An Eigenfilter Approach to Design Two-Dimensional Zero-Phase FIR Filters via McClellan Transform

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Abstract

In this paper, we describe a new method for designing two-dimensional filters with the McClellan transform. It is well known that McClellan transform can simplify the two-dimensional filter design problem into a one dimensional problem. Therefore, the two separate problems to design 2-D filter are: design of a 1-D prototype filter and design of a 2-D transformation function. A method using eigenfilters to design the 2-D transformation function is proposed, which unifies and simplifies the design problem. The efficiency of the method is shown through design examples.

1 Introduction

The two dimensional filter design has been of growing interest due to the various applications in the areas such as image processing, radar, sonar and seismic data processing. The FIR filter has been receiving more attention than the IIR filter, since linear or zero phase and stability constraints can be satisfied easily. The design approaches can be generally classified into two categories. One is the transform method which is related to the McClellan transform, that is, design a 1-D prototype filter and transform it to 2-D filter by the McClellan or generated McClellan transform [2]. The other one is based on the min-max or least square approaches [10], [11].

The transform technique has been widely used due to its simplicity and efficiency. The key problem in the transform method is to devise techniques for fast and efficient calculation of the coefficients of the first-order or high-order original or generated McClellan transform so that a given contour in the (ω_1, ω_2) plane can be best matched. An almost circular symmetric filter was first introduced by McClellan [1]. In [2] ,the design of 2-D filters with elliptical magnitude response of arbitrary orientation was given by Psarakis and Moustakides. In [3] and [4], design methods for quadrantal and centrally symmetric 2-D FIR fan filters were proposed. All these methods are separated and can be used to design only one specific shaped filter or design a particular transform to match a specific (ω_1, ω_2) plane contour. In [5], Psarakis and Moustakides proposed a unified optimization method to design the scaled transform directly, but the final optimization problem is not a easy problem.

The purpose of this research is to find a unified approach for simple and fast designing the coefficient of McClellan transform such that it can best match the any kind of (ω_1, ω_2) plane contour. We design the unscaled transform first, which finally leads to simple eigenfilter approach. Scaled transform with same isopotentials was generated at the end of the design by simple scaling process. The design process described in this paper is focus on the first-order original method. Same method can be used for high-order or generated McClellan transform design. Section two contains a brief review of the theory behind the McClellan transform. Section three describes our method to choose the optimum coefficient of the original McClellan transform for best matching the (ω_1, ω_2) plane contour. Section four gives some examples to show the simplicity and efficiency of this design method.

2 Review of the McClellan Transform

The detail of the McClellan transform can be found in several references[5],[6]. Here, the basic background is stressed for the continuity and the better understanding of later sections.

Let $h_1(n) = h_1(-n)$ be the impulse response of a 1-D zero phase odd length filter with nonzero samples from $-N \leq n \leq N$. Because of even symmetry, the Fourier transform can be expressed as

$$H_1(\omega) = h_1(0) + 2\sum_{n=1}^{N} h(n)cos\omega n. \tag{1}$$

Expressing $cos\omega n$ as a Chebychev polynomial,

$$cos\omega n = T_n(cos\omega) \tag{2}$$

where the Chebechev polynomials are given by the recursion

$$T_0(x) = 1, \quad T_1 = x, \quad \cdots, \quad T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) \tag{3}$$

(1) becomes

$$H_1(\omega) = h(0) + 2\sum_{n=1}^{N} T_n(cos\omega) \tag{4}$$

The transform uses the relation

$$cos(\omega) = F(\omega_1, \omega_2) \tag{5}$$

to generate the 2-D filter with frequency response

$$H(\omega_1, \omega_2) = h(0) + 2\sum_{n=1}^{N} T_n(cos\omega)|_{cos(\omega)=F(\omega_1,\omega_2)} \tag{6}$$

Where ω is the 1-D frequency, (ω_1, ω_2) is the 2-D frequency pair, and $F(\omega_1, \omega_2)$ can be either original McClellan transform F_{OM} for quadrntal 2-D filter design or generalized McClellan transform [2] F_{GM} for centrally symmetric filter design, as following:

$$F_{OM}(\omega_1, \omega_2) = \sum_{i=0}^{N}\sum_{j=0}^{N} t_{ij}cos(i\omega_1)cos(j\omega_2) \tag{7}$$

$$F_{GM}(\omega_1, \omega_2) = \sum_{i=0}^{N}\sum_{j=0}^{N} t_{ij}cos(i\omega_1)cos(j\omega_2) + \sum_{k=0}^{N}\sum_{l=0}^{N} t_{kl}sin(k\omega_1)sin(l\omega_2) \tag{8}$$

A necessary condition that $F(\omega_1, \omega_2)$ must satisfy is

$$-1 \leq F(\omega_1, \omega_2) \leq 1 \quad \forall\omega_1, \omega_2 \in [-\pi, \pi] \tag{9}$$

Usually, we like some points in (ω_1, ω_2) to satisfy the equalities in order to cover the entire 1-D frequency band.

The basic property used in the transform method is that the isopotentials of transform $F(\omega_1, \omega_2)$ are also the isopotentials of the frequency response of the 2-D filter. This is due to an isopotential of the transform corresponds to a constant $cos(\omega)$.

In this paper, we concentrate on the first order original McClellan transform

$$F(\omega_1, \omega_2) = t_{00} + t_{10}cos(\omega_1) + t_{01}cos(\omega_2) + t_{11}cos(\omega_1)cos(\omega_2) \tag{10}$$

The similar method can be used in the high order or generalized McClellan transform.

3 Design Criterion

Nguyen and Swamy *et al* [8] derived a normalized procedure that can be used to convert any real 2-D function $F(\omega_1, \omega_2)$ into a valid 2-D tangential filter with the same isopotentials.

$$f(\omega_1, \omega_2) = \frac{2F(\omega_1, \omega_2) - (F_{max} + F_{min})}{F_{max} - F_{min}} \tag{11}$$

Therefore, we can scale $F(\omega_1, \omega_2)$ at the very end of the design to $f(\omega_1, \omega_2)$.

For any 2-D filter with a contour C as a separate curve between the passband and the stopband region, we would like the transform $F(\omega_1, \omega_2)$ to be a constant in the specific contour C as expression(12).

$$F(\omega_1, \omega_2)|_{(\omega_1, \omega_2) \in C} = constant \tag{12}$$

In most case, the right side of above equation to be exactly constant is impossible. However, we can optimize the coefficients in (10) to best match the contour C. Using Psarakis and Moustakides proposed optimization approach [5]. The mean value of $F(\omega_1, \omega_2)$ on the cut off contour C is

$$\bar{F} = \frac{1}{L} \oint_c F(\omega_1, \omega_2) ds \tag{13}$$

where

$$L = \oint_c ds \tag{14}$$

the variance is

$$Var(F) = \oint_c [F(\omega_1, \omega_2) - \bar{F}]^2 ds \tag{15}$$

The set of coefficients $(t_{00}, t_{10}, t_{01}, t_{11})$ which minimizes the variance $Var(F)$ is best approximation of (12). Equation(13) and (15) can be further written as

$$\bar{F} = t_{00} + A_{10}t_{10} + A_{01}t_{01} + A_{11}t_{11} \tag{16}$$

and

$$Var(F) = \oint_c [D_{10}(\omega_1, \omega_2)t_{00} + D_{01}(\omega_1, \omega_2)t_{01} + D_{11}(\omega_1, \omega_2)t_{11} \tag{17}$$

Where

$$A_{10} = \frac{1}{L} \oint_c cos(\omega_1) ds \tag{18}$$

$$A_{01} = \frac{1}{L} \oint_c cos(\omega_2) ds \tag{19}$$

$$A_{11} = \frac{1}{L} \oint_c cos(\omega_1) cos(\omega_2) ds \tag{20}$$

and

$$D_{10}(\omega_1, \omega_2) = cos(\omega_1) - A_{10} \quad (21)$$
$$D_{01}(\omega_1, \omega_2) = cos(\omega_2) - A_{01} \quad (22)$$
$$D_{11}(\omega_1, \omega_2) = cos(\omega_1)cos(\omega_2) - A_{11} \quad (23)$$

Define the vectors

$$D^T(\omega_1, \omega_2) = [D_{10}(\omega_1, \omega_2)\ D_{01}(\omega_1, \omega_2)\ D_{11}(\omega_1, \omega_2)] \quad (24)$$

and

$$t^T = [t_{10}\ t_{01}\ t_{11}] \quad (25)$$

Then, $Var(F)$ can be expressed as

$$Var(F) = t^T Q t \quad (26)$$

here

$$Q = \oint_c D(\omega_1, \omega_2) D^T(\omega_1, \omega_2) ds \quad (27)$$

Notice that Q is real symmetric and positive-definite matrix. By Rayleigh's principle [9], the eigenvector corresponding to the smallest eigenvalue of matrix Q is the designed coefficients of the transform (10).

4 design examples

The detailed design procedure is as follows:

1. Using (14), (18)-(24) and (27) to calculate the matrix Q. By Rayleigh's principle, obtain the coefficients (t_{10}, t_{01}, t_{11}). Notice that all the calculation can be completed by *Mathematica*.
2. Using (11) to scale the $F(\omega_1, \omega_2)$ to $f(\omega_1, \omega_2)$, which is valid transform. calculate the $\bar{f}$ and get ω_0 which equals to $arccos(\bar{f})$.
3. Using some 1-D filter design approach, design a zero-phase FIR filter with cut off frequency ω_0.
4. Substitute the $cos\omega$ in (4) with $f(\omega_1, \omega_2)$ to get the designed 2-D frequency filter

Diamond Shape 2-D Low-Pass Filter Design

In this example, we follow the design procedure to design a diamond shape 2-D low-pass filter with the cut-off contour as Figure 1(a). From step 1, we get the coefficients (t_{10}, t_{01}, t_{11}) as $(0.613929, 0.613929, -0.496169)$. After the scaling based on equation (11), we have the scaled transform $0.4041 + 0.5cos\omega_1 + 0.5cos\omega_2 - 0.4041cos\omega_1 cos\omega_2$. The isopotentials of this transform is shown as Figure 1(b).

From step two, by (16) we know the ω_0 is 0.4224. By step three, we use Parks-McClellan method in the *Matlab* to get the equaripple low-pass filter with cut-off frequency ω_0 0.4224. Finally, substitute the $cos\omega$ in (4) with $f(\omega_1, \omega_2)$, we get the designed 2-D diamond shape filter as Figure 2.

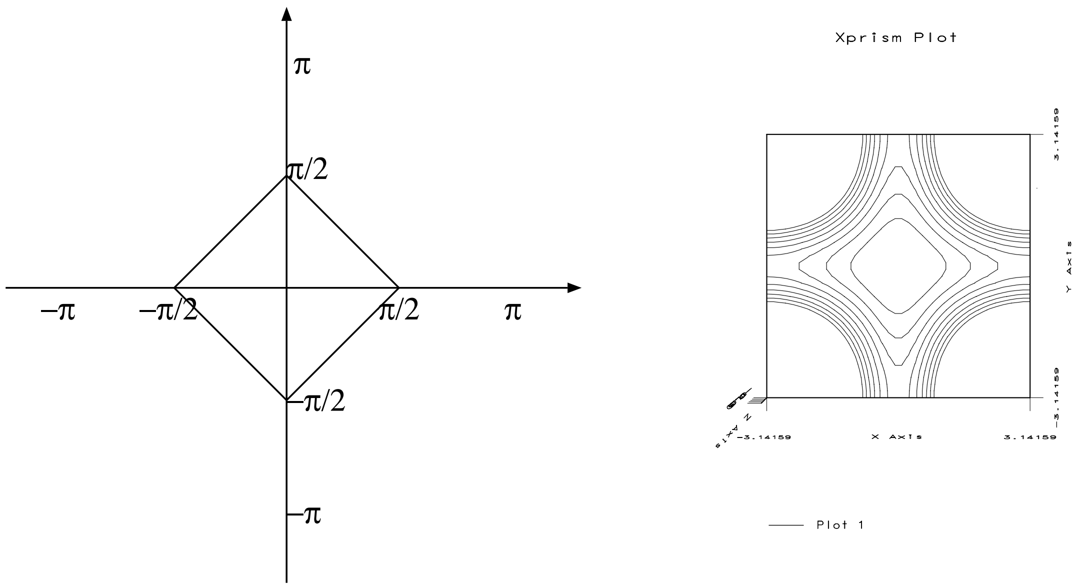

Figure 1: (a)left: Desired contour. (b)right: Isopotentials of designed transformation.

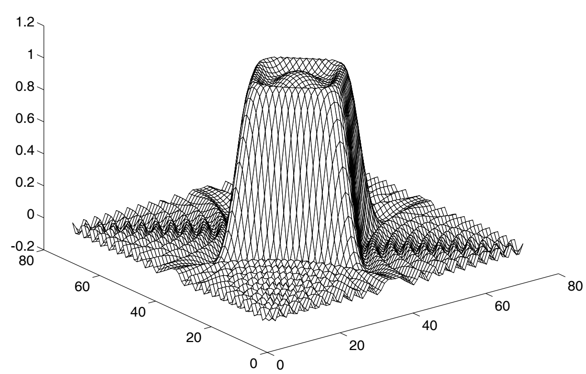

Figure 2: The designed 2-D diamond shape low-pass filter

Quadrantal Fan Filter Design

In this example, we design a quadrantal fan filter with desired contour as Figure 3(a). The coefficients (t_{10}, t_{01}, t_{11}) to minimize the equation(26) has been found as $(0.529267, -0.819986, -0.217946)$. The scaled transform is $-0.16153 + 0.392266 cos\omega_1 - 0.607732 cos\omega_2 - 0.161531 cos\omega_1 cos\omega_2$, which has the isopotentials as Figure 3(b). The final designed fan filter as Figure 4.

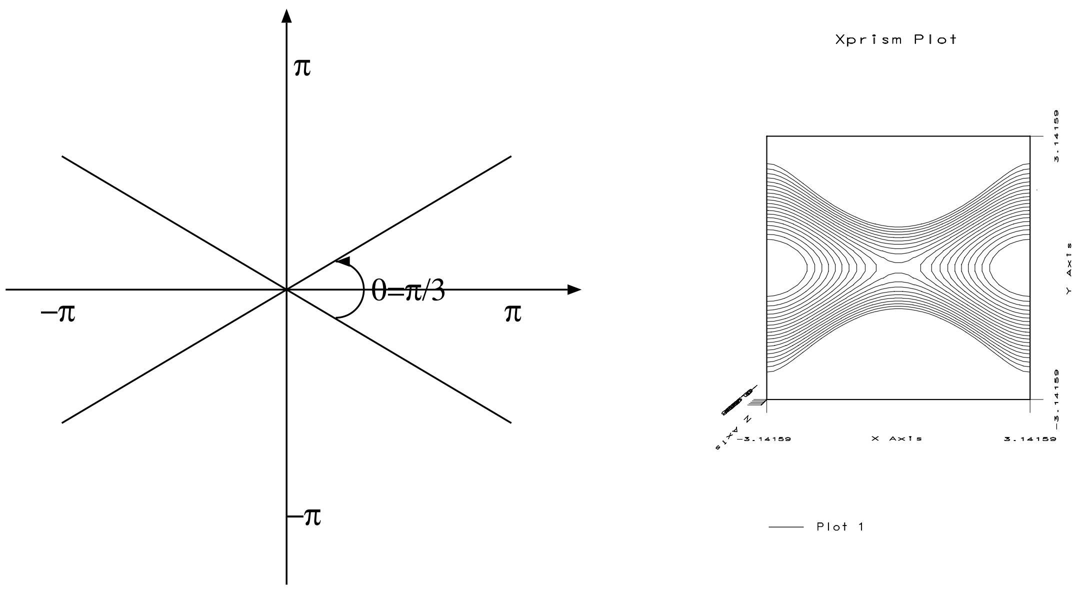

Figure 3: (a)left: Desired contour. (b)right: Isopotentials of designed transformation.

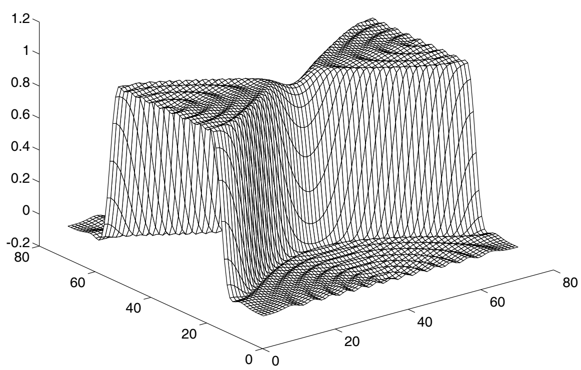

Figure 4: The designed fan filter

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