qquad \text{if } i \in [1, I - 1]. \quad (3.6)$$

Therefore, T-InFO uses the same upstream priority scheme as InFO: it instructs a link to give priority to arriving traffic flows from an upstream link, and instructs that the maximum flow be sent downstream, limited to the link's t-capacity.

Lemma 3.1. *Under T-InFO, if $p(i, s, t) > 0$, then $q(i, s, t) = \tilde{c}(i, s)$.*

Proof. For $p(i, s, t) > 0$, substitute (3.5) into (3.2) to obtain the desired result. $\square$

As mentioned, an upstream priority scheme alone is not optimal at the network level. Therefore, (3.6) presents a routing scheme used to better distribute traffic in the network. It diverts flows at every diverge in proportion to the t-capacities of the emerging links. The performance of T-InFO is shown in the next section.

3.5 Optimality of T-InFO

Let $\mathcal{N}_i^T(t) \doteq$ the number of residents that can evacuate from the bounded risk set i under T-InFO.

Theorem 3.2. *T-InFO maximizes the number of residents that can evacuate from any bounded risk set at any time, i.e., $\mathcal{N}_i^T(t) = \mathcal{N}_{i,j}^U(t)$, for all t and a select j.*

Proof. Consider a bounded risk set i and a time t when $P_i(t) > 0$. Locate the most downstream risk-level set internal to the set that still has remaining residents, i.e., find the minimum $j \geq i$ that satisfies: $\langle p \rangle_j(t) > 0$. By our selection of j, the part of the set downstream of j is empty, so the number evacuated from this downstream part is obviously P_{ij}^-. The number evacuated from the upstream part of the set is now calculated. Our selection of j implies that $\langle p \rangle_j(t) > 0$. As $\langle p \rangle_j(t')$ is a non-increasing function in t', $\langle p \rangle_j(t) > 0 \Rightarrow \langle p \rangle_j(t') > 0, \forall t' \in [0, t]$. By condition (3.3), $\langle p \rangle_j(t') > 0 \Rightarrow p(j, s, t') > 0$, and by Lemma 3.1, $p(j, s, t') > 0 \Rightarrow q(j, s, t') = \tilde{c}(j, s)$, $\forall s$ and $t' \in [0, t]$. Since $\sum_{l \in \mathcal{L}_j} q(l, t) =$

$\langle q \rangle_j(t)$ and $\sum_{l \in \mathcal{L}_j} \tilde{c}(l) = \langle \tilde{c} \rangle_j$, the previous means that $\langle q \rangle_j(t') = \langle \tilde{c} \rangle_j$. Hence, the number of residents evacuated from the upstream part by time t is $\langle \tilde{c} \rangle_j \cdot t$. The combined total is: $\mathcal{N}_i^T(t) = \langle \tilde{c} \rangle_j \cdot t + P_{ij}^-$. Note that this equals or exceeds the left-hand-side of the inequality in (3.4). But an upper bound cannot be exceeded, so $\mathcal{N}_i^T(t) = \mathcal{N}_{i,j}^U(t)$ for the selected j and whenever $P_i(t) > 0$.

When $P_i(t) = 0$, T-InFO would have evacuated everyone from the set, i.e., $\mathcal{N}_i^T(t) = P_i(0) = P_{i,i}^+ + P_{i,i}^-$. Again, $\mathcal{N}_i^T(t) \geq \mathcal{N}_{i,j}^U(t)$. Therefore, $\mathcal{N}_i^T(t) = \mathcal{N}_{i,j}^U(t)$ for the selected j for all times. $\qquad \square$

Theorem 3.2 implies that $\mathcal{N}_{i,j}^U(t)$ is a least upper bound and that no other strategy can achieve better results than T-InFO.

Corollary 3.3. *T-InFO minimizes the evacuation time of every bounded risk set.*

Proof (by contradiction). Consider a strategy X that can finish evacuating some bounded risk set at a time t^X, sooner than T-InFO's finishing time, t^T. Then this strategy would have evacuated everyone from the set by $t^X \leq t^T$, when T-InFO had not. That is, X would have evacuated more people than T-InFO by this time. But this contradicts Theorem 3.2. $\qquad \square$

Like InFO for the freeways, T-InFO is optimal in two ways: (i) it maximizes the number of evacuees from any bounded risk set at all times, and (ii) it minimizes the evacuation time of every bounded risk set. When the bounded risk set is the complete system, T-InFO is optimal for the system — the complete evacuation tree. Therefore, by the definition presented in Section 2.3, T-InFO is a comprehensive strategy at the network level.

In addition to being optimal, T-InFO has many advantages. Note that to implement (3.5) and (3.6), two pieces of data are needed: the t-capacities, $\tilde{c}(i,s)$, and the arriving traffic flows, $q(i+1,s,t)$. The $\tilde{c}(i,s)$ can be pre-determined using the readily known system capacity function $c(i,s)$. The time-dependent values $q(i+1,s,t)$ can be obtained in real-time from traffic counters/sensors which are widely deployed in cities. Hence, the strategy does not require any predictive data, and is adaptive to changing conditions on the road. The above — the need for only the $\tilde{c}(i,s)$ and the $q(i+1,s,t)$ — further shows that T-InFO has minimal data requirement.

Note that the controller at each on-ramp and at every major diverge can operate T-InFO independently, without coordination. Hence, T-InFO is decentralized. Finally, like InFO, T-InFO is socially acceptable as it gives priority to upstream, most-at-risk residents. These benefits presented here make T-InFO a realistic strategy — it stands a good chance of being implemented in an evacuation.

3.6 Extensions of T-InFO

3.6.1 Time-dependent Capacity

T-InFO is shown here to remain optimal even when the system's capacity changes. The result means that the strategy is robust to the occurrence of unexpected events such

as traffic incidents. Also, this result accommodates the relaxation of the second assumption in Section 3.2.2.

When incidents arise, the capacities of different sections of the system will change. Let $c(i,s)$ now be modified such that it can vary with time. The time-dependent capacity function is denoted as $c(i,s,t)$. Consequently, the t-capacity function is: $\tilde{c}(i,s,t) = \min(\, c(i,s,t)\,,\ \sum_{e \in \mathcal{E}_{i,s}} \tilde{c}(e,t)\,)$. Of course, for $i = 1$, $\tilde{c}(i,s,t) = c(i,s,t)$. Note that (3.5) and (3.6) will now be adjusted by replacing $c(i,s)$ with $c(i,s,t)$. Now let $\langle \tilde{c} \rangle_i(t) = \sum_{l \in \mathcal{L}_i} \tilde{c}(l,t)$ be the new risk-level t-capacity for i.

Theorem 3.4. *With time-dependent capacities, T-InFO continues to maximize the number of evacuees from every bounded risk set at all times, and minimize the time to evacuate every set.*

Proof. Clearly, the proof of Lemma 3.1 continues to hold if $\tilde{c}(i,s)$ is replaced by $\tilde{c}(i,s,t)$. Now note that an upper bound for the number of evacuees from bounded risk set j is $\tilde{C}_j(t) = \int_0^t \langle \tilde{c} \rangle_j(x)dx$, instead of $\langle \tilde{c} \rangle_j \cdot t$. It is now easy to see that all the steps in the logic leading to (3.4) and in the proof of Theorem 3.2 continue to be true if $\langle \tilde{c} \rangle_j \cdot t$ is replaced by $\tilde{C}_j(t)$. Thus, Theorem 3.2 holds. Again, if a strategy maximizes the number of evacuees from a bounded risk set, it too must minimize the evacuation time of the set; i.e., Corollary 3.3 holds. □

In practice, very little information about the incident is needed. The information should contain two things: the incident's severity and its location. The former can indicate how many lanes may be closed and help determine the reduction in capacity. The latter can help identify the parts of the network that are affected by the incident, i.e., parts upstream of the incident location that are used by routes passing through the location. Once the location is known, controllers at the "affected parts" of the network will be notified of the incident's severity (and thus, the estimated reduction in t-capacity downstream).

Now, when the second assumption of Section 3.2.2 is violated, there may be queues in the system at the start of the evacuation. But this is just another case of time-dependent capacity: the link capacity(ies) can be simply set equal to zero wherever there are queues. If this is done until queues dissipate in the system, then it can be seen from the above that T-InFO remains optimal.

3.6.2 Driver Adaptation

T-InFO is found here to be robust to a form of driver adaptation that is likely to happen when the disaster is not imminent; i.e., if people try to minimize their evacuation time by moving to an upstream, higher priority on-ramp while T-InFO is implemented. People may do this if they know that they have sufficient time to evacuate. It is shown below that under T-InFO, with this form of behavior, an equilibrium arises and preserves the evacuation time of the system.

Definitions. All risk levels that share the same minimum cut are now grouped. These groups are then numbered and labeled in the upstream direction from 1 to K, where $K \doteq$ the total number of groups/minimum cuts in the system. The minimum cut shared by the

risk-level sets in group k is now known as the "t-bottleneck for group k". Its corresponding t-capacity is labeled $\{\tilde{c}\}_k$. Note that $\{\tilde{c}\}_k > \{\tilde{c}\}_{k+1}$. Note also that under this redefinition of the problem, each group always has some available t-capacity for use to discharge their residents.

Now, use $\mathcal{G}_k$ to define the set of all risk-level sets that make up group k. Let $t^T \doteq$ the original, non-adaptive evacuation time of the complete system under T-InFO. Then define k^* as the most downstream group in the system to finish the evacuation at this time. Furthermore, let $\{p\}_k = \sum_{i \in \mathcal{G}_k} \langle p \rangle_i(0)$ be the total original population in group k. Superscript "A" is now used to denote the result after adaptation.

Upstream adaptation condition. The assumptions are: ramp switching takes no time, and all switches occur at the start of the evacuation (therefore no ramp is left empty intermittently). Then after the adaptation, the resulting risk-level populations $\langle p \rangle_i^A$ are said to be a user equilibrium under T-InFO if remaining residents at the end of each queue cannot reduce their evacuation time by switching to a ramp with equal or higher risk.

If upstream adaptation takes place when T-InFO is implemented, only the ramps in the most upstream risk-level set of a group will be used for evacuating residents of the group. (This should be clear: when T-InFO is implemented, a non-empty ramp that is not in the most upstream position in a group would finish after upstream ramps in the group due to upstream priority. Therefore, it could not be in equilibrium.) In view of this, each group is now assumed without loss of generality to have only one risk-level set. Only groups are mentioned from now on.

Lemma 3.5. *If group k^* acts as a barrier such that downstream residents do not move into groups $k^* \ldots K$ in their adaptation, then an equilibrium population distribution for group $k \geq k^*$ is: $\{p\}_k^A = t^T \cdot (\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1})$. In such an equilibrium, groups $k^* \ldots K$ would finish evacuating concurrently, at time t^T.*

Proof. Note that under T-InFO, all unfinished groups would discharge simultaneously. Also, an unfinished group k would discharge its residents at a rate of $\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1}$ whenever group $k+1$ is discharging. Therefore, after adaptation, if a group k $(k \geq k^*)$ is the first to finish evacuating among groups $k^* \ldots K$, then it must have finished at some time $t^{\min} = \{p\}_k^A / (\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1})$ under T-InFO. This is true since group $k+1$, which has been discharging since $t = 0$, has not finished at $t^{\min}$. When the equilibrium condition is substituted into the numerator of $t^{\min}$, the result is: $t^{\min} = t^T$. $\square$

Lemma 3.6. *When the equilibrium of Lemma 3.5 is met, the equilibrium population in the bounded risk set comprising groups $k \ldots K$, $\forall k \geq k^*$, is no less than the original population in the set, i.e.: $\sum_{j=k}^{K} \{p\}_j^A \geq \sum_{j=k}^{K} \{p\}_j$.*

Proof (by contradiction). Recall that a group $k \geq k^*$ in Lemma 3.5 would finish evacuating at time t^T. Also, note that T-InFO would instruct ramps to send a total flow of $\{\tilde{c}\}_k$ from the bounded risk set comprising $k \ldots K$ whenever group k has not finished evacuating. Therefore, $\forall k \geq k^*$, $\sum_{j=k}^{K} \{p\}_j^A = \{\tilde{c}\}_k \cdot t^T$, where the right-hand-side is an upper bound for the number that can evacuate from the bounded risk set in time t^T. Now, if the lemma is false, there would be a $k \geq k^*$ such that $\sum_{j=k}^{K} \{p\}_j > \sum_{j=k}^{K} \{p\}_j^A$. But this would mean that the upper bound is violated, which is impossible. $\square$

Lemma 3.6 guarantees that the equilibrium of Lemma 3.5 can be achieved. The final theorem extends these results to the complete system.

Theorem 3.7. *Under T-InFO, there exists a user equilibrium with $\sum_{j=k}^{K}\{p\}_j^A \geq \sum_{j=k}^{K}\{p\}_j$, $\forall k$, that does not change the evacuation time of the complete system.*

Proof. If $k^* = 1$ the result is obvious. Otherwise, divide the problem into an upstream part that comprises groups $k^* \ldots K$, and a downstream part comprising groups $1 \ldots k^* - 1$. If the equilibrium in Lemma 3.5 exists, people downstream do not backtrack past k^*. Also, the upstream part finishes evacuating in time t^T, after the finishing time of any group in the downstream part. During the discharge of the upstream part, T-InFO keeps the t-bottleneck for group k^* saturated. Therefore, all t-capacities in the downstream part are effectively reduced by $\{\tilde{c}\}_{k^*}$. However, note that the populations in the downstream part are conserved.

Now, let the downstream part become a reduced version of the original problem (variables of the reduced problem will be denoted with a prime), with the t-capacity for group k, $\forall k < k^*$, being transformed as follows: $\{\tilde{c}\}_{k'} = \{\tilde{c}\}_k - \{\tilde{c}\}_{k^*}$. Note that this reduced problem is of the same type as the one in Lemmas 3.5 and 3.6. Therefore, the same conclusions can be drawn: groups $k'^* \ldots k^* - 1$ will finish evacuating in time $t'^T < t^T$. Also, $\sum_{j=k}^{K'}\{p\}_j^A \geq \sum_{j=k}^{K'}\{p\}_j$, implying that $\sum_{j=k}^{k^*-1}\{p\}_j^A \geq \sum_{j=k}^{k^*-1}\{p\}_j$, $\forall k \in [k'^*, k^* - 1]$. Combined with Lemma 3.6, the result is $\sum_{j=k}^{K}\{p\}_j^A \geq \sum_{j=k}^{K}\{p\}_j$, $\forall k \geq k'^*$. Again, if $k'^* = 1$, the complete problem is solved. Otherwise, repeat these steps until the last group to discharge is group 1. □

Hence, people's risk-increasing movements do not at all affect the performance of T-InFO. This result of robustness implies that T-InFO remains the strategy of choice even when people are believed likely to be moving around outside of the tree.

3.6.3 Strategy Insights

Results similar to those in Section 2.8 are presented here. In particular, it is shown that some relaxed versions of T-InFO also optimize the evacuation process in a tree.

Let risk-level sets sharing the same t-bottleneck be grouped and numbered from 1 to K, as in the previous section. So $\{\tilde{c}\}_k$ is the t-capacity for group $k \leq K$. Under T-InFO, a group is always discharging a flow equal to t-capacity as long as it has remaining residents. So, if group $k + 1$ has residents, it discharges at $\{\tilde{c}\}_{k+1}$, and the residual t-capacity for the group immediately downstream (i.e., group k) is $\{\tilde{c}\}_k - \{\tilde{c}\}_{k+1}$. Let $\{r\}_k(t) \doteq$ the aggregate ramp discharge flow into the links in group k at time t; $\{q\}_k(t) \doteq$ the flow leaving group k (and therefore passing the t-bottleneck for group k) at t; and $\{p\}_k(t) \doteq$ the aggregate number of remaining residents within group k at t. Then a relaxed version of (3.5) is:

$$\{r\}_k(t) = \begin{cases} 0 \,, & \text{if } \{p\}_k(t) = 0 \,; \\ \{\tilde{c}\}_k - \{q\}_{k+1}(t) \,, & \text{if } \{p\}_k(t) > 0 \text{ and } k \in [1, K-1]; \\ \{\tilde{c}\}_k \,, & \text{if } \{p\}_k(t) > 0 \text{ and } k = K \,. \end{cases} \tag{3.7}$$

Since $\{\tilde{c}\}_k > \{\tilde{c}\}_{k+1}$, $\forall k$, any amount of flow released from group k (i.e., $\{q\}_k(t)$) will not cause congestion downstream, since groups $1 \ldots k-1$ will always be instructed to release only the residual t-capacity. Note that the above formula shows that order does not matter among risk-level sets within the same group. Therefore, as long as the corresponding t-bottleneck is saturated whenever there are remaining residents in the group, the strategy remains optimal. (This can be shown as follows. If $p(i,s,t)$, $q(i,s,t)$, and $\tilde{c}(i,s)$ are replaced by $\{p\}_k(t)$, $\{q\}_k(t)$, and $\{\tilde{c}\}_k$, respectively, then Lemma 3.1 would continue to be true. Also, if the risk-level sets considered in the proof of Theorem 3.2 are replaced by groups, then the theorem can be shown to be true. Consequently, Corollary 3.3. is true.)

Therefore, risk-level sets in the same group can be re-ordered in any manner. As long as their combined discharge saturates the corresponding t-bottleneck, T-InFO can be implemented to optimize the evacuation. This knowledge allows emergency officials some flexibility in their management approach. They can prioritize on-ramps which may be downstream of other unfinished ramps in the same group, perhaps to serve special vehicles.

Chapter 4

Concluding Remarks

Traffic needs to be managed in an evacuation. If this is not done for a freeway, an internal bottleneck can easily reduce outflows from the system to well below evacuation capacity. In a network, besides being constrained by internal bottlenecks, traffic can self-distribute unevenly. As a result, some routes can get heavily congested, while others are under-utilized.

The strategies proposed in this dissertation can help streamline evacuation traffic to avoid these kinds of problems. For a freeway, it turns out that the InFO upstream priority scheme can completely eliminate any reduction in evacuation capacity even when internal bottlenecks arise. The scheme can be implemented using input control such as metering lights at the on-ramps. Of course, in an emergency evacuation in which driver compliance cannot be guaranteed, it is recommended that traffic cops be staged at the on-ramps to implement InFO.

For more general networks shaped like a tree, the T-InFO input-and-routing control strategy helps resolve both, the internal bottleneck problem and the uneven traffic distribution problem. The solution takes advantage of an adaptive behavior commonly exhibited by drivers in reality: an attempt to minimize evacuation time by using the on-ramp with the shortest queue. For implementation, the recommendation is again the staging of traffic personnel at the on-ramps and at major diverges.

Other benefits help make InFO and T-InFO realistic for emergency management. They can be summarized briefly. InFO and T-InFO are comprehensive strategies for evacuation management, always maximizing evacuation flows and minimizing evacuation time. The strategies are also adaptive to real-time traffic including driver route-changing behaviors, and robust to capacity changes caused by incidents and initial queues in the system. Finally, both strategies bypass the need for central coordination during operation: independent instructions can be sent to a controller at an on-ramp or major diverge to implement InFO or T-InFO. Also, InFO and T-InFO are socially acceptable because they always allow most-at-risk residents to evacuate first. Therefore they can be beneficial even when there is no internal d-bottleneck or t-bottleneck. These benefits mean that the strategies are useful for evacuation management.

Therefore, the results here help fill a void in current evacuation literature, which has so far only proposed strategies requiring demand predictions or unrealistic controls.

Furthermore, the results present building blocks for future research: the management of more general networks, ones with multi-centric risks, include merges, and/or lead to multiple safety destinations. The insights learned about traffic priority and distribution will no doubt be of significance.

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Appendix A

Homogeneous Freeway Analysis: Transients

A.1 Initial Transient

Figure A.1 shows a detailed time-space diagram of the downstream end of the homogeneous freeway at the beginning of the evacuation. The case $c/d = 3$ is depicted. Traffic states indicated on this figure correspond to those labeled on the fundamental diagram presented in Figure 2.2. The front of the queue is shown by a thick line. The initial transient, t_0, is described in the next paragraph.

Since $d < c$, all ramps initially discharge evacuees at rate d onto the empty freeway. An observer stationed at ramp i would only see discharging flows (from ramp i) and no passing flows for the first l/u time units. Then evacuees from ramp $i + 1$ arrive and pass the observer at rate d (meanwhile ramp i continues to discharge at d). This persists until $t = 2l/u$ when evacuees from ramp $i + 2$ also arrive; hence, a total flow of $2d$ now passes the observer as ramp i continues to discharge at d. Since the total flow now (sent downstream into link i) is $3d = c$, the freeway is saturated. The arrival of traffic from ramp $i + 3$ at $t = 3l/u$ sets off congestion on the freeway; the first shock wave propagates upstream from ramp i. From then on, drivers from the ramps and the freeway would take turns merging into the downstream freeway. So, $t_0 = 3 \cdot l/u$ in this example. In the general case, $t_0 \approx (c/d)(l/u)$.

The figure also shows briefly what happens beyond the initial transient. At $t = t_0$, the front of the queue emerges at ramp 1, and the freeway begins to discharge at capacity for the next T^{mid} time units. The front of the queue remains at ramp 1 for t_1 time units until the ramp finishes discharging. When ramp 1 empties, a shock wave is generated and travels backward to ramp 2 which now becomes the front of the queue. Ramp 2 then spends t_2 time units discharging as the front of the queue. When ramp 2 empties, another shock propagates upstream to ramp 3, and so on. This persists until the final transient of the evacuation.
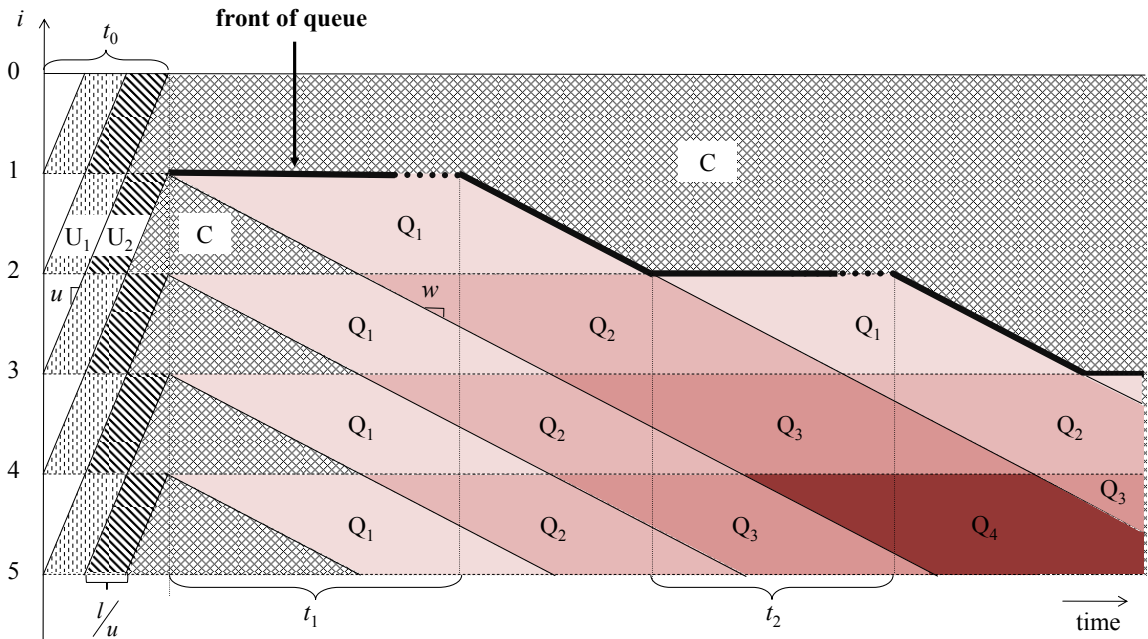
Figure A.1: Downstream Freeway at Initial Transient, $c/d = 3$ and $\alpha < 1$

A.2 Final Transient

The upstream end of the freeway during the final moments of the evacuation is depicted in Figure A.2. The thick solid line once again shows the front of the queue, while the thick dotted line shows the back of the queue. In this example, the back of the queue begins receding forward at $t = t_{Br}$ and meets the front of the queue at $t = t_M$.

Note that ramp I always emits more flow than $I-1$ if $\alpha < 1/2$. This is true because there is no traffic upstream of ramp I competing for capacity on link I. Hence, for $t \leq t_{Br}$, if ramp I discharges at rate q_I, then ramp $I-1$ would discharge at $q_{I-1} = \alpha \, q_I/(1-\alpha) < q_I$. For $t > t_{Br}$, $q_I = d$ and $q_{I-1} = \alpha(1 - \alpha)c < q_I$ (since it is assumed that $\alpha c \leq d$, so $\alpha(1 - \alpha)c \leq d$) in this example. Thus, I empties before $I-1$, and the back of the queue moves forward, slowly, if $\alpha < 1/2$.

As discussed, the final transient, t_f, marks the return of the under-capacity state U, and consists of a few ($\leq c/d$) unfinished ramps discharging simultaneously at rate d. Thus, it cannot last longer than p/d. As shown in Figure A.2, the last ramp to finish is $I^* = I - 1$, not I, since the back of the queue recedes forward. However, I offers a good approximation as $I \to \infty$ since the number of ramps that are unfinished at $t = t_{Br}$ is negligible relative to I. Therefore, ramps can be assumed to generally finish in order, from downstream to upstream.
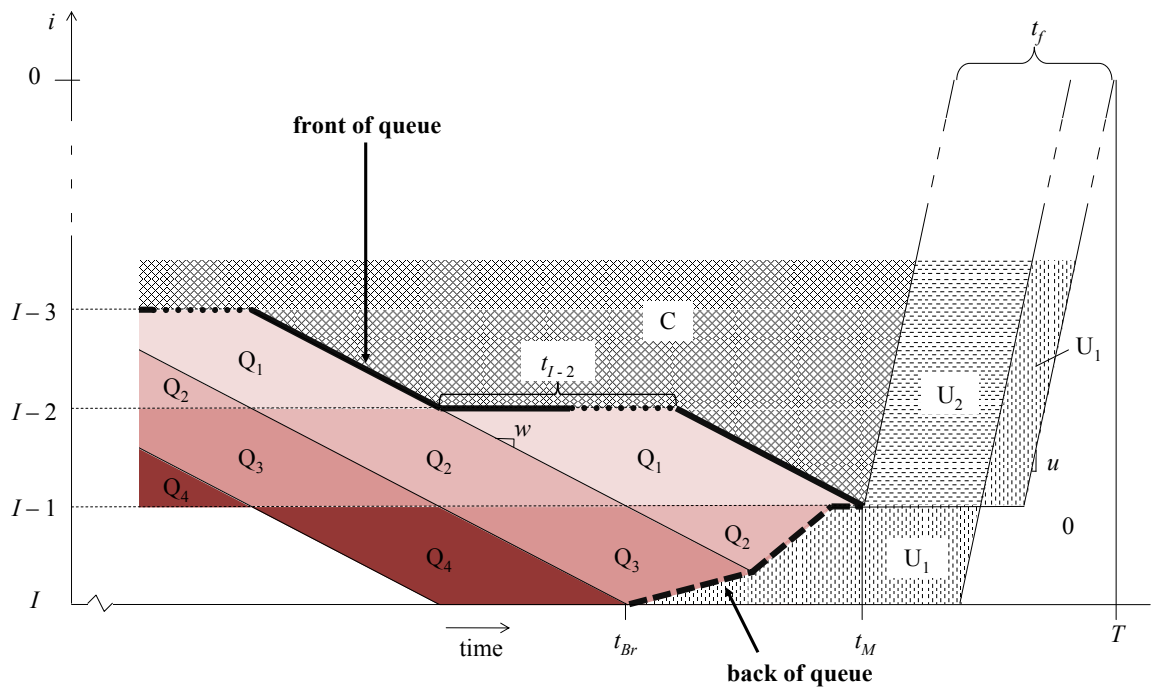
Figure A.2: Upstream Freeway at Final Transient, $c/d = 3$ and $\alpha < 1$

Appendix B

Decomposition of a Tree

This appendix analyzes another special case evacuation tree: when people have preferred evacuation routes and would not accept management's routing instructions. In this case, traffic splits at all the diverges are fixed. The results here show that the tree can be solved as a collection of freeway problems, and the InFO strategy thus apply.

Definitions for a generic evacuation tree like the one shown in Figure B.1a are now defined. Let the safety destination nodes/routes be numbered and labeled from 1 to S and indexed with the variable s, such that a route s leads to safety node s. The links on a route s are numbered in the upstream direction from 1 to I_s. Then, link (i, s) denotes the ith link on route s (so $i \leq I_s$). Nodes have the same labels as their immediate upstream links, and the threat node has the label $(I_s + 1, s)$, $\forall s$. Furthermore, $\beta(i, s)$ is the proportion of flow that enters link (i, s) from node $(i + 1, s)$. Finally, let $q(i, s)$ be the flow leaving link (i, s) (and therefore entering node (i, s)) at any time.

Theorem B.1. *The proportion of flow on link (i, s) destined for safety node s is $\prod_{j=1}^{i-1} \beta(j, s)$. The proportion of that link flow that is destined for the other safety node(s) is $1 - \prod_{j=1}^{i-1} \beta(j, s)$.*

Proof (by induction). The result is obviously true for $i = 1$ since link $(1, s)$ is at the exit. Assuming that the result for $i = i - 1$ is true, i.e., $\prod_{j=1}^{i-2} \beta(j, s)$ of the flow on link $(i - 1, s)$ is destined for safety node s. Now, note that $\beta(i - 1, s)$ of the flow from node (i, s) enters link $(i - 1, s)$. Therefore, $\beta(i - 1, s) \cdot \prod_{j=1}^{i-2} \beta(j, s) = \prod_{j=1}^{i-1} \beta(j, s)$ of the flow from link (i, s) is destined for s. Hence, the result is true for $i = 1$. Finally, note that $1 - \prod_{j=1}^{i-1} \beta(j, s)$ is the remaining proportion of flow from link (i, s). It must be routed to the other safety nodes due to flow conservation. □

From now on, $\prod_{j=1}^{i-1} \beta(j, s)$ is called the "cumulative route split" from link (i, s) to s. Thus, if a link (i, s) is used by n routes, there would be n cumulative route splits from the link. This implies that exclusive traffic flows exist on each link, i.e.: the cumulative route split $\prod_{j=1}^{i-1} \beta(j, s)$ applied to the link flow $q(i, s)$ results in a flow that exclusively serves traffic destined for s.

Now, for a link that is used by n routes (and therefore has n exclusive flows), "horizontally slice" it into n sub-links, such that each sub-link has the capacity to serve one
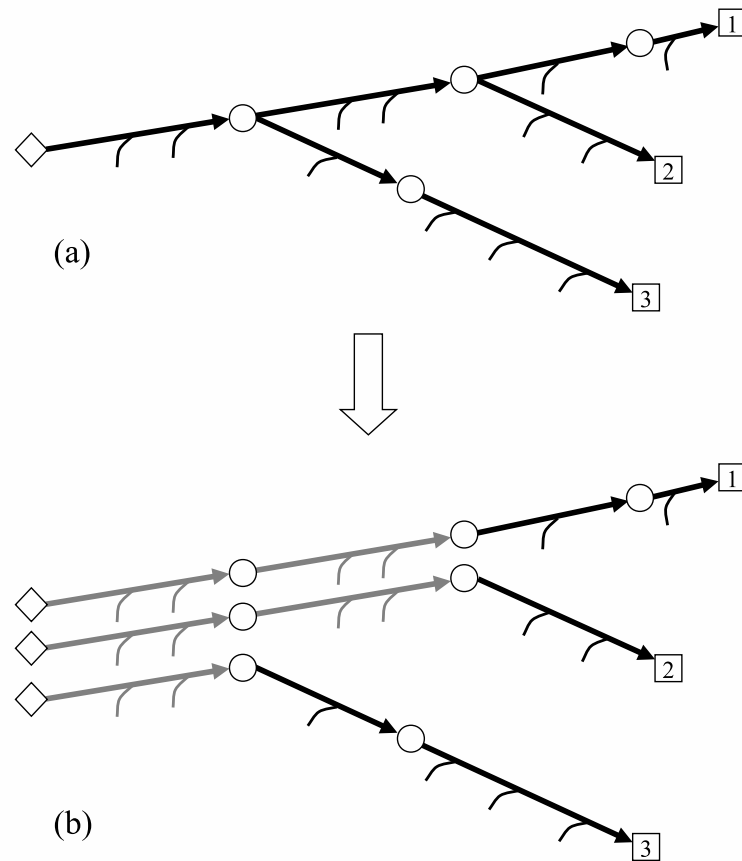
Figure B.1: Decomposition from (a) An Evacuation Tree to (b) Virtual Freeways

of the n exclusive traffic flows. Along a route, the (exclusive) sub-links that serve traffic aiming for the same safety node are then connected. Using this procedure, the original evacuation tree with n routes can be imagined to decompose into a collection of n "virtual freeways". This idealization is shown in Figure B.1b.

The above discussion implies that the original network can be solved as a collection of freeway problems. Therefore, InFO is optimal. In practice, traffic controllers would simply implement InFO at the ramps along a link. In this case, however, the flow to release from each on-ramp should be limited to the aggregate d-capacity. This aggregate is found for each link by summing the link's various d-capacities corresponding to the different routes.