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VARIANCE AND REGRESSION ANALYSES OF MOYER MODEL PARAMETER DATA AND THEIR VARIATION WITH PRIMARY PROTON ENERGY

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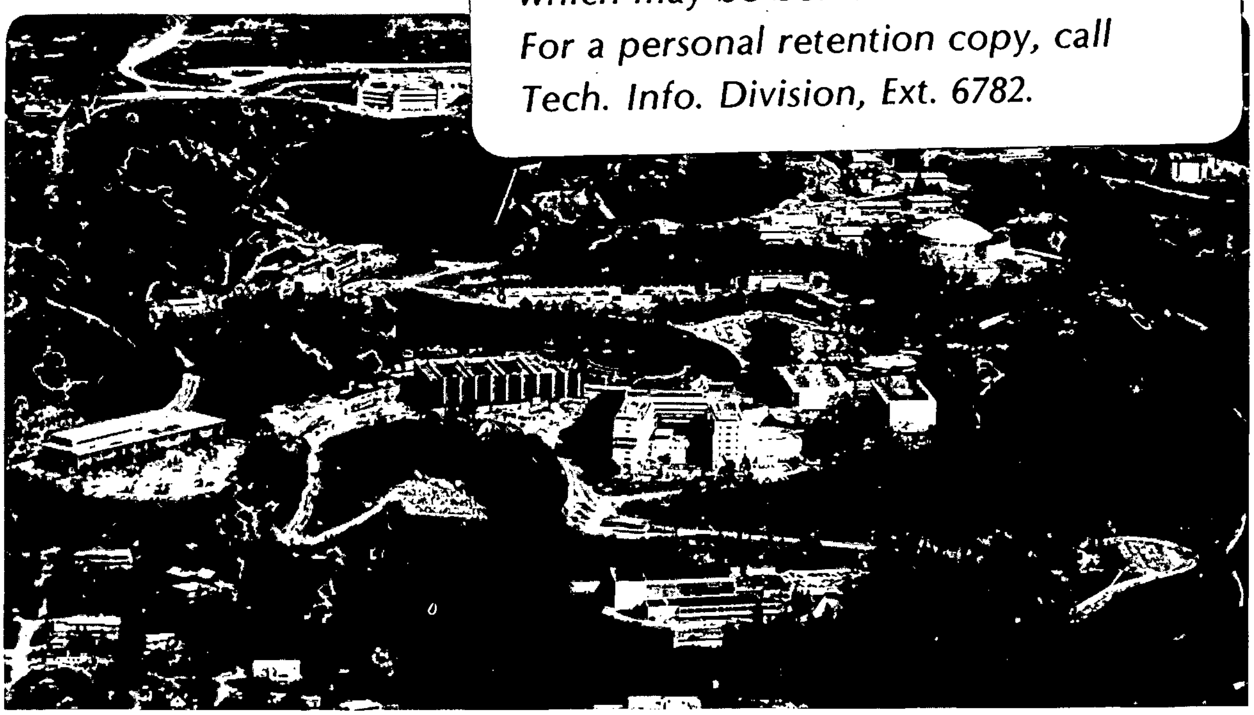

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VARIANCE AND REGRESSION ANALYSES OF MOYER MODEL PARAMETER DATA AND THEIR VARIATION WITH PRIMARY PROTON ENERGY

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ABSTRACT

Experimental values of the Moyer Model Parameter, H_0, are summarized and presented as a function of proton energy, E_p. The variation of $H_0(E_p)$ with E_p is studied by regression analysis. Regression Analysis of the data under log-log transformation gives the best value for the exponent m of 0.77 ± 0.26, but a t-test did not reject m = 1 (p ± 20 percent). Since m = 1 was not excluded, and a Fisher's F-test did not exclude linearity, a linear regression analysis was performed. A line passing through the origin was not rejected (Student's t-test, p = 30 percent) and has the equation: $H_0(E_p) = (1.61 \pm 0.19) \times 10^{-13}$ Sv m^2/GeV to be compared with a value of $(1.65 \pm 0.21) \times 10^{-13}$ Sv m^2/GeV published by Stevenson et al. (St 82).

VARIANCE AND REGRESSION ANALYSES OF MOYER MODEL PARAMETER DATA AND THEIR VARIATION WITH PRIMARY PROTON ENERGY

"Experience Joined With Common Sense,
to Mortals is a Providence"

"The Spleen"
Matthew Green 1696–1737

INTRODUCTION

Stevenson et al. (St 82) have recently reviewed the available experimental determinations of the Moyer model parameter, H_0 (Mo 62).

H_0 is defined by the equation:

$$H = (1/r^2)\ H_0 \exp(-\beta\theta) \exp(-d/\lambda) \tag{1}$$

where H is the dose equivalent on the shield surface per interacting proton and the symbols r, θ and d are explained in Figure 1. The angular distribution parameter, β, and the attenuation length, λ, are well determined both by theoretical and experimental means (Pa 73, St 82).

The empirically determined values of H_0 are summarized in Table 1.

H_0 is a function of the primary proton energy and it is the statistical analysis of the data of Table 1, both by analysis of variance and by regression analysis, in order to determine the functional form of this variation, that this paper describes.

Table 1. Summary of published values of moyer model parameters $H_0(E_p)$.

Primary Proton Energy, $[E_p]$ (GeV)	Moyer Parameter, $[H_0(E_p)]$ ($Sv\ m^2$)		Source
7.4	1.4	10–12	Sh69, St69
7.4	2.1	10–12	Sh69, St69
10.0	0.96	10–12	Ho 66
13.7	2.5	10–12	Gi 68
13.7	3.1	10–12	Gi 68
21.0	1.6	10–12	Ho 79
23.0	3.5	10–12	Ma 79
25.5	3.3	10–12	Gi 68
25.5	5.0	10–12	Gi 68
25.5	6.6	10–12	Ro69, St82
30.0	3.4	10–12	Aw 70

2. ENERGY VARIATION OF H_o

It is important to understand the variation of H_o with proton energy, both so that the experimental determinations of H_o at various proton energies may be combined to permit accurate interpolation and perhaps, more importantly, to allow extrapolation to higher energies. Such a need arose, for example in the design of shielding for the 50 Gev Beijing Proton Synchrotron (Ch 80, Li 79).

Since the principle use of the Moyer Model is in the calculation of transverse shielding, we are interested in the global production of neutrons at large angles to the interaction target, as determined outside substantial shielding. At energies below 1 Gev there is evidence that the global production of neutrons is roughly proportional to neutron energy (for a summary see Pat 73). If an exponential variation of the form:

$$H_o(E_p) = kE_p^m \qquad (2)$$

is assumed, a value of m = 1 sets an upper limit to the variation of neutron production with proton energy and this is therefore a conservative assumption for extrapolating the experimental determinations of H_o to higher energies.

There has been some speculation in the literature as to the value of the coefficient m. Lindenbaum pointed out that the production of shower particles varied as $E^{0.25}$ and suggested a value of m = 0.50 for fast nucleons, intermediate between that for shower particles and low energy neutrons (Li 61, Pa 73). The data obtained from Monte-Carlo calculations of the Hadron Cascades generated in matter by high-energy protons suggest a value of m = 0.75 (Fe 72).

Until recently there were insufficient experimental data to empirically investigate the relationship between H_o and E_p but the experimental data of Table 1, shown in Figs. 2 and 3, now make this possible.

2. STATISTICAL ANALYSIS OF DATA

The number of data points severely limits the analysis and our purpose here is to first show that the assumption of linearity between the random variables $H_o(E_p)$ and E_p or between the random variables $\log_{10}H_o(E_p)$ and $\log_{10}E_p$ is not excluded by the experimental data.

The sequence of statistical tests described is:

(a) Fisher's F-Test of the hypothesis of linearity of the data and of the data under log-log transformation

(b) Regression analysis under the assumptions

$$H_o(E_p) = a + bE_p$$

and $\log_{10} H_o(E_p) = C + m \log_{10}E_p$

(c) Student's t-Test of log-log transformed data for m = 1

(d) Student's t-Test for a = o

(e) Linear regression analysis with $H_o(E_p) = b'E_p$

(f) Analysis of variance techniques to calculate 95 percent confidence bands to regression lines.

These tests are summarized here but are described in more detail by Lieu et al. (Li 82).

3.1 ANALYSIS OF VARIANCE

Since the data of Table 1 have more than one determination of $H_o(E_p)$ at energies of 7.4, 13.7 and 25.5 Gev the assumption of linearity of the data may be tested by analysis of variance techniques (Fi 70, Sn 80).

(a) Analysis of Variance of Log-Log Transformed Data

Table 2 gives the analysis of variance data.

Table 2. Analysis of variance (log-log transformed data).

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
Regression	0.30136	1	0.30136
About Regression	0.23608	k-2 = 5	0.04722
Within Group	0.06578	N-k = 4	0.01644
Residual	0.30186	N-2 = 9	0.03354
Total	0.60322	N-1 = 10	

Working hypotheses: HYP(0): $E(y|x) = k + mx$, or regression is linear

(Where $y = \log_{10} H_o(E_p)$ and $m = \log_{10}E_p$)

HYP(1): not linear

Test statistic: $F = \dfrac{\text{About-Regression Mean Square}}{\text{Within-Group Mean Square}}$

$$= \frac{0.04772}{0.01644} \quad \text{(from Table 2)}$$

$$= 2.90$$

Level of significance: $\alpha = 0.05$

Critical region: $F > F_{1-\alpha,k-2,N-2}$. With N = 11, k = 7 this becomes:

$$F > F_{0.95,5,4} = 6.26$$

Decision: since $F \not> 6.26$ the data do not support rejection of the hypothesis that the relation between $\log_{10}H_o(E_p)$ and $\log_{10}E_p$ is linear.

(b) Analysis of Variance of Untransformed Data

Table 3 gives the analysis of variance data.

Table A3. Analysis of variance.

Source	Sum of Squares	Degrees of Freedom	Mean Square
Regression	1.2540×10^{-23}	1	1.2540×10^{-23}
About Regression	0.8790×10^{-23}	k-2 = 5	0.1756×10^{-23}
Within Group	0.5872×10^{-23}	N-k = 4	0.1468×10^{-23}
Residual	1.4652×10^{-23}	N-2 = 9	0.1628×10^{-23}
Total	2.7192×10^{-23}	N-1 = 10	

Working hypotheses: HYP(0): $E(y|x) = a + bx$ [where $y = H_o(E_p)$ and $x = (E_p)$]

HYP(1): Not linear

Test statistic: $F = \frac{\text{ABOUT REGRESSION MEAN SQUARE}}{\text{WITHIN GROUP MEAN SQUARE}}$

$$= \frac{0.1756 \times 10^{-23}}{0.1468 \times 10^{-23}} \quad \text{(from Table 3)}$$

$$= 1.20$$

Level of significance: $\alpha = 0.05$

Critical region: $F > F_{0.95,\ 5,4} = 6.26$

Decision: Since $F = 1.20 \not> 6.26$ the data do not support the rejection of the hypothesis of linearity between $H_o(E_p)$ and E_p.

(c) Summary

A Fisher's F-Test of both the untransformed and the log-log transformed data show that both data sets are consistent with the assumption of linearity.

3.2 REGRESSION ANALYSIS

Regression analysis of the log-log transformed data gives:

$$H_o(E_p) = 3.07\ E_p^{0.769} \tag{3}$$

with estimated value of the slope, $\hat{m} = 0.769$ and the estimated variance of $\hat{m}$, $S_{\hat{m}} = \pm 0.257$. Similarly linear regression of the data gives:

$$H_o(E_p) = 5.22 \times 10^{-13} + (1.37 \times 10^{-13})Ep.$$

The estimated variance on the intercept, $S_{\hat{a}}$, $= \pm 9.86 \times 10^{-13}$.

3.3 STUDENT'S t-TEST

With $\hat{m} = 0.769$ and $S_{\hat{m}} = 0.257$ a t-Test does not reject $m = 1.0$ ($P = 20$ percent) (Li 82).

With $\hat{a} = 5.22 \times 10^{-13}$ and $S_{\hat{a}} = 9.86 \times 10^{-13}$ a t-Test does not reject $a = 0$ ($P = 30$ percent) (Li 82).

3.4 LINEAR REGRESSION ANALYSIS FORCED THROUGH THE ORIGIN

Since the data are compatible with the assumption of linearity (Fisher's F-Test), with the assumption of $m = 1$ in log-log transformation (Student's t-Test) and with the assumption of $a = o$ when untransformed (Student's t-Test), it is reasonable to fit the data by a line forced through the origin giving:

$$H_o(E_p) = 1.608 \times 10^{-13}\ Ep$$

with $S_{\hat{b}}$, $= 0.47 \times 10^{-13}$

3.5 CALCULATION OF CONFIDENCE BANDS

Figures 2 and 3 show the 95 percent confidence bands to the lines calculated by regression analysis. For details of these calculations see Lieu et al. (Li 82).

4. CONCLUSIONS

The experimental data may be fitted by straight lines either in linear or log-log transformation. The best value of the coefficient, m, is 0.77 ± 0.26, but linearity is not excluded by a Student's t-test ($p = 20$ percent). A straight line forced through the origin has a slope $(1.61 \pm 0.19) \times 10^{-13}$ Sv m^2/Gev [$(1.00 \pm 0.12) \times 10^{-13}$ Sv m^2/J] to be compared with a value of $(1.65 \pm 0.21) \times 10^{-13}$ Sv m^2/Gev [$(1.03 \pm 0.13) \times 10^{-13}$ Sv m^2/J] given by Stevenson et al., (St 82).

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LIST OF FIGURES

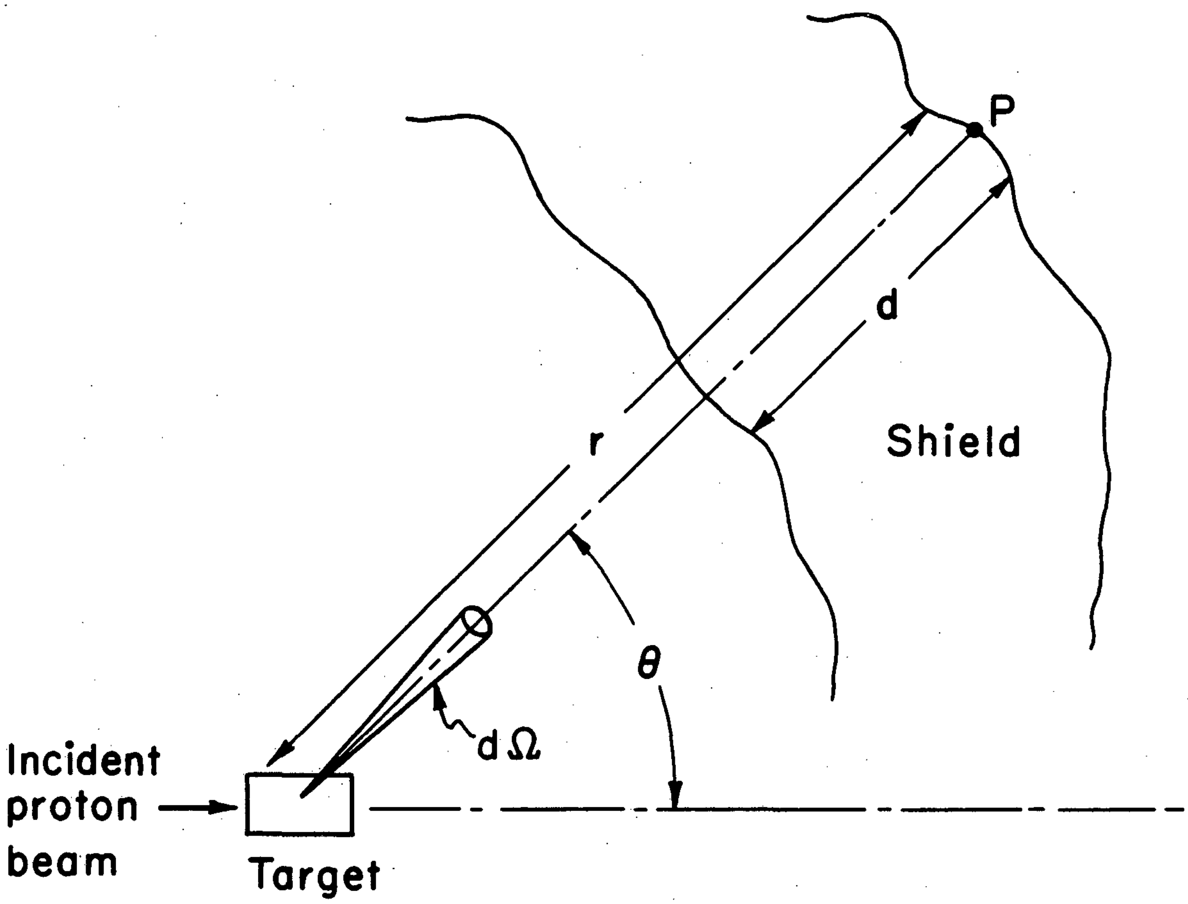

Fig. 1 XBL 733-2478

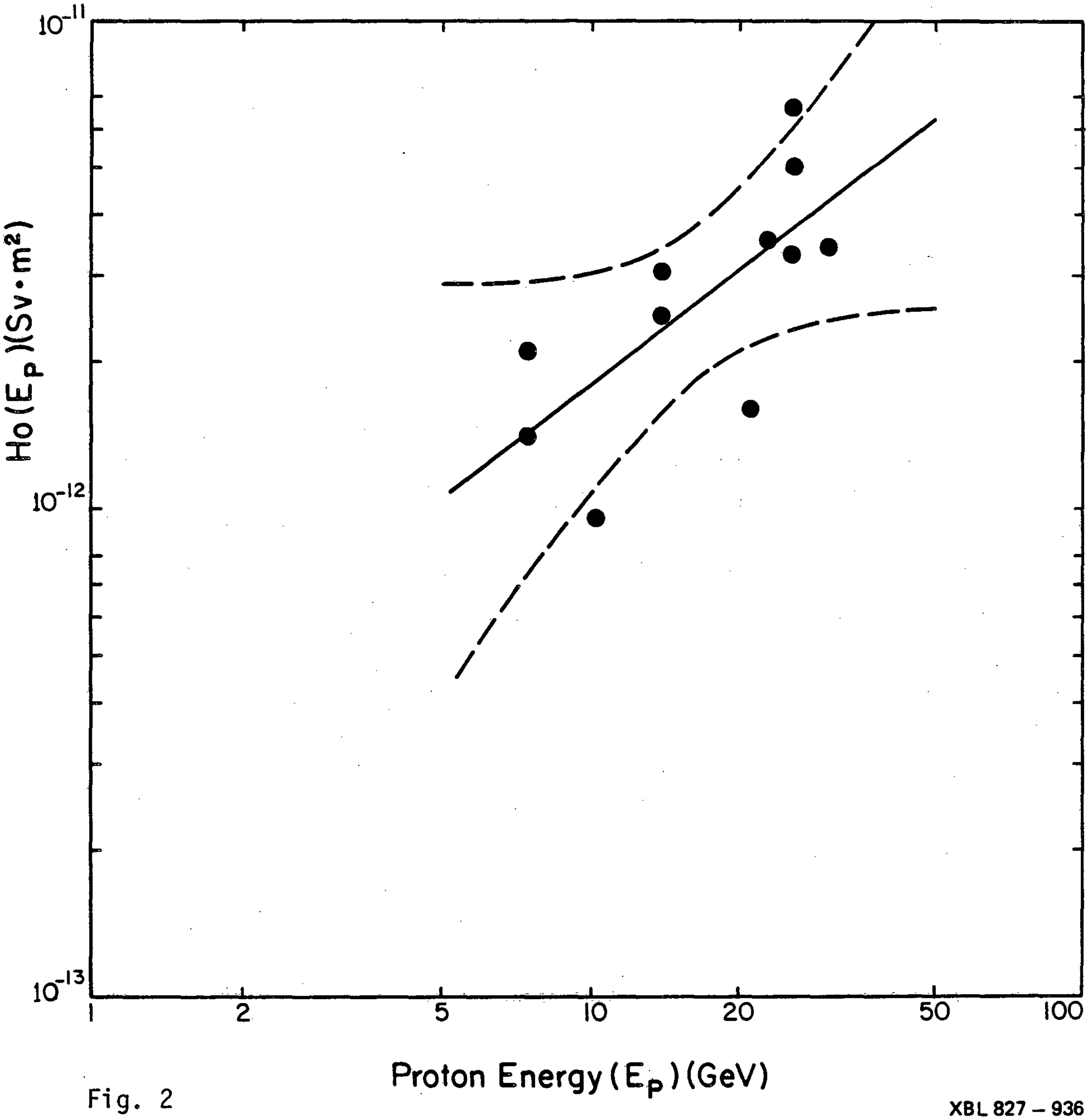

Fig. 2

XBL 827 – 936

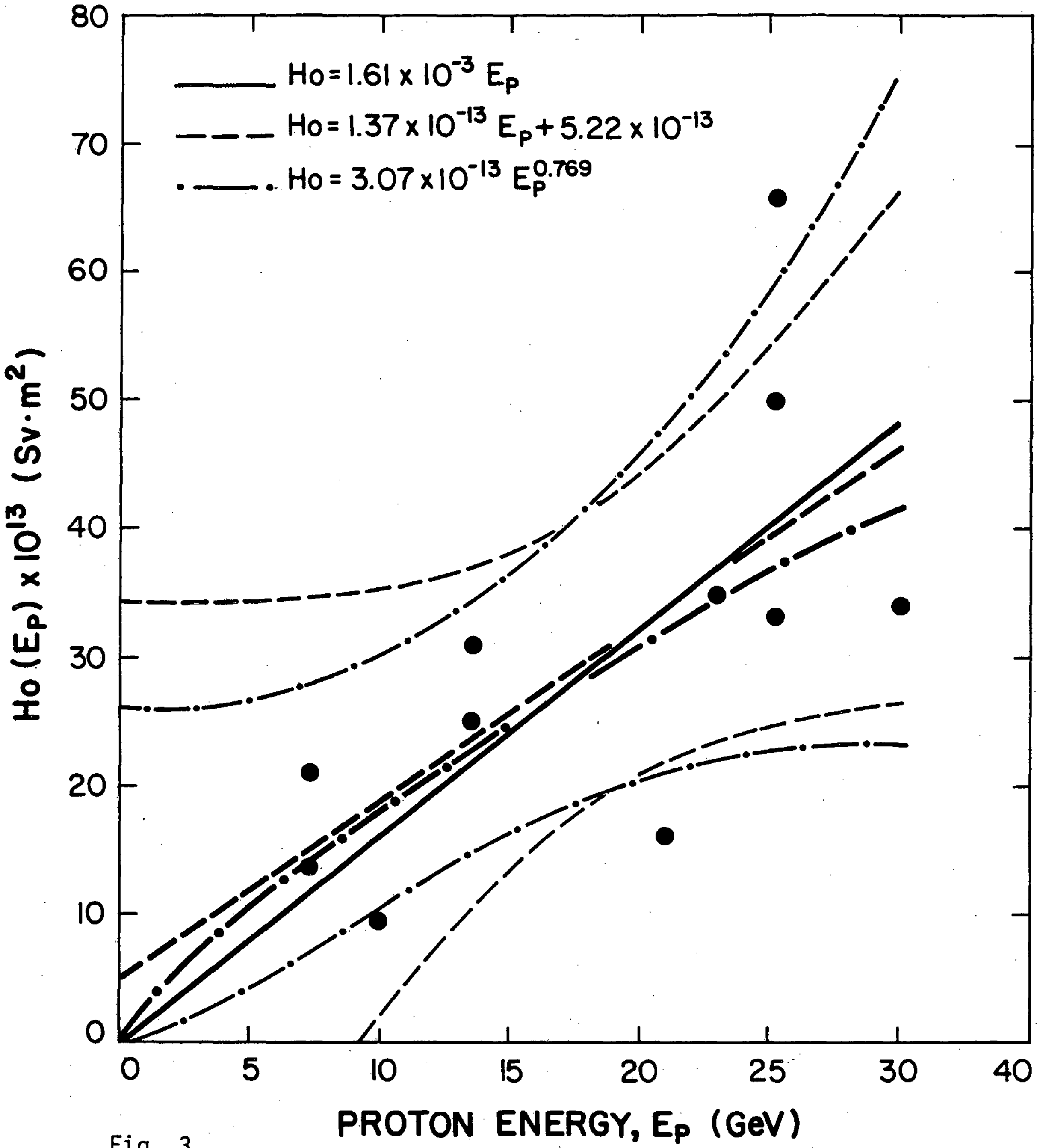

Fig. 3

XBL 827 — 937

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