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The optical model of the nucleus, which has proved useful in the study of coherent scattering processes, is here applied to the inelastic scattering of pi mesons by nuclei. Various aspects of meson and nucleon kinematics as modified by the presence of the optical model potential are discussed. Formulae and numerical results which relate free pion-nucleon cross sections to cross sections for pion-nucleon collisions within nuclei are also given. They indicate that the coherent effects described by the optical model play an important role in the inelastic scattering of mesons.

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The interactions in which pi mesons are scattered by atomic nuclei are qualitatively of two kinds - those which leave the nuclear state unaltered (elastic or coherent) and those which excite the nucleus, or are inelastic. The elastic scattering results from interference of coherent waves scattered from all parts of the nucleus, and reflects the properties of the nuclear medium as a whole. These properties are expressed by means of a potential well, conventionally termed the "optical model potential",[1] which serves to describe how a pion propagates through a nucleus without loss of energy.

In addition to these elastic processes, the scattering of high energy pions is often accompanied by energy transfers large enough to excite one or more nucleons above the nuclear Fermi energy. Such inelastic events are commonly attributed to a sequence of pion collisions with single nucleons within the nucleus, with interaction strengths given by the known free pion-nucleon cross sections. If coherent effects are ignorable, then inelastic cross sections may be obtained by the "Monte Carlo" technique which computes the classical trajectories of a beam of pions that scatter incoherently from an assembly of nucleon targets.[2,3] In this scheme, the exclusion principle forbids collisions which place the recoiling nucleon in a momentum state already occupied by a similar nucleon. The consequence is to limit the number of inelastic collisions a pion may suffer; a pion with, say, 100 Mev kinetic energy is seldom scattered more than once or twice in its journey through a heavy nucleus. There is, of course, no limitation of this kind on the elastic scattering processes.

We wish to suggest that the coherent processes in the nuclear medium, as evidenced by the elastic cross sections and characterised by the optical model potential, may affect significantly the calculation of inelastic cross sections by the Monte Carlo method. Not only is the effective kinetic energy at which pion-nucleon collisions occur displaced, but the dispersive nature of the medium (the variation of the potential with pion energy) alters the magnitudes of pion flux and the density of states. Expressed differently, the optical model potential may give the meson an "effective mass" which varies with energy and is quite different from its actual mass. This viewpoint is, of course, familiar in the study of transport phenomena involving electrons in solids.

In the present work we shall pursue the implications of this point. That is, we shall use the optical model potential, as inferred from dispersion relations, to determine the modified kinematics of meson propagation in nuclei. The results will be applied to a discussion of pion scattering by neutrons and protons within nuclei and some comparisons with experiment will be given. We find that the dispersion in the nuclear medium affects markedly the energy spectrum of mesons emitted at a given angle, as well as the magnitude and angular distribution of cross sections for incident pion energies up to a few hundred Mev.

Although calculated results are presented for this entire energy range, we must emphasize that near the "resonance" energy for pion nucleon scattering, the very strong interactions limit the accuracy of the approximations upon which our simple model is based. In this region, the elementary theory described here must be regarded as qualitative at best. Nevertheless, we expect that in the resonance region as elsewhere, no theory which aims at quantitative agreement with experiments on nuclear scattering of mesons can afford to neglect the role of coherent effects.

2. The Optical Model

From a formal point of view, the optical model treats the elastic scattering of pions by nuclei exactly. We contemplate, in the present paper, pion energies quite small compared to nuclear rest energies. Hence we neglect at the outset the recoil of the nucleus in an elastic interaction. Then such interactions are described rigorously if we choose a suitable potential V and write a Sherödinger equation for the meson wave function $\psi(\underset{\sim}{r})$:

$$\omega \psi(\underset{\sim}{r}) + \int \langle \underset{\sim}{r} | V | \underset{\sim}{r}' \rangle \, d^3\underset{\sim}{r}' \, \psi(\underset{\sim}{r}') = E \psi(\underset{\sim}{r}). \tag{1}$$

The energy ω is $(p^2 + \mu)^{1/2}$ where p is the momentum operator for the pion and μ its rest mass. The potential is expected to be non-local and, in consequence of absorbtive processes, non-Hermitian.

From a practical standpoint, it is desirable that the form of the potential be simple and, moreover, that it be related to the dynamics of pion-nucleon interactions. For these reasons, the potential is written in an approximate form[4]

$$\langle \underset{\sim}{r} | V | \underset{\sim}{r}' \rangle = \rho\left(\frac{\underset{\sim}{r} + \underset{\sim}{r}'}{2}\right) W(\underset{\sim}{r} - \underset{\sim}{r}'), \tag{2}$$

where $\rho(\underset{\sim}{r})$ is the nuclear density. As a further simplification, $\rho(\underset{\sim}{r})$ is normalized to unity when $\underset{\sim}{r}$ lies within the nucleus and is zero otherwise.

The "well depth" V_o of the optical model potential is the Fourier transform of W:

$$V_o(K) = \int e^{-i \underset{\sim}{K} \cdot \underset{\sim}{r}} \, W(\underset{\sim}{r}) \, d^3\underset{\sim}{r}. \tag{3}$$

Thus, when the potential acts upon a plane wave exp $(i\underset{\sim}{K}\cdot\underset{\sim}{r})$, we have, if the nuclear radius is large compared to K^{-1}:

$$V e^{i\underset{\sim}{K}\cdot\underset{\sim}{r}} \approx \rho(\underset{\sim}{r})\, V_0(K)\, e^{i\underset{\sim}{K}\cdot\underset{\sim}{r}} . \tag{4}$$

Approximate solutions to (1) may then be sought in which an incident plane wave is coupled to a plane wave within the nucleus:

$$\psi(\underset{\sim}{r}) \sim e^{i\underset{\sim}{q_0}\cdot\underset{\sim}{r}}, \quad \underset{\sim}{r} \text{ outside the nucleus}$$

$$\psi(\underset{\sim}{r}) \sim e^{i\underset{\sim}{K}\cdot\underset{\sim}{r}} \quad \underset{\sim}{r} \text{ inside the nucleus.} \tag{5}$$

The energy-momentum relations now have the form

$$E = (q_0^2 + \mu^2)^{1/2} = \epsilon_0 + \mu \tag{6}$$

and

$$E = (K^2 + \mu^2)^{1/2} + V_0(K) = \epsilon(K) + \mu . \tag{7}$$

Taken together, Eq. (6) and (7) determine the momentum change suffered by a pion upon entering a nucleus. The well depth V_0 may be regarded, equivalently, as a function of the incident pion kinetic energy $\epsilon_0 = E - \mu$, or of q_0, or of K. It is, in general, complex; the real and imaginary parts of V_0 are specified by the notation

$$V_0 = V_R - i\, V_I . \tag{8}$$

On the basis of the Serber model,[2] one can relate V_o to the pion-nucleon interaction. Indeed, to a reasonable approximation, V_o is proportional to f(K), the amplitude for forward scattering of pions by nucleons.[4] Further, $V_o(K)$ is an analytic function of the complex variable $(K^2 + \mu^2)^{1/2}$ in the upper half plane, and may be explicitly calculated from the dispersion relations of Goldberger.[5] The quantities V_R and V_I so calculated are shown (with the inclusion of a correction for true absorbtion of pions[3,4] in Fig (1). For the purposes of the present paper, we accept as correct the optical model potential as given by Fig. (1).

It is clear from (7) and (8) that the pion momentum K within the nucleus is complex. Setting $K = k + i\eta$, we see that η and V_I have the same sign and, from Fig. 1, both are positive. The plane wave exp $(i K.r)$ is exponentially damped as the pion propagates through the nucleus. The quantity $(2\eta)^{-1}$ is then identified as the mean free path λ_t for the absorbtive effects characterized by V_I. In Fig. (2) the quantity η is given as a function of the pion momentum k within the nucleus. This is taken from Table I of reference (4). In Figs. (1) and (2) we have increased the quantities V_R and V_I of reference (4) by a factor of 1.6. This corresponds to changing the nuclear radius parameter r_o from 1.4 to 1.2, which now seems a better value.

3. Pion Kinematics in Nuclei

The calculation of a cross section entails, first of all, a knowledge of the velocity of the particles which make up the incident beam. In a quantum treatment, one ordinarily constructs wave packets of fairly well defined energy and makes use of the mean group velocity $dE(K)/dK$ of the waves. As is well known, the group velocity coincides with the classically defined particle velocity when $E(K)$ denotes the energy of a free particle. We must now consider in what sense this procedure is applicable to the pion-nucleon collisions which occur in a nuclear medium.

In order to use the model discussed in the previous section, we must require that the breadth of a meson packet be small compared to a nuclear diameter. Otherwise, one cannot specify whether the meson is truly inside or outside of the nucleus during its time of interaction. A further limitation is that the packet breadth not exceed a mean free path for absorbtion, for it is difficult to speak of the propogation of a particle when it is already half absorbed, initially. On the other hand, the packet must maintain its localization at least until it has an opportunity for a collision. Consequently, the spread of a packet in the time needed to cover a mean free path length must be small compared to the breadth itself.

We consider, for simplicity, a one dimensional packet $\psi(x,t)$, wholly within the nucleus, which at $t = 0$ is represented as

$$\psi(x,0) = \int_{-\infty}^{\infty} a(K-Q)\, e^{iQx}\, dQ \tag{9}$$

For illustrative purposes, we select a Gaussian packeting function

$$a(K-Q) = (\Delta p\sqrt{2\pi})^{-1} \exp\left\{-\frac{(K-Q)^2}{2(\Delta p)^2}\right\}, \tag{10}$$

so that

$$\psi(x,0) = e^{iKx}\, e^{-\frac{x^2}{2(\Delta x)^2}} \tag{11}$$

with $\Delta x = (\Delta p)^{-1}$.

To obtain the time dependence of ψ, we first deform the path of integration into the upper half plane so that the variable Q passes through all values $Q_1 + iQ_2$ which, by Eq. 7 appear as momenta corresponding to a real energy. The wave function is now represented as a superposition of eigen-waves in the nucleus. Consequently,

$$\psi(x,t) = \int a(K-Q)\, e^{i\,(Qx - E(Q)t)}\, dQ. \tag{12}$$

We approximate E(Q) within the region of integration by the initial terms of a Taylor series about K :

$$E(Q) = E(K) + (Q-K)\frac{dE}{dK} + \frac{1}{2}(Q-K)^2\frac{d^2E}{dK^2}. \tag{13}$$

The integral remains of gaussian type so that

$$\psi(x,t) = e^{i[Kx - E(K)t]} \exp\left\{-\frac{\left(x - \frac{dE}{dK}t\right)^2}{2\alpha}\right\} \tag{14}$$

with

$$\alpha = (\Delta p)^{-2} + it\frac{d^2E}{dK^2}.$$

Equation (14) suggests that dE/dK be interpreted as the velocity of the packet, but this is only possible when dE/dK is real. If not, we separate the derivative into real and imaginary parts, and define a "group velocity" v_G by

$$\frac{dE}{dK} = v_G + iv_G'. \tag{15}$$

Thus, v_G represents an approximate propogation velocity of the packet if we may suppose v_G' small and if, further, the packet does not spread so rapidly as to lose its meaning. Even if $dE/dK \approx v_G$ and $d^2E/dK^2 \approx v_G'$ were real, the width Δp in momentum space must still be narrow enough to permit the expansion (13).

The latter leads to

$$|\psi(x,t)|^2 = exp\left\{ -\frac{(x-v_G t)^2}{(\Delta_t x)^2} \right\}, \tag{16}$$

where the spread $\Delta_t x$ at time t is given by

$$(\Delta_t x)^2 = \left[(\Delta x)^2 + \frac{d v_G}{d k} \frac{t}{\Delta x}\right]^2. \tag{17}$$

Imposing the conditions that Δx be less than the mean free path λ, and that $(v_G' t/\Delta x)$ be less than Δx if $t < \lambda/v_G$, we are lead to a consistency requirement:

$$\frac{1}{v_G} \frac{d v_G}{d k} < \lambda \tag{18}$$

To obtain numerical values from these quantities, we rewrite Eq. (7) as

$$E(K) = E(k + i\eta) \approx E(k) + i v_G \eta = \epsilon_0 + \mu \tag{17}$$

From Eqs. (7) and (8), we obtain $V_I = v_g \eta$ and $\omega(k) + V_R = \epsilon_o + \mu$. Strictly speaking, the mean-free-path and thus η should now be recalculated from the cross sections of Section IV and V_I should be obtained from this. Since it is not important for our conclusions, we shall use the values of η taken from Fig. (2). In Figure (3) are given the resulting values of V_I and V_R as functions of ϵ_o.

In Fig. (4) are shown the momentum k vs ϵ_o and (for comparison) the free particle momentum q_o vs $\epsilon_o = (q_o^2 + \mu^2)^{1/2} - \mu$).

In Fig. (5), the group velocity defined by (15) is displayed as a function of the pion momentum within the nucleus. The limitations upon the values of v_G are not met in the resonance region where $V_R(K)$ has a positive slope. This interval is, in optical terminology, the region of "anomalous dispersion". It is the region in which absorbtion is large, where the group velocity exceeds the particle velocity and may exceed the velocity of light. As in the optical case, the identification of v_G with a velocity of energy transport fails under such circumstances.

Although the given theory breaks down in the region of anomalous dispersion, it is desirable that some qualitative guide to the magnitudes of particle fluxes and inelastic cross sections be obtained for energies near resonance. An analagous situation has been considered long ago by Brillouin:[6] the propagation of the wave front of a half infinite light wave through a dispersive medium. Brillouin made use of the known analytic form of the index of refraction (due to resonating bound electrons) - a form similar to that of the optical model well depth. He showed that the wave front propagates with a fairly well-defined velocity for all frequencies. This "signal velocity" agrees with the group velocity for normal dispersion, but parts company with it, remaining close to but less than the speed of light in the anomalous region.

Motivated by this example, we shall likewise "smooth out" the group velocity curve, obtaining the dashed curve in Fig. (5) which approximates the speed of light in the anomalous region. There is, indeed, no necessity for a packet to have a well-defined velocity such as possessed by the wave front in Brillouin's example. We hope, however, that this adjustment of v_G approximates a realistic velocity with which to calculate particle fluxes and that a qualitative indication of dispersive effects in inelastic scattering near resonance may thereby be obtained.

We note, finally, that a conservation equation for meson current may be obtained for wave packets described as in (12). The meson probability density $P(x)$ is proportional to $\psi^*\psi$, so that

$$P(x) \approx \int dQ\,dQ'\, a(K-Q)\,a^*(K-Q')\, e^{i[Qx - E(Q)t]} \times e^{-i[Q'x - E^*(Q')t]}. \tag{19}$$

The Q and Q' integrations may be carried out along the real axis. The application of $i\frac{\partial}{\partial t}$ to $P(x)$ brings under the integral sign in (19) a factor of $E(Q) - E(Q')$, while an application of $-i\nabla$ yields a factor of $(Q - Q')$. Separating E into real and imaginary parts, $E = E_1 + iE_2$, we have

$$\begin{aligned} E(Q) - E^*(Q') &= E_1(Q) - E_1(Q') + i\,[E_2(Q) + E_2(Q')] \\ &\approx (Q - Q')\,v_G(K) + 2i\,V_I(K), \end{aligned}$$

subject to the limitations previously discussed. Hence, we infer that

$$\frac{\partial}{\partial t} P(x) + \frac{\partial}{\partial x}[v_G(K)\,P(x)] = -2\,V_I(K)\,P(x), \tag{20}$$

which shows, for sufficiently narrow packets, how the instantaneous change in density in a small region is due to a flow of probability with velocity v_G and to an absorbtion of probability into the medium.

IV The Cross Section for Pion-Nucleon Scattering within Nuclei

Consider first the scattering of a meson of momentum k_0 against a free nucleon at rest. Let the scattered pion have momentum K and the nucleon a momentum

$$\underset{\sim}{Q} = \underset{\sim}{k_0} - \underset{\sim}{K}.$$

The final energy is determined from

$$\omega(K) + (Q^2 + M^2)^{1/2} = \omega(k_0) + M. \tag{21}$$

The differential scattering cross section is

$$\left(\frac{d\sigma}{d\Omega}\right)_{free} = (2\pi)^4 k_0^{-1} \omega(k_0) \int |t|^2 \delta\left(\omega(K) + (p^2+M^2)^{1/2} - \omega(k_0) - M\right) K^2 dK$$

Here t is the "scattering operator"[7] and an average and sum over initial and final nucleon spin states is implied. Carrying out the integration over K gives

$$\left(\frac{d\sigma}{d\Omega}\right)_{free} = (2\pi)^4 F_f \; |t|^2, \tag{22}$$

where

$$F_f = \frac{\omega(k_0)}{k_0} \cdot \frac{K^2}{B_f},$$

$$B_f = \hat{K} \cdot \left[\underset{\sim}{K}/\omega - (p^2+M^2)^{-1/2} (\underset{\sim}{k_0} - K)\right] \tag{23}$$

In these equations the value of K is defined by Eq. (21).

To obtain the cross section when the nucleon is bound in a nucleus, we assume that t is the same as for a free nucleon. This may be justified in a qualitative way: First, if the scattering interaction is weak, so t is given in Born approximation from some potential, the assumption is evidently valid. On the otherhand, if the pion-nucleon interaction is very strong and of short range (as actually seems to be true), we argue that high energy virtual transitions determine t. For these, however, the optical model potential should be relatively unimportant. We shall actually take

$$t_{bound}(\theta) = t_{free}(\theta). \tag{24}$$

That is, the free and bound quantities t are considered equal for the same scattering angle θ. This is somewhat arbitrary, but seems consistent with our semi-quantitative discussion.

It is useful to define the "effective mass" of a pion as μ^*, where

$$\frac{1}{\mu^*} = 2\,\frac{dE}{dk^2} . \tag{25}$$

Thus $v_g = \frac{k}{\mu^*}$, if W_G is negligible in Eq. (15). The energy E is defined by Eq. (7). In Fig. (6) we display μ^* as a function of the pion momentum.

The energy conservation equation (21) is now replace by

$$E(k) + (Q^2 + M^2)^{1/2} = \omega(k_o) + M, \tag{26}$$

where again $\underset{\sim}{Q} = \underset{\sim}{k}_o - \underset{\sim}{k}$. We suppose the imaginary part of E to be neglected and write the cross section for scattering from a bound nucleon [$(v_g)_o$ is the group velocity of the incident pion] as:

$$\left(\frac{d\sigma}{d\Omega}\right)_{bound} = \frac{(2\pi)^4}{(v_g)_o} \int |t|^2\, \delta\left(E(k) + (Q^2+M^2)^{1/2} - \omega(k_o) - M\right) k^2 dk.$$

Evaluation of the integral above leads to (the scattering angle θ is the angle between $\underset{\sim}{k}_o$ and $\underset{\sim}{k}$)

$$\frac{d\,\sigma(\theta)}{d\,\Omega} = (2\pi)^4\, F\, |t|^2 , \tag{27}$$

in analogy to Eq. (22). Here

$$F = \frac{k^2}{(v_g)_\mu}\,\frac{1}{B}, \tag{28}$$

$$B = \hat{k}\cdot\left[\underset{\sim}{k}/\mu^* - \frac{1}{M}(\underset{\sim}{k}_o - \underset{\sim}{k})\right]$$

The expression for B may be improved by replacing M by the effective nucleon mass M^* in nuclear matter. For the computation of the next section we have used Brueckner's and Wada's[8] value $M^* = .6\,M$ at very low energies and joined this to that obtained from the nucleon optical model potential at higher energies.

The relation (27) has also been obtained in the course of a discussion of quantum mechanical transport phenomena.[9]

Equation (22) and (27) may be combined to give

$$\left(\frac{d\sigma}{d\Omega}\right)_{bound} = \frac{F}{F_f}\left(\frac{d\sigma}{d\Omega}\right)_{free}. \tag{29}$$

V. Calculated Cross Sections

In Fig. (6) we show the effective mass μ^* (as given by Eq. (25)) as a function of the momentum k of a pion in the nuclear medium. This is to be compared to $\omega = (k^2 + \mu^2)^{1/2}$, which is also shown.

The function F_f of Eq. (23) is given in Fig. (7) for several incident momenta k_0 and scattering angles θ. In Fig. (8) we give corresponding values for the function F of Eq. (28). From these we may relate the "free" and "bound" scattering cross sections, as given by Eq. (29).

From Eq. (29) we have obtained the cross section for inelastic scattering of 135 Mev. pions on Carbon. Multiple scattering corrections were neglected, except for a simple exponential attenuation of the incident wave. In Fig. (9) the resulting calculated cross section is compared with that measured by Kessler and Lederman.[10] It is evident from Fig. (9) that the absolute magnitude of the cross section is not very well obtained from our calculation. This is very sensitive, however, to the value of the mean free path. It is also not at all evident that our corrections are reflected in the experimental cross sections. It is unfortunate for our comparison that the energy at which this experiment was done corresponds so closely to the resonance energy for scattering in nuclei. At this energy we do not anticipate any quantitative reliability for our calculation.

VI Discussion of Results

We have seen that dispersive corrections to pion scattering in nuclei are by no means negligible. Indeed, near the resonance energy for pion-nucleon scattering these are too large to be handled adequately by the conventional, simple theory which has been used.

Our discussion has assumed that each inelastically scattered wave interacts continuously with a quiescent nuclear medium. When the particles in the medium are strongly correlated, the local disturbance caused by an inelastic scattering may modify the medium, causing our arguments to be invalid. The conditions under which this is expected to occur have been discussed before.[11] From this discussion we may expect our assumption of an undisturbed medium to be valid when the deBroglie wavelength of the scattered particle is large compared to the distance over which nucleons are correlated in nuclei. For a degenerate Fermi gas model this distance is roughly $\hbar/P_F$, where P_F = 270 Mev/c. Thus for pion energies below resonance, for which the dispersive corrections are most important, we expect our assumption of an undisturbed nuclear medium to be valid.

Finally, our evaluation of the optical model potential has been based on the first approximation, which gives the potential as linear function of the forward scattering amplitude for pions on free nucleons. The second approximation is also rather simple at high energies (see, for example, Eq. (83), reference (11).

To write this out, we must first introduce the nuclear pair correlation function $G(r)$: Let $P_1(\underset{\sim}{Z}_1)$ be the probability of finding nucleon "1" at $\underset{\sim}{Z}_1$ within the nucleus and let $P_2(\underset{\sim}{Z}_1,\underset{\sim}{Z}_2)$ be the joint probability of finding nucleon "1" at Z_1 and nucleon "2" at Z_2. Then

$$P_2(\underset{\sim}{z}_1,\underset{\sim}{z}_2) = P_1(\underset{\sim}{z}_1)\,P_2(\underset{\sim}{z}_2)\left[1+G(|\underset{\sim}{z}_1-\underset{\sim}{z}_2|)\right] \tag{30}$$

defines G. Let

$$R_c = \int_0^\infty G(r)\,dr, \tag{31}$$

$$\Delta_R = -V_R R_C \frac{E}{K_o}$$
$$\Delta_I = -V_I R_C \frac{E}{K_o}, \tag{32}$$

where E is given by Eq. (6). Then the optical model potential, correct to second order is

$$V_{op} = [\,V_R - iV_I\,]\,(1 + i\Delta_R + \Delta_I). \tag{33}$$

The quantities V_R and V_I here and in Eq. (32) are those of Eq. (8) and Figs (1a) and (1b).

Except near the resonance energy, the quantities Δ_R and Δ_I do not give a very important correction to Eq. (33).

For example, assuming a degenerate Fermi gas model of the nucleus, we estimate $R_C \approx -1/3\ r_o$ (where r_o is the mean radius per nucleon). For this value the resulting quantities Δ_R and Δ_I are plotted in Fig. (10).

We are indebted to Professor L. M. Lederman for informing us that he also has considered the effect of the optical potential on inelastic pion corrections.[12]

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FIGURE CAPTIONS

1. (a). V_R is shown as a function of the pion momentum k within the nucleus. (b) V_R and V_I are shown as functions of $(\omega(k) - \mu)$.

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2. The imaginary part of the momentum K (Eq. (71)) is shown as a function of k.

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3. V_R and V_I are shown as functions of $\epsilon_o = \omega(q_o) - \mu$, the kinetic energy of the pion outside of the nucleus.

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4. The momentum k of a pion in the nucleus is shown as a function of the energy ϵ_o (Eq. (71)). For comparison, the momentum q_o of a free pion is also shown as a function of ϵ_o (Eq. (6)).

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5. The pion group velocity V_G is shown as a function of the momentum k within nucleus (Eq. (15)). The corresponding free particle velocity k/ω is also shown. The dashed curves represent conjectured values, consistent with $V_G \leq c$.

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6. The effective mass μ^* of the pion is shown as a function of the momentum k within the nucleus. Also shown is the relativistic "free particle mass" ω.

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7. The function F_f of Eq. (23) is shown for several values of k_o and the scattering angle θ.

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8. (a,b). The function F of Eq. (28) is shown for several values of k_o and scattering angle θ.

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9. Comparison is made of calculated inelastic scattering of 125 Mev pions with measurements of Kessler and Lederman on Carbon.

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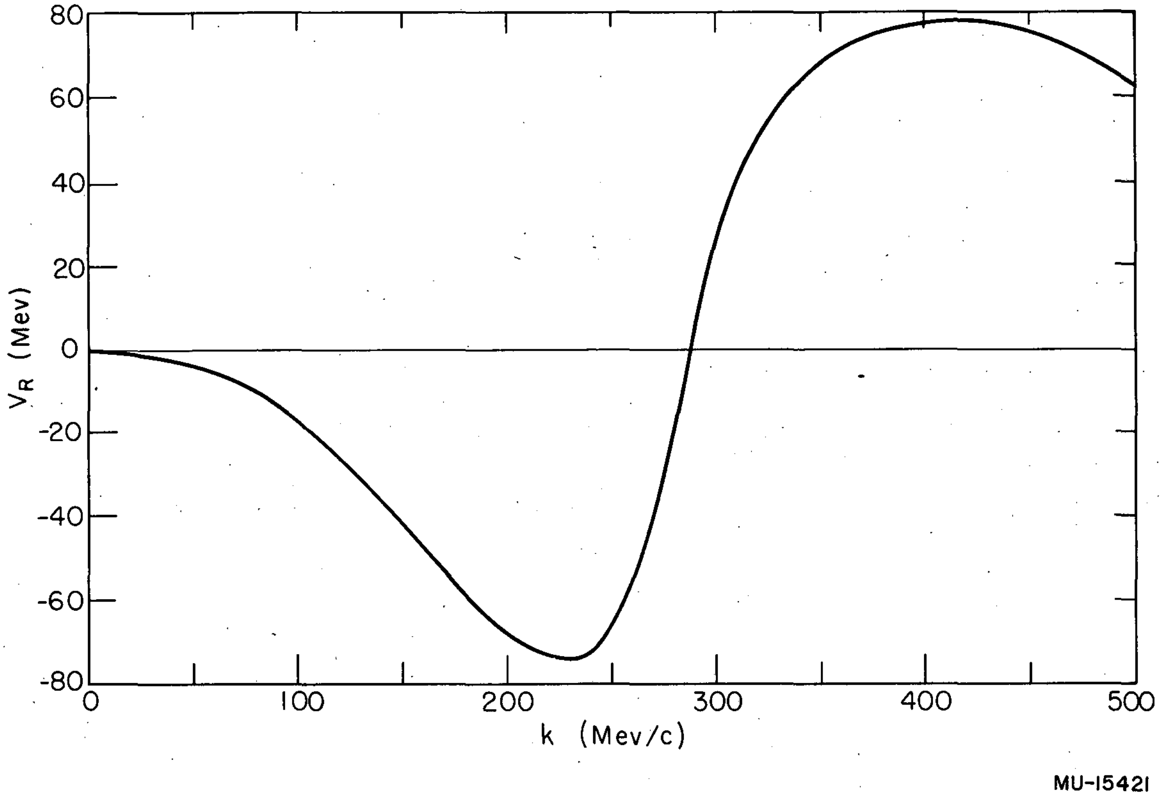
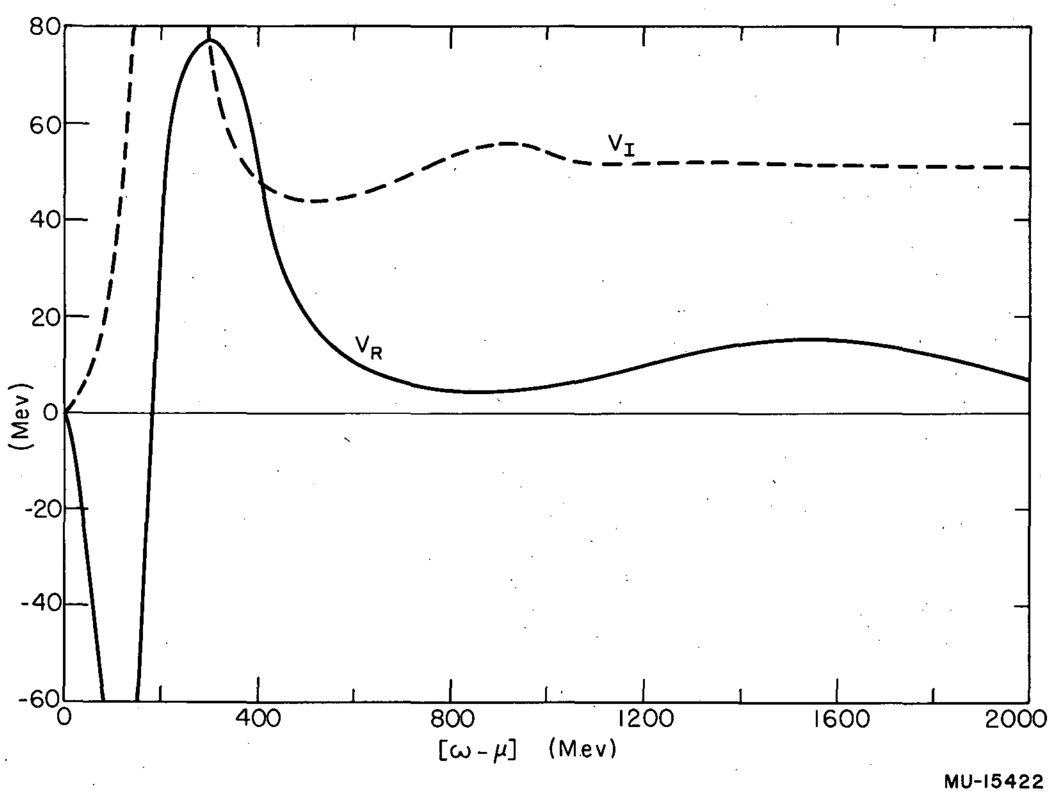
10. The function Δ_R and Δ_I of Eq. (32) are given as functions of ϵ_o.



80
60
40
20
0
-20
-40
-60
-80
V_R (Mev)
0
100
200
300
400
500
k (Mev/c)
MU-15421

(a)



80
60
40
20
0
-20
-40
-60
(Mev)
V_I
V_R
0
400
800
1200
1600
2000
[ω - μ] (Mev)
MU-15422

Fig. 1 **(b)**

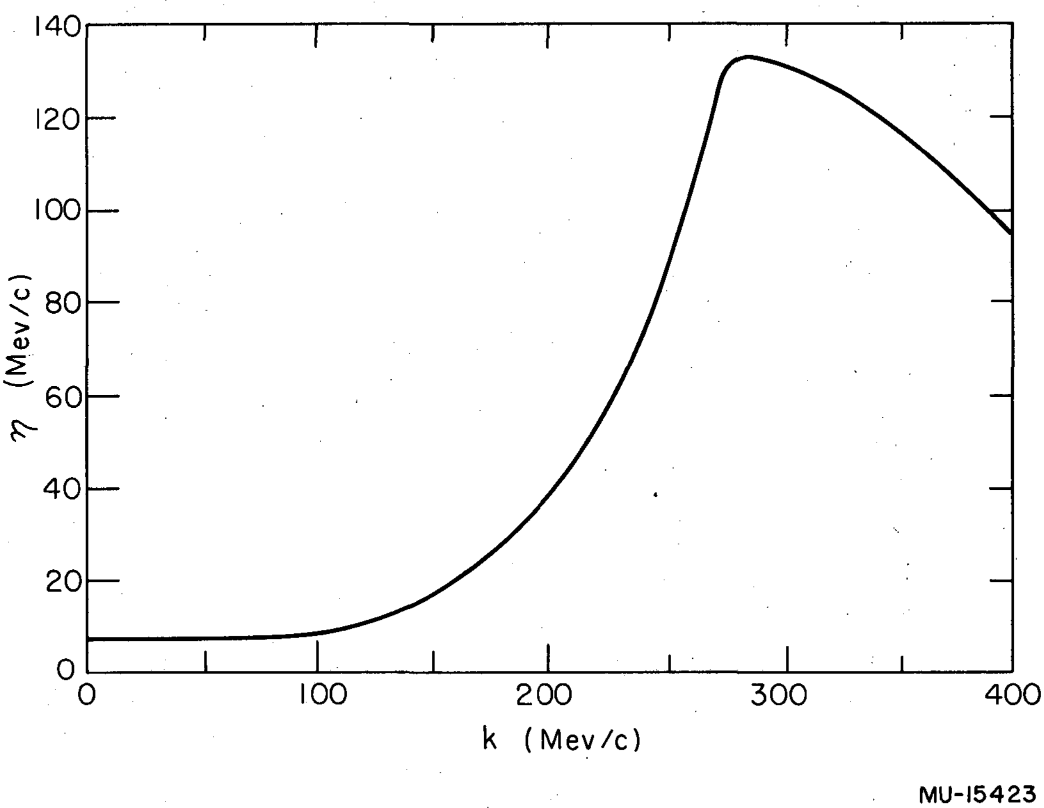


Fig. 2

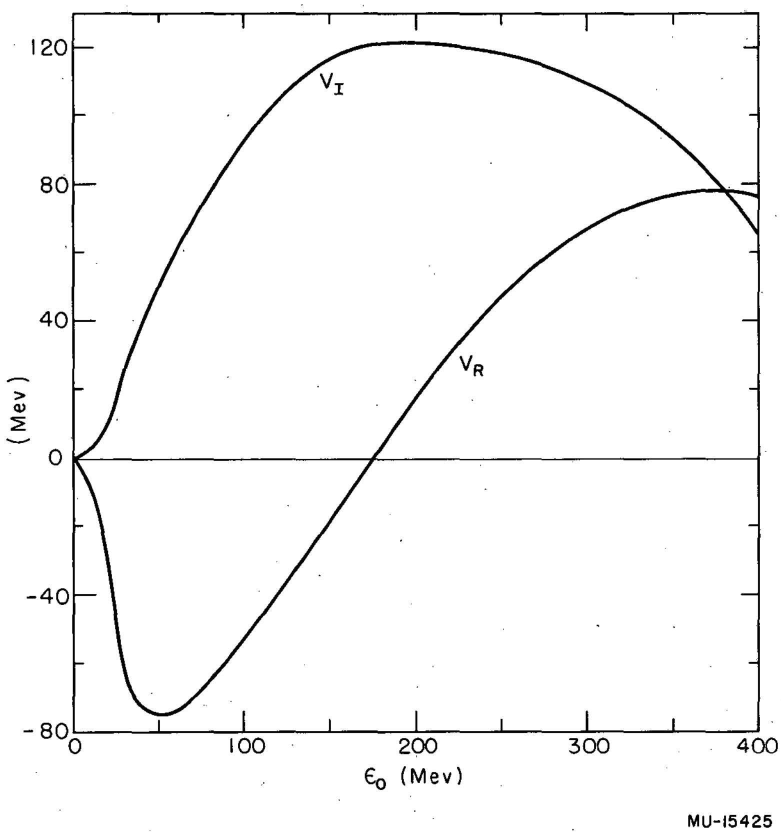

120
80
40
0
-40
-80
(Mev)
V_I
V_R
0
100
200
300
400
ϵ_0 (Mev)
MU-15425

Fig. 3

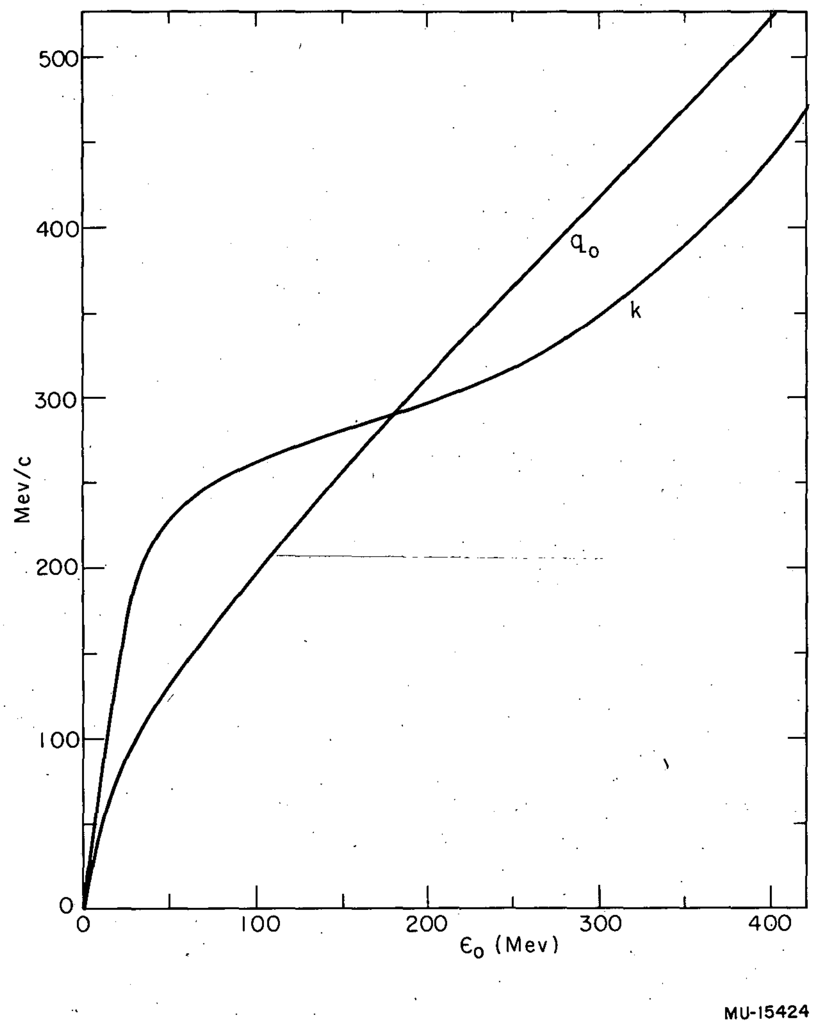

500
400
300
200
100
0
Mev/c
q_o
k
0
100
200
300
400
ϵ_o (Mev)
MU-15424

Fig. 4

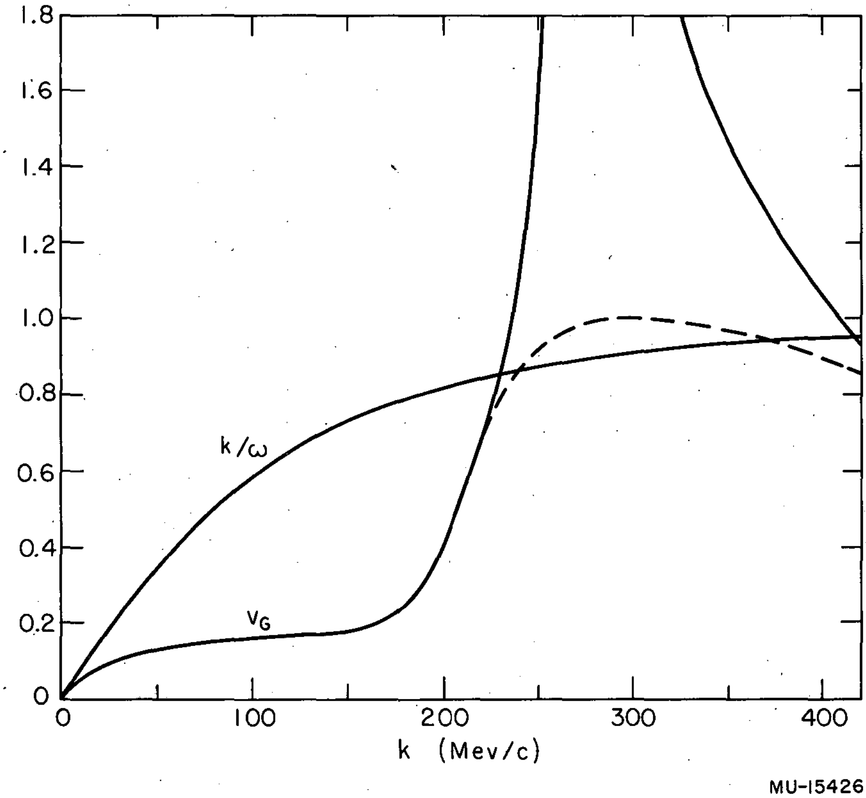


Fig. 5

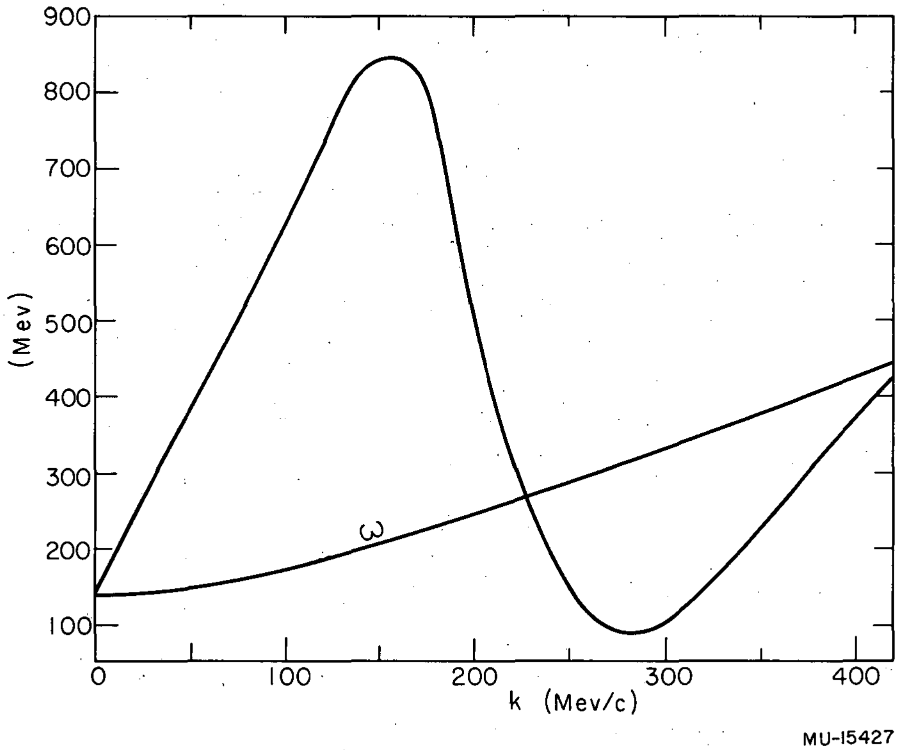
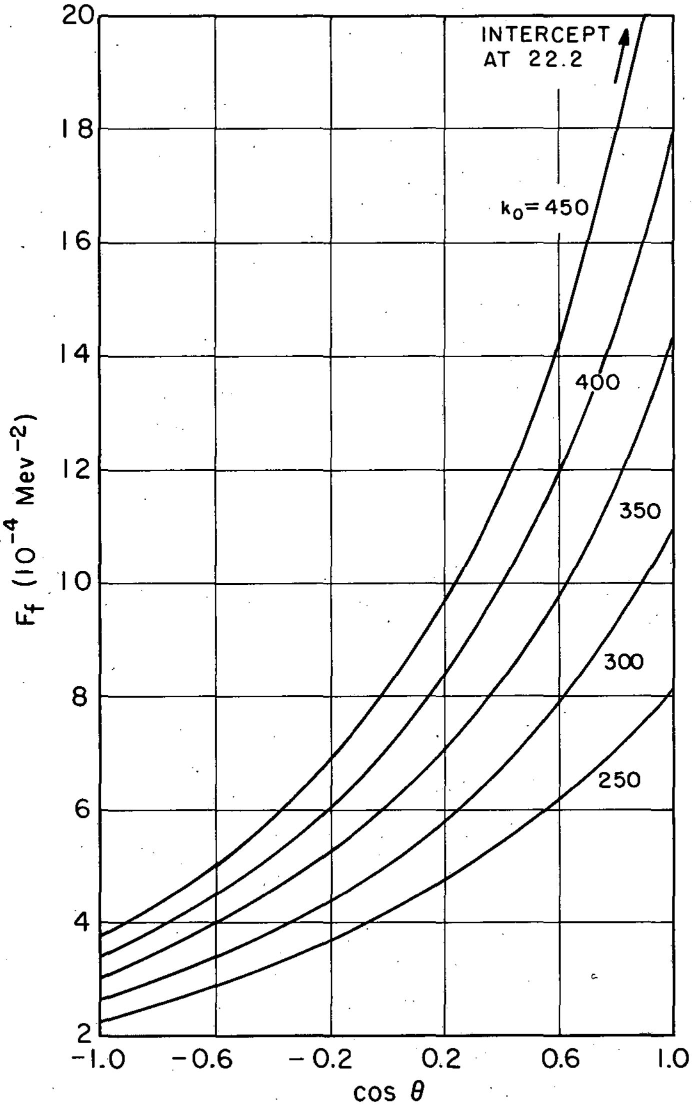
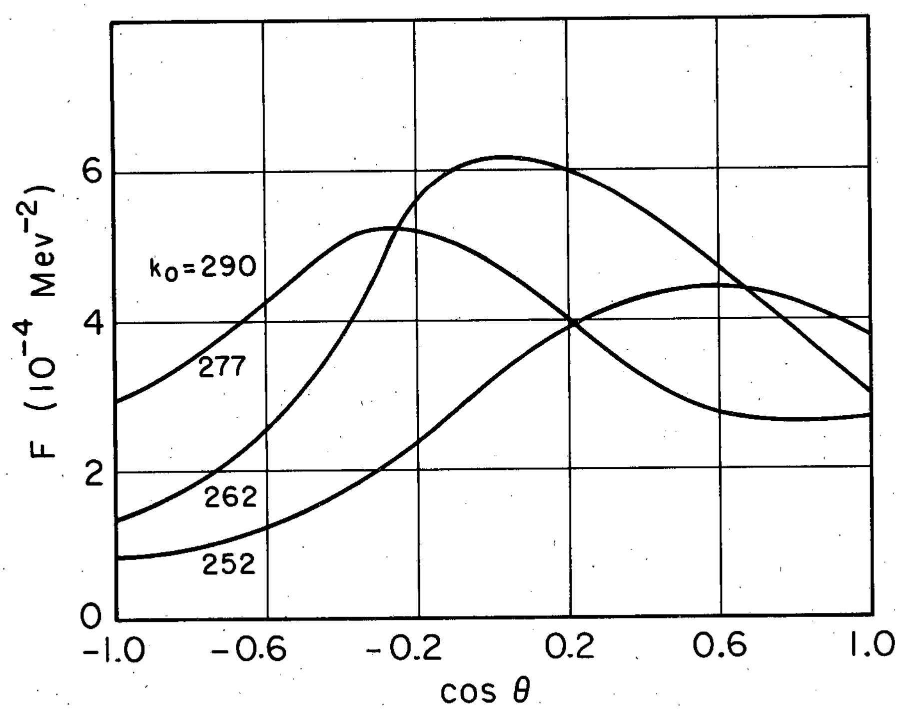

900
800
700
600
500
400
300
200
100
(Mev)
ω
0
100
200
300
400
k (Mev/c)
MU-15427

Fig. 6



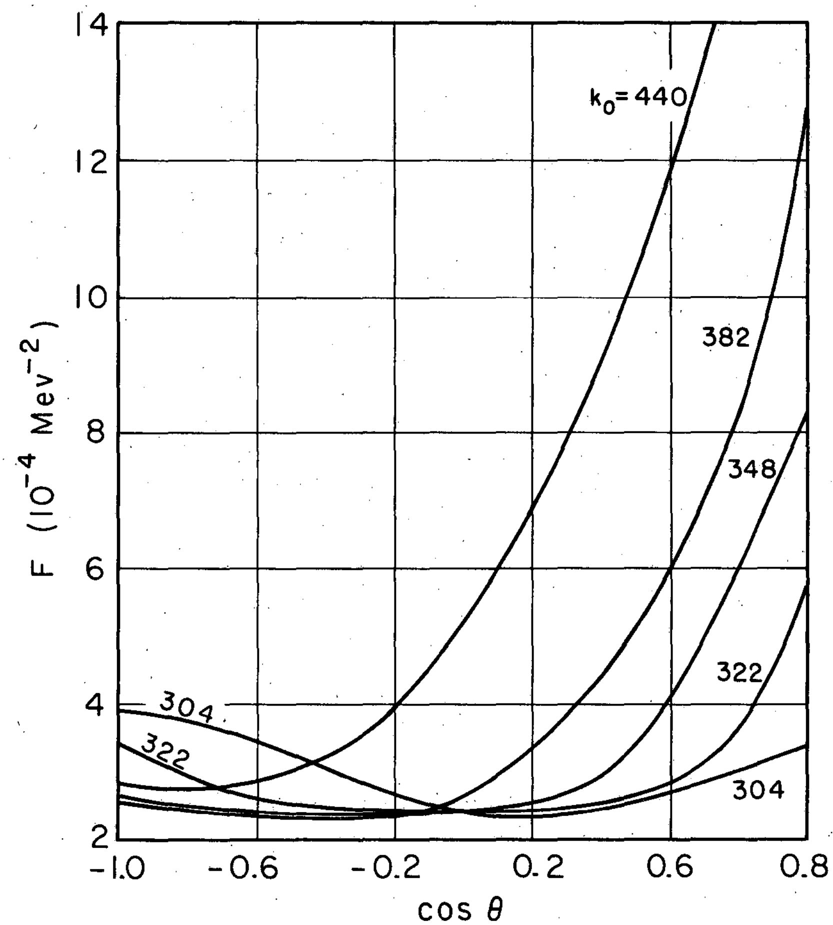
MU-15428

Fig. 7.



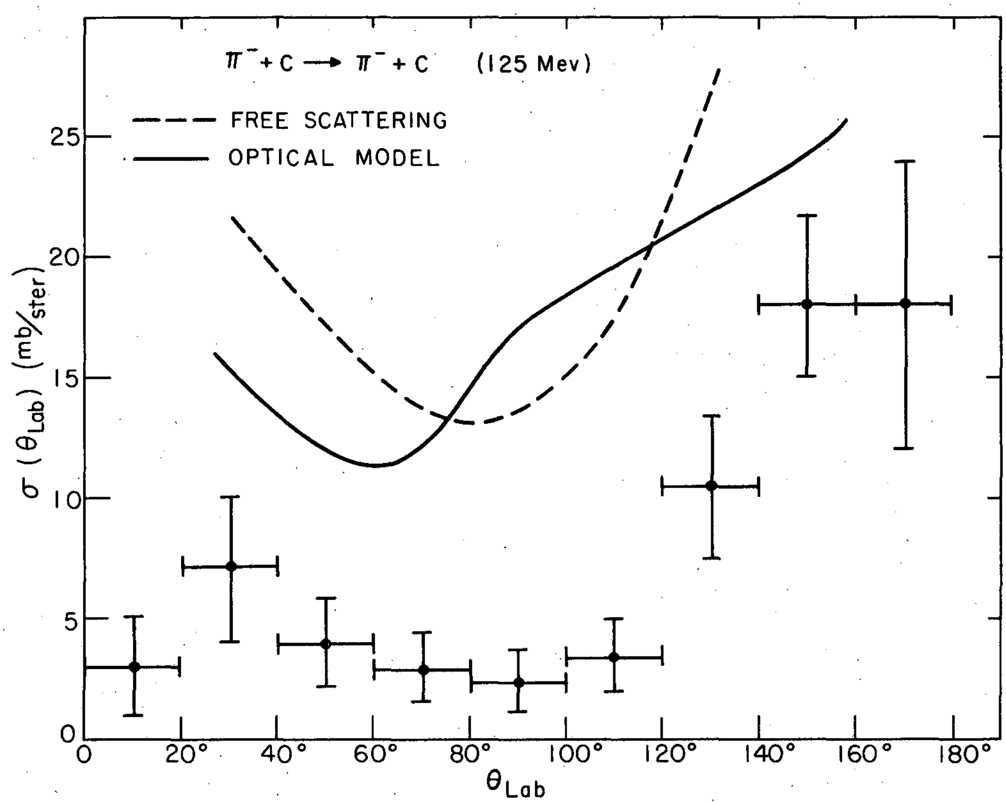
MU-15429

(a)



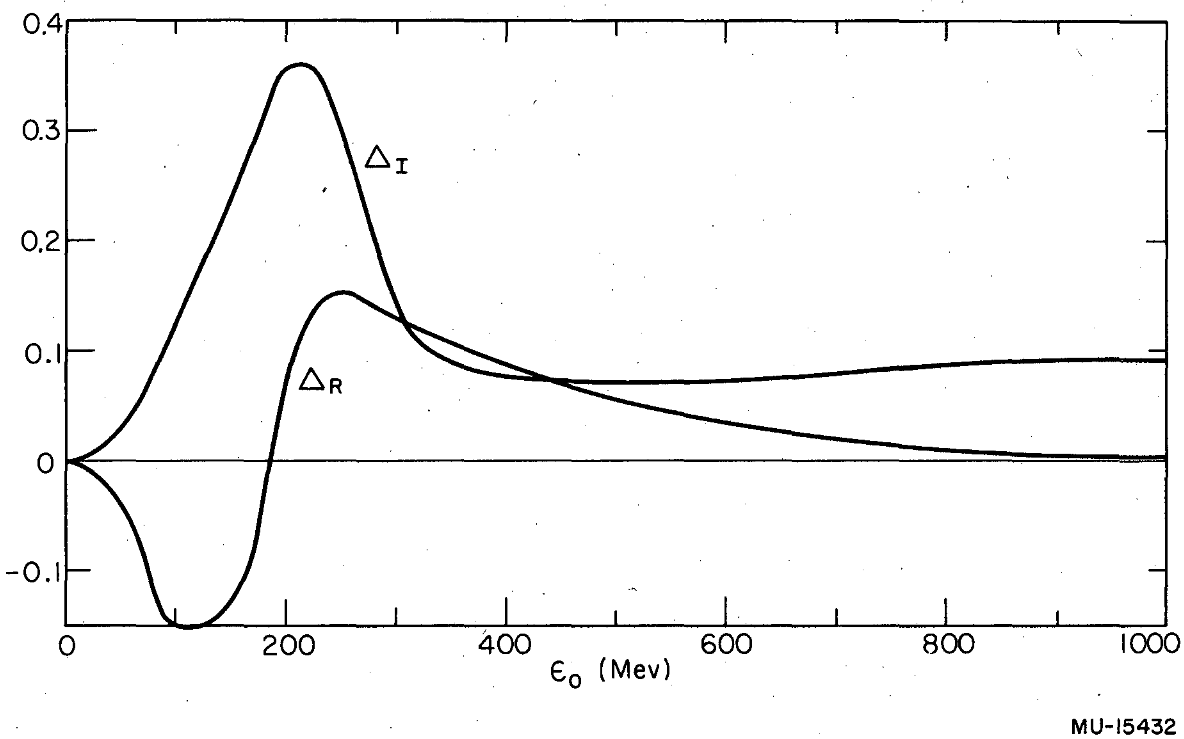
MU-15430

Fig. 8. (b)



MU-15431

Fig. 9



0.4
0.3
0.2
0.1
0
-0.1
0
200
400
600
800
1000
ϵ_0 (Mev)
Δ_I
Δ_R
MU-15432

Fig. 10