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June 1968

Berkeley, California

DISCLAIMER

To be submitted to Physical Review

UCRL-18291
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

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ABSTRACT

For $\rho\Delta$ production the main amplitude of an $M = 1$ pion vanishes at $t = 0$, and for ρ production the $M = 1$ pion has a zero in the physical region near $|t| = t_0$ of the order of $m_\pi^{\ 2}$ in the $N\bar{N}$ vertex. We fit the sharp peak of the $\rho\Delta$ production data at 8 GeV/c with the $M = 1$ pion interfering with the A_1. Pseudo-threshold relations at $t = (m_\Delta - m_N)^2$ are essential to this fit. Even better fits are obtained with a square root zero in all vertices involving the pion at t_0, as assumed in the early np charge-exchange fits. We also achieve a fit to the ρ production data at 8 GeV/c, 4.2 GeV/c, and 2.75 GeV/c with this model. Here the daughter of the A_1 serves to compensate for the zero in the $\pi N\bar{N}$ vertex. In both reactions, to limit the number of free parameters, we restrict ourselves to t-channel amplitudes for the zero helicity ρ by fitting $\rho_{00} \frac{d\sigma}{dt}$.

INTRODUCTION

The reactions $\pi N \to \rho\Delta$ and $\pi N \to \rho N$ present an interesting challenge to Regge-pole phenomenology. Both reactions should be strongly influenced by the pion exchange, but the $M = 1$ pion used in fits to pion photoproduction[1] and np charge exchange[2] present difficulties. The $M = 1$ pion couples to the $N\bar{N}$ vertex at $t = 0$, and has a zero in the physical region at $|t| = t_0$ of order m_π^2. Both the nonzero coupling to the $N\bar{N}$ vertex and the displaced zero are essential features of the photoproduction and np charge-exchange fits. Moreover, a recent photoproduction sum rule[3] of Bietti and Roy-Chu supports the conspiring pion with a zero in its residue at $|t| = 1.5\ m_\pi^2$. Through factorization these features of the $M = 1$ pion have a profound effect on the pion contribution to ρ production and $\rho\Delta$ production.

Recent data[4] for $\pi^+ p \to \rho^0 \Delta^{++}$ at 8 GeV/c exhibit a sharp peak in the forward direction with a width approximately m_π^2. This peak is easily explained in the absorption model by the pion pole with its coupling taken from the widths of the ρ and the Δ, and an absorption factor to bring the magnitude down to fit the data at larger $|t|$. However, in the Regge-pole model, a recent argument of Le Bellac[5] shows that the helicity nonflip coupling of the $M = 1$ pion must vanish at $t = 0$ for $\rho\Delta$ production. The ingredients of the argument are that the residue for zero helicity ρ, $\beta_{\pi\rho \to N\bar{N}}$, has a $(t)^{\frac{1}{2}}$ singularity, while the $M = 1$ pion must couple to the $N\bar{N}$ vertex. As we shall show in Section IB, this residue must actually vanish for a parent trajectory (i.e., $M = 1$ pion), so the $(t)^{\frac{1}{2}}$ must be in the $\pi\pi\rho$ vertex. By factorization this implies a full factor of t in the sense residue of the π for $\pi\rho \rightarrow \overline{N}\Delta$. There seems to be a clear disagreement between the data and the dominance of the $M = 1$ pion.

We propose a pure Regge-pole exchange model which can reproduce the data with the $M = 1$ pion. To obtain the sharp peak we introduce the A_1 trajectory to interfere with the $M = 1$ pion. We further assume that the A_1 couples to the $\pi\rho$ and $\overline{N}\Delta$ channels at $t = 0$. Note that if the A_1 contribution vanishes at $t = 0$ but is finite for some other range of t, analyticity still requires an infinite daughter sequence.

The idea of an interference model is to allow the pion contribution, which is vanishing near the edge of the physical region, to grow rapidly and interfere destructively with the slowly varying contribution of another singularity. It is important to realize that t_{min} for the $\rho\Delta$ production at 8 GeV/c is about $1.4\ m_\pi^2$, so that even if there is a zero in the pion amplitude near this point (in addition to the zero at $t = 0$), interference is still possible.

When there is a square root zero in the $\pi N\overline{N}$ vertex at t_0, factorization and analyticity require this square root zero to be in all pion vertices. This zero occurs at about $1.5\ m_\pi^2$. When the zero in the $\pi N\overline{N}$ vertex is a full zero, analyticity does not demand that the zero exist in the other vertices. In this case most of the interference occurs before t_{min} and affects our fits mostly through the smearing due to the width of the ρ and the Δ. In both cases, we find that the pseudothreshold singularity in the sense amplitude for the A_1 is essential to the fit. This is true even though we have properly used the pseudothreshold relations to eliminate the pole in the formulae for the cross section. The intricacies of our fits are explained in Section II.

We do not claim that the A_1 offers the only explanation of the data within the Regge hypothesis; other poles or cuts might do as well. It would be attractive on physical grounds to have the sharp peak due to the rapid variation of the pion amplitudes, and an interference effect seems to be the only way to achieve this for the $M = 1$ pion. But, as emphasized above, even in this model the analyticity properties at pseudothreshold are at least as important as the pion pole in obtaining the sharp peak.

The data[6] for $\pi^+ p \to \rho^+ p$ at 8 GeV/c exhibit a broad peak (width $\approx 15 m_\pi^2$) with a slight turnover in the forward direction. Again the absorption model provides a fit, starting with an elementary pion that vanishes at $t = 0$. The $M = 1$ pion also vanishes at $t = 0$, but unaided it would give a dip at $t_0 \approx 1$ to $2\ m_\pi^2$ (due to the zero in the $N\bar{N}$ vertex) not observed in the data. The A_1 and its daughter give a way out of this difficulty. The daughter contributes to the same amplitudes as the pion and contributes at $t = 0$, since its residue is singular. As we explain in detail below, the interference of the π and the daughter compensates for the structure in the π residue at t_0, while reinforcing the dip at $t = 0$.

In testing this model, we have endeavored to introduce a minimum of unknown parameters. As more data become available, a more detailed fit may be justified. To this end we restricted ourselves to a fit of $\rho_{00} \frac{d\sigma}{dt}$, as this involves the pion pole but not all the helicity amplitudes. In $\rho\Delta$ production for $|t| < 0.1(\mathrm{GeV})^2$, ρ_{00} is about 0.85,[4] so that clearly these amplitudes must be mostly responsible for the sharp peak. The pseudothreshold relations at $t = (m_\Delta - m_N)^2 \approx 4m_\pi^{\ 2}$ further restrict the parameters.

For ρ production, ρ_{00} is about 0.5 and increasing for small t.[6] Since the cross section varies by two decades for $|t|$ between zero and $0.5(\mathrm{GeV})^2$ the fit is not strongly affected by the exact shape of ρ_{00} assumed. In addition, we fit the cross section of the ρ production at 4.2 and 2.75 GeV/c to check that the energy dependence can be accounted for. The intercept of the A_1 trajectory at $t = 0$, $\alpha_A(0)$, is a parameter common to both fits, and a value of $\alpha_A(0) = 0.2$ to 0.4 is suitable. By using the known couplings to the pion pole and only one exponential parameter for each trajectory, we reduce the number of parameters to three for $\rho\Delta$ and four for ρ production, plus $\alpha_A(0)$.

Clearly better fits could be obtained by allowing more free parameters, but the test of this model would be less convincing.

In Section I, we present the formalism for $\rho\Delta$ production (part A) and ρ production (part B) and in Section II, a discussion of the fits to the data.

I. FORMALISM AND PARAMETRIZATION

A. Formalism for $\rho\Delta$ Production

We define our s and t channels as

$$s: \quad \pi^+ p \to \rho^0 \Delta^{++}$$

$$t: \quad \pi^+ \rho^0 \to \bar{p}\ \Delta^{++}.$$

Since the $N\Delta$ pseudothreshold $[t = \Delta^2 = (m_N - m_\Delta)^2]$ is near the s-channel physical region, the pseudothreshold relations[7] among the t-channel helicity amplitudes that contribute to the density matrix ρ_{00} should be included in our formalism. For the kinematic singularity free amplitudes $\bar{f}^t$,

$$f^t_{\lambda_\Delta \lambda_{\bar{N}};00} = \frac{[\Phi(s,\ t)]^{\frac{1}{2}|\lambda_\Delta - \lambda_{\bar{N}}|}}{[t - (m_N + m_\Delta)^2](t - \Delta^2)^{\frac{1}{2}}}\ \bar{f}^t_{\lambda_\Delta \lambda_{\bar{N}};00} \quad , \qquad (1)$$

the constraint can be written

$$\bar{f}^t_{\frac{1}{2}-\frac{1}{2};00} = \frac{\Delta}{C(s)}\ \bar{f}^t_{\frac{1}{2}\frac{1}{2};00} + \frac{\sqrt{3}}{4\ \Delta}\, C(s)\ \bar{f}^t_{\frac{3}{2}\frac{1}{2};00}$$

$$\bar{f}^t_{\frac{3}{2}\frac{1}{2};00} = \frac{\sqrt{3}\ \Delta}{C(s)}\ \bar{f}^t_{\frac{1}{2}\frac{1}{2};00} - \frac{C(s)}{4\ \Delta}\ \bar{f}^t_{\frac{3}{2}-\frac{1}{2};00} \qquad (2)$$

when $t = \Delta^2$. In these relations we have $\Phi(s,t) = [2\, t^{\frac{1}{2}}\, P_N(t)\, P_\Delta(t)\, \sin\theta_t]^2$, the Kibble function, and $C(s) = (4t(P_N\, P_\Delta\, Z_t)|_{t=\Delta^2})^2$. These constraints remove the apparent pole at $t = \Delta^2$ in the product $\rho_{00}\,\frac{d\sigma}{dt}$. This latter quantity is given by

$$\rho_{00}\,\frac{d\sigma}{dt} = \frac{1}{64\,\pi\, s\, k^2}\,\frac{1}{t - \Delta^2}\,\frac{1}{[t - (m_N + m_\Delta)^2]}\,\{|\bar{f}^t_{\frac{1}{2}\frac{1}{2};00}|^2$$

$$+ \Phi(s,t)[|\bar{f}^t_{\frac{1}{2}\,-\frac{1}{2},00}|^2 + |\bar{f}^t_{\frac{3}{2}\,\frac{1}{2},00}|^2]$$

$$+ [\Phi(s,t)]^2\, |\bar{f}^t_{\frac{3}{2}\,-\frac{1}{2};00}|^2\} \quad . \tag{3}$$

The only trajectories that couple to the $\pi^+\rho^0$ vertex for zero helicity ρ have unnatural spin parity $[P = -(-)^J]$, $I = 1$, and negative G parity. The only mesons listed in the Rosenfeld tables[8] with these quantum numbers are $\pi(0^-)$, $A_1(1^+)$, and $\pi_A(1640)$. In our model, we include the π and the A_1 and the first daughter of A_1 denoted by d which could be identified with $\pi_A(1640)$. It is important to notice that the existence of the daughter sequence for an unequal mass channel is a consequence of analyticity at $t = 0$. Once the A_1 is assumed to couple to the $\pi N \to \rho\Delta$ amplitude, the daughter sequence is needed to cancel the singularities of the lower-order terms in the expansion of the A_1 contribution. Thus for the process $\pi N \to \rho\Delta$, the contribution of d to the cross section is cancelled at $t = 0$. However, in the process $\pi N \to \rho N$, the d trajectory will satisfy type-II[9] conspiracy in conjunction with the A_1 trajectory, and its contribution can interfere with the pion contribution.

The amplitude $f^t_{\frac{3}{2}\,-\frac{1}{2};00}$ turned out to be unimportant in fitting the data, and in order to introduce as few parameters as possible into our model we set it to zero. The pseudothreshold constraints in Eq. (2) can then be satisfied by the simple relations

$$\bar{f}^t_{\frac{1}{2}\,-\frac{1}{2};00} = \frac{\Delta}{C(s)}\,\bar{f}^t_{\frac{1}{2}\frac{1}{2};00}$$

$$\bar{f}^t_{\frac{3}{2}\,\frac{1}{2};00} = \sqrt{3}\,\bar{f}^t_{\frac{1}{2}\,-\frac{1}{2};00} \tag{4}$$

when

$$t = \Delta^2.$$

Again, in order to introduce as few parameters as possible we assume that the contributions of each pole to the above three amplitudes have the same t-behavior apart from obvious factors such as the nonsense factors in the spin-flip amplitudes. This restriction results in making the spin-flip amplitudes in Eq. (4) proportional. We thus write the cross section as

$$\rho_{00} \frac{d\sigma}{dt} = \frac{1}{64 \pi s k^2} K(t) \{|f_1|^2 + 4 \Phi(s, t)|f_2|^2\} \quad , \tag{5}$$

where

$$K(t) = (m_\pi^2 - \Delta^2)[m_\pi^2 - (m_N + m_\Delta)^2]^2/(t - \Delta^2)[t - (m_N + m_\Delta)^2]^2 \quad .$$

The Reggeized amplitudes f_1 and f_2 are written as

$$\bar{f}^t_{\frac{1}{2}\frac{1}{2};00} \propto f_1 = \sum_i \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} \gamma_1^i(t) (s/s_0)^{\alpha_i}$$

$$\bar{f}^t_{\frac{1}{2}\,-\frac{1}{2};00} \propto f_2 = \sum_i \frac{\Delta}{C(s)} \frac{1 \pm e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} \gamma_2^i(t) (s/s_0)^{\alpha_i} \tag{6}$$

with the following parametrization:

$$\alpha_\pi(t) = -m_\pi^2 + t\,\alpha'_\pi; \quad \alpha'_\pi = 1(\mathrm{GeV})^{-2}$$

$$\alpha_A(t) = \alpha_A(0) + \frac{1 - \alpha_A(0)}{m_A^2}\,t; \quad m_A^2 \approx 1.12(\mathrm{GeV})^2$$

$$\gamma_1^\pi(t) = t\,G\left(\frac{t + t_0}{m_\pi^2 + t_0}\right) e^{b_\pi (t - m_\pi^2)} \quad \alpha'_\pi$$

$$\gamma_2^\pi(t) = \Delta^2 \frac{\alpha_\pi(t)}{\alpha_\pi(\Delta^2)}\, G\left(\frac{t + t_0}{m_\pi^2 + t_0}\right) e^{b_\pi (t - m_\pi^2)} \quad \alpha'_\pi$$

$$\gamma_1^A(t) = \gamma_0^A\, \alpha_A(\alpha_A + 1)\, e^{b_A t}$$

$$\gamma_2^A(t) = \gamma_1^A(t). \tag{7}$$

The factor G can be related to the known width of the ρ and Δ by continuing the cross section to the pion pole $(t = m_\pi^2)$:

$$G = 4\pi^2\, \frac{m_\rho\, m_\Delta}{m_\pi^2} \left(\frac{6\,\Gamma_\rho\, \Gamma_\Delta}{q_\rho\, q_\Delta}\right)^{\frac{1}{2}}, \tag{8}$$

where

$$q_\rho = \frac{1}{2}(m_\rho^{\,2} - 4m_\pi^{\,2})^{\frac{1}{2}} \quad ,$$

$$q_\Delta = \frac{1}{2m_\Delta}\,[m_\Delta^{\,2} - (m_N + m_\pi)^2]^{\frac{1}{2}}\,[m_\Delta^{\,2} - (m_N - m_\pi)^2]^{\frac{1}{2}} \quad . \tag{9}$$

The factor of t in the pion sense residue, $\gamma_1^{\pi}(t)$, is a consequence of the $M = 1$ pion, first pointed out by Le Bellac (see Introduction). For the case of a square root zero at t_0 in the $\pi N\bar{N}$ vertex, analyticity and factorization also require a zero at t_0 for all residues involving the pion. Fits to np charge exchange (assuming the A_1 coupling to the $N\bar{N}$ is small) suggest that t_0 is about $m_\pi^{\,2}$. With a full zero in the $\pi N\bar{N}$ vertex, there need not be a zero in the $\pi\bar{N}\Delta$ vertex. However, one might expect a zero with a different displacement from $t = 0$ due to the $N\Delta$ mass difference. In any case, for the "full zero" fit, we place the zero at larger negative t so that it does not affect the sharp forward peak.

B. Formalism for ρ Production

We define our s and t channels as

$$s:\quad \pi^+ + p \to \rho^+ + p$$

$$t:\quad \pi^+ + \rho^- \to \bar{p} + p \;.$$

The Reggeized t-channel helicity amplitudes which contribute to the density matrix ρ_{00} are

$$\bar{\Phi}_1^{\,t} = f^t_{\frac{1}{2}\frac{1}{2};00}/K_1(t) = \sum_i \frac{(1 \pm e^{-i\pi\alpha_i})}{\sin \pi\alpha_i} \gamma_1^{\,i}(t)\,(s/s_0)^{\alpha_i}$$

$$\bar{\Phi}_2^{\,t} = f^t_{\frac{1}{2}\,-\frac{1}{2};00}/\sin\theta_t\, K_2(t) = \sum_i \frac{(1 \mp e^{-i\pi\alpha_i})}{\sin \pi\alpha_i} \gamma_2^{\,i}(t)\,(s/s_0)^{\alpha_i - 1} , \tag{10}$$

with kinematical factors defined in the s-physical region by

$$K_1(t) = \frac{m_\pi}{(-t)^{\frac{1}{2}}} \frac{m_\rho^{\,2} - m_\pi^{\,2}}{\{[t - (m_\rho + m_\pi)^2][t - (m_\rho - m_\pi)^2]\}^{\frac{1}{2}}}$$

$$K_2(t) = \frac{m_\pi}{(-t)^{\frac{1}{2}}} (1 - t/4m_N^{\,2})^{\frac{1}{2}}$$

$$|\sin\theta_t|^2 = \cos^2\theta_t - 1 = \frac{-t(s-u)^2}{(4m_N^{\,2} - t)[t - (m_\rho + m_\pi)^2][t - (m_\rho - m_\pi)^2]} - 1 . \tag{11}$$

The γ's are reduced residues with ghost-killing factors and factors of t included. Their exact parametrization is given below. The cross section is given by

$$\rho_{00}\frac{d\sigma}{dt} = \frac{1}{64\,\pi\, s\, k^2}\left[(K_1)^2\,|\bar{\Phi}_1{}^t|^2 + (\cos^2\theta_t - 1)(K_2)^2|\bar{\Phi}_2{}^t|^2\right] \quad . \tag{12}$$

There is a conspiracy condition at $t = 0$ between the kinematical singularity-free amplitudes, $\bar{\Phi}_1{}^t$ and $\bar{\Phi}_2{}^t$:

$$\bar{\Phi}_1{}^t(t = 0) = \bar{\Phi}_2{}^t(t = 0) \quad . \tag{13}$$

This relation is extremely important, since it removes the singularity at $t = 0$ just outside the physical region which comes from the factors $(K_1{}^2,\ K_2{}^2 \propto \frac{1}{t})$ in the expression for $\rho_{00}\frac{d\sigma}{dt}$ above (at 8 GeV, $t_{min} = 0.06\,m_\pi{}^2$).

Only unnatural spin-parity trajectories with negative G-parity can contribute to these amplitudes, since the $\pi\rho$ vertex has zero helicity for the ρ and G-parity minus one. Moreover, the standard consideration of spin-parity shows that $\Phi_1{}^t$ has a singlet $N\bar{N}$ vertex with $G = -(-1)^I P$, and $\Phi_2{}^t$ has a triplet $N\bar{N}$ vertex with $G = (-)^I P$. The only known[8] mesons with these quantum numbers are for $I = 1$, $\pi(0^-)$, $A_1(1^+)$, $\pi_A(1640)$, and for $I = 0$, $H(1^+)$. The conspiracy relation written for Regge parameters is

$$\gamma_1{}^i(0) = \gamma_2{}^j(0) \quad \text{and} \quad \alpha_i(0) = \alpha_j(0) - 1 \quad . \tag{14}$$

A factor of t in the π residue $\gamma_1^{\pi}(t)$ follows immediately from this relation if the pion is a parent trajectory, i.e., in the absence of a trajectory one unit above the pion with opposite signature and parity. On the other hand, the A_1 need not vanish at $t = 0$, since it contributed to Φ_2, and its daughter trajectory d one unit below can satisfy this constraint,

$$\gamma_2^{A}(0) = \gamma_1^{d}(0) \quad . \tag{15}$$

Perhaps $\pi_A(1040)$ is the first member of the daughter trajectory with $J^P = 0^-$. At any rate, in the interest of simplicity no additional trajectories are assumed for the $\pi_A(1640)$ or the $H(1^+)$. Their $t = 0$ intercept would be well below the π, if they exist at all.

The coupling of the pion at the pion pole is known in terms of $g^2_{N\bar{N}\pi}/4\pi = 14.5$ and $g^2_{\rho\pi\pi}/4\pi = 2.4$ ($\Gamma_\rho = 120$ MeV). We parameterize the pion residues with this constraint at $t = m_\pi^2$ and with a zero at $t = -t_0$ in the $\pi N\bar{N}$ vertex located according to a consistent fit of π^+ photoproduction and np charge exchange with an $M = 1$ pion.[10] The exact location of this zero is not crucial to our fit. As in the case of $\rho\Delta$ production, the pion trajectory is parameterized with slope 1 GeV^{-2}, the A_1 trajectory is linear passing through the A_1 mass and intersecting $t = 0$ at $\alpha_A(0)$, and the daughter of A_1 has a parallel trajectory. The Regge parameters are:

$$\gamma_1^{\pi}(t) = t\, G' \frac{t + t_0}{m_\pi^2 + t_0} (\alpha_\pi + 1)\, e^{b'_\pi (t - m_\pi^2)}\, \alpha'_\pi$$

$$\gamma_2^{A}(t) = \gamma^A(0) \frac{\alpha_A(t)[\alpha_A(t) + 1]}{\alpha_A(0)[\alpha_A(0) + 1]}\, e^{b'_A t}$$

$$\gamma_2^{d}(t) = \gamma^A(0) \frac{\alpha_d(t)[\alpha_d(t) + 1]}{\alpha_d(0)[\alpha_d(0) + 1]}\, e^{b'_d t} , \qquad (16)$$

where

$$G' = \frac{\pi}{2} g_{N\pi\pi}\, g_{\rho\pi\pi} \cdot \frac{m_\rho^2 - 4m_\pi^2}{m_\rho^2 - m_\pi^2} \cdot \frac{m_\rho}{m_\pi} . \qquad (17)$$

It should be noted that in the case of a square root zero in the $\pi N\bar{N}$ vertex, the above residue still must have a full zero at $t = -t_0$, i.e., a square root in each vertex $\pi\rho\pi$ and $\pi N\bar{N}$. In this case, t_0 is the same parameter as the t_0 for the $\rho\Delta$ production. Besides t_0 and $\alpha_A(0)$, there are four independent parameters for ρ production $[\gamma^A(0),\ b'_\pi,\ b'_A,\ b'_d]$ and three independent parameters for $\rho\Delta$ production $[\gamma_0^A,\ b_\pi,\ b_A]$.

II. DISCUSSION ON FITS TO $\pi^+ p \to \rho^0 \Delta^{++}$ AND $\pi^+ p \to \rho^+ p$

There are recent data at 8 GeV/c for $\rho\Delta$ production[4] and ρ production[6] cross sections as a function of $t' = |t - t_{min}|$. The variable t' is used to remove the smearing of the data due to the variation in the mass of the resonances. (For ρ production $t_{min} \approx m_\pi^2/15$ at 8 GeV/c and can be neglected). This is important for the $\rho\Delta$ production, since $t_{min} = 1.4\, m_\pi^2$ at 8 GeV/c for central masses, and the peak is very sharp with a width of about m_π^2. We also consider ρ-production data[11,12] at 4.2 GeV/c and 2.75 GeV/c and wider angle $\rho\Delta$-production data[4] out to $|t| = 0.25\,(\mathrm{GeV})^2$ at 8 GeV/c.

In both processes we fit $\rho_{00}\frac{d\sigma}{dt}$. The reason for this is simply to limit the huge number of parameters involved in the cross section. In order to do this we have assumed a smooth interpolation to the measured values of ρ_{00} (see Fig. 1a, 1b). We felt that the uncertainty in the form of ρ_{00} was not too high a price to pay for the simplification achieved. When we consider that $\frac{d\sigma}{dt}$ has three times as many amplitudes and several more trajectories (ω, φ, A_2 for ρ production, and A_2 for $\rho\Delta$ production), it is apparent that many fits to the cross section would be possible. Even with the uncertainty in ρ_{00}, the fit to $\rho_{00}\frac{d\sigma}{dt}$ is a more severe test of our model.

The recent consistent fit[10] of np charge exchange, $p\bar{p} \to n\bar{n}$, and π^+ photoproduction gives an indication of the location of the zero in the $\pi N\bar{N}$ vertex. There are two types of fits according to the type of zero assumed in the $\pi N\bar{N}$ vertex (denoted $V_{\pi N\bar{N}}$): (1) "Square-root-zero" fit $V_{\pi N\bar{N}} \propto (t + t_0)^{\frac{1}{2}}$; (2) "Full zero" fit $V_{\pi N\bar{N}} \propto t + t_0$. The best fit in the square-root-zero case gives $t_0 = 1.5 m_\pi^2$ and in the full-zero case gives $t_o = 2.5\ m_\pi^2$. However, the location of these zeros should not be taken too seriously, since the A_1 and its daughter were not included in these fits. It is possible, however, to assume that the $A_1N\bar{N}$ coupling and the $A_1\gamma\pi$ coupling are small, so that the value of t_o is changed little by the addition of the A_1 trajectory. (Note that the $A_1\gamma\pi$ coupling is not related to our $A_1\rho\pi$ coupling by vector dominance, since here we consider zero helicity ρ). In this spirit, we fixed t_o at $1.5m_\pi^2$ in all π vertices for the "square-root-zero" fit and at $2.5\ m_\pi^2$ in the $\pi N\bar{N}$ vertex for the "full-zero" fit. However, our fits are not sensitive to the precise location of the zero.

In our fits the parameter $\alpha_A(0)$ plays a crucial role. The interference of π with A_1 for $\rho\Delta$ production and π with d for ρ production goes to zero for $\alpha_A(t) = \alpha_\pi(t)$. On the other hand the energy dependence of ρ production is that of α_{eff} between 0.15 and zero (see Fig. 2). We found that the right energy dependence and sufficient interference could be achieved for $\alpha_A(0) = 0.2$ to 0.4. Since a value of $\alpha_A(0) = 0.3$ offered the best results, the discussion of the fits will be given for this intercept.

A. Fits to $\pi^+p \to \rho^o\Delta^{++}$

We obtained two distinct types of fits for this reaction, which can be classified by the type of zero assumed for the $\pi N\bar{N}$ vertex:

For the "square-root-zero" fit (Fig. 3, solid line) the zero in the π residues was fixed at $t_o = 0.03\ (\text{GeV})^2$, although the position of this zero could be shifted by about m_π^2 without destroying the fit. For the "full-zero" fit (Fig. 3, dotted line) no zero is necessary in $\pi\rho\pi$ and

$\pi\bar{N}\Delta$ vertices, but we found that a zero at $t_o = 0.15$ was convenient, although not vital to the fit. This zero is well outside the region of the sharp peak, and the form of our residue is one choice from many possible smooth parametrizations with or without zeros. Also note that the "square-root-zero" fit is more peaked due to a larger interference of the π amplitude with A_1 in f_1 (Fig. 4).

In both fits, the peak was largely produced by the rapid decrease of the A_1 amplitude in f_1. The rapid decrease is due to the pseudothreshold pole in this amplitude. It is an interesting feature of the pseudothreshold singularity that it can cause a sharp peak, even though the residue of the "pole" is zero in the cross section. This effect is achieved because the helicity-flip amplitudes are supressed in the physical region by the half-angle factors $\Phi(s,t)$, allowing the rapidly decreasing sense amplitude to dominate the cross section.

Finally, the peak was enhanced further by averaging the results over Breit-Wigner mass spectra for the ρ and the Δ. This averaging decreases the width by about 20%.

B. Fits to $\pi^+p \rightarrow \rho^+p$

For both the square root and the full zero in the $\pi N\bar{N}$ vertex, there is a full zero in the π residue for ρ production. The solid line in Fig. 5 corresponds to a "square-root-zero" fit with $t_o = 0.03 (\text{GeV})^2$. There is a slight indication of an inflection in the ρ data that is reproduced for $t_o = 0.05$ to 0.07 ("full-zero" fit, dotted line in Fig. 5), but nothing is conclusive.

From a Chew-Low extrapolation plot of $(t - m_\pi^2)^2 \frac{d\sigma}{dt}$, one can see that the data at 8 GeV/c are consistent with zero cross section at $t = 0$. In our fit this feature (the dip at $t = 0$) was reproduced qualitatively by the cancellation of the πd interference term with the A_1 contribution at $t = 0$. The Chew-Low extrapolation of the data at 8 GeV/c gives an acceptable value for the width of the ρ. We obtain $\Gamma_\rho \approx 90$ MeV for the linear extrapolation from points with $|t| < 0.15$ $(\mathrm{GeV})^2$, and higher values could be obtained particularly with the inclusion of quadratic terms in the extrapolation.

It has been pointed out that the smallness of the contribution of the pion pole to the cross section may be in contradiction to the success of the Chew-Low extrapolation. We would like to present the following arguments against such objections: Recently K. Miller has emphasized that the accuracy of the extrapolation of an analytic function $F(t)$, $F(t) = (t - m_\pi^2)^2 \frac{d\sigma}{dt}$, from the data to a point t_1 $(t_1 = m_\pi^2)$ depends both on the accuracy of the data and the strength of the bound assumed for the function.[13] Indeed, if no bound is assumed on the boundary of a complex domain which contains the data and the point t_1, the error in the extrapolation to t_1 may be arbitrarily large. In the Chew-Low extrapolation this bound is implemented by the restriction to low-order polynomials (linear or quadratic). In Regge theory such a bound is assumed for the amplitudes, hence for $F(t)$, but not necessarily for the contributions of the individual Regge poles. The size and the shape of the residue of the pion Regge poles in the region of the data are irrelevant to the accuracy of the extrapolation as long as $F(t)$ is analytic and bounded.

We may illustrate this point somewhat differently. The function $\frac{d\sigma}{dt}$ has a double pole at $t = m_\pi^2$ and can be written as $\frac{g^4}{(t - m_\pi^2)^2} + B(t)$. With the proper bound and analyticity of $\frac{d\sigma}{dt}$, the determination of g^4 by extrapolation is accurate as long as this residue is not very small. (Note that in our model, as well as in other models such as absorption, one fixes the value of g^4 by the experimental ρ-width and the πN coupling constant). It is clear that if we rewrite $\frac{d\sigma}{dt}$ as $\frac{\beta^2(t)}{(t - m_\pi^2)^2} + B'(t)$ with $\beta^2(m_\pi^2) = g^4$ such that $\beta(t)$ is small in the region of the data, we have not changed any properties of $\frac{d\sigma}{dt}$ relevant to the Chew-Low extrapolation.

A more complete investigation of this model should include fits with the A_1 to np charge exchange, $p\bar{p} \to n\bar{n}$, and $\pi^\pm$ photoproduction. In fact, there exists a discrepancy for the "square-root-zero" case between the fits to NN scattering and photoproduction that could be removed by a moderate A_1 and d contribution. For $t_o = 0.03\,(\text{GeV})^2$, this discrepancy[10] occurs between the values of $g^2/4\pi$ used in NN scattering ($g^2/4\pi = 13$) and photoproduction ($g^2/4\pi = 16.8$). Also for the "full-zero" fit, the present value of $t_o = 0.05$ is in disagreement with the value $t_o = 0.03$ of the photoproduction sum rule. Again the inclusion of A_1 and d in the high-energy fits, and the daughter trajectory in the sum rule could bring agreement. Since the Regge contribution to the sum rule is $\beta^\pi N^\alpha + (t - m_\pi^2)\, \beta^d N^{\alpha_d}$, we would not expect the new term to displace the zero far from $t_o = 0.03$. Also the photoproduction fits will not be strongly affected, since the A_1 and its daughter do not contribute in leading order at $t = 0$.

In conclusion, we would like to emphasize that this model is not intended to be the unique Regge-pole solution. Indeed, considerably better fits for $\rho\Delta$ production could be obtained if A_1 were replaced by an interfering amplitude which was more in phase with the pion. Then the pion would more effectively give the peak by interference. Moreover, it is possible that the explanation of the data discussed here lies in an $M = 0$ pion whose contribution is modified by other singularities such as cuts. The main point of this paper has been to present a reasonable pure pole model which would show that factorization and the hypothesis of an $M = 1$ pion is not in contradiction with the recent ρ-production and $\rho\Delta$-production data.

ACKNOWLEDGMENT

We wish to thank Professor Geoffrey Chew, Professor J. D. Jackson and Dr. Chung-I Tan for their helpful advise in numerous discussions.

FOOTNOTES AND REFERENCES

* This work was supported in part by the United States Atomic Energy Commission.

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Table I. Parameters for $\pi^+p \to \rho^0\Delta^{++}$ fits with the A_1 intercept $\alpha_A(0) = 0.3$. The pion contribution has been normalized by the constant $G = 1950$ in Eq. (8).

Parameters	"Square root zero" fit	"Full zero" fit
Zero in π residue at $\|t\|=t_o$:	0.03 (fixed)	0.15 (fixed)
A_1 residue γ_o^A :	0.20 G	-0.22 G
π exponential b_π:	12.0 GeV^{-2}	7.5 GeV^{-2}
A_1 exponential b_A:	4.1 GeV^{-2}	4.8 GeV^{-2}
χ^2/No. of data points:	38/44	42/47

Table II. Parameters for $\pi^+p \to \rho^+p$ fits with the A_1 intercept at $\alpha_A(0) = 0.3$. The pion contribution normalized by the constant $G' = 570$ in Eq. (17).

Parameters	"Square root zero" fit	"Full zero" fit
Zero in $\pi N\bar{N}$ vertex at $\|t\|=t_o$:	0.03 (fixed)	0.05 (fixed)
A_1 residue $\gamma^A(0)$:	0.39 G'	0.32 G'
π exponential b'_π:	5.1 GeV^{-2}	5.1 GeV^{-2}
A_1 exponential b'_A:	1.3 GeV^{-2}	-2.0 GeV^{-2}
d exponential b'_d:	4.7 GeV^{-2}	3.8 GeV^{-2}
χ^2/No. of data points:	20/28	24/28

FIGURE CAPTIONS

Fig. 1a. Density matrix ρ_{00} at 8 GeV/c for $\pi^+ p \to \rho^o \Delta^{++}$,[4] where the line is the interpolation used in our fit.

Fig. 1b. Density matrix ρ_{00} at 8 GeV/c for $\pi^+ p \to \rho^+ p$.[6] The smooth line interpolation was used in our fit, with $\rho_{00} = 0.45$ for $|t| > 0.1$ [I. Derado, J. A. Poirier, N. N. Biswas, N. M. Cason, V. P. Kenney, and W. D. Shephard, Phys. Letters 24B, 112 (1967)]. The exact shape of ρ_{00} is not crucial to our fits.

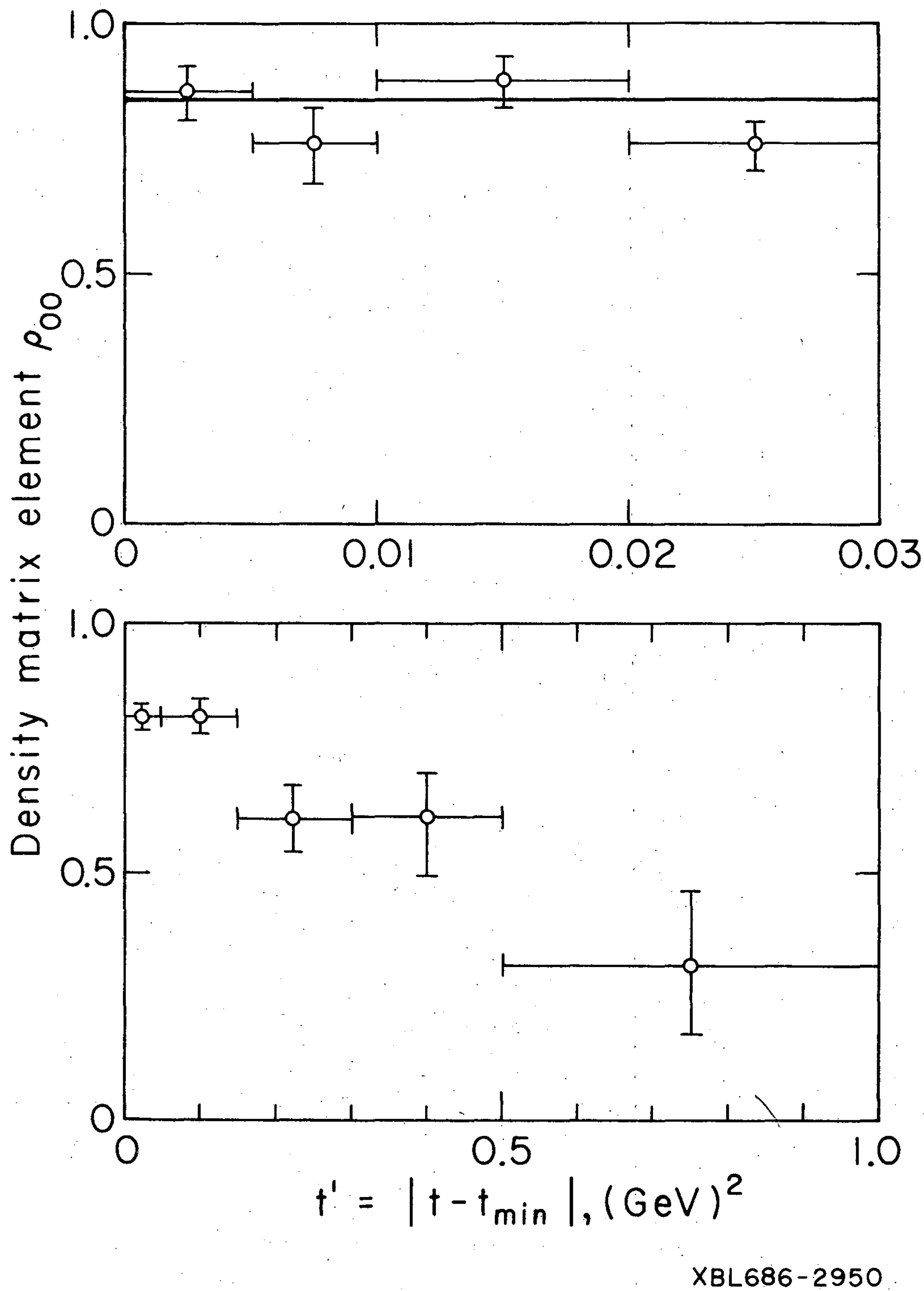
Fig. 2. The energy dependence of our predicted longitudinal cross section for $\pi^+ p \to \rho^+ p$, i.e., $\int dt \left[\rho_{00} \frac{d\sigma}{dt}\right]$, compared with the experimental measurements for the total cross section. [O. Morrison, CERN TC Division Report. CERN/TC/Physics 66-20 (1966).] Solid line is normalized to go through the point at 8 GeV/c.

Fig. 3. "Square-root-zero" fit (solid line) and "full-zero" fit (dotted line to $\rho_{00} \frac{d\sigma}{dt}$ for $\pi^+ p \to \rho^o \Delta^{++}$ at 8 GeV/c.[4]

Fig. 4. Amplitudes for the "square-root-zero" fit to $\rho\Delta$ production. The phase of the A_1 in f_1 is measured relative to the pion in f_1, and all contributions to the magnitude of f_2 are included in the line labelled F_2. Hence, $\rho_{00} \frac{d\sigma}{dt} (\pi^+ p \to \rho^o \Delta^{++}) = |\pi + \mathrm{Re}\, A_1|^2 + |\mathrm{Im}\, A_1|^2 + |F_2|^2$.

Fig. 5. "Square-root-zero" fit (solid line) and "full-zero" fit (dotted line) to $\rho_{00} \frac{d\sigma}{dt} (\pi^+ p \to \rho\, p)$ at 8 GeV/c.[6]

Fig. 6. The "square-root-zero" fit to $\rho_{00} \frac{d\sigma}{dt} (\pi^+ p \to \rho^+ p)$ (dotted line) at 4.2 GeV/c[11] with the pion contribution $|\pi|^2$, the A_1 contribution $|A_1|^2$, and the πd interference term $\rho_{00} \frac{d\sigma}{dt} (\pi^+ p \to \rho^+ p) = |\pi|^2 + \pi d + |A_1|^2$.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0
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0.02
0.03
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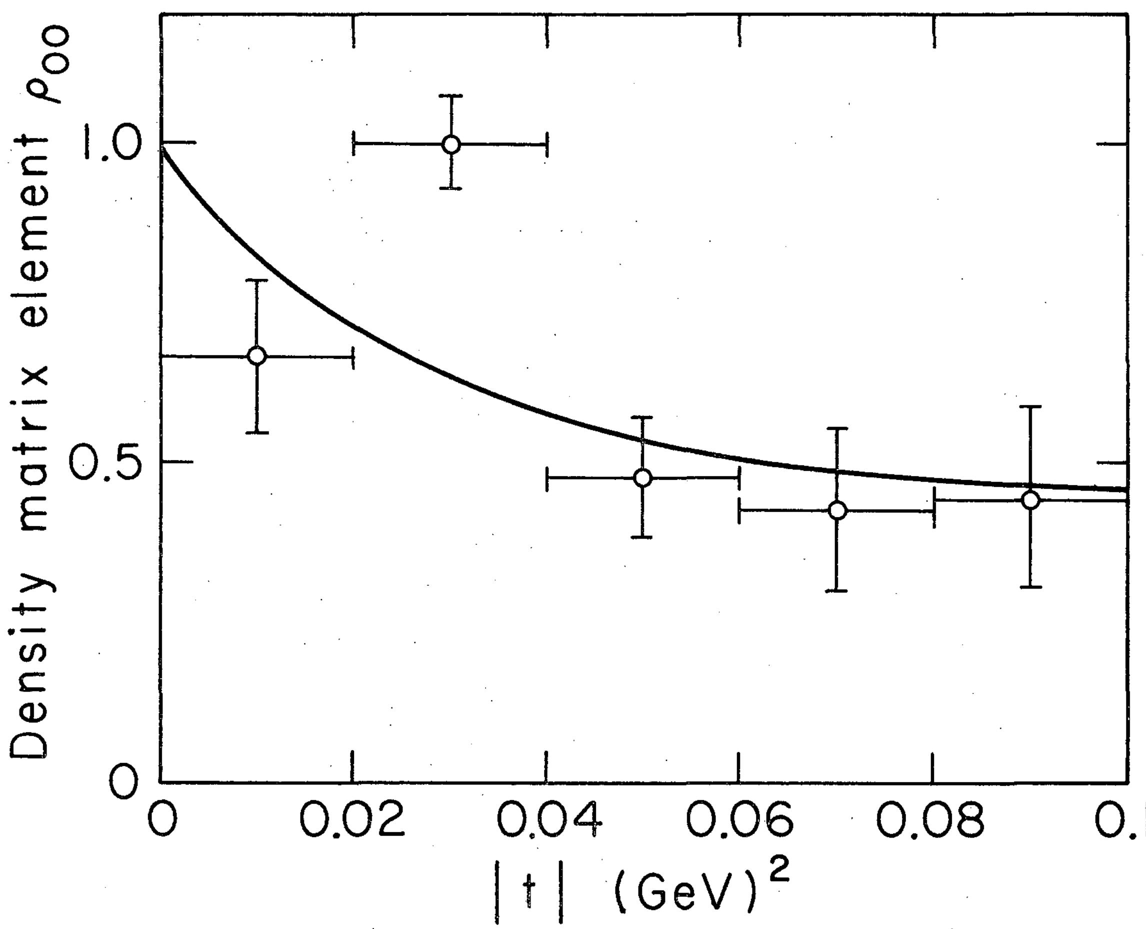
1.0
t' = |t - t_min|, (GeV)²
XBL686-2950

Fig. 1a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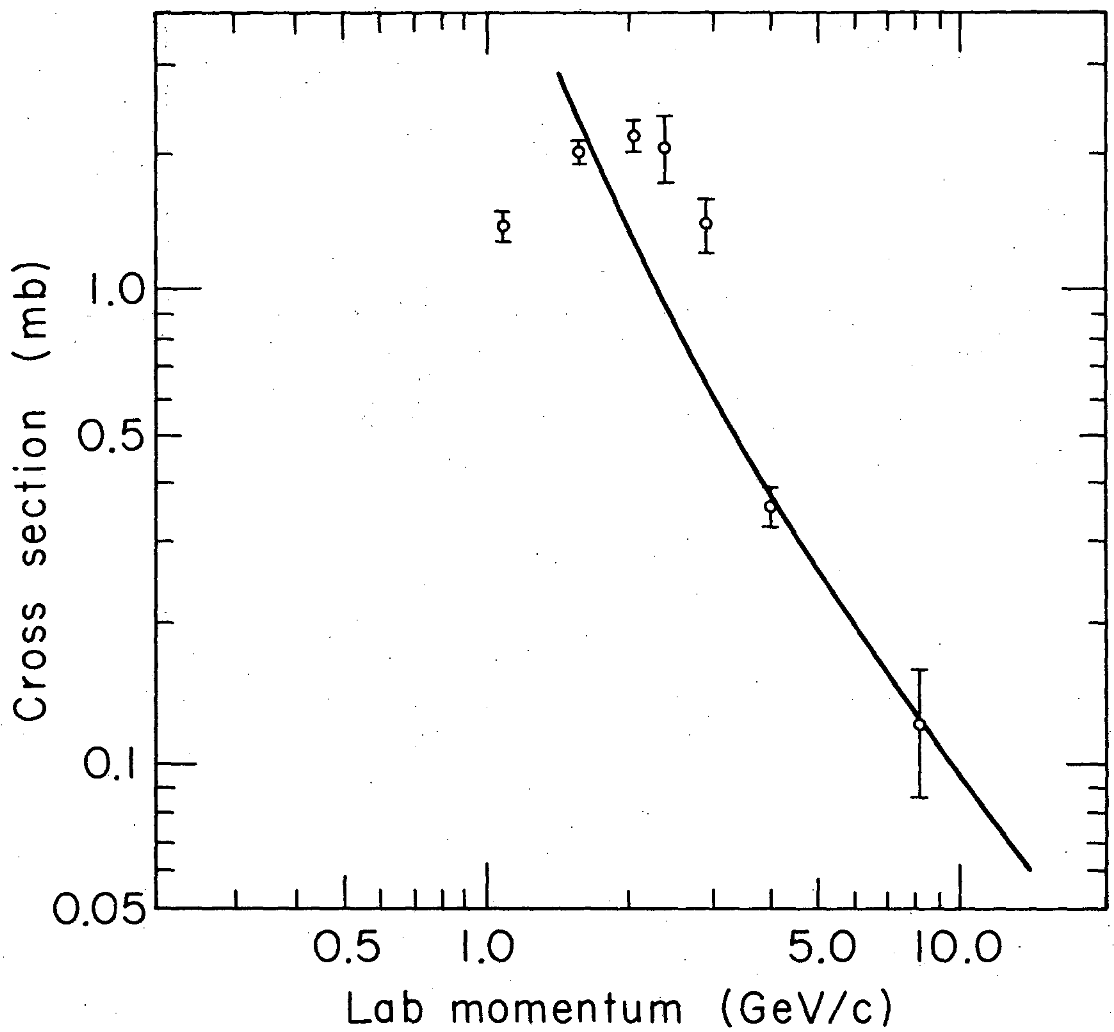
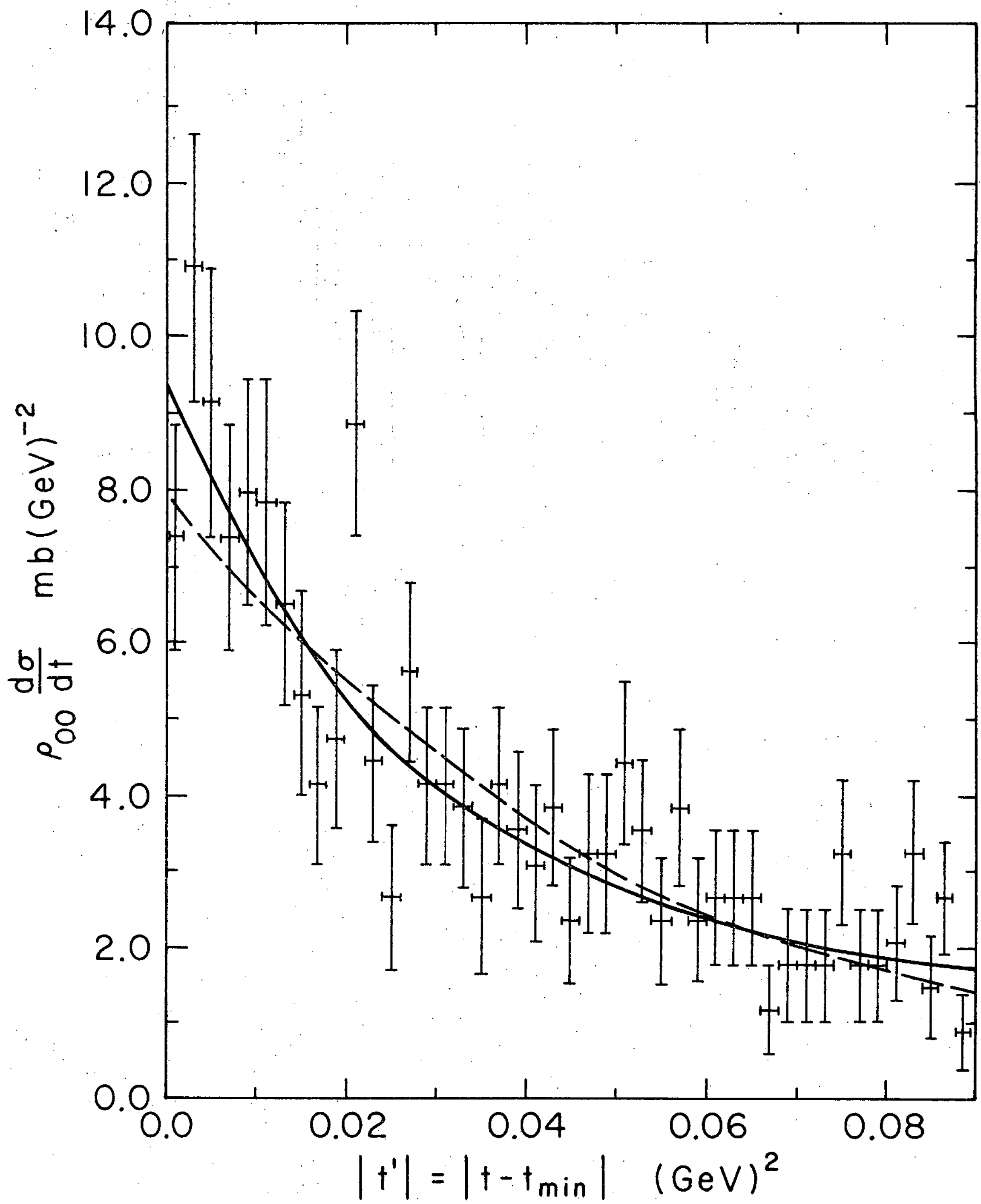
|t| (GeV)^2
XBL686-2945

Fig. 1b



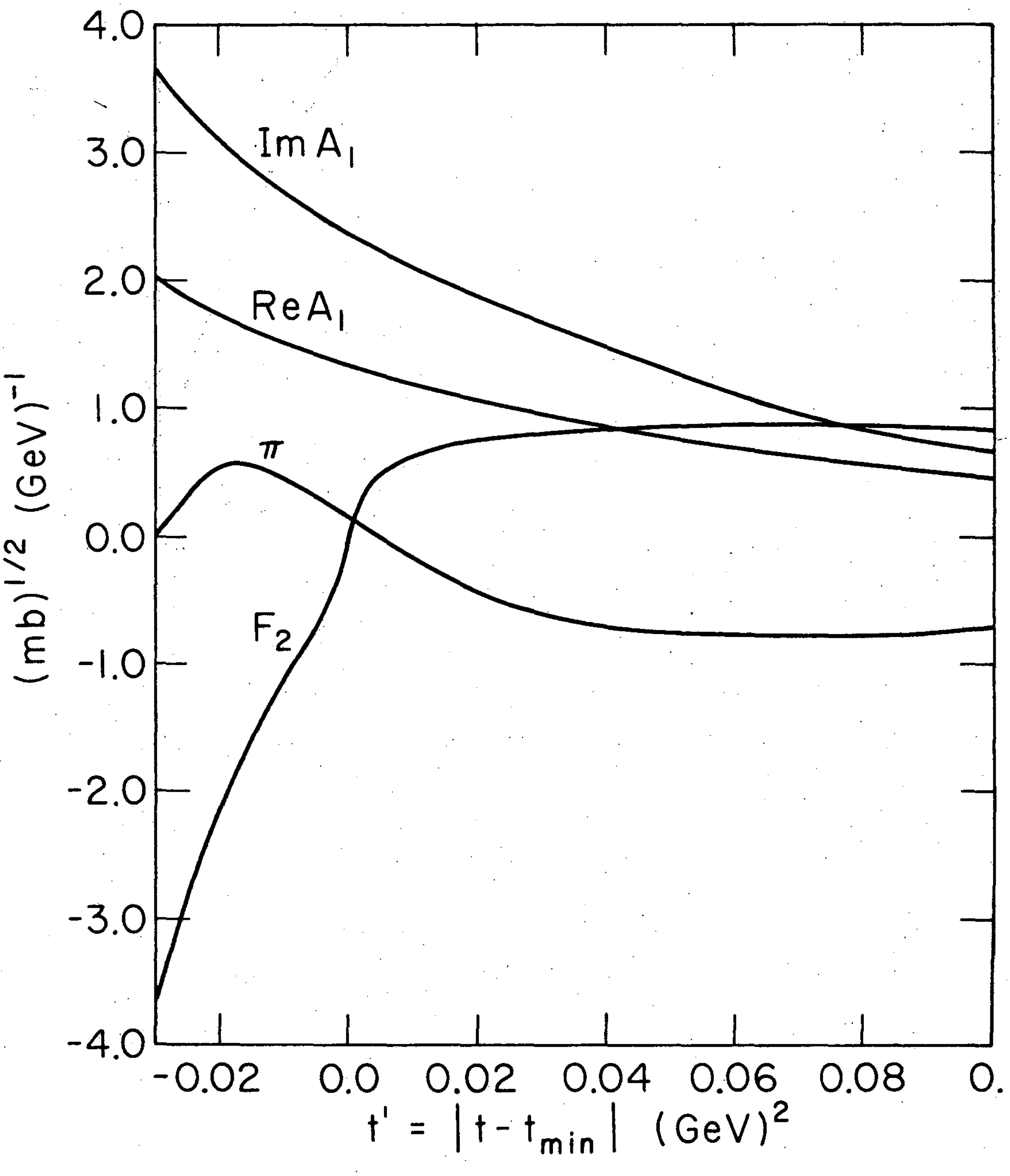
XBL686-2949

Fig. 2



XBL686-2981

Fig. 3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3.0
2.0
1.0
0.0
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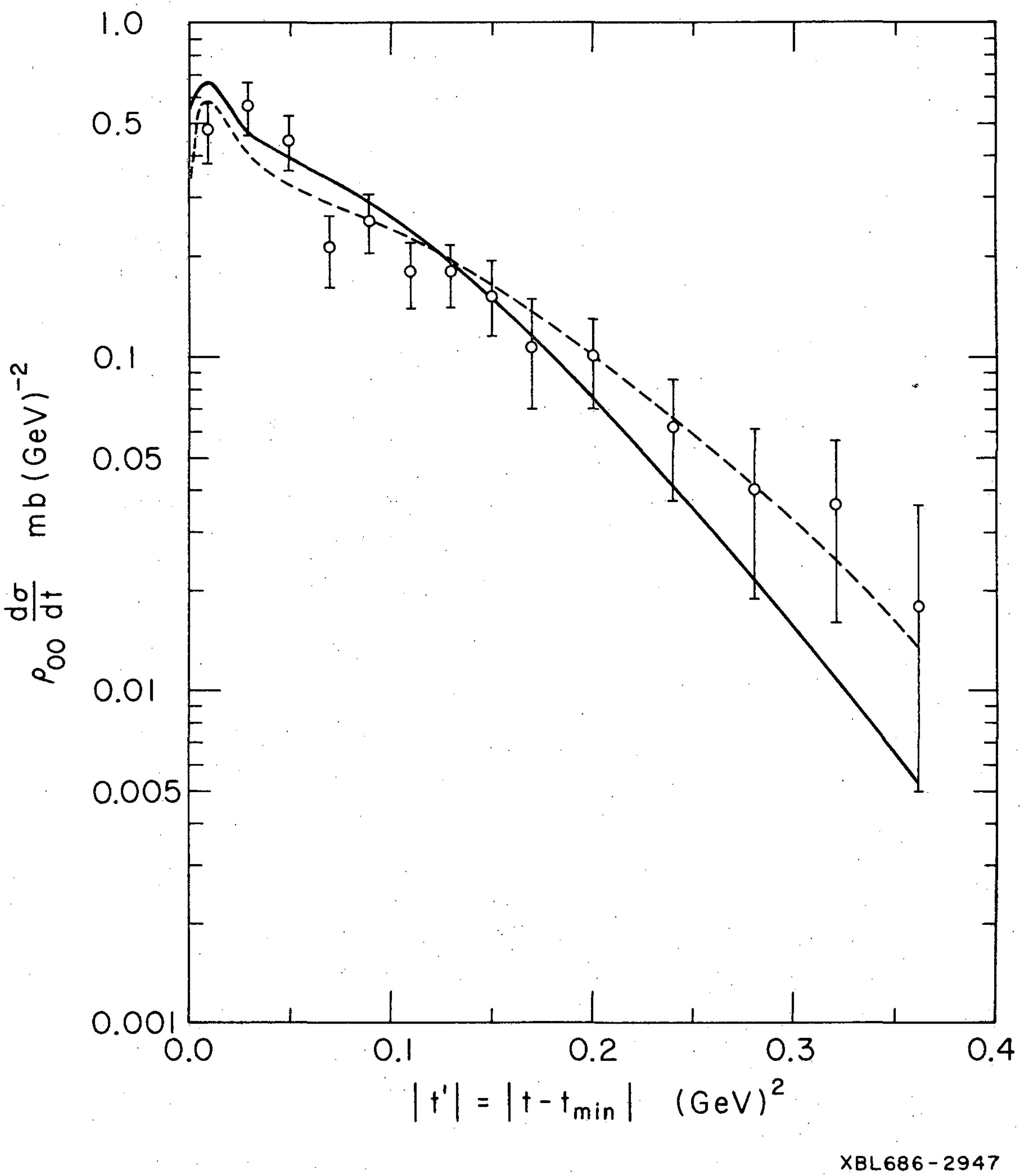
-2.0
-3.0
-4.0
Im A1
Re A1
π
F2
(mb)^1/2 (GeV)^-1
-0.02
0.0
0.02
0.04
0.06
0.08
0.1
t' = |t - t_min| (GeV)^2
XBL686-2946

Fig. 4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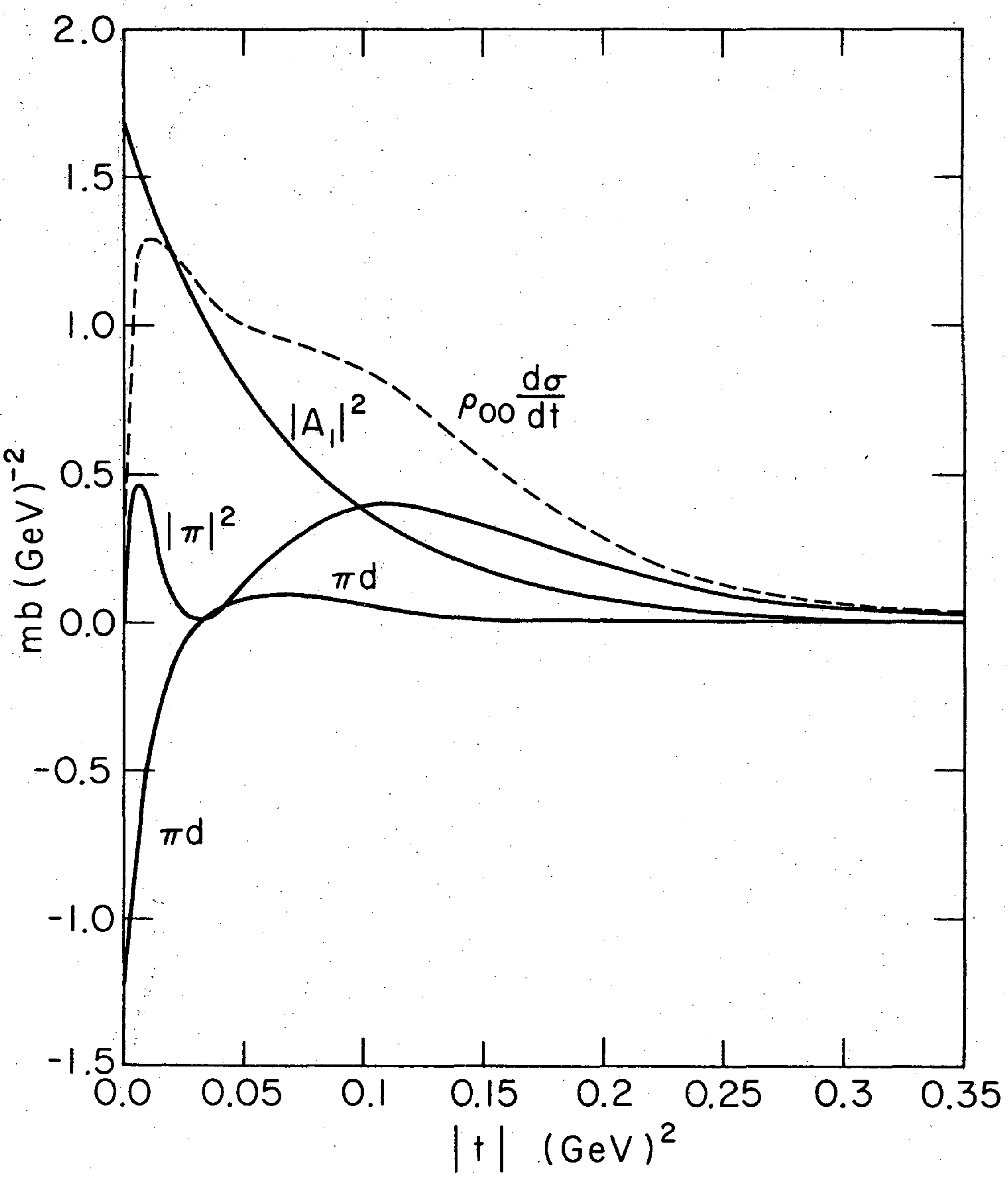
0.05
0.01
0.005
0.001
0.0
0.1
0.2
0.3
0.4
$\rho_{00} \frac{d\sigma}{dt}$ mb (GeV)$^{-2}$
$|t'| = |t - t_{min}|$ (GeV)^2
XBL686-2947

Fig. 5



XBL686-2948

Fig. 6