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EFFECTS OF ELECTROMAGNETIC FIELDS ON POLARIZED PARTICLES

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Berkeley, California

Depolarization resonances

$$0 = n + m\nu_r + l\nu_z - \gamma \frac{g_s}{2} - 1 \qquad n, m, l = 0, \pm 1, \pm 2, \ldots$$

$\gamma\left(\frac{g_s}{2} - 1\right) = 1.8$ non relativistic

$\nu_r = 3.25$

$\nu_z = 4.3$

$5.$
3.25
1.75

$(1.8) = F = n + 3.25m + 4.3l$

possible resonances

$n = 1 \quad m = -1 \quad l = 1 \qquad F = -1 - 3.25 + 4.3 = 2.05$

$n = 5 \quad m = -1 \quad l = 0 \qquad F = 1.75$

$n = 6 \quad m = 0 \quad l = -11 \qquad F = 1.7$

$8 \quad m = -2 \quad l = 0 \qquad F = 1.5$

8
6.5

$10 \quad m = 0 \quad l = -2 \qquad F = 1.4$

1.4
8.6

$n = 3.25m + 1.8 \qquad 5.05, 8.3, 11.55, 14.80, 18.05$

$n = 4.3l + 1.8 \qquad 6.1, 10.4, 15.2, 19.5$

3.25
5

5.05	4.3
3.25	1.8
8.30	6.1
3.25	4.3
11.55	10.4
3.25	4.3
14.80	15.2
3.25	4.3
18.05	19.5

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I. INTRODUCTION

Electromagnetic fields interact with the electromagnetic multipole moments of a particle and may therefore produce changes in its polarization state. The magnetic moment interaction will generally be the dominant one. In order to include possible depolarization effects in high energy accelerators or in a subsequent beam handling by lenses or bending fields, it is necessary to discuss the problem relativistically. For particles with spin 1/2 the rate of spin precession in homogeneous fields has been investigated (To51, Me55, Ca58) by explicit use of the Dirac equation, where in some cases a Pauli term was included to account for an anomalous magnetic moment. No relativistic equation in wave mechanics is available to describe particles with a spin larger than 1/2. Bargmann et al. (Ba59) did derive from classical relativistic electrodynamics a covariant classical equation of motion for particles with an arbitrary spin which is valid in homogeneous electromagnetic fields. These results are expected to be correct quantum mechanically, because the expectation value of the vector operator representing the spin will (according to a general theorem by Ehrenfest) necessarily have the same time dependence as one obtains from a classical equation of motion. In Sec. II the change of the polarization state in homogeneous electromagnetic fields is discussed on a somewhat more elementary basis than using the above mentioned method. In Sec. III we consider time dependent fields with special emphasis on fields that occur in cyclic accelerators. In Sec. IV results of depolarization calculations for the Berkeley 88" Cyclotron are given.

II. TIME INDEPENDENT HOMOGENOUS ELECTROMAGNETIC FIELDS

The polarization state of a beam of particles with arbitrary spins is quantitatively described by means of a density matrix ρ or equivalently by a set of expectation values t_{KM} of spin-tensor operators T_{KM}. The rank K is an integer number in the range between 0 and 2s, while M takes on the (2K + 1) integer values between -K and +K. (Ed54). This treatment is nonrelativistic and therefore the terms vector (T = 1) and tensor ($T \geq 2$) polarization have an obvious meaning only in the rest system of the particles. Let us first assume that this rest system is not accelerated, then the classical equation for the spin-motion of a particle in a magnetic field $\vec{B}^{(r)}$ (measured in the rest system) will have the customary form

$$\frac{d\vec{s}}{d\tau} = \frac{q}{2mc} g_s \vec{s} \times \vec{B}^{(r)} \qquad (1)$$

where τ is the time in the rest system and the particle has rest mass m, charge q and a magnetic moment $\vec{\mu}$. The factor g_s is defined by

$$\vec{\mu} = \frac{q\hbar}{2mc} g_s \vec{s} \quad . \qquad (2)$$

Note that g_s is not the normal Landé factor g, but related to it by

$$g_s = \frac{m}{m_p} \frac{e}{q} g \qquad (3)$$

where m_p and e are the rest mass and charge of the proton. According to Eq. (1) the spin precesses in an inertial rest frame around the magnetic field with an angular velocity

$$\vec{\omega}_L^{(r)} = -\frac{q}{mc}\frac{g_s}{2}\vec{B}^{(r)} \qquad (4)$$

Equations (1 and 4) are no longer correct, if the rest system is accelerated. Figure 1 shows a rest system at time t(R) and t + δt(R') respectively. The coordinate axes of the R and R' system are chosen to be parallel to the axis of an inertial system L (the laboratory system may be considered as an example). Classically the axis of R and R' would then be parallel, but this is not true relativistically. In fact the axis of the rest system rotate with an angular velocity $\vec{\omega}_T^{(r)}$ given by

$$\vec{\omega}_T^{(r)} = \frac{\gamma-1}{\gamma}\frac{\vec{v}\times\vec{a}^{(r)}}{v^2} \quad , \qquad (5)$$

where $\vec{v}$ is the velocity of the R frame as measured in the system L. $\vec{a}^{(r)}$ is the acceleration of the rest system and γ is as usual an abbreviation for $\left[1-(v/c)^2\right]^{-1/2}$. One notes that $\vec{\omega}_T = 0$ for low velocities ($\gamma = 1$) as expected. L. W. Thomas (Th27) first pointed out that such a rotation takes place, if the coordinate system is accelerated. The Thomas precession can be derived without reference to the cause of this acceleration (Mo60, p. 53), (Ja62, p.364). The R' system is connected with the L system by a Lorentz transformation with the velocity $-(\vec{v} + \delta\vec{v})$ and the L with the R system correspondingly by one with velocity $\vec{v}$. Performing these two successive transformations one sees that the transformation from R to R' can be made by a single Lorentz transformation plus a rotation. The rotation is just $\vec{\omega}_T \cdot \delta t$. In our case the acceleration is produced by the electric field in the rest system $\vec{E}^{(r)}$

$$\vec{a}^{(r)} = \frac{q}{m} \vec{E}^{(r)} \tag{6}$$

Generally we are interested in the spin precession frequency in a rest frame rotating with an angular velocity ($\vec{\omega}^{(r)}_{orb}$) such, that one of the coordinate axis is always coincident with the direction of motion of the particle. Because the time rate of change of any vector $\vec{a}$ in a coordinate system rotating with angular velocity $\vec{\omega}$ is the sum of $-\vec{\omega} \times \vec{a}$ and the time rate of change in a non-rotating system, we receive for such a rotating rest frame

$$\frac{d\vec{s}}{d\tau} = \vec{\omega}^{(r)}_{s} \times \vec{S} \tag{7}$$

with

$$\vec{\omega}^{(r)}_{s} = \vec{\omega}^{(r)}_{L} - \vec{\omega}^{(r)}_{T} - \vec{\omega}^{(r)}_{orb} \tag{8}$$

The well known expressions (see for example (Mo60, p. 142))

$$\begin{aligned} \vec{B}^{(r)} &= \gamma \left\{ \vec{B} + \frac{\vec{v}}{v^2} (\vec{v} \cdot \vec{B}) (\frac{1}{\gamma} - 1) - \frac{1}{c} \vec{v} \times \vec{E} \right\} \\ \vec{E}^{(r)} &= \gamma \left\{ \vec{E} + \frac{\vec{v}}{v^2} (\vec{v} \cdot \vec{E}) (\frac{1}{\gamma} - 1) + \frac{1}{c} \vec{v} \times \vec{B} \right\} \end{aligned} \tag{9}$$

allow us to replace the rest frame fields with those measured in the laboratory system. We can also express the rest frame time τ through the laboratory time t by

$$d\tau = \frac{1}{\gamma} dt \tag{10}$$

For example $\omega^{(r)} = \frac{d\phi^{(r)}}{d\tau} = \gamma\omega$, where we use the notation $\omega \equiv \frac{d\phi^{(r)}}{dt}$. Even though ω is now written without the superscript (r), one should keep in mind, that the angle is measured in the rest system. Spin directions deduced from the precession frequency $\vec{\omega}_s$ are therefore given in a rest frame, which as one axis (for example the z-axis) parallel to the velocity of the particles. Table I shows the direction and magnitude of the spin precession frequencies for several cases of practical interest deduced from the above equations. These results were first obtained by Bargmann et al. (Ba59) with a covariant four vector description of the spin. This method is more general than the one given here and the reader will find a lucid representation in the lecture notes "Relativistic Kinematics" by R. Hagedorn (Ha63, p. 124). Whenever $\vec{\omega}_s$ is perpendicular to $\vec{v}$, an initially longitudinal polarization will be converted to a transverse one and vice versa (remember angles are measured in the rest frame!). This is the case for the examples a) c) and e) in Table I. At very high velocities the precession rate becomes zero for the velocity filter (e), and it depends only on the anomalous part $(\frac{g_s}{2} - 1)$ of the magnetic moment in the cases a) and c). At low velocities this is still true for the deflection in a magnetic field but an electric field will not influence the direction of the spin any more and we find therefore $\vec{\omega}_s = -\vec{\omega}_e$. Deuterons do have a low anomalous magnetic moment, $\frac{g_s}{2} - 1 \simeq -1/7$. If the purpose of the electromagnetic field is to change the polarization state of a polarized deuteron beam it is therefore more convenient at low energies to use an electric or a crossed electric and magnetic field, where the precession angular velocity is $-\vec{\omega}_e$ and $6/7\vec{\omega}_c$ respectively, than a magnetic deflection where $\vec{\omega}_s$ is only $-1/7\vec{\omega}_c$. In example b) the spin precesses around the velocity with a rate which is independent of γ. This arrangement was first used by Hillman et al. (Hi56) to measure "left-right" or "up-down" asymmetrics in scattering experiments with fixed counter positions. The definition of g_s

Table I. Spin precession frequency $\vec{\omega}_s$ for several homogeneous field configurations.

Orbital motion of the particle and field configuration in the laboratory system		$\vec{\omega}_s$	$\vec{\omega}_T^{(r)}$	$\vec{\omega}_L^{(r)}$
a) $\vec{B}$ $\vec{v}$	circular motion with angular velocity $\vec{\omega}_c = -\frac{q\vec{B}}{\gamma mc}$	$\gamma \frac{g_s}{2} - 1 \, \vec{\omega}_c$	$\gamma(\gamma-1)\vec{\omega}_c$	$\gamma^2 \frac{g_s}{2} \vec{\omega}_c$
b) $\vec{v}$ $\vec{B}$	rectilinear uniform motion	$\frac{g_s}{2} \vec{\omega}_c$	0	$\gamma \frac{g_s}{2} \vec{\omega}_c$
c) $\vec{E}$ $\vec{v}$	circular motion with angular velocity $\vec{\omega}_e = \frac{q}{\gamma m u^2} \vec{v} \times \vec{E}$	$\left[\gamma(\frac{g_s}{2} - 1) - \frac{g_s}{2} \; \vec{\omega}_e\right]$	$\gamma(\gamma-1)\vec{\omega}_e$	$(\gamma^2-1)\frac{g_s}{2} \vec{\omega}_e$
d) $\vec{v}$ $\vec{E}$	accelerated rectilinear motion	0	0	0
e) $\vec{E} = -\frac{\vec{v}}{c} \times \vec{B}$ $\vec{v}$ $\vec{B}$	rectilinear uniform motion	$\frac{1}{\gamma} \frac{g_s}{2} \vec{\omega}_c$	0	$\frac{g_s}{2} \vec{\omega}_c$

by Eq. (2) would seem to indicate that the above treatment would be correct only for charged particles, but it is easy to see that the precession frequencies are also valid for lim q = 0.

To describe the polarization state of a beam of particles with arbitary spins, we have to specify the magnitude of all the tensor components t_{KQ} up to rank K = 2s. Let us assume that this set of numbers is given at time t = 0 in the rest frame R of the particles. Again it is convenient to assume that the z-axis of R is along the velocity of the particles. We have then to determine the new tensor components t'_{KQ} after time t in a rest frame system R', which shall have the same relative orientation to the beam as R. According to the foregoing considerations we will find at time t still the components t_{KQ} (t=0) in a coordinate system R", which is turned by an angle

$$\vec{\Omega} = \int \vec{\omega}_s dt \tag{12}$$

against R'. In the examples considered (Table I) $\vec{\omega}_s$ is independent of the time and $\vec{\Omega}$ is simply $\vec{\omega}_s t$. The tensor components t'_{KQ} in R' are given by

$$t'_{KQ} = \sum_{Q'} t_{KQ'} \, D^K_{Q'Q} \, (\alpha\beta\gamma) \tag{13}$$

where $(\alpha\beta\gamma)$ are the Euler angles producing a rotation $\vec{\Omega}$ and the $D^K_{Q'Q}$ are the elements of the rotation matrix (Ed54, p. 54). The usual definitions of spin-one tensor operators tranforming like spherical harmonics (see Eq. (13)) are (La55)

$$T_{00} = 1 \qquad T_{10} = \sqrt{\frac{3}{2}}\; S_z$$

$$T_{2\pm2} = \frac{1}{2}\sqrt{3}\,(S_x \pm iS_y)^2 \qquad T_{2\pm1} = \mp\frac{1}{2}\sqrt{3}\Big((S_x \pm iS_y)S_z + S_z(S_x \pm iS_y)\Big)$$

$$T_{20} = \frac{1}{\sqrt{2}} (3S_z^2 - 2)$$

Here S_i are the spin one angular momentum operators. In order to simplify the geometry we will consider only the two cases where $\vec{\Omega}$ is either parallel or perpendicular to the velocity of the particles. All cases given in Table I fulfill this condition. (The problem of finding the Euler angles in a general case is solved for example in (Go50) p. 107). In the first case which corresponds to example b) in Table I $\alpha = \Omega$, $\beta = \gamma = 0$ and Eq. (13) gives

$$t'_{KQ} = t_{KQ}\, e^{iQ\Omega} \tag{14}$$

In the second case let us specify the direction of $\vec{\Omega}$ by the polar angles $\theta = \pi/2$ and ϕ, then the Euler angles are $\alpha = 3\pi/2 + \phi$ $\beta = \Omega$ $\gamma = \alpha$. For spin 1 particles Eq. (13) has with the abbreviations $s \equiv \sin\Omega$ $c \equiv \cos\Omega$ the following explicit form

$$\begin{aligned}
t'_{11} &= t_{11}\, \tfrac{1}{2}(1+c) - t_{1-1}\, e^{i2\phi}\, \tfrac{1}{2}(1-c) + it_{10}\, e^{i\phi}\, \tfrac{1}{\sqrt{2}}s \\
t'_{10} &= it_{11}\, e^{-i\phi}\, \tfrac{1}{\sqrt{2}}s + it_{1-1}\, e^{i\phi}\, \tfrac{1}{\sqrt{2}}s + t_{10}c \qquad (15) \\
t'_{22} &= t_{22}\, \tfrac{1}{4}(1+c)^2 + t_{2-2}\, e^{i4\phi}\, \tfrac{1}{4}(1-c)^2 \\
&\quad + it_{21}\, e^{i\phi}\, \tfrac{1}{2}s(c+1) + it_{2-1}\, e^{i3\phi}\, \tfrac{1}{2}s(c-1) \\
&\quad - t_{20}\, e^{i2\phi} \sqrt{\tfrac{3}{8}}\, s^2
\end{aligned}$$

$$t'_{21} = it_{22}\, e^{i\phi}\, \tfrac{1}{2}s(c+1) + it_{2-2}\, e^{i3\phi}\, \tfrac{1}{2}s(c-1)$$

$$+ t_{21}\, \tfrac{1}{2}(2c-1)(c+1) + t_{2-1}\, e^{i2\phi}\, \tfrac{1}{2}(2c+1)(c-1)$$

$$+ it_{20}\, e^{i\phi} \sqrt{\tfrac{3}{2}}\, sc$$

$$t'_{20} = - t_{22}\, e^{-i2\phi} \sqrt{\tfrac{3}{8}}\, s^2 - t_{2-2}\, e^{i2\phi}\sqrt{\tfrac{3}{8}}\, s^2 \qquad (15)$$

$$+ it_{21}\, e^{-i\phi} \sqrt{\tfrac{3}{2}}\, sc + it_{2-1}\, e^{i\phi}\sqrt{\tfrac{3}{2}}\, sc$$

$$+ t_{20}\, \tfrac{1}{2}(3c^2-1)$$

$$t'_{K-Q} = (-)^Q\, t'^{*}_{KQ}$$

It is seen as expected that the first pair of equations and the one for t'_{1-1}, which follows from the last equation, describe the rotation of the polarization vector expressed in terms of a rank one tensor. In most cases of practical interest the above formulas are still further simplified. Let us first consider the problem of generating with electromagnetic fields those tensor components which are zero in an incoming beam. For example a deuteron beam extracted parallel to the magnetic field of the ionizer in a polarized source can have only t_{10} and t_{20} different from zero. Let us now pass this beam through a velocity filter (see case e of Table I). We can adjust the electromagnetic fields such, that the magnitude of any of the components of t'_{KQ} will be a maximum. It follows from Eq. (15) that $|t'_{22}|$ has a maximum value of $\sqrt{3/8}\, t_{20}$ for $\Omega = \pi/2$. This Ω creates also a maximal transverse polarization (Maximum for $|t_{11}|$). The component $|t'_{21}|$ reaches the same maximum value as $|t'_{22}|$ but at $\Omega = \pi/4$. A rotation of the velocity filter around the beam axis has the same effect as a subsequent longitudinal magnetic field would have (see Eq. (14)). The deflection of the beam through a fixed angle by a magnetic or electric field would give a fixed magnitude of Ω. For the purpose of changing the polarization state such an arrangement is therefore quite inferior to one with a velocity filter. If a polarized deuteron beam is accelerated in a cyclotron the polarization state is rotationally symmetric about the magnetic field of the accelerator. Let us choose this direction as the y axis and again z parallel to the velocity of the deuterons. The polarization state is now given by t_{20}, $t_{22} = \sqrt{3/2}\, t_{20}$ and a pure imaginary t_{11}. In a coordinate system where the z-axis would be the symmetry axis we would find the components $\hat{t}_{20} = -2t_{20}$, $\hat{t}_{10} = i\sqrt{2}\, t_{11}$ and all other components zero, see Eqs. (15). Again one can produce a $|t'_{21}| = \sqrt{3/2}\, t_{20}$ with $\vec{\Omega}$ in the x-direction and magnitude $\hbar/4$. Longitudinal vector polarization and a $t'_{20} = -2t_{20}$ will be produced with $\Omega = \pi/2$.

The formulation of spin motion as used in this section allows us to treat approximately cases where the electromagnetic fields in the laboratory system are no longer homogeneous. In such fields the particles in a beam with finite cross section will experience different precession angles $\vec{\Omega}$. This results in a "depolarization" of the beam. To be more specific let us consider the influence of a quadrupole lens pair on a beam of particles having only a longitudinal polarization. The action of the pair on the beam trajectory shall be considered as equivalent to a "thin lens" action (see Fig. 2(a)). The spin of those particles which enter the lens with an angle α will be turned by η toward the optic axis of the quadrupole. It follows from Fig. 2(a) and Table I that

$$\eta = \alpha(1+\frac{\alpha}{\beta})e$$

with

$$e = 1 + \gamma(\frac{g_s}{2} - 1) \qquad \text{(for a magnetic lens)} \qquad (16)$$

$$e = 1 + \gamma(\frac{g_s}{2} - 1) - \frac{g_s}{2\gamma} \qquad \text{(for an electric lens).}$$

If our beam of particles has a symmetric intensity distribution about the optical axis, then the average over all rays will give only a longitudinal polarization. The relative decrease of the longitudinal spin component can therefore be called the "depolarization" D. If we assume the lens enhance as uniformly illuminated up to an angle $\alpha = \alpha_o$, then

$$D = \left[\frac{1}{2}\alpha_o(1+\frac{\alpha}{\beta})e\right]^2 \tag{17}$$

With $\alpha_o = \pm 4°$ and a = b the depolarization D is 4% for a nonrelativistic proton beam passing through a magnetic quadrupole lens pair. Under the same conditions the depolarization for tritons would be 35% and for deuterons 0.4%. For a polarized triton beam it would be advisable to use electric quadrupole lenses. Evidently the depolarization can be smaller in a succession of lenses. This is the case for trajectories like the ones drawn by full lines in Fig. 2(b). In fact D would be zero at the point P_3 using the above approximation. The dotted trajectories of Fig. 2(b) would give at P_3 depolarization again given by Eq. (17). This behavior is clearly demonstrated in the work of Cohen et al. (Co59) who investigated the depolarization of protons during acceleration in a 50 MeV injector linac. This linac is strong focusing by means of quadrupole magnets enclosed within each of the 124 drift tubes. Cohen et al. integrated Eq. (11) numerically with a computer, following several proton trajectories. The calculation showed, that the proton moment never deviated more than 10° from the direction at injection.

III. TIME DEPENDENT ELECTROMAGNETIC FIELDS

We will describe the polarization state of the particles in a frame of reference which is attached to one particle and which has one of the coordinate axes always pointing in the direction of motion. In this rest system we can have time dependent fields $\vec{E}^{(r)}$ and $\vec{B}^{(r)}$ if the particles are moving through inhomogeneous laboratory fields or if the laboratory fields are themselves time dependent. In addition to introducing a time dependence the inhomogenity would give us interaction terms with the electric quadrupole moment and still higher moments of the particles. Good (Go62) derived classical relativistic equations of motion for the spin (in the rest system) including the effects of first order field gradients on the quadrupole moment. In most practical cases it is not necessary to consider these effects. To see this, let us make a rough estimate of the frequency ω resulting from the interaction of the electric quadrupole moment Q with an electric field gradient $\frac{\partial E}{\partial z}$. We have $\omega \simeq \frac{1}{\hbar}\, Qe \,\frac{\partial E}{\partial z}$ which amounts to $\omega \approx 10^{-6}\ s^{-1}$ for deuterons in a field gradient of $2 \cdot 10^{5}\ Vcm^{-1}$. Such a low frequency will introduce new resonances very close to magnetic moment resonances or will merely broaden or shift them by small amounts. In the rest system of the particles the effects of time dependent fields may therefore again be treated by solving the equation of motion, Eq. (7). In the case where the electric field in the laboratory frame is zero, Cohen (Co62) did solve the corresponding Eq. (11) directly with a computer. In some special cases, the change of the spin state can be given in a closed analytic form. The results were obtained for low velocities of the particles and can be used in the present considerations if we replace all quantities by the corresponding ones measured in the rest system. We will summarize the results for two cases. We consider the situation, where the magnetic field in the rest system can be represented as the vector sum of a static magnetic field $\vec{B}_o^{(r)}$ and a sufficiently weak oscillating perturbing field. We consider first the case where the frequency $\omega^{(r)}$ of the perturbing field is constant. The transition probability of a particle with spin 1/2 from the state $m_s = 1/2$ to $m_s = -1/2$ or vice versa in the time interval τ (Ra56)

$$P_{\frac{1}{2}\,-\frac{1}{2}} = P_{-\frac{1}{2}\,\frac{1}{2}} = \frac{\left|b^{(r)}\right|^2}{\left|a^{(r)}\right|^2} \sin^2\left(\frac{1}{2}\, a^{(r)}\tau\right) \tag{18}$$

with

$$b^{(r)} = \omega_L^{(r)} \frac{B_p^{(r)}}{B_o^{(r)}} \tag{19}$$

$$a^{(r)} = \left\{ \left(\omega^{(r)} - \omega_s^{(r)}\right)^2 + \left(b^{(r)}\right)^2 \right\}^{1/2} \tag{20}$$

Here $\omega_L^{(r)}$ and $\omega_s^{(r)}$ spin precession frequencies (see Eqs. (4, 8) and Table I). $B_p^{(r)}$ is the amplitude of the perturbing magnetic field component perpendicular to $B_o^{(r)}$ and rotating in the same sense as the magnetic moment. The fields in the rest system are given by Eq. (9) in terms of laboratory fields. In the second case $\omega^{(r)} - \omega_s^{(r)}$ may vary linearly with time:

$$\omega^{(r)} - \omega_s^{(r)} = T^{(r)}\tau \tag{21}$$

The equation of motion for the spin is in this case a confluent hypergeometric equation, as has been shown by Froissart (Fr60) and by Kim (Ki63). If $m_s = 1/2$ at time $\tau = -\infty$ the probability $P_{\frac{1}{2}\,-\frac{1}{2}}$ of finding $m_s = -1/2$ at the time $\tau = \infty$

is obtained from the proper asymptotic solution to this equation.

$$P_{\frac{1}{2}\,-\frac{1}{2}} = P_{-\frac{1}{2}\,\frac{1}{2}} = 1 - e^{-\frac{\pi}{2}\left|b^{(r)}\right|^2 / \left|T^{(r)}\right|}, \qquad (22)$$

Here $b^{(r)}$ is defined as in Eq. (19). For small transition probabilities $P_{\frac{1}{2}\,-\frac{1}{2}} \simeq \frac{\pi}{2} \frac{\left|b^{(r)}\right|^2}{\left|T^{(r)}\right|}$. The time $\Delta\tau$ needed to cross the resonance can be estimated from Eq. (18). Ninety percent of the above transition probability stems from the time interval between $-\Delta\tau$ to $\Delta\tau$ given by

$$\Delta\tau \simeq \frac{\pi b^{(r)}}{2T^{(r)}} \frac{1}{\sqrt{P_{\frac{1}{2}\,-\frac{1}{2}}}} \qquad (23)$$

or for small transition amplitudes by $\Delta\tau \simeq \sqrt{\pi/2\left|T^{(r)}\right|}$. Eq. (22) can therefore be used approximately where $\left(\omega^{(r)} - \omega_s^{(r)}\right)$ is a linear function of time only in a time interval of $\pm\,\Delta\tau$.

The probability P_{pq} for a transition of a particle with arbitrary spin s from a state $m_s = p$ to one with $m_s = q$ can be expressed in terms of the transition probability of a particle with spin 1/2. This is done by taking advantage of the fact, that $\vec{s}$ can be considered as composed of 2s angular momenta of spin 1/2 (see for example Bl45 or Ra56). The following expressions together with the symmetry relations

$$P_{pq} = P_{qp} = P_{-p\,-q} \qquad (24)$$

give all transition probabilities needed for particles with spin 1 and 3/2 respectively.

$$P_{11} = (1-P)^2 \qquad P_{01} = 2P(1-P)$$

$$P_{1-1} = P^2 \qquad P_{00} = (2P-1)^2 \tag{25}$$

$$P_{\frac{3}{2}\frac{3}{2}} = (1-P)^3 \qquad P_{\frac{1}{2}\frac{1}{2}} = 4(1-P)P^2 - 4(1-P)^2P + (1-P)^3$$

$$P_{\frac{3}{2}\frac{1}{2}} = 3P(1-P)^2 \quad P_{\frac{1}{2}-\frac{1}{2}} = 4(1-P)^2P - 4(1-P)P^2 + P^3$$

$$P_{\frac{3}{2}-\frac{1}{2}} = 3P^2(1-P) \tag{26}$$

$$P_{\frac{3}{2}-\frac{3}{2}} = P^3$$

P is an abbreviation for $P_{\frac{1}{2}-\frac{1}{2}}$ as calculated respectively from Eq. (18) or Eq. (22) depending on the actual experimental condition. For P = 1 it is seen that $P_{pq} = \delta_{p-q}$. This leads therefore to the complete reversal of the spin direction relative to the static magnetic field. An application of this result, known in nuclear induction as adiabatic fast passage was suggested by Abragam et al. (Ab58) as an efficient method of changing the polarization state of the atomic beam in polarized ion sources. The treatment in terms of transition probabilities gives all the required information for a beam of particles with a rotationally symmetric polarization state about the magnetic field $\vec{B}_o$. If we designate the new polarization state (symmetrical about the z-axis) by a prime, it follows from Eq. (25) that for spin 1 particles

$$t'_{10} = (1-2P)t_{10}$$

$$t'_{20} = (1-6P(1-P))t_{20} \qquad (27)$$

We do not treat here the case where the polarization that is not symmetrical, but it may be worthwhile to point out briefly where changes would have to be made. The spin state $\psi = \sum_m a_m |s,m\rangle$ will be changed through the transition to a state $\psi' = \sum_{mp} a_m C_{mp} |s_1 p\rangle$. Knowledge of the transition amplitudes C_{mp} instead of the above used transition probabilities $P_{mp} = |C_{mp}|^2$ will allow the calculation of the new density matrix ρ'. The C_{pq} have to be calculated only for a spin of 1/2. The ones for a general spin s are given in terms of spin 1/2 amplitudes by Ramsay (Ra56, p. 429). The next section will show the practical application of the above equations in the case of cyclic accelerators.

IV. DEPOLARIZATION IN CYCLIC ACCELERATORS

We will discuss first the proposed schemes of injecting polarized particles into cyclotrons and discuss in more detail the depolarization of particles during the acceleration process. At the end of the section we summarize the results obtained by different authors for the depolarization in synchrocyclotrons and synchrotrons. Particles from polarized ion sources are injected into cyclotrons either as an atomic beam or as ionized particles. In the first method as used by Keller et al. (Ke61) and Beurtey et al. (Be63) the atomic beam is sent toward the center of the cyclotron in the median plane between the magnetic poles. The symmetry axis of the polarization state is parallel to the magnetic guide field of the accelerator. Ionization with electron bombardment takes place in the strong magnetic field at the center. To achieve a reasonable current and to prevent a too large contamination of the beam with unpolarized ions formed in the ionization region from the residual gas, the atomic beam should have a small divergence and the partial pressure in the cyclotron of gases likely to contaminate the beam should be low. The last condition is evidently easier to fulfill for deuterons than for protons. The second method is used successfully in the Birmingham sector focused cyclotron (Po65, Po66). The ionization of the atomic beam takes place outside the cyclotron where the contamination with unpolarized particles is easier to control. After acceleration to about 10 keV the beam is injected through an axial hole of the cyclotron magnet into the central region where the particles are deflected by an electric field toward their orbit. About 3.5% of the injected (unpulsed) beam is successfully accelerated. Experiments are being done in Saclay (Be65) to inject a charged polarized beam in the median plane of the cyclotron by compensating the magnetic force at each point of the trajectory with an electric field.

We will now consider in more detail the possible depolarization during the acceleration. The details of the calculation will depend on the type of accelerator, but the underlying ideas which were mainly developed by Froissart et al. (Fr60) and Kim et al. (Ki63) are equally applicable to other types of cyclic accelerators as well as to the calculation of the depolarization during the injection or extraction. For the numerical illustrations the University of California 88-inch cyclotron in the Lawrence Radiation Laboratory, Berkeley will be used (Ke62). Figure 3 shows the magnet pole arrangement of this three sector isochronous cyclotron. A typical equilibrium orbit of particles with a constant energy is also shown in this figure. The motion of the particles can be determined by the knowledge of the magnetic field $\vec{B}(r, \phi, z = 0)$ in the median plane.

We write for the fourier-expansion of this field along a circle with radius r

$$B(r, \phi, 0) = \sum_{n=0}^{\infty} A_n(r) \cos\left[n(\phi-\phi_n(r))\right] \tag{28}$$

A cyclotron with perfect N-fold symmetry (N is the number of sectors) has only nonvanishing coefficients A_n for $n = kN$, $k = 1,2$, etc. Figure 4 shows these coefficients and the phase angles ϕ_n for a typical magnetic field of the Berkeley 88"-cyclotron. Particles of a constant energy will make oscillatory deviations from the equilibrium orbit. The total deviation of an actual orbit from a circle in the median plane can therefore be written approximately (Sm59 or Ha62) as

$$\begin{aligned} z &= z_m \cos(\nu_z \omega_c t + \delta_z) \\ \Delta r &= \frac{r}{N^2-1} \frac{A_n}{A_o} \cos N\omega_c t + \Delta r_m \cos(\nu_r \omega_c t + \delta_r) \end{aligned} \tag{29}$$

The first term in the equation for Δr represents the deviation of the equilibrium orbit from a circle. The other terms describe the vertical and horizontal betatron oscillations. Their amplitudes z_m and Δr_m and phases δ_z, δ_r depend on the initial conditions of the particle. The angular frequencies $\nu_z \omega_c$ and $\nu_r \omega_c$ of the betatron oscillations can be calculated from the magnetic field coefficients A_n (Sm59). Figure 5 shows examples of their magnitude for a proton and a deuteron beam as functions of radius.

To be able to calculate the depolarization of the particles, we need the frequencies and amplitudes of the magnetic fields which the particle experiences in its orbit and which are perpendicular to the main magnetic field. In the median plane the magnetic field has no such components, but particles which are above or below this plane will experience B_r and B_ϕ field components.

$$B_r = \frac{\theta B}{\theta r} z + \frac{\theta^2 B}{\theta r^2} z\Delta r + \ldots$$

$$B_\phi = \frac{1}{r}\frac{\theta B}{\theta \phi} z + \frac{1}{r}\frac{\theta^2 B}{\theta\phi\theta r} z\Delta r + \ldots \tag{30}$$

These Taylor expansions follow from curl $\vec{B} = 0$ and from the above mentioned vanishing of B_r and B_ϕ in the median plane. All derivatives in Eq. (30) are understood to be taken at $z = 0$, $\phi = \omega_c t$. Inserting the expressions (28) and (29) for z, Δr and B into the above equations we see that the particles experience horizontal perturbing fields with angular frequencies $\omega_{nm\ell}$ given by

$$\omega_{nm\ell} = \omega_c(n+m\nu_r+\ell\nu_z) \quad n,m,\ell = 0, \pm 1, \pm 2, \ldots \tag{31}$$

where an expansion of Eq. (30) to higher powers in z will give rise to the values of m and ℓ larger than one. It is convenient to introduce $\Delta\nu_{nm\ell}$ as the deviation of $\omega_{nm\ell}$ from the spin precession angular velocity in units of ω_c

$$\Delta\nu_{nm\ell} = n + m\nu_r + \ell\nu_z - \gamma\frac{g_s}{2} - 1 \tag{32}$$

Which choice of (n,m,ℓ) will give rise to resonant depolarization $(\Delta\nu_{nm\ell} = 0)$ can be judged easily from Figs. 4 and 5 and from the value of $\gamma\left(\frac{g_s}{2} - 1\right)$ of the particle. For nonrelativistic protons $\gamma\left(\frac{g_s}{2} - 1\right) = 1.8$, we expect possible resonances to occur for $(1,1,-1)$, $(2,0,-1)$, $(3,-1,-1)$ etc. The transition probability depends (see Eq. (18) or (22)) on the amplitude of the perturbing field $B_p^{(r)}(\omega_{nm\ell})$ and on the time spent near the resonance.

$$\vec{B}_p^{(r)}(\omega_{nm\ell}) = \vec{B}\phi(\omega_{nm\ell}) + \gamma\vec{B}_r(\omega_{nm\ell}) \qquad (33)$$

In a three sector accelerator the largest amplitudes are present for n = 0 or 3 (see Fig. 4). We expect therefore that the contribution from the (3,-1,-1) - resonances are the most important ones for the depolarization of protons in a three sector accelerator. The following explicit expressions for these fourier-components are obtained from Eqs. (28) to (30)

$$B\phi(\omega_{3,-1,-1}) = \frac{1}{8}\,\frac{z_m \Delta r_m}{r}\;9A_3\,\frac{\partial\phi_3}{\partial r} + i3\frac{\partial A_3}{\partial r}$$

$$B_r(\omega_{3,-1,-1}) = \frac{1}{8}\,z_m\Delta r_m\left[\frac{\partial^2 A_3}{\partial r^2} - 9A_3\left(\frac{\partial\phi_3}{\partial r}\right)^2 - i\left(6\frac{\partial A_3}{\partial r}\frac{\partial\phi_3}{\partial r} + 3A_3\frac{\partial^2\phi_3}{\partial r^2}\right)\right] \qquad (34)$$

A similar consideration shows that (0,0,-1) - resonances give the most important contribution to the depolarization of deuterons. The perturbing field in this case is

$$B_p^{(r)}(\omega_{0,0,-1}) = \frac{1}{2}\,\gamma z_m\,\frac{\partial A_o}{\partial r} \qquad (35)$$

Figure 6 shows $\Delta v_{3,-1,-1}$ for two proton energies and $\Delta v_{0,0,-1}$ for one deuteron energy as function of radius. In all examples the particles have to cross depolarization resonances at least twice. The transition probability P given by Eq. (22) can be written for an isochronous cyclotron in the following form

$$P = \left(\frac{B_r^{(r)}(\omega_{nm\ell})}{A_o(R)} \frac{g_s}{2}\right)^2 \frac{\pi^2 U}{\frac{d}{dr}(\Delta v_{nm\ell})} \frac{r}{R^2} \frac{\gamma^{(R)+1}}{\gamma^{(r)}} (1-K(r)) \qquad (36)$$

with $P \ll 1$

U average number of turns until extraction

R extraction radius

r resonance radius

$$\gamma^2(r) = 1 + (\gamma^2(R)-1)\left(\frac{rA_o(r)}{RA_o(R)}\right)^2$$

$$K = -\frac{r}{A_o}\frac{dA_o}{dr}$$

The transition takes place (see Eq. (23)) in the interval

$$\Delta v_{nm} = \pm \frac{B_p^{(r)}(\omega_{nm\ell})}{A_o(r)} \frac{g_s}{2} \frac{\pi}{\sqrt{2P}} \qquad (37)$$

Table II shows that the magnitudes of the depolarizations of protons and deuterons accelerated in the Berkeley 88-inch cyclotron are small. An average energy gain of 100 keV per turn was assured and 1 cm for the amplitudes of the betatron oscillations. The resonance width (Eq. (37)) is also shown in Table II. It is seen that for most resonances Eq. (21) is fulfilled over the region given by Eq. (24). At small radii this condition is more marginal. The contributions to the depolarization from regions where the resonance condition is not fulfilled can be estimated from Eq. (18). For protons with 55 MeV final energy P is approximately 5×10^{-5} in the interval from r = 4 inch to r = 18 inch. Small values for the calculated depolarization were found by Kim et al. (Ki63) for deuterons accelerated in the Birmingham sector focussed cyclotron and by Khoe et al. (Kh63) for protons accelerated in the cyclotron at Washington University in St. Louis. (see also So64). Experimentally polarized deuterons were

Table II. Depolarization of protons and deuterons accelerated with the Berkeley 88-inch cyclotron

Particle	final energy MeV	type of resonance (n,m,ℓ)	approx. resonance radius (inch)	$P \times 10^2$	resonance width $\pm\Delta\nu_{nm\ell}$	Total decrease of vector polarization	Total decrease of tensor polarization t_{20}
p	55	(3,-1,-1)	2.5	.02	.08	.1%	
			24	.04	.01		
p	10	(3,-1,-1)	6	.02	.1	.06%	
			17	.01	.03		
d	65	(0,0,-1)	4	.08	.07	.34%	1.0%
			13	.01	.03		
			21	.01	.04		
			24	.01	.04		
			27	.02	.03		
			36	.04	.04		

successfully accelerated in a conventional cyclotron up to 22 MeV in Saclay (Be63).

In synchrocyclotrons the depolarizations are generally more severe, mainly because of the large number of turns during acceleration. Indeed the calculated depolarization of protons is substantial for the Rochester 130 inch synchrocyclotron (Lo62). For the Dubna synchrocyclotron small proton but large deuteron depolarizations are predicted (Pl64). Because the magnitude of the depolarization depends quadratically on the amplitude of the vertical betatron oscillation (see Eqs. (36) and (30)) a reduction of this amplitude possibly by a careful injection of a good quality beam in the median plane may avoid too large effects.

Protons depolarize generally completely during acceleration in synchrotrons (Fr60, Co62, Ze64). Cohen (Co62) proposes a method using pulsed quadrupole magnets to traverse the dangerous resonances quickly enough to avoid excessive depolarizations.

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FIGURE CAPTIONS

Fig. 1. A partical trajectory (shown as full line) and two rest frames (R and R') as seen from an inertial system (L).

Fig. 2. Depolarization in electromagnetic lenses.

Fig. 3. Schematic of the Berkeley 88" cyclotron, showing the spiral ridges, and equilibrium orbit, and the deflector.

Fig. 4(a) Amplitudes of the first three harmonics of the magnetic field of the Berkeley 88-inch cyclotron. The field shape corresponds approximately to 55 MeV protons extracted at a radius of 38.8". (b) Phase angle of the harmonics. For other energies the magnitude of A_n will be different, but the relevant ratio $\frac{A_n}{A_o}$ will be approximately the same. Data from (C166).

Fig. 5. Calculated radial (v_r) and vertical (v_z) betatron frequencies for the Berkeley 88-inch cyclotron. (Data from C166)

55 MeV protons o-o-o and ●-●-●

65 MeV deuterons △-△-△ ~~▲-▲-▲~~

Fig. 6. Deviation $\Delta v_{nm\ell}$ of the perturbing field frequencies from the spin precession frequencies, measured in units of the cyclotron frequency, for the Berkeley 88-inch cyclotron.

o-o-o $\Delta v_{3,-1,-1}$, 55 MeV protons

●-●-● $\Delta v_{3,-1,-1}$, 10 MeV protons

△-△-△ $\Delta v_{0,0,-1}$, 65 MeV deuterons

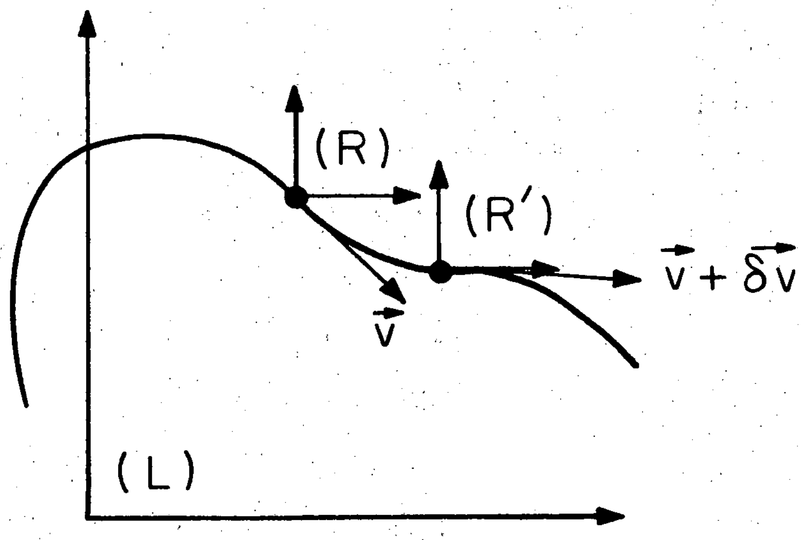

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Fig. 1

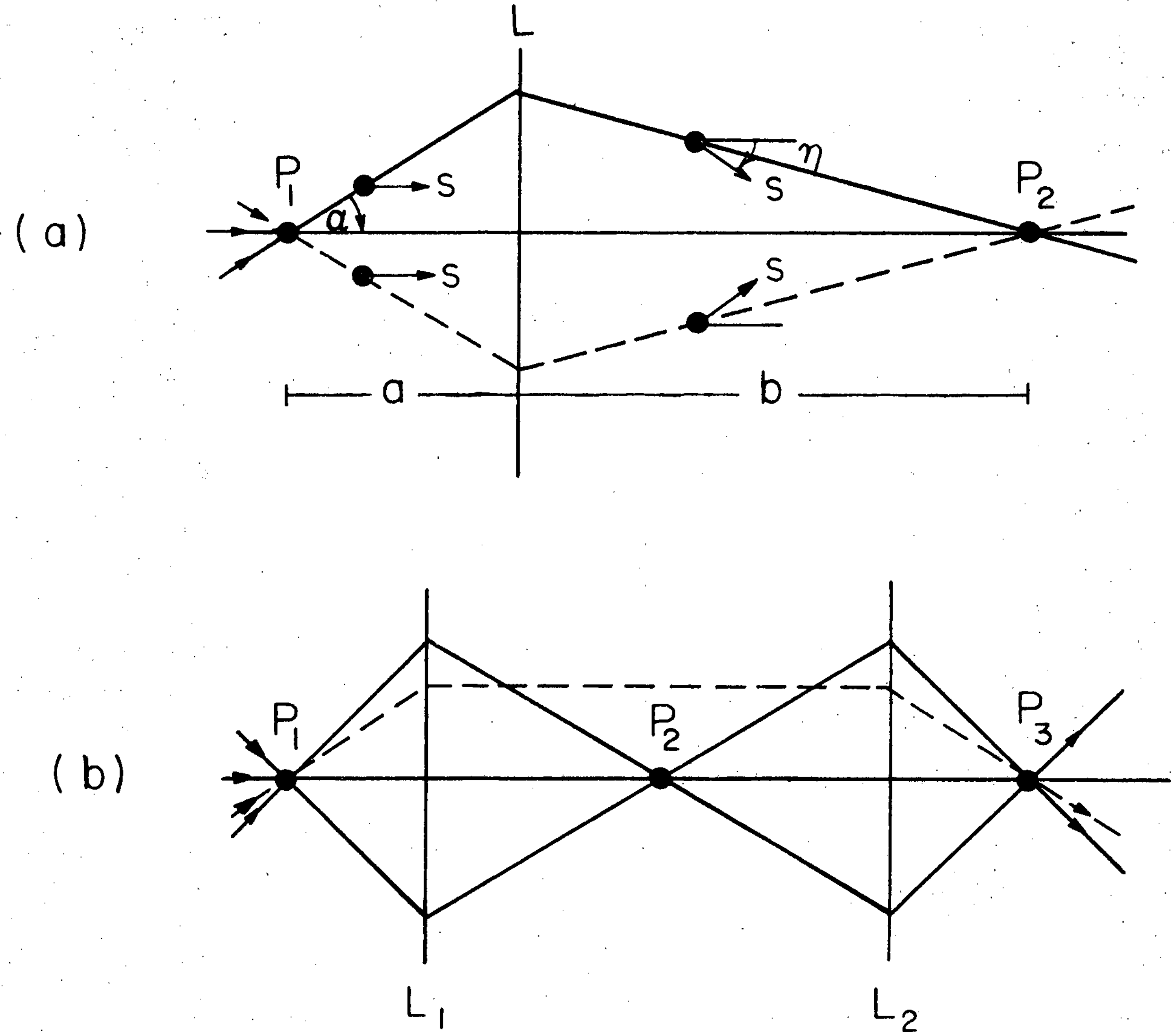

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Fig. 2

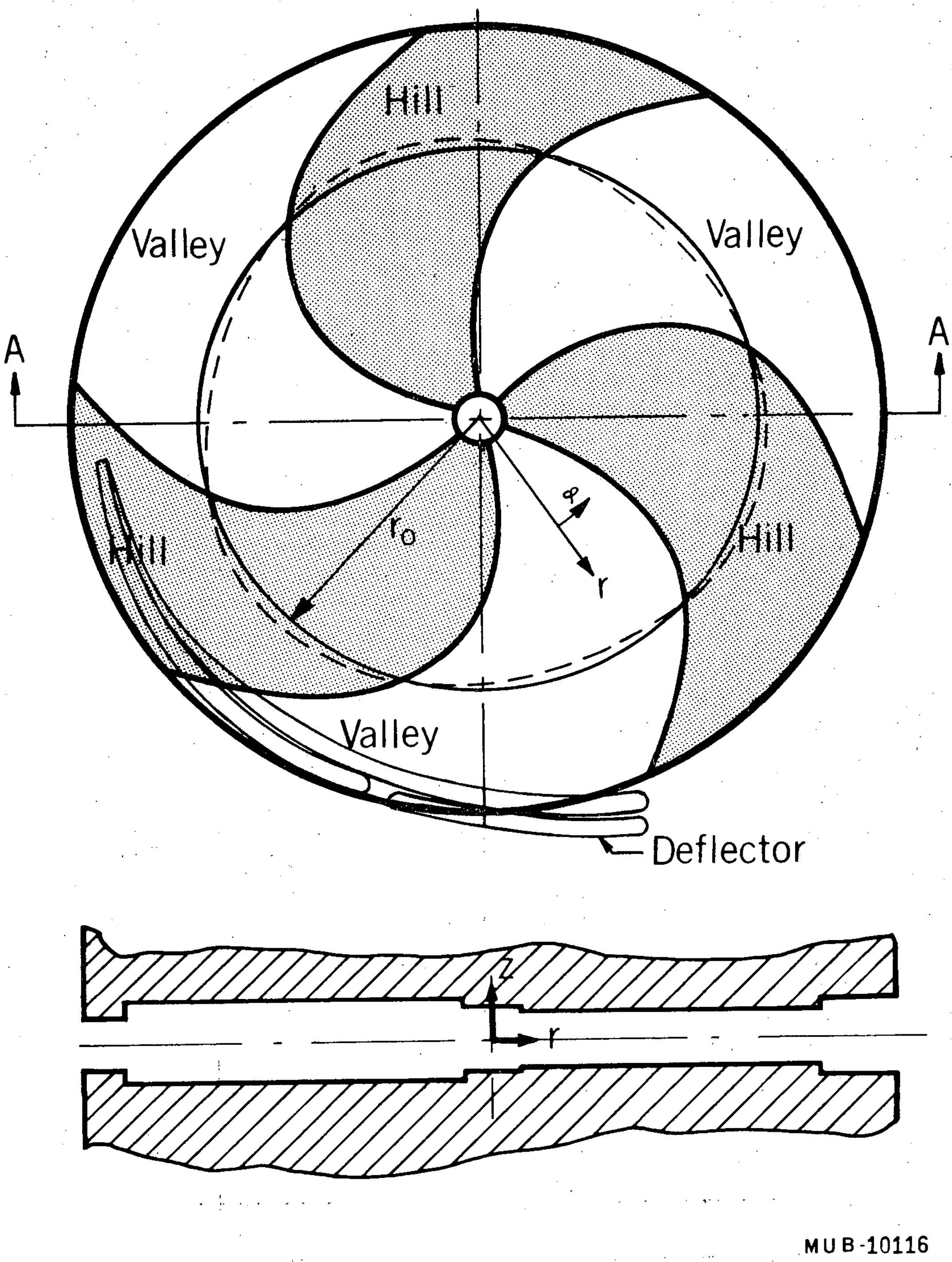

Hill
Valley
Valley
A
A
r
φ
r
Hill
Hill
Valley
Deflector
z
r
MUB-10116

Fig. 3

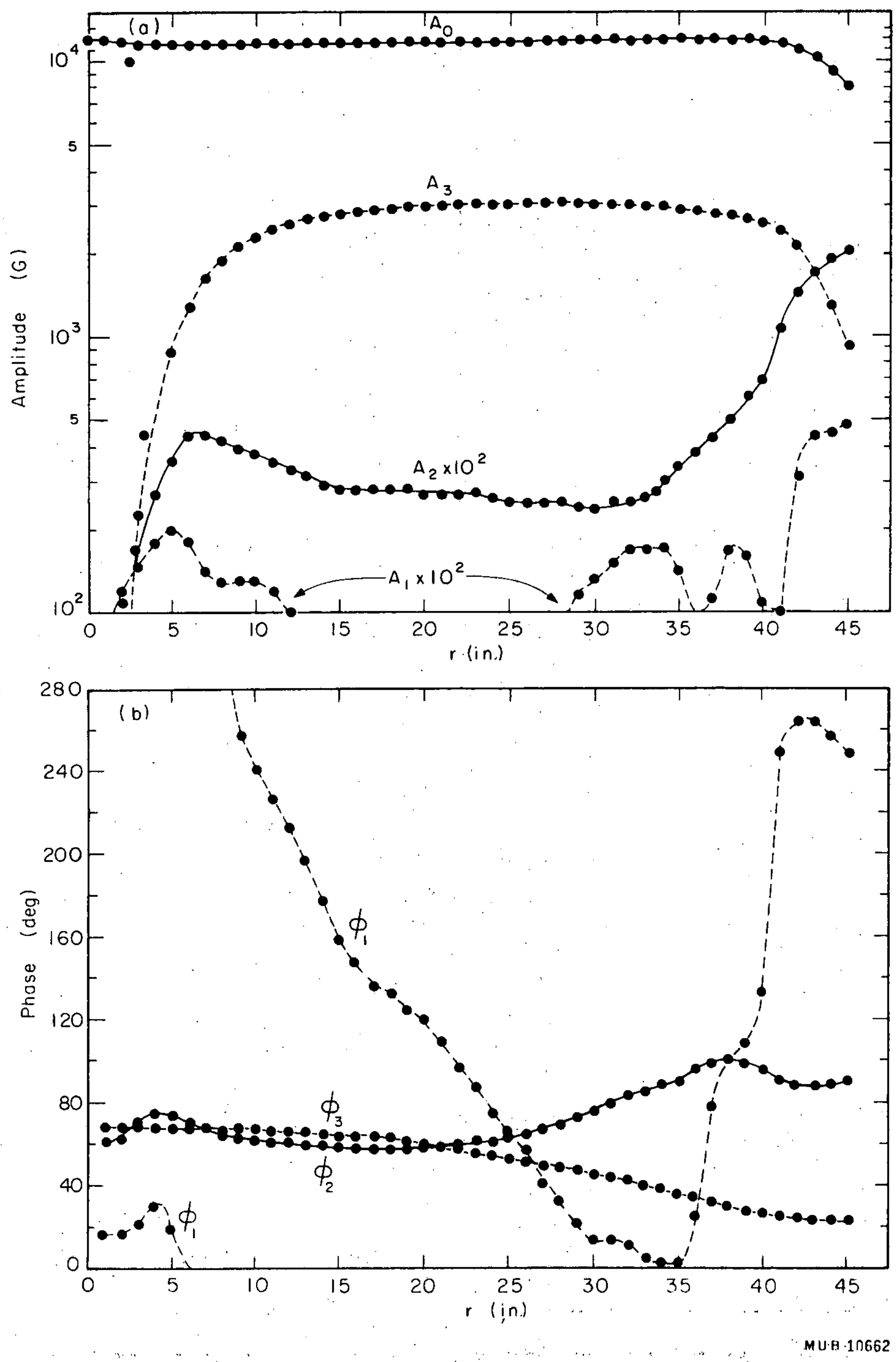
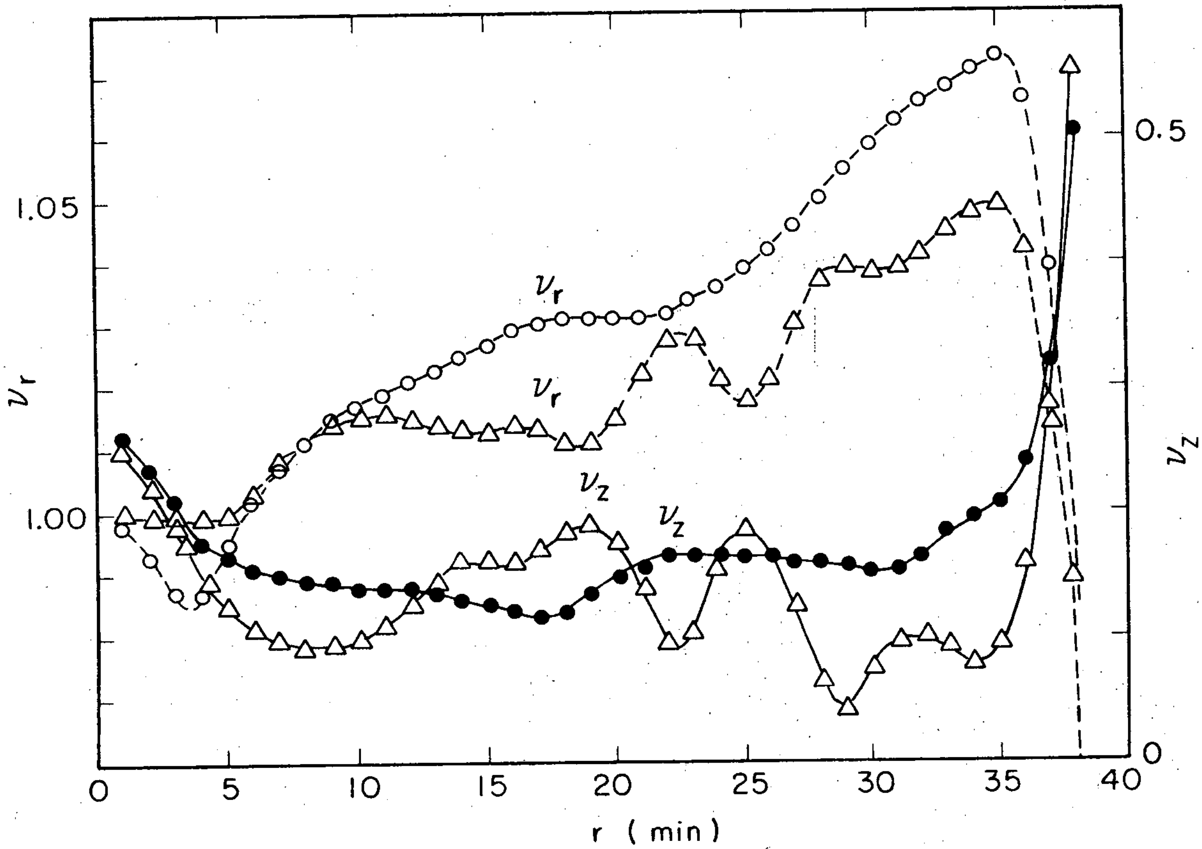

(a)
A0
10^4
5
A3
Amplitude (G)
10^3
5
A2 x 10^2
A1 x 10^2
10^2
0 5 10 15 20 25 30 35 40 45
r (in.)
280
(b)
240
200
160
120
80
40
0
Phase (deg)
φ1
φ3
φ2
φ1
0 5 10 15 20 25 30 35 40 45
r (in.)
MUB 10662

Fig. 4



MUB-10666

Fig. 5

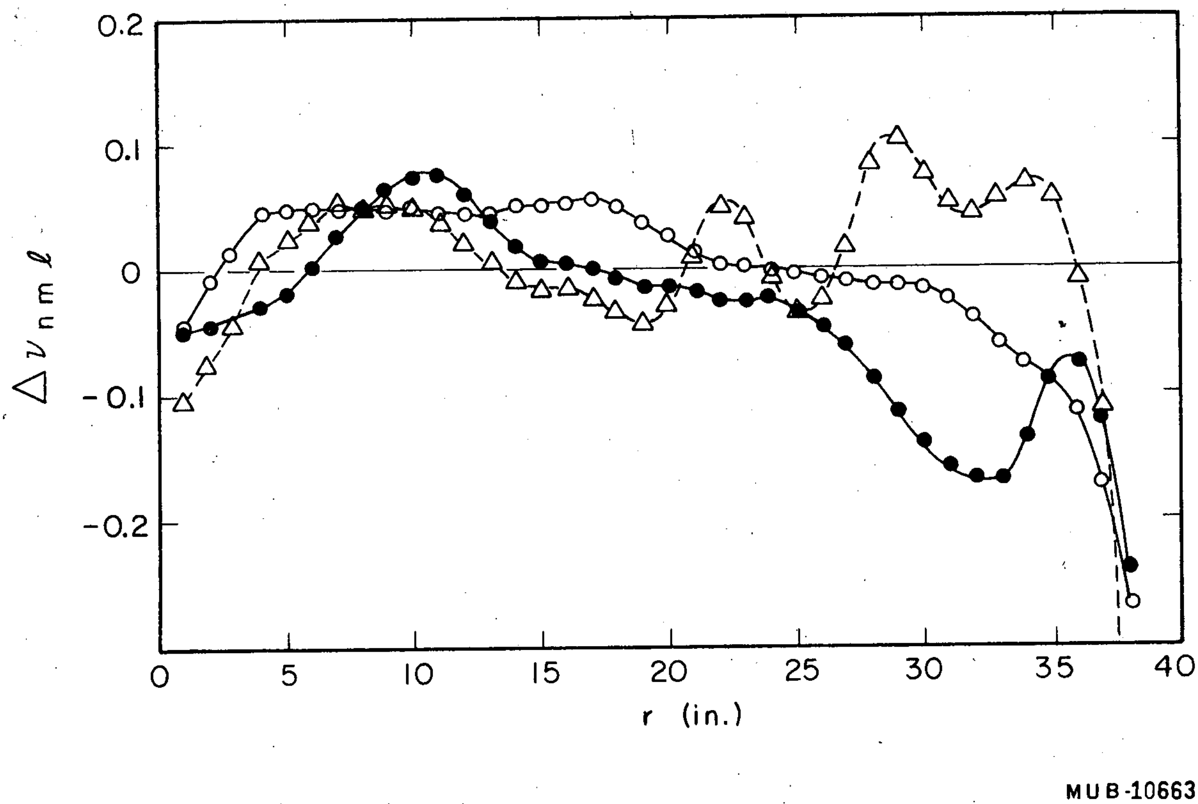


Fig. 6