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PHENOMENOLOGICAL ANALYSIS OF THE C-VIOLATING DECAY $\pi^0 \rightarrow 3\gamma$

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ABSTRACT

The most general amplitude for the C-violating decay $\pi^0 \to 3\gamma$ is given, subject to the restrictions of Lorentz invariance, gauge invariance, and Bose statistics. It is found that, to lowest order in centrifugal barrier factors, the Dalitz-plot density for the decay is uniquely determined. If we assume that the decay involves a C-violating coupling as strong as that responsible for the ordinary two-photon decay, the estimated branching ratio $\Gamma(\pi^0 \to 3\gamma)/\Gamma(\pi^0 \to 2\gamma)$ is 10^{-7} to 10^{-8}, including the effects of centrifugal barriers.

I. INTRODUCTION

The charge-conjugation parity of a system of n photons is $(-1)^n$. Thus a particle that decays via C-conserving interactions cannot decay into both an even and an odd number of photons. For a spin-zero particle such as the π^0 meson or the singlet positronium "particle", C invariance is the only selection rule forbidding decays into both two and three photons. C-violating <u>weak</u> interactions may be expected to give some three-photon decay of π^0's and of singlet positronium, but with branching ratios far too small to be detected. Thus the experimental detection of a three photon rate would be evidence for a C-violating, <u>nonweak</u> interaction.

In giving a phenomenological analysis of such decays, one may construct decay amplitudes and Dalitz-plot densities satisfying certain conditions of simplicity and (or) generality, and also satisfying as many invariance conditions as are consistent with the inherent C violation of the decay. In particular, we may consider T-conserving, P-violating decays, and T-violating, P-conserving decays. Schechter has given such an analysis with particular emphasis on a possible T-conserving three-photon decay of singlet positronium.[1] Berends has more recently discussed T-violating decays.[2] Experimentally, a result for singlet positronium has recently been reported,[3] but owing to possible complications in the chemistry, the interpretation of this experiment is not clear. For $\pi^0 \to 3\gamma$ decay, recent experiments have established that its branching ratio relative to the two-photon decay is less than 5×10^{-6} (90% confidence level).[4]

In this paper we give a more complete phenomenological discussion than those in references 1 and 2. Our remarks, except for those concerning estimated branching ratios, apply equally to decays of π^0 and of singlet positronium, but we shall consider for definiteness the case of the π^0. In view of the theoretical speculations about possible C-violating electromagnetic currents[5] or C-violating terms in semi-strong interactions,[6] the case of the π^0 is of greater interest.

II. DECAY AMPLITUDE

The decay amplitude M must be expressed in terms of the four-momentum p_μ of the π^0, and the four-momenta $k_\mu^{(i)}$ and field variables $A_\mu^{(i)}$ ($i = 1, 2, 3$) of the three photons; M must be linear in the field variable of each photon. Rotation invariance, proper Lorentz invariance, gauge invariance, and Bose statistics further require that (1) M must be a Lorentz-invariant pseudoscalar (T-violating, P-conserving) or scalar (T-conserving, P-violating); (2) the photon field variables must appear in the form $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; and (3) M must be symmetric under interchange of any two of the indices 1, 2, 3.

In constructing all possible forms of M, it will be very convenient to confine our attention to the three-vector forms of the variables, expressed in the π^0 center-of-mass system. All the Lorentz invariant forms can clearly be rewritten in three-vector form, and conversely the covariant form of any three-vector expression can easily be written, as shown below [Eqs. (5)].

Expressed in terms of three-vectors, M is a sum of terms, each of which is either a scalar or a pseudoscalar, is trilinear in $\underset{\sim}{F}_i$ (where $\underset{\sim}{F}_i = \underset{\sim}{E}_i$ or $\underset{\sim}{B}_i$), is composed of various powers of the photon energy and momentum variables ω_i and $\underset{\sim}{k}_i$ (i = 1, 2, 3), and is symmetric in 1, 2, and 3. The basic energy-momentum conservation and gauge-invariance identities that hold are

$$\underset{\sim}{k}_1 + \underset{\sim}{k}_2 + \underset{\sim}{k}_3 = 0, \tag{1a}$$

$$\omega_1 + \omega_2 + \omega_3 = m, \tag{1b}$$

and

$$\underset{\sim}{k}_1 \cdot \underset{\sim}{\epsilon}_1 = \underset{\sim}{k}_2 \cdot \underset{\sim}{\epsilon}_2 = \underset{\sim}{k}_3 \cdot \underset{\sim}{\epsilon}_3 = 0. \tag{1c}$$

From them follow the identities

$$\underset{\sim}{k}_j \cdot \underset{\sim}{\epsilon}_i = -\underset{\sim}{k}_k \cdot \underset{\sim}{\epsilon}_i, \tag{2a}$$

$$\underset{\sim}{k}_i \cdot \underset{\sim}{k}_j = \frac{1}{2}(\omega_k^2 - \omega_i^2 - \omega_j^2), \tag{2b}$$

and

$$\underset{\sim}{k}_i \times \underset{\sim}{k}_j = \underset{\sim}{k}_j \times \underset{\sim}{k}_k = \underset{\sim}{k}_k \times \underset{\sim}{k}_i \equiv \underset{\sim}{q}, \tag{2c}$$

where i, j, k are any cyclic permutation of 1, 2, 3.

The possible forms of M are

$$A = \Sigma\, g\, (\underset{\sim}{F}_1 \cdot \underset{\sim}{F}_2 \times \underset{\sim}{F}_3),$$

$$B = \Sigma\, g\, (\underset{\sim}{v} \cdot \underset{\sim}{F}_1)(\underset{\sim}{F}_2 \cdot \underset{\sim}{F}_3),$$

$$C = \Sigma\, g\, (\underset{\sim}{v} \cdot \underset{\sim}{F}_1)(\underset{\sim}{v} \cdot \underset{\sim}{F}_2 \times \underset{\sim}{F}_3),$$

and

$$D = \Sigma\, g\, (\underset{\sim}{v} \cdot \underset{\sim}{F}_1)(\underset{\sim}{v} \cdot \underset{\sim}{F}_2)(\underset{\sim}{v} \cdot \underset{\sim}{F}_3), \tag{3}$$

where g and $\underset{\sim}{v}$ denote general (pseudo)scalar and (pseudo)vector functions of ω_i and $\underset{\sim}{k}_i$, and Σ denotes summation over all permutations of 1, 2, 3. We assume further that g and $\underset{\sim}{v}$ are expressible as polynomials in ω_i and $\underset{\sim}{k}_i$.

Now the functions g can always be reexpressed in terms of the ω_i only. As for the functions $\underset{\sim}{v}$, they may be taken as equal to a (pseudo)scalar function, which we absorb in g, multiplied either by one of the $\underset{\sim}{k}_i$ or else by the pseudovector $\underset{\sim}{q} = \underset{\sim}{k}_1 \times \underset{\sim}{k}_2$. Furthermore, from relations (1c) and (2a) and the relations $\underset{\sim}{q} \cdot \underset{\sim}{E}_1 = -\omega_1 \underset{\sim}{k}_2 \cdot \underset{\sim}{B}_1$ and $\underset{\sim}{q} \cdot \underset{\sim}{B}_1 = \omega_1 \underset{\sim}{k}_2 \cdot \underset{\sim}{E}_1$, it follows that for $\underset{\sim}{v} \cdot \underset{\sim}{F}$ we need only consider $\underset{\sim}{k}_2 \cdot \underset{\sim}{F}_1$, $\underset{\sim}{k}_3 \cdot \underset{\sim}{F}_2$, and $\underset{\sim}{k}_1 \cdot \underset{\sim}{F}_3$. By similar arguments we can show further that all type-C terms can be rewritten as combinations of types B and D. Thus we need only consider the forms

$$A = \Sigma g (\underset{\sim}{F}_1 \cdot \underset{\sim}{F}_2 \times \underset{\sim}{F}_3),$$

$$B = \Sigma g (\underset{\sim}{k}_2 \cdot \underset{\sim}{F}_1)(\underset{\sim}{F}_2 \cdot \underset{\sim}{F}_3),$$

and

$$C = \Sigma g (\underset{\sim}{k}_2 \cdot \underset{\sim}{F}_1)(\underset{\sim}{k}_3 \cdot \underset{\sim}{F}_2)(\underset{\sim}{k}_1 \cdot \underset{\sim}{F}_3). \qquad (4)$$

All possible Lorentz-invariant forms can be rewritten as sums of these forms. Also all three-vector terms can be rewritten in covariant form by using the identities

$$\omega_1 = \frac{1}{m} k_\mu^{(1)} (k_\mu^{(2)} + k_\mu^{(3)}),$$

$$\underset{\sim}{k}_2 \cdot \underset{\sim}{F}_1 = -\frac{1}{m} p_\mu k_\nu^{(1)} f_{\mu\nu}^{(1)},$$

$$\underset{\sim}{F}_1 \cdot \underset{\sim}{F}_2 = -\frac{1}{m^2} p_\rho p_\sigma f_{\rho\mu}^{(1)} f_{\sigma\mu}^{(2)},$$

and

$$\underset{\sim}{F}_1 \cdot \underset{\sim}{F}_2 \times \underset{\sim}{F}_3 = \frac{1}{m^2} p_\rho p_\sigma f_{\rho\mu}^{(1)} f_{\sigma\nu}^{(2)} f_{\mu\nu}^{(3)}, \qquad (5)$$

where

$$f_{\mu\nu} = F_{\mu\nu} \quad \text{for } \underset{\sim}{F} = \underset{\sim}{E}$$

$$= G_{\mu\nu} \quad \text{for } \underset{\sim}{F} = \underset{\sim}{B},$$

$$G_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma},$$

and the metric is (1, -1, -1, -1).

Thus we must consider all possible terms of type (4). We first consider only the scalar (P-violating) terms; the pseudoscalar (P-conserving) terms can be obtained from them by a simple substitution. The scalar possibilities are:

$$A1 = \Sigma g\,(\underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2 \times \underset{\sim}{B}_3)$$

$$A2 = \Sigma g\,(\underset{\sim}{B}_1 \cdot \underset{\sim}{B}_2 \times \underset{\sim}{B}_3)$$

$$B1 = \Sigma g\,(\underset{\sim}{k}_1 \cdot \underset{\sim}{E}_3)(\underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2)$$

$$B2 = \Sigma g\,(\underset{\sim}{k}_1 \cdot \underset{\sim}{E}_3)(\underset{\sim}{B}_1 \cdot \underset{\sim}{B}_2)$$

$$B3 = \Sigma g\,(\underset{\sim}{k}_1 \cdot \underset{\sim}{B}_3)(\underset{\sim}{E}_1 \cdot \underset{\sim}{B}_2)$$

$$C1 = \Sigma g\,(\underset{\sim}{k}_2 \cdot \underset{\sim}{B}_1)(\underset{\sim}{k}_3 \cdot \underset{\sim}{B}_2)(\underset{\sim}{k}_1 \cdot \underset{\sim}{E}_3)$$

$$C2 = \Sigma g\,(\underset{\sim}{k}_2 \cdot \underset{\sim}{E}_1)(\underset{\sim}{k}_3 \cdot \underset{\sim}{E}_2)(\underset{\sim}{k}_1 \cdot \underset{\sim}{E}_3). \qquad (6)$$

We note that in A2 and C2 only the totally antisymmetric part of g contributes, and thus g is of the form

$$g_{123} = (\omega_1 - \omega_2)(\omega_2 - \omega_3)(\omega_3 - \omega_1)\, h_{123},$$

where h is a totally symmetric function. Similarly in all other terms except B3, only the part of g that is antisymmetric in its first two indices contributes, and thus g is of the form

$$g_{123} = (\omega_1 - \omega_2)\, f_{123},$$

where f is symmetric in its first two indices. The forms (6) may be reexpressed in terms of the photon polarization vectors $\underset{\sim}{\epsilon}$ according to the relations

$$\underset{\sim}{E} = \omega\, \underset{\sim}{\epsilon}$$

and

$$\underset{\sim}{B} = \underset{\sim}{k} \times \underset{\sim}{\epsilon}.$$

Doing this and performing various algebraic manipulations, one can show that the forms (6) are entirely equivalent to the decay amplitude

$$M = \Sigma[(\omega_1 - \omega_2 - \omega_3) f_{231} + \omega_2 \omega_3 f'_{231}] (\omega_2 - \omega_3) \omega_1 (\underset{\sim}{k}_2 \cdot \underset{\sim}{\epsilon}_1)(\underset{\sim}{\epsilon}_2 \cdot \underset{\sim}{\epsilon}_3)$$

$$+ h_{123} (\omega_1 - \omega_2)(\omega_2 - \omega_3)(\omega_3 - \omega_1)(\underset{\sim}{k}_2 \cdot \underset{\sim}{\epsilon}_1)(\underset{\sim}{k}_3 \cdot \underset{\sim}{\epsilon}_2)(\underset{\sim}{k}_1 \cdot \underset{\sim}{\epsilon}_3). \tag{7}$$

Here f, f', and h are arbitrary polynomials in ω_1, ω_2, ω_3; f and f' are symmetric in their first two indices; and h is totally symmetric. Expression (7) is the most general scalar decay amplitude. The most general pseudoscalar decay amplitude may be obtained from (7) by the substitutions $\underset{\sim}{\epsilon}_i \rightarrow \frac{1}{\omega_i} \underset{\sim}{k}_i \times \underset{\sim}{\epsilon}_i$.

III. DALITZ-PLOT DENSITY

Taking the matrix elements of (7) or of its pseudoscalar counterpart, one finds that all such elements consist of the factor $\underset{\sim}{q} \cdot \hat{\underset{\sim}{n}}$ (where $\hat{\underset{\sim}{n}}$ is the unit normal to the decay plane) multiplied by a quantity that vanishes when $\omega_1 = \omega_2 = \omega_3$ and also when any two ω's are equal and the third is zero. Therefore the Dalitz-plot density is

$$d_{123} = \Sigma \, |M|^2 = \text{const.} \cdot a_{123} \cdot b_{123}, \tag{8a}$$

where

$$a_{123} = |\underset{\sim}{q} \cdot \hat{\underset{\sim}{n}}|^2 = |\underset{\sim}{k}_i \times \underset{\sim}{k}_j|^2 = \omega_i^2 \omega_j^2 \sin^2 \theta_{ij}$$

$$= \frac{1}{2}(\omega_1^2 \omega_2^2 + \omega_2^2 \omega_3^2 + \omega_3^2 \omega_1^2) - \frac{1}{4}(\omega_1^4 + \omega_2^4 + \omega_3^4), \tag{8b}$$

and b_{123} is a nonnegative function of ω_1, ω_2, ω_3 which is symmetric in 1, 2, 3 and vanishes for $\omega_1 = \omega_2 = \omega_3$ or for any two ω's equal and the third zero. Limiting ourselves to expressions up to the fourth power in ω, we find only the unique function

$$b_{123} = (\omega_1 - \omega_2)^2 (\omega_3 - \omega_1 - \omega_2)^2 + (\omega_2 - \omega_3)^2 (\omega_1 - \omega_2 - \omega_3)^2 + (\omega_3 - \omega_1)^2 (\omega_2 - \omega_3 - \omega_1)^2. \tag{8c}$$

The Dalitz-plot density obtained from (8a, b, c) is the same as that given in Ref. 2. The energy and angle distributions in terms of the Dalitz-plot density are

$$\frac{\partial^2 \sigma}{\partial \omega_1 \partial \omega_2} \propto d(\omega_1, \omega_2, m - \omega_1 - \omega_2)$$

and

$$\frac{\partial^2 \sigma}{\partial \omega_1 \partial \cos\theta_{12}} \propto \frac{\omega_1 \omega_2}{m - \omega_1(1 - \cos\theta_{12})} d(\omega_1, \omega_2, m - \omega_1 - \omega_2)$$

where

$$\omega_2 = \frac{m(m - 2\omega_1)}{2m - 2\omega_1(1 - \cos\theta_{12})} .$$

The density (8a, b, c) is plotted in Fig. 1. As is seen, the density vanishes at the center of the plot, where $\omega_1 = \omega_2 = \omega_3 = m/3$, and along the edges, where the photons are collinear; in fact, all possible Dalitz-plot densities for $\pi^0 \to 3\gamma$ vanish at these points.

Since the ω_i are small compared to characteristic normalizing masses of m or larger, it is to be expected that the greatest contribution to the decay will come from the density given by (8c), and that we can neglect terms containing more powers of ω_i, i.e., more centrifugal barrier factors.

The density (8a, b, c) could, of course, have been obtained from simpler arguments than given above. For example, we could simply have noted that the lowest number of powers of ω_i and $\underset{\sim}{k}_i$ in any expression of type (6) is four, which appear only in the expressions

$$A1a = \Sigma\ (\omega_1 - \omega_2)\ (\underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2 \times \underset{\sim}{B}_3)$$

$$B3a = \Sigma\ (\underset{\sim}{k}_1 \cdot \underset{\sim}{B}_3)\ (\underset{\sim}{B}_1 \cdot \underset{\sim}{E}_2 - \underset{\sim}{B}_2 \cdot \underset{\sim}{E}_1)$$

and their pseudoscalar counterparts

$$A1a' = \Sigma\ (\omega_1 - \omega_2)\ (\underset{\sim}{B}_1 \cdot \underset{\sim}{B}_2 \times \underset{\sim}{E}_3)$$

$$B3a' = \Sigma\ (\underset{\sim}{k}_1 \cdot \underset{\sim}{E}_3)\ (\underset{\sim}{E}_1 \cdot \underset{\sim}{B}_2 - \underset{\sim}{E}_2 \cdot \underset{\sim}{B}_1).$$

These expressions all yield Dalitz-plot density (8a, b, c), with eight centrifugal-barrier factors. However from this argument alone we do not know whether (8a, b, c) is unique, or whether there exist lower-order expressions. For example, in the calculation of Ref. 2, the same expressions (8a, b, c) are obtained from the form

$$\frac{1}{4}\Sigma[(\omega_2\,\omega_1)\omega_3 - (\underset{\sim}{k}_2 - \underset{\sim}{k}_1) \cdot \underset{\sim}{k}_3]\,(\underset{\sim}{k}_2 \cdot \underset{\sim}{E}_3)\,(\underset{\sim}{E}_1 \cdot \underset{\sim}{B}_2 + \underset{\sim}{E}_2 \cdot \underset{\sim}{B}_1),$$

which has six powers of ω_i and k_i appearing explicitly. For this case, one might have expected to find 12 centrifugal-barrier factors in $\Sigma|M|^2$, but it turns out that four powers of ω can be factored out in the form $(\omega_1 + \omega_2 + \omega_3)^4 = m^4$, leaving the density (8a, b, c) with only eight centrifugal-barrier factors.

Returning momentarily to the question of Lorentz invariance, we note that the density (8a, b, c) can be obtained from Lorentz-invariant forms containing seven powers of four-momenta, such as the expression in Ref. 2, or the expressions

$$\frac{1}{m^3} \Sigma\, p_\rho\, p_\sigma\, p_\tau\, k_\rho^{(1)}\, F_{\mu\nu}^{(1)}\, F_{\mu\sigma}^{(2)}\, G_{\nu\tau}^{(3)} \quad \text{(pseudoscalar)}$$

and

$$\frac{1}{m^3} \Sigma\, p_\rho\, p_\sigma\, p_\tau\, k_\rho^{(1)}\, F_{\mu\nu}^{(1)}\, F_{\mu\sigma}^{(2)}\, F_{\nu\tau}^{(3)}. \quad \text{(scalar)}$$

All expressions containing only five powers of four-momenta give vanishing matrix elements (there is an error in the derivation of Eq. (7) of Ref. 1; the expression actually vanishes).

IV. BRANCHING RATIO

The decay rate in terms of $\Sigma |M|^2$ is

$$\Gamma = \frac{(2\pi)^4}{2m} \int \Sigma\, |M|^2\, d\rho\, ,$$

where

$$d\rho = \frac{d^3 \underset{\sim}{k}_1}{2\omega_1 (2\pi)^3} \frac{d^3 \underset{\sim}{k}_2}{2\omega_2 (2\pi)^3} \frac{d^3 \underset{\sim}{k}_3}{2\omega_3 (2\pi)^3} \delta^4 (k^{(1)} + k^{(2)} + k^{(3)} - p)$$

or

$$\Gamma = \frac{1}{8m(2\pi)^3} \int_0^{m/2} d\omega_1 \int_{(m/2) - \omega_1}^{m/2} d\omega_2\, \Sigma\, |M|^2.$$

In the usual phenomenological analysis of decay rates,[7] one assumes that, with the above normalization, $\Sigma |M|^2$ is equal to unity apart from (1) appropriate coupling-constant factors, (2) centrifugal-barrier factors dictated by arguments like those in Sec. II and III of this paper, and (3) the appropriate power of m for dimensional reasons. Following this recipe, we obtain

$\Gamma = m \times$ (coupling constant) $\times$ (dimensionless phase space) $\times$ (centrifugal-barrier factor).

For an estimate of the centrifugal-barrier factor we may take $(1/n)^p$ where n is the number of photons and p the number of centrifugal-barrier factors. For the two-photon decay of π^0, this gives

$$\Gamma^{2\gamma} = m\alpha^2 \frac{1}{16\pi} \left(\frac{1}{2}\right)^4 = 2.4\ \Gamma^{\text{meas.}},$$

where $\Gamma^{\text{meas.}}$ is the measured total decay rate. Thus it does not make much difference whether we compare our estimated three-photon rate with the estimated two-photon rate or with the measured two-photon rate.

For the three-photon rate we get

$$\Gamma^{3\gamma} = m\alpha^3 \frac{1}{8(4\pi)^3} \left(\frac{1}{3}\right)^8 = 1.4 \times 10^{-7}\ \Gamma^{\text{meas.}}.$$

For an alternate estimate of the centrifugal-barrier factor, we may average (8a, b, c) over the Dalitz plot, which gives

$$\frac{\int_0^{m/2} d\omega_1 \int_{(m/2)-\omega_1}^{m/2} d\omega_2\, d(\omega_1, \omega_2, m-\omega_1-\omega_2)}{\int_0^{m/2} d\omega_1 \int_{(m/2)-\omega_1}^{m/2} d\omega_2} = \frac{1}{2^7 \cdot 3^2 \cdot 5 \cdot 7} = 0.162\left(\frac{1}{3}\right)^8.$$

Thus we conclude that the branching ratio $\Gamma(\pi^0 \to 3\gamma)/\Gamma(\pi^0 \to 2\gamma)$ is in the region 10^{-7} to 10^{-8}, provided it involves a C-violating coupling that is as strong as the coupling responsible for the ordinary two-photon decay.

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FOOTNOTES AND REFERENCES

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FIGURE LEGEND

Fig. 1. Calculated Dalitz-plot density given by Eqs. (8a, b, c). Shown are contours of equal density, in units of the average density.

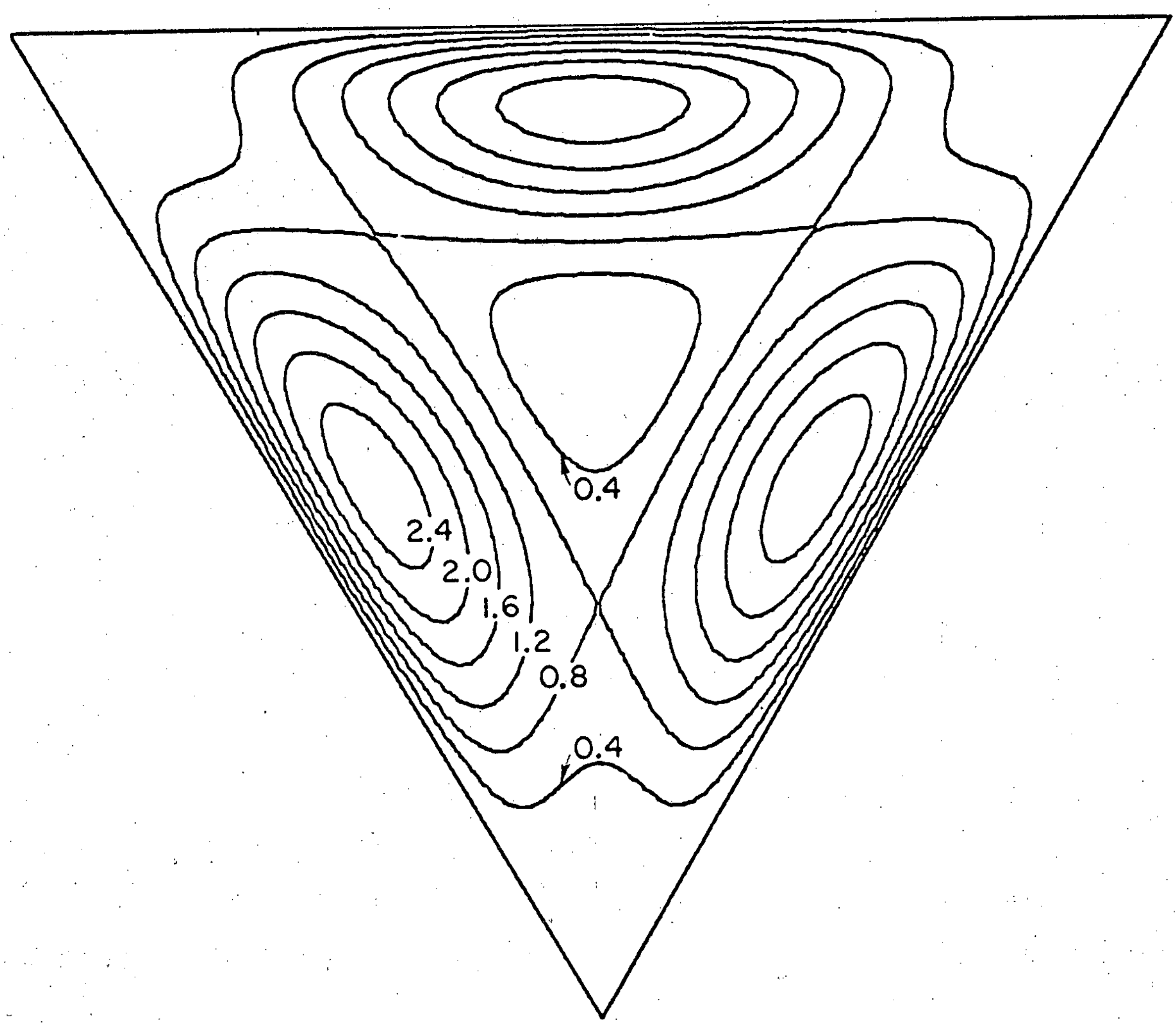

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Fig. 1