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AND STOCK PRICES: A NOTE

BY

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UNCERTAIN INFLATION, REAL RISK AND STOCK PRICES: A NOTE

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Working Paper No. 85-99

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UNCERTAIN INFLATION, REAL RISK AND STOCK PRICES: A NOTE

ABSTRACT

It is plausible to assume that increased inflation uncertainty is closely related to increased real economic productivity risk. Based on this assumption, it can be demonstrated that the observed real decline in stock market prices (returns) over the last two decades are the result of increased inflation uncertainty. This finding is contrary to some recent empirical evidence, such as Pindyck (1984), which claims that increased inflation uncertainty has had negligible effects on stock market returns (prices).

Burton Malkiel (1979) attributes the decline in real stock prices during the recent inflationary period to an increasingly uncertain business environment, particularly increased inflation uncertainty.¹ Pindyck (1984) claims that the overall increase in economic uncertainty is represented by an increased variance of the real gross marginal rate of return on capital (i.e., before tax corporate profit rate). Given the volatility of inflation during the post-1960 period and the results from earlier macroeconomic studies about the adverse effect of inflation uncertainty on real economic activity,² Malkiel's and Pindyck's arguments are mutually consistent if the observed increased variance of the real gross marginal return on capital were engendered by increased inflation uncertainty.

But Pindyck allegedly eliminates the possibility of the inflation uncertainty effects on real economic productivity risk and, hence, stock price (return) risk:

"any increase in the variance of inflation would have had a negligible direct effect on the variance of the net real return on equity (p. 340)."

In fact, Pindyck suggests that the stock prices increase when inflation uncertainty increases (Table 1, p. 345). Pindyck's latter result contradicts the well-documented empirical finding that stock market returns are negatively correlated with inflation.³

In brief, we will show that the adverse effect of inflation uncertainty on the riskiness of the real gross return on capital is the principal cause for depressed stock prices. Using Pindyck's empirical results, we will show that the stock prices should decrease when inflation uncertainty increases.⁴

I. ASSET RETURNS AND INFLATION: THE THEORY⁵

It will be assumed that returns on assets and price changes follow continuous-time stochastic (Wiener) processes which are time-homogeneous Markov processes. The inflation rate over the time interval dt is described by:

$$(1) \quad dP/P = \pi dt + \sigma_{\pi} dz_{\pi}$$

where P denotes the price level; π is the instantaneous expected inflation rate; σ_{π} is the instantaneous standard deviation of the Wiener process for price changes, that is, σ_{π} represents inflation uncertainty;⁶ $dz_{\pi} = y_{\pi} \sqrt{dt}$; z_{π} is a standard Wiener process or Brownian motion; and y_{π} is, by construction, a standardized normal random variable which is identically and independently distributed over time.

Given the nominally (instantaneous) risk-free interest rate before taxes, R , the net real interest rate after tax over the time interval dt , ξ_b , is described by:⁷

$$(2) \quad \xi_b = \{(1-\theta)Rdt - dP/P\} / (1 + dP/P) \\ = \{(1-\theta)R - \pi + \sigma_{\pi}^2\} dt - \sigma_{\pi} dz_{\pi}$$

where θ is the personal income tax rate; $(1-\theta)Rdt$ is the net nominal interest after taxes (because "nominal" interests are taxed).

The real gross marginal return on capital over the time interval dt , m , is also described by the stochastic differential equation (3-a):

$$(3-a) \quad m = \alpha dt + \sigma_m dz_m$$

where α is the instantaneous expected real gross rate of return on capital; σ_m is its instantaneous standard deviation; and $dz_m = y_m \sqrt{dt}$ where z_m is a standard Wiener process and y_m is a standardized normal random variable.

It is important to note while Pindyck is aware that $E(dz_\pi dz_m) = \rho dt \neq 0$, he implicitly assumes that σ_m is strictly independent of σ_π . The theoretically possible interrelationship between σ_m and σ_π creates the primary basis for differences in interpreting the empirical findings. Hence, equation (3-a) should be rewritten using the relationship described by equation (3-b) to take into account the interrelationship between the variances of real gross returns on capital (σ_m) and inflation (σ_π):

$$(3-b) \quad \sigma_m dz_m = m_\pi \sigma_\pi dz_\pi + \sigma_x dz_x$$

where $dz_x = y_x \sqrt{dt}$; y_x is also a standardized normal random variable; $E[y_\pi y_x] = 0$ by construction; and $m_\pi = \text{COV}(\sigma_m dz_m, \sigma_\pi dz_\pi) / \sigma_\pi^2 dt = \rho \sigma_m / \sigma_\pi$; ρ is the instantaneous correlation coefficient between the real gross return on capital and inflation; that is, m_π measures how the real gross return on capital responds to uncertain inflation.⁸ Combining equations (3-a) and (3-b) produces the theoretically correct equation (3-c):

$$(3-c) \quad m = \alpha dt + m_\pi \sigma_\pi dz_\pi + \sigma_x dz_x$$

Note that we have not assigned a priori a value for m_π ; instead the value of m_π will be inferred from empirical evidence. Therefore, equation (3-a) (used by Pindyck) is a "special" case where m_π in equation (3-c) is zero.

The net after-tax cost of one dollar borrowing is $(1-\tau_s)Rdt - dP/P$;⁹ where τ_s is the "statutory" corporate income tax rate. Assume that a 1 percent increase in the price level is assumed to reduce net profits per unit of capital by an amount equal to λ .¹⁰ Letting q denote the price of the share; and b denote corporate borrowing per unit of capital, the firm's real net earnings per dollar of equity over the time interval dt , ψ_s , is:

$$(4-a) \quad (1-b) q \psi_s = (1-\tau_e)\alpha dt - b[(1-\tau_s)Rdt - dP/P] \\ + (1-\tau_e)m_{\pi} \sigma_{\pi} dz_{\pi} + (1-\tau_e)\sigma_x dz_x \\ - \lambda dP/P$$

where τ_e is the "effective" corporate income tax rate. By substituting equation (1) for (dP/P) , equation (4-a) becomes equation (4-b):

$$(4-b) \quad (1-b) q \psi_s = \{(1-\tau_e)\alpha - b(1-\tau_s)R + (b-\lambda)\pi\}dt \\ + \{(1-\tau_e)m_{\pi} + (b-\lambda)\}\sigma_{\pi} dz_{\pi} \\ + (1-\tau_e)\sigma_x dz_x$$

Let δ be the dividend payout ratio and θ_c be the effective tax rate on capital gains. The tax rate on dividend income, $\delta\psi_s$, is θ ; and the tax rate on capital gains, $(1-\delta)\psi_s q$ and $(dP/P)q$, is θ_c . Thus, the real after personal tax return on equity over the time interval dt , ξ_s , is:

$$(5-a) \quad \xi_s = \psi_s [(1-\theta)\delta + (1-\theta_c)(1-\delta)q] - \theta_c (dP/P)$$

Let $a = \{(1-\theta)\delta + (1-\theta_c)(1-\delta)q\}/(1-b)q$. By substituting equation (1) for (dP/P) and equation (4-b) for ψ_s , equation (5-a) becomes equation (5-b):

$$\begin{aligned}
 (5-b) \quad \xi_s &= \{a[(1-\tau_e)\alpha - b(1-\tau_s)R + (b-\lambda)\pi] - \theta_c\pi\}dt \\
 &+ \{a[(1-\tau_e)m_\pi + (b-\lambda)] - \theta_c\}\sigma_\pi dz_\pi \\
 &+ a(1-\tau_e)\sigma_x dz_x \\
 &= r_s dt + c_\pi \sigma_\pi dz_\pi + c_x \sigma_x dz_x
 \end{aligned}$$

where r_s is the instantaneous expected real rate of equity return after personal tax (i.e., the first set of bracketed terms in RHS of the first equality); and $c_\pi = a[(1-\tau_e)m_\pi + (b-\lambda)] - \theta_c$ measures how the net real equity return responds to uncertain inflation, that is,
 $c_\pi = \text{COV}(\xi_s, dP/P)/\sigma_\pi^2 dt$.

But if one were not to consider explicitly the interrelationship between the real productivity risk (σ_m) and inflation uncertainty (σ_π) (i.e., $m_\pi = 0$), ξ_s would have become:

$$(5-c) \quad \xi_s = r_s dt + \{a(b-\lambda) - \theta_c\}\sigma_\pi dz_\pi + a(1-\tau_e)\sigma_m dz_m$$

II. REAL UNCERTAINTY AND INFLATION UNCERTAINTY: EMPIRICAL ANALYSIS

Using equation (5-c), Pindyck estimated the variance of stock returns to be:¹¹

$$(6-a) \quad (1/dt)\sigma^2(\xi_s) = .000004\sigma_\pi^2 + .518\sigma_m^2 - .0029\rho\sigma_\pi\sigma_m$$

where the parametric values in equation (5-c) are assumed to be $\theta = .30$, $\theta_c = .05$, $\tau_s = .48$, $\tau_e = .40$, $b = .30$, $\lambda = .26$, $\delta = .43$, and $a = 1.2$ (with $q = 1$).

Pindyck argues that since the coefficient of σ_{π}^2 is so small (i.e., .000004), the effect of uncertain inflation on common stock risk is negligible. This would be true if σ_m were unrelated to σ_{π} . However, using Pindyck's own results, if one considers σ_m and σ_{π} to be interrelated as in equation (3-c), equation (6-a) would become equation (6-b):¹²

$$(6-b) \quad (1/dt)\sigma^2(\xi_s) = [.000004 + .518m_{\pi}^2 + .0029m_{\pi}] \sigma_{\pi}^2 + .518 \sigma_x^2$$

Therefore, the level of impact of inflation uncertainty on the variance of stock returns depends on the parametric value of m_{π} . Using Pindyck's results (Table A1, p. 348: $\rho = -.22$, $\sigma_{\pi}^2 = 4.6 \times 10^{-6}$, and $\sigma_m^2 = .0030$), m_{π} , equal to $\rho\sigma_m/\sigma_{\pi}$, is inferred to be -5.6183 . As might be anticipated from earlier macroeconomic studies (see footnote 2), the implied magnitude of m_{π} , using Pindyck's own results, is large.

Unfortunately, and in addition, this yields an internal inconsistency in Pindyck's analysis. On the one hand, he evaluates stock risk, assuming the equivalent of equation (3-a), (i.e., $m_{\pi} \equiv 0$); and, on the other hand, his empirical findings can be construed to imply a large negative value for m_{π} . In fact, by replacing -5.6183 for m_{π} in equation (6-b), equation (7) is generated:

$$(7) \quad (1/dt)\sigma^2(\xi_s) = 16.335 \sigma_{\pi}^2 + .518 \sigma_x^2$$

Equation (7) suggests that the impact of inflation uncertainty (i.e., the coefficient of σ_{π}^2) is approximately 30 times larger than that of real production uncertainty. Hence, it would appear that a "slight" restatement and reinterpretation of Pindyck's own model and findings would generate the opposite conclusion: inflation

uncertainty by affecting real productivity risk has a significant impact on stock risk and stock prices.

III. INFLATION UNCERTAINTY AND THE STOCK PRICE

Given the real interest rate and real stock returns, investor's optimization is solved for the fraction of wealth invested in equity, β^* (see Pindyck's appendix):

$$(8-a) \quad \beta^* = \frac{E[\xi_s - \xi_b]}{\gamma(1/dt)\text{VAR}(\xi_s - \xi_b)} + \frac{(1/dt)\text{VAR}(\xi_b) - (1/dt)\text{COV}(\xi_s, \xi_b)}{(1/dt)\text{VAR}(\xi_s - \xi_b)}$$

where γ is the Pratt-Arrow measure of relative risk aversion. From equations (2) and (5-b) it can be shown that:

$$(1/dt)\text{VAR}(\xi_s - \xi_b) = (c_\pi + 1)^2 \sigma_\pi^2 + a^2(1-\tau_e)^2 \sigma_x^2$$

$$(1/dt)\text{VAR}(\xi_b) = \sigma_\pi^2$$

$$(1/dt)\text{COV}(\xi_s, \xi_b) = -c_\pi \sigma_\pi^2$$

where $c_\pi = a\{(1-\tau_e)m_\pi + (b-\lambda)\} - \theta_c$. By substituting these covariances into equation (8-a):

$$(8-b) \quad \beta^* = \frac{\gamma^{-1}E[\xi_s - \xi_b] + (c_\pi + 1)\sigma_\pi^2}{(c_\pi + 1)^2 \sigma_\pi^2 + a^2(1-\tau_e)^2 \sigma_x^2} = \beta^*(q, \sigma_\pi^2, \dots)$$

Since β^* is a function of q and σ_π^2 , it can be shown as

$$(9) \quad d \log q / d\sigma_\pi^2 = B[-1/\gamma + (c_\pi + 1) - \beta(c_\pi + 1)^2] / \{(c_\pi + 1)^2 \sigma_\pi^2 + a^2(1-\tau_e)^2 \sigma_x^2\}$$

where B is a constant in Pindyck's equation (A12) (p. 349).

Using the parametric values in Pindyck's Table A1, equation (9) can be used to re-examine how an increase in the variance of inflation affects the stock price. Contrary to Pindyck's conclusions, the modified result reported in our Table I, columns 2 and 3 show that stock price changes are negatively related to changes in inflation uncertainty; and the inflation uncertainty effect is larger than that of the variance of the real gross marginal return on capital. Moreover, even if λ is assumed to be zero, (i.e., if it is assumed that there is no tax on nominal capital gains), the magnitude of the adverse effect of inflation uncertainty on stock prices (column 3 in Table I) is substantial.

From equation (8-b), solving for the expected market risk premium for common stock at equilibrium, $E[\xi_s - \xi_b]$, yields equation (10):

$$(10) \quad \gamma^{-1} E[\xi_s - \xi_b] = \beta \{ (c_\pi + 1) \sigma_\pi^2 + a^2 (1 - \tau_e)^2 \sigma_x^2 \} - (c_\pi + 1) \sigma_\pi^2$$

Hence,

$$(11) \quad d \gamma^{-1} E[\xi_s - \xi_b] / d \sigma_\pi^2 = \beta (c_\pi + 1)^2 - (c_\pi + 1)$$

Pindyck's results (Table A1) suggests that $a = 1.2$ (with $q = 1$); $\tau_e = .40$; $m_\pi = -5.6183$; $b = .30$; $\lambda = .26$; and $\theta_c = .05$. These parametric values imply $c_\pi = -4.05$, which signifies that the required risk premium for common stock increases when inflation uncertainty increases. For example, assuming that $\gamma = 6$ and $\beta = .67$:

$$(12) \quad dE[\xi_s - \xi_b] / d\sigma_\pi^2 \approx 56$$

The result in equation (12) strongly suggests that the risk premium for common stock has increased as a response to increased inflation uncertainty, resulting in relatively depressed stock prices.¹³

IV. CONCLUDING REMARK

Our results strongly indicate that the adverse effect of inflation uncertainty on the real gross marginal return of capital is likely to be the "principal" cause for the decline in stock prices during the recent inflationary period.

FOOTNOTES

¹Malkiel states:

"inflation is far from neutral as it appears in many textbooks. High levels of inflation are associated with considerable variability in the inflation rate and with large relative price changes, which make long-run future planning especially hazardous. Thus, even if total profits increase pari passu with inflation, the dispersion of profits among business increases with the rate of inflation (p. 297)."

Also, Irwin Friend (1982), working in a tangentially related area, finds that inflation uncertainty impacts negatively on stock returns.

²See Lucas (1973) and Friedman (1977), among others.

³See Lintner (1975), Bodie (1976), Jaffe and Mandelker (1976), Nelson (1976), Fama and Schwert (1977), and Friend and Hasbrouck (1982), among others.

⁴This result has been reported independently by Dokko and Edelstein (1985) in a related but different context. They claim that the adverse effect of inflation uncertainty on corporate before tax profits is the principal cause for an increase in the real required return for common stock, resulting in relatively depressed stock prices.

⁵For convenience, the development of the model follows that of Pindyck (1984).

⁶Inflation uncertainty is viewed as a dispersion measure of the distribution from which a point forecast for expected inflation has been drawn.

⁷The stochastic continuous-time version of the Fisher equation was originally derived by Stanley Fischer (1975).

⁸Uncertainty about the future, induced by inflation uncertainty, is likely to affect the firm's investment decision (see footnote 1). Similarly, consumers may alter consumption-saving decisions because of perceived changes in inflation uncertainty. Since the generating function for the real gross return on capital can be viewed as a reduced form of the aggregate production function and the aggregate demand function, a "two-factor" generating process such as equation (3-c) is likely to be an appropriate model. Empirical evidence indicates that a higher inflation level is closely associated with a wider dispersion of relative price changes [see Vining and Elwertowski (1976), and Parks (1978)], a result that was theoretically anticipated by Friedman (1977). m_{π} in our model takes into account the possibility of uneven relative price changes (i.e., the non-neutrality of inflation).

⁹The after tax real interest rate to the firm in the stochastic continuous-time model should have been $\{(1-\tau_s)R - \pi + \sigma_{\pi}^2\}dt - \sigma_{\pi} y_{\pi} \sqrt{dt}$. Pindyck ignored σ_{π}^2 in the expected real interest rate to the firm, which is, however, inconsequential to the results.

¹⁰This assumption, also, made by Pindyck, is arbitrary. In order to examine why stock returns and inflation are negatively related to each other, a clear distinction should be made between the inflationary effects on "before tax" profits and "after tax" profits. It will be shown that even though λ is assumed to be zero, that is, even when

there is no tax on nominal capital gains, stock prices decrease when inflation uncertainty increases.

¹¹Equation (6-a) is Pindyck's (p. 340) equation (13). From equation (5-c), $(1/dt)\sigma^2(\xi_s) = \{a(b-\lambda)-\theta_c\}^2\sigma_\pi^2 + a^2(1-\tau_e)^2\sigma_m^2 + 2\{a(b-\lambda)-\theta_c\}a(1-\tau_e)\rho\sigma_\pi\sigma_m$.

¹²From equation (3-b), $\sigma_m^2 = m^2\sigma_\pi^2 + \sigma_x^2$; and $\rho\sigma_\pi\sigma_m = m\sigma_\pi^2$.

¹³Poterba and Summers (1984) suggest that if the variances for inflation uncertainty and real uncertainty are weakly serially correlated, they have only a small impact on stock prices. Dokko and Edelstein (1985) show that inflation uncertainty measures are, in general, strongly serially correlated, while real uncertainty measures are not serially correlated. This finding implies that inflation uncertainty may have an important impact on stock prices.

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TABLE I†

EFFECTS OF INFLATION UNCERTAINTY ON STOCK MARKET PRICES

γ , relative risk aversion measure	$d \log q / d\sigma_{\pi}^2$			$d \log q / d\sigma_m^2$
	$\lambda = .26$		$\lambda = .00$	
	(1)	(2)	(3)	
1.0	6	-4,769	-3,166	-161
2.0	325	-6,199	-4,385	-220
3.0	491	-6,941	-5,092	-251
4.0	593	-7,395	-5,554	-270
5.0	661	-7,702	-5,880	-282
6.0	711	-7,923	-6,121	-292
7.0	748	-8,089	-6,308	-298
8.0	777	-8,220	-6,456	-303
9.0	800	-8,324	-6,577	-312
10.0	819	-8,410	-6,677	-348

†Column (1) assumes that σ_m and σ_{π} are independent of each other with $m_{\pi} = 0$ (i.e., a recalculation of Pindyck estimates for the effect of inflation uncertainty on the stock price); column (2) is estimated using equation (9); column (3) is estimated using equation (9) and assuming there is no tax on nominal capital gains (i.e., $\lambda=0$); and column (4) is replicated from Pindyck's Table 1 for reference purposes.