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James-Stein Type Estimators in Large Samples with Application to the Least Absolute Deviations Estimator

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Abstract: We explore the extension of James-Stein type estimators in a direction that enables them to preserve their superiority when the sample size goes to infinity. Instead of shrinking a base estimator towards a fixed point, we shrink it towards a data-dependent point. We provide an analytic expression for the asymptotic risk and bias of James-Stein type estimators shrunk towards a data-dependent point and prove that they have smaller asymptotic risk than the base estimator. Shrinking an estimator toward a data-dependent point turns out to be equivalent to combining two random variables using the James-Stein rule. We propose a general combination scheme which includes random combination (the James-Stein combination) and the usual nonrandom combination as special cases. As an example, we apply our method to combine the Least Absolute Deviations estimator and the Least Squares estimator. Our simulation study indicates that the resulting combination estimators have desirable finite sample properties when errors are drawn from symmetric distributions. Finally, using stock return data we present some empirical evidence that the combination estimators have the potential to improve out-of-sample prediction in terms of both mean square error and mean absolute error.

Key Words: Shrinkage; Asymptotic risk; Combination estimator

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1. INTRODUCTION

Shrinkage techniques for the linear regression model have been studied extensively since the seminal works by Stein (1955) and James and Stein (1960), who proved that the usual estimator for the mean of a multivariate normal distribution is inadmissible and there exists an improved estimator with smaller risk when the dimension of the multivariate normal vector is greater than two.

Even though this discovery was surprising, its usage has been restricted to small sample situations because the advantage of smaller risk tends to disappear as the sample size grows. Schmoyer and Arnold (1989) proposed a James-Stein (JS) type estimator that can achieve risk improvement in large samples, at a cost of imposing a very restrictive assumption on the prior information. We follow an approach taken by Green and Strawderman (1991) for fixed sample size n to shrink a given base estimator towards a data-dependent point; here, however, unlike Green and Strawderman, our data dependent point can be either asymptotically biased or correlated with the base estimator, and we consider what happens as $n \to \infty$. The resulting shrinkage estimator in its general form asymptotically dominates both the base estimator and the data-dependent point in terms of quadratic loss. The data-dependent point can be another estimator under some mild restrictions.

Shrinkage estimators of this type have been studied in depth in a series of papers by Saleh and Sen (1985a, 1985b, 1986, 1987a, 1987b), Sen and Saleh (1987) and Saleh and Han (1990). These authors considered shrinking unrestricted estimators towards restricted estimators as a smooth version of the pre-test estimator and provided a range for the degreee of shrinkage that ensures risk dominance. In this paper we investigate shrinkage estimation in a more general setup that permits our results to be applied to a wide range of estimators used in econometrics and statistics, substantively extending the validity of the work of Saleh and his collaborators. Further, we provide explicit expressions for the optimal shrinkage parameter values and propose consistent estimators for these values, yielding feasible minimum asymptotic risk estimators.

To illustrate our results we choose the Least Absolute Deviations estimator as the base estimator and the Least Squares estimator as the data-dependent point. Our estimator in this case is an optimal mix of the information contained in the two estimators. This example can be viewed as a multivariate extension

of Laplace (1818), who combined the sample median and the sample mean by minimizing the asymptotic variance.

2. ASYMPTOTIC RISK IMPROVEMENT

Suppose data are generated according to $y_t = X_t'b^0 + e_t \quad t = 1, 2, ..., n$, where $b^0 \in R^k$ and e_t are independent and identically distributed with mean 0 and variance s^2. Let b_n be an estimator for b^0, and let Q_n be a symmetric positive semi-definite $k \times k$ matrix. The quadratic loss is $L(b_n, b^0) \equiv (b_n - b^0)'Q_n(b_n - b^0)$ and its expectation is the risk, denoted by $R(b_n, b^0)$. Suppose $n^{1/2}(b_n - b^0)$ and $L(b_n, b^0)$ converge in distribution to some integrable random variables Z and Ψ respectively. The asymptotic bias of $\{b_n\}$ is then defined by $AB(\{b_n\}) \equiv E(Z)$ and the asymptotic risk of $\{b_n\}$ is given by $AR(\{b_n\}, b^0) \equiv E(\Psi)$. We consider a $k \times 1$ vector g_n towards which the base estimator b_n is shrunk. Classical JS type estimators are obtained by setting g_n to a fixed number. In this paper, we allow g_n to be data-dependent. We now provide formal conditions for our analysis.

Assumption 1. $n^{-1}Q_n \xrightarrow{p} Q$ where Q is a nonstochastic symmetric positive definite matrix.

Assumption 2. $\begin{bmatrix} n^{1/2}(b_n - b^0) \\ n^{1/2}(g_n - b^0) \end{bmatrix} \xrightarrow{d} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \sim N(x, \Sigma)$ where $x \equiv \begin{bmatrix} 0 \\ q \end{bmatrix}$, $\Sigma \equiv \begin{bmatrix} A & \Delta \\ \Delta' & B \end{bmatrix}$ and A, B, and Σ are symmetric positive definite matrices.

Once an analyst restricts attention to, for instance, some particular estimators for the base estimator and the data-dependent point, Assumption 2 can be replaced by a set of more primitive conditions. This is especially straightforward when b_n and g_n are M – estimators. We note that the local alternatives used in Saleh and Sen (1985a, 1985b), Saleh and Sen (1986), Saleh and Sen (1987a, 1987b) provide one example of the bias term ? in Assumption 2. In this paper we entertain the flexible situation where the bias can be either zero or non-zero.

The natural JS type shrinkage is

$$d_{c_1}^{JS}(b_n, g_n) \equiv \{1 - c_1 / \| b_n - g_n \|_n^2\}(b_n - g_n) + g_n \tag{1}$$

where c_1 is a constant and $\| b_n - g_n \|_n^2 \equiv (b_n - g_n)' Q_n (b_n - g_n)$. When n is fixed and g_n is independent of the base estimator, the estimator in (1) is identical to the one in Green and Strawderman (1991). Saleh and Sen (1987a) and Saleh and Han (1990) have studied this type of general JS estimator extensively in a special case in which the base estimator is the Unrestricted Least Squares Estimator and the data-dependent point is the Restricted Least Squares Estimator. Further details are given in Saleh and Sen (1985a, 1985b, 1986, 1987b) and Sen and Saleh (1987).

Before proceeding, we define some variables used in our analysis. Since there exists a matrix P such that $PP' = \Sigma$, we define $Z \equiv P^{-1}U$ so that $Z \sim N(m, I_{2k})$ where $m \equiv P^{-1}x$. Then it can be shown that $(U_1 - U_2)' Q (U_1 - U_2) = Z' M_1 Z$ and $U_1' Q (U_1 - U_2) = Z' M_2 Z$ where $M_1 \equiv P' J_1' Q J_1 P$, $J_1 \equiv (I_k, -I_k)$, $M_2 \equiv P' J_2' Q J_1 P$, and $J_2 \equiv (I_k, 0)$. This transformation allows us to use the results in Ullah (1990) for the ratio of quadratic forms of normal random variables. We now provide conditions ensuring that the shrinkage estimator in (1) dominates the base estimator and specify the optimal value for c_1.

Theorem 1. Suppose that Assumptions 1 and 2 hold and $k > 4$. Then

(i) Let $c_1^* \in \text{argmin}\ AR(\{d_{c_1}^{JS}(b_n, g_n)\}, b^0)$. Then $c_1^* = n / w$ where

$$w \equiv \int_0^\infty | N_{0t} |^{-1/2} \exp\{-\frac{1}{2} m' N_{1t} m\} dt,$$

$$n \equiv \int_0^\infty | N_{0t} |^{-1/2} \{tr(M_2 N_{0t}^{-1}) + m' N_{2t} m\} \exp\{-\frac{1}{2} m' N_{1t} m\} dt,$$

with $N_{0t} \equiv I + 2tM_1$, $N_{1t} \equiv 2tM_1 N_{0t}^{-1}$ and $N_{2t} \equiv N_{0t}^{-1} M_2 N_{0t}^{-1}$.

(ii) $AR(\{d_{c_1^*}^{JS}(b_n, g_n)\}, b^0) = -n^2 / w + k$ where $k \equiv tr(QA)$.

(iii) $AR(\{d_{c_1^*}^{JS}(b_n, g_n)\}, b^0) \leq AR(\{b_n\}, b^0)$ where the equality holds only when $n = 0$.

(iv) $AR(\{d_{c_1}^{JS}(b_n, g_n)\}, b^0) \leq AR(\{b_n\}, b^0)$ if $c_1 \in [\min\{0, 2n / w\}, \max\{0, 2n / w\}]$ where the equality holds only when $n = 0$.

(v) $AB(\{d^{JS}_{c_1^*}(b_n, g_n)\}) = -c_1^* M_3 (mw + w'_m)$

where $M_3 \equiv J_1 P$ and $w'_m \equiv -\int_0^\infty |N_{0t}|^{-1/2} \exp\{-\frac{1}{2} m' N_{1t} m N\} N_{1t} m dt$.

We call the shrinkage estimator in (1) with the optimal c_1^* the James-Stein Mix (JSM). It is obvious from Theorem 1 that in order to have a non-empty range of c_1 for risk dominance, we require three conditions: (i) $0 < w < \infty$, (ii) $|n| < \infty$, and (iii) $n \neq 0$. As will be shown in Lemma 4, $k > 2$ is a sufficient condition for (i) and $k > 4$ is sufficient for (ii). Therefore, when the base estimator is correlated with the data-dependent point, a sufficient condition for asymptotic risk dominance is $k > 4$. In contrast to the finite sample fixed point shrinkage analysis where condition (iii) is automatically satisfied, it must be imposed in our asymptotic setup, where we call it the Relative Efficiency Condition (REC). The REC does not allow the choice of an asymptotically efficient estimator as the base estimator unless either we select a super-efficient estimator as our data-dependent point or the data-dependent point has an asymptotic bias. When b_n is asymptotically efficient and g_n is asymptotically unbiased and not super-efficient, one can easily show that $Cov(U_1, U_1 - U_2) = 0$, which implies that $n = 0$, and there is no risk improvement.

To gain additional insight, consider the special case in which the base estimator is not correlated with the data-dependent point $(\Delta = 0)$. Then, it can be shown that $n = (k-2)s^2 w$. In this special case we only need the condition $k > 2$, which is the same as in the finite sample analysis. Note that this also implies that $c_1^* = (k-2)s^2$, which is precisely the same shrinkage factor as in the finite sample analysis. Hence, the deviation of the ratio n/w from $(k-2)s^2$ depends on the degree of the asymptotic correlation between the base estimator and the data-dependent point.

Whereas the JSM is a combination of two random variables using a random weight, conventional combination estimators use a nonrandom weight. This nonrandom combination has been studied mainly for independent estimators by, for example, Cohen (1976) and Green and Strawderman (1991). Laplace (1818) combined the sample median and the sample mean by minimizing asymptotic variance.

Here we consider combining multi-dimensional correlated estimators by minimizing asymptotic risk. The usual combination gives

$$d_{c_2}^{NR}(b_n, g_n) \equiv \{1 - c_2\}(b_n - g_n) + g_n \tag{2}$$

where c_2 is a constant. Using the same arguments as in Theorem 1, we obtain the following results.

Theorem 2. Suppose that Assumptions 1 and 2 hold. Then

(i) Let $c_2^* \in \text{argmin } AR(\{d_{c_2}^{NR}(b_n, g_n)\}, b^0)$. Then $c_2^* = r / a$ where

$a \equiv tr[(A - \Delta - \Delta' + B + qq')Q]$ and $r \equiv tr[(A - \Delta)Q]$.

(ii) $AR(\{d_{c_2^*}^{NR}(b_n, g_n)\}, b^0) = -r^2 / a + k$.

(iii) $AR(\{d_{c_2^*}^{NR}(b_n, g_n)\}, b^0) \leq AR(\{b_n\}, b^0)$ where the equality holds only when $r = 0$.

(iv) $AR(\{d_{c_2}^{NR}(b_n, g_n)\}, b^0) \leq AR(\{b_n\}, b^0)$ if $c_2 \in [\min\{0, 2r / a\}, \max\{0, 2r / a\}]$

where the equality holds only when $r = 0$.

(v) $AR(\{d_{c_2^*}^{NR}(b_n, g_n)\}, b^0) \leq AR(\{g_n\}, b^0)$ where the equality holds only when

$\gamma = 0$ with $\gamma = E[(U_1 - U_2)'QU_2]$.

(vi) $AB(\{d_{c_2^*}^{NR}(b_n, g_n)\}) = c_2^* q$.

We call the combination in (2) with the optimal c_2^* the Nonrandom Mix (NRM). If both $r \neq 0$ and $\gamma \neq 0$, the analog of the REC in the present context, then the asymptotic risk of the NRM is strictly smaller than that of both the base estimator and the data-dependent point.

So far, we have investigated the JSM and the NRM separately. It will be desirable to combine them together in one estimator so that the random and non-random contributions to the asymptotic risk reduction are determined simultaneously in an optimal way. Such a general combination scheme is naturally given by

$$d_l^{OW}(b_n, g_n) \equiv \{1 - l_1 - l_2 / \| b_n - g_n \|_n^2\}(b_n - g_n) + g_n, \tag{3}$$

where $l = (l_1, l_2)'$ is a constant vector. We now require the following additional assumption.

Assumption 3. $(U_1 - U_2)'Q(U_1 - U_2)$ is nondegenerate.

The following theorem provides results on the combination scheme (3).

Theorem 3. Suppose that Assumptions 1, 2 and 3 hold and $k > 4$. Then

(i) $AR(\{d_l^{OW}(b_n, g_n)\}, b^0)$ is strictly convex in l.

(ii) Let $l^* \in \operatorname{argmin} AR(\{d_l^{OW}(b_n, g_n)\}, b^0)$. Then

$l_1^* = (aw-1)^{-1}(rw-n)$ and $l_2^* = (aw-1)^{-1}(an-r)$ where a, r, n and w are given in Theorems 1 and 2.

(iii) $AR(\{d_{l^*}^{OW}(b_n, g_n)\}, b^0)$

$= (aw-1)^{-2}\{-ar^2w^2 - (2arn - a^2n^2 + r^2)w + (an^2 - 2rn)\} + k.$

(iv) $AR(\{d_{l^*}^{OW}(b_n, g_n)\}, b^0) \leq AR(\{b_n\}, b^0)$ where the equality holds only when $r = 0$ and $n = 0$.

(v) $AB(\{d_{l^*}^{OW}(b_n, g_n)\}) = l_1^* q - l_2^* M_3(mw + w_m')$.

We call the estimator in (3) with the optimal weight l^* the Optimal Weighting Mix (OWM) and $(a, r, n, w)'$ the Combination Control Parameters (CCPs). We now prove that the OWM asymptotically dominates both the JSM and the NRM.

Corollary 1. Suppose that Assumptions 1, 2 and 3 hold and $k > 4$. Then

(i) $AR(\{d_{l^*}^{OW}(b_n, g_n)\}, b^0) \leq AR(\{d_{c_1^*}^{JS}(b_n, g_n)\}, b^0)$ where the strict inequality holds if l_1^* is not equal to zero.

(ii) $AR(\{d_{l^*}^{OW}(b_n, g_n)\}, b^0) \leq AR(\{d_{c_2^*}^{NR}(b_n, g_n)\}, b^0)$ where the strict inequality holds if l_2^* is not equal to zero.

The optimal weights l_1^*, l_2^* can be viewed as the contribution of the nonrandom mix and the random mix respectively. In the special case where $A=s^2I$, $B=t^2I$, $\Delta=0$, $Q=I$, and $q=0$, we can show that $l_1^* = s^2/(s^2+t^2)$ and $l_2^* = 0$; i.e. the random mix makes no contribution. On the other hand, if $A=s^2I$, $B=t^2I$, $\Delta=0$, $Q=I$, and $q\neq 0$, then it can be shown that $l_2^*\neq 0$ if and only if $(k-2)E[1/(k-2+2P)]\neq k(s^2+t^2)/[q'q+k(s^2+t^2)]$ where P has a Poisson distribution with mean $q'q/2(s^2+t^2)$.

An exhaustive comparison of biases and risks of the shrinkage estimators beyond that presented in Corollary 1 would be difficult to present in the space allowed because these are functions of many parameters. Thus, we consider a special case in which

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ q\,i \end{bmatrix}, \begin{bmatrix} s^2 I_k & d\,I_k \\ d\,I_k & t^2 I_k \end{bmatrix} \right),$$

where q is a scalar and i is the $k\times 1$ unit vector. Further we set $s^2=t^2=1$. Then, the risks and biases are functions of only q, d, and k. Some graphical comparisons of risks and biases are displayed in Figure1 for selected values for q,d and k. It emerges that in this special case, there exists an ordering in the risks and biases; that is BASE < JSM < OWM < NRM in terms of the asymptotic biases, and OWM < NRM < JSM < BASE in terms of the asymptotic risks.

3. ESTIMATION

Even though the OWM has some appealing properties, it contains the four unknown combination control parameters $(a, r, n, w)'$. We now discuss how to estimate the optimal CCPs consistently. For simplicity, we consider only the case where there is no asymptotic bias. A similar analysis can be conducted when ? ? 0; this is especially straightforward when ? is under the user's control, as can often be arranged. Suppose that $\hat{A}_n, \hat{B}_n, \hat{\Delta}_n$, and $\hat{Q}_n$ are consistent estimators for A, B, Δ, and Q respectively, and consider the following estimators for the combination control parameters:

$$\hat{a}_n \equiv tr[(\hat{A}_n - \hat{\Delta}_n - \hat{\Delta}_n' + \hat{B}_n)\hat{Q}_n] \tag{4}$$

$$\hat{r}_n \equiv tr[(\hat{A}_n - \hat{\Delta}_n')\hat{Q}_n] \tag{5}$$

$$\hat{w}_n \equiv \int_0^\infty |\hat{N}_{0tn}|^{-1/2}\, dt \tag{6}$$

$$\hat{n}_n \equiv \int_0^\infty |\hat{N}_{0tn}|^{-1/2}\, tr\left[\hat{M}_{2n}\hat{N}_{0tn}^{-1}\right] dt \tag{7}$$

where $\hat{N}_{1tn} \equiv I + 2t\hat{M}_{1n}$, $\hat{M}_{1n} \equiv \hat{P}_n' J_1' \hat{Q}_n J_1 \hat{P}_n$, $\hat{M}_{2n} \equiv \hat{P}_n' J_2' \hat{Q}_n J_1 \hat{P}_n$ and $\hat{P}_n$ is the Cholesky decomposition matrix of $\hat{\Sigma}$. Before showing the consistency results, we first establish that the control parameters are finite.

Lemma 4. Suppose that Assumption 2 holds. Then

(i) $|a| < \infty$.

(ii) $|r| < \infty$.

(iii) $|w| < \infty$ if $k > 2$.

(iv) $|n| < \infty$ if $k > 4$.

We now establish that the estimators defined in (4) – (7) are consistent.

Theorem 4. Suppose that Assumptions 1, 2 and 3 hold and $k > 4$. Then

(i) $\hat{a}_n \xrightarrow{p} a$.

(ii) $\hat{b}_n \xrightarrow{p} b$.

(iii) $\hat{w}_n \xrightarrow{p} w$.

(iv) $\hat{n}_n \xrightarrow{p} n$.

Once we obtain consistent estimators for the CCPs, a natural way to approximate the OWM is given by

$$d_{\hat{l}_n}^{OW}(b_n, g_n) \equiv \{1 - \hat{l}_{1n} - \hat{l}_{2n} / \| b_n - g_n \|_n^2\}(b_n - g_n) + g_n \tag{8}$$

where $\hat{l}_{1n} \equiv (\hat{a}_n \hat{w}_n - 1)^{-1}(\hat{r}_n \hat{w}_n - \hat{n}_n)$ and $\hat{l}_{2n} \equiv (\hat{a}_n \hat{w}_n - 1)^{-1}(\hat{a}_n \hat{n}_n - \hat{r}_n)$. We call the estimator in (8) the Optimal Weighting Scheme (OWS) Estimator. An interesting question is whether we can still achieve optimality (minimum asymptotic risk) with this estimator. The following corollary answers this question.

Corollary 2. Suppose that Assumptions 1, 2 and 3 hold and $k > 4$. Then

(i) $\hat{l}_{1n} \xrightarrow{p} l_1^*$ and $\hat{l}_{2n} \xrightarrow{p} l_2^*$.

(ii) $n^{1/2}(d^{OW}_{\hat{l}_n}(b_n, g_n) - b^0) \xrightarrow{p} d^{OW}_{l^*}(U_1, U_2)$.

(iii) $AR(\{d^{OW}_{\hat{l}_n}(b_n, g_n)\}, b^0) = AR(\{d^{OW}_{l^*}(b_n, g_n)\}, b^0)$.

The OWS estimator has the same limiting distribution as the OWM, and therefore achieves the asymptotic minimum bound. The same analysis as for the OWS estimator applies to the JSM and the NRM. We call the resulting estimators the James-Stein Combination (JSC) Estimator and the Nonrandom Combination (NRC) Estimator respectively.

4. APPLICATION

In this section we discuss how our method can be utilized in combining two possibly correlated estimators. We choose the Least Absolute Deviations (LAD) estimator as the base estimator and the Ordinary Least Squares (OLS) estimator as the data-dependent point. The resulting estimator is an optimal combination of the two estimators. There has been some interesting research on this issue. As previously mentioned, Laplace (1818) combined the sample median and the sample mean. Taylor (1974) suggested a two step procedure; first apply the LAD estimator to identify outliers to be trimmed and then apply the OLS estimator. Arthanari and Dodge (1981) combined the objective functions of the LAD estimator and the LS estimator. On the other hand, the use of the JS technique for combining the Unrestricted LAD and Restricted LAD estimators was extensively investigated by Saleh and Sen (1987b). To the best of our knowledge combining the LAD and OLS regression estimators has not been studied previously.

As shown in Bates and White (1993), both the LAD estimator and the OLS estimator are members of a RCASOI (Regular Consistent Asymptotically Second Order Indexed) class under some regularity conditions. For any member b_n in a RCASOI class, there is a score representation (s_n^0) and Hessian representation (H_n^0) such that $b_n - b^0 = H_n^{0-1} s_n^0 + o_p(n^{-1/2})$. In particular, we have the following representation for the two estimators; $s_n^{LS} = 2\sum_{t=1}^{n} X_t e_t$, $H_n^{LS} = 2\sum_{t=1}^{n} E(X_t X_t')$, $s_n^{LAD} = -2\sum_{t=1}^{n} X_t(1_{[e_t \le 0]} - 1/2)$ and $H_n^{LAD} = 2f(0)\sum_{t=1}^{n} E(X_t X_t')$ where $f(0)$ is the value of the density of e_t at zero. Given these representations it is not difficult to show the required joint asymptotic normality (Assumption 2), which follows from

$$n^{1/2}\begin{bmatrix}(b_n^{LAD} - b^0)\\ (b_n^{LS} - b^0)\end{bmatrix} = \begin{bmatrix} n^{-1}H_n^{LAD} & 0_{k\times k} \\ 0_{k\times k} & n^{-1}H_n^{LS}\end{bmatrix}^{-1} n^{-1/2}\begin{pmatrix} s_n^{LAD} \\ s_n^{LS}\end{pmatrix} + o_p(1).$$

Together with some moment conditions on X_t and e_t, the identical and independent distribution assumption is sufficient (though not necessary) to deliver the desired result. The asymptotic covariance between the two estimators is given by $\Delta = \{4f(0)\}^{-1} E(X_t X_t')^{-1} E(S_{1t} S_{2t}') E(X_t X_t')^{-1}$ where $S_{1t} \equiv -2X_t(1[e_t \le 0] - 1/2)$ and $S_{2t} \equiv 2X_t e_t$. We estimate the asymptotic covariance by the plug-in principle: $\hat{\Delta}_n \equiv \{4\hat{f}_n(0)\}^{-1}[n^{-1}\sum_{t=1}^{n} X_t X_t']^{-1}\, n^{-1}\sum_{t=1}^{n} \hat{S}_{1t}\hat{S}_{2t}'\, [n^{-1}\sum_{t=1}^{n} X_t X_t']^{-1}$ where $\hat{f}_n(0)$ is an estimate for the density at zero and $\hat{S}_{1t}, \hat{S}_{2t}$ are estimates for S_{1t}, S_{2t} using $\hat{e}_t \equiv y_t - X_t' b_n$. We study the behavior of various combinations of LAD and OLS in the following sections.

5. SIMULATION

We conduct Monte Carlo experiments designed to investigate the finite sample properties of our JS type estimators. For purposes of comparison, we also include certain interesting stable estimators (the Ridge estimator, the Garrotte estimator and the Non-Negative Garrotte estimator). Stable estimators have been shown to deliver good prediction performance (Breiman 1995, 1996). The definitions for the

stable estimators are given in Table 1. In the simulation we combine the LAD and the OLS estimators described in Section 4 to obtain examples of the NRC, JSC and OWS estimators.

Table 1. Definition of Estimators

Estimator	Definition
Ridge (b^R)	$b^R \in \text{argmin} \parallel y - Xb \parallel^2$ s.t. $b'b < s$
Garrotte (b^G)	$b^G \in \text{argmin} \parallel y - Zg \parallel^2$ s.t. $Z_{ij} = X_{ij} b_j^{LS}$ and $g'g < s$
Non-Negative Garrotte (b^N)	$b^N \in \text{argmin} \parallel y - Zg \parallel^2$ s.t. $Z_{ij} = X_{ij} b_j^{LS}$ and $g'i < s, g \geq 0$

NOTE: Values for s are determined by k-fold cross-validation.

The data for the simulation are generated as $y_t = X_t' b^0 + e_t$ where $t = 1, 2, ..., n$, $b^0 \in R^k$, $n = 500$ and $k = 5$. We set $b^0 = (1,1,1,1,1)'$. The number of replications is 1,000. We obtain the LAD estimator using the efficient L_1 algorithm developed by Barrodale and Roberts (1974). The simulation was carried out on a 266MHz PC using MATLAB. The random number generator used in the simulation is that from the MATLAB Statistics Toolbox.

We choose four symmetric distributions and two non-symmetric distribution for e_t. Symmetric distributions are the Uniform distribution within [-4,4], the standard normal distribution, the Student t-distribution with 3 degrees of freedom, and the Cauchy distribution with interquartile range 1. These represent moderate, heavy and very heavy tailed distributions. For the non-symmetric distributions, we choose the shifted Chi-square distribution centered at zero with 12 degrees of freedom and the shifted Rayleigh distribution centered at zero with parameter 4. The first entry of X_t is one, and the remaining explanatory variables X_t are generated using the joint normal distribution $N(0, \Sigma)$, where the covariances are set to 0.5 and the variances are one. We estimate the required density $\hat{f}_n(0)$ using a kernel method with Gaussian kernel. For each replication we compute the quadratic loss value for each estimator. We approximate the risk by averaging the loss values over all replications. The results are collected in Table 2.

Table 2. Finite Sample Risk Comparison over Different Error Distributions $(n=500)$

	Uniform	Normal	Student-t	Cauchy	c^2	Rayleigh
OLS	26.306	5.006	14.789	463971952.194	121.770	35.089
LAD	77.029	7.791	9.345	12.789	398.972	99.337
NRC	20.595	5.035	8.908	12.531	126.610	35.982
JSC	72.765	6.306	9.104	12.707	396.751	96.982
OWS	20.617	5.035	8.910	12.525	126.925	35.830
RIDGE	25.504	4.974	14.471	433311022.768	115.666	33.612
GAR	27.241	5.103	15.216	463971921.290	126.777	36.226
NNGAR	26.311	5.006	14.789	463970894.151	123.116	35.090

It is well known that the performance of the median is worse than the sample mean when the error is distributed uniformly. As expected, the risk of the LAD estimator (77.029) is greater than the risk of the OLS estimator (26.306) in this case. All combination methods give negative weight to the LAD estimator. As a result, both the OWS estimator and the NRC estimator dominate the OLS estimator. When the regression error is normal, the OLS estimator is asymptotically efficient. Not surprisingly, the OLS estimator displays the best performance except for the Ridge estimator. However, the deterioration of the OWS estimator and the NRC estimator relative to the OLS estimator is not large (-0.57 %). As expected for the Student-t distribution, the risk of the LAD estimator (9.345) is smaller than the risk of the OLS estimator (14.789). All combination estimators have smaller risk than both the LAD estimator and the OLS estimator. The improvement of the combination estimators over the LAD estimator and the OLS estimator is about 2 - 5 % and 38 - 40 % respectively. The Cauchy distribution represents a very heavy tailed distribution. The risk performance of the OLS estimator is much worse than that of the LAD estimator (463971952 and 12.789 respectively). Nevertheless, combining the LAD estimator with the OLS estimator makes an improvement over the LAD estimator. The improvements over the LAD estimator and the OLS estimator are about 0.6 - 2 % and 100 % respectively.

The LAD estimator is out-performed by the OLS estimator in terms of risk (398.972 and 121.770) for the Chi-square distribution. For the OWS estimator and the NRC estimator, the weight on the LAD

estimator is very small (about 0.080 - 0.096). On the other hand, the JSC estimator gives a large positive weight to the LAD estimator (0.99), which clearly shows the inferiority of the JSC estimator when the regression error is not symmetric. The failure can be explained by the bias in the constant coefficient, which makes the distance between the two estimators very large (384.778). This in turn makes the JS weight too large. All combination estimators are better than the LAD estimator, but worse than the OLS estimator. When the error has the Rayleigh distribution, the result is basically the same as for the Chi-square distribution. However, the skewness is smaller than for the Chi-square distribution, and as a result, the bias in the constant term is much smaller. The OWS estimator and the NRC estimator now give a small negative weight to the LAD estimator.

The performance of the stable estimators are shown in the same table. The Ridge estimator gives smaller risk than the OLS estimator over all error distributions considered in the simulation including the normal distribution. This is a well-known standard result based on the trade-off between variance and bias. On the other hand, the other stable estimators (the Garrotte estimator and the Non-Negative Garrotte estimator) are not better than the OLS estimator, which might at first seem surprising. However, Breiman (1995, 1996) showed that the Garrotte estimator and the Non-Negative Garrotte estimator give smaller prediction mean squared error than the OLS estimator when irrelevant variables appear in the model. Here none of our variables are irrelevant. Despite this, the additional risk associated with the Garrotte and Non-Negative Garrotte is small.

Overall, we see that our combination estimators, and especially the NRC and OWS, perform consistently well across our various examples. Each of the other estimators behaves much less reliably.

6. EMPIRICAL STUDY: OUT-OF-SAMPLE PREDICTION

In this section we investigate the out-of-sample predictive ability of the combination estimators using actual data. Let y be a $T \times 1$ vector of out-of-sample actual values and let e be a $T \times 1$ vector of the prediction errors where T is the number of out-of-sample observations. In order to evaluate forecasting performance, we use the following forecasting error measurements: prediction mean squared error $PMSE(e) \equiv e'e/T$ and prediction mean absolute error $PMAE(e) \equiv T^{-1}\sum_{t=1}^{T}|e_t|$. We also use R^2 type prediction measures: $R^2 \equiv 1 - PMSE(e)/S^2(y)$ and $R_A^2 \equiv 1 - PMAE(e)/MAE(y)$ where

$S^2(y)$ is the sample variance of y and $MAE(y)$ is the mean absolute error of y. The data set contains daily stock market returns for ADC TeleCom Co. and HomeStake Co., stocks that have been randomly chosen from the DATASTREAM database. The sample period covers January 1, 1990 through March 31, 1996 which gives us 1630 observations. We model daily excess returns, computed by subtracting the 3-month US T-bill rate from daily returns. Table 3 provides summary statistics.

Table 3. Summary Statistics for Daily Excess Stock Returns (in percent)

	Mean	Median	Max	Min	Std. Dev.	Skew.	Excess Kurtosis
ADC TelCom	0.15	-0.01	11.92	-22.13	2.94	-0.17	3.93
HomeStake	0.01	-0.02	11.25	-12.45	2.52	0.08	1.96

Our forecasting model for excess returns is

$$r_t = a + \sum_{i=1}^{k_1} b_i r_{t-i} + \sum_{i=1}^{k_2} g_i r_{m\,t-i} + e_t$$

where r_{mt} is the daily excess returns on the S&P500 index and $k_1 = k_2 = 1$. The simple efficient market hypothesis requires that $a = b = g = 0$, so that the best predictor is zero. We call this the Random Walk predictor and we include this in our comparison study. We use a fixed rolling window method to estimate the coefficients, and set the size of estimation window to be 520, which is about a two year sample period. We repeat the entire exercise identically for each of the 8 estimators and for each of the target variables.

The outcomes are summarized in Figures 2 and 3. We can represent an estimator as a point in PMAE-PMSE space. In these diagrams, we prefer estimators located closer to the origin because the PMAE and the PMSE can be treated as "bad" commodities. We represent combination and stable estimators by their first initial in PMAE-PMSE space except that the NRC estimator is denoted by "c" and the Random Walk predictor denoted by "w". For example, "r" stands for the Ridge estimator, "g" for the Garrotte estimator, and so on. In the case of ADC TeleCom (Figure 2), all combination estimators outperform both the LAD and the OLS estimators in terms of PMSE, but the improvement over the LAD estimator is very small. The performances of the NRC and OWS are almost identical.

They also achieve better performance than the stable estimators. Interestingly, all our estimators beat the Random Walk predictor in terms of PMSE, but the Random Walk predictor beats all estimators in terms of PMAE, which indicates that there seems to exist some forecastibility. Nevertheless, if the analyst's loss function is the sum of absolute deviations, then the random walk predictor is preferred.

The out-of-sample prediction results for HomeStake stock are given in Figure 3. The Non-negative Garrotte, Garrotte, Ridge and JSC estimators outperform all other estimators in terms of both PMSE and PMAE. The behavior of the Random Walk predictor is similar to the results seen for ADC TeleCom stock. The combinations estimators generally achieve better performance than the LAD and OLS estimators but the magnitude of improvement is very small.

Prediction performance measured by prediction R^2 is summarized in Table 4. The prediction R^2 is not necessarily positive because out-of-sample predictions are not guaranteed to be orthogonal to out-of-sample residuals. The prediction R^2 compares the performance of a predictor to the imaginary situation where we know in advance the sample mean of the target variable over the entire out-of-sample period and use it as our predictor. Therefore, a positive prediction R^2 indicates that the predictor is better in terms of PMSE than the sample mean assumed known in advance. According to the summary statistics in Table 4, the return on ADC TeleCom Co. is more difficult to predict than that for HomeStake Co.. Nevertheless, all combination estimators and the LAD estimator give positive prediction R^2's.

Table 4. Out-of-Sample Prediction Performance

	ADC TeleCom Co.		HomeStake Co.	
	R^2	R_A^2	R^2	R_A^2
OLS	-0.001410	0.000879	0.005622	-0.00955
LAD	0.001613	0.003402	0.004698	-0.00674
NRC	0.001871	0.002967	0.006569	-0.00878
JSC	0.001782	0.003366	0.006987	-0.00663
OWS	0.001858	0.002964	0.006606	-0.00875
RIDGE	-0.000540	0.003894	0.006282	-0.00561
GAR	-0.001280	0.002069	0.007043	-0.00616
NNGAR	-0.000320	0.002136	0.010452	-0.00405
Random Walk	-0.001999	0.004883	-0.000147	-0.00149

7. CONCLUSION

We have proposed an extension of JS type estimators in a direction that preserves their risk improvement when the sample size goes to infinity. This extension supports use of JS type estimators when one has a moderate or large number of observations. This is important because large data sets are becoming more and more widely available. We permit the data-dependent point towards which we shrink our base estimator to be asymptotically biased or asymptotically correlated with the base estimator, in contrast to previous work. Many interesting estimators are valid candidates for use as a data-dependent point. Our results thus suggest that a wide range of estimation and forecasting improvements over standard techniques are readily available using our approach.

APPENDIX

Proof of Theorem 1. One can also show using Assumption 2 that $n^{1/2}(d^{JS}_{c_1}(b_n, g_n) - b^0) \xrightarrow{d} \{1 - c_1 / \| U_1 - U_2 \|^2\}(U_1 - U_2) + U_2 \equiv d^{JS}_{c_1}(U_1, U_2)$. This implies that $L(d^{JS}_{c_1}(b_n, g_n), b^0) \xrightarrow{d} \| d^{JS}_{c_1}(U_1, U_2) - 0 \|^2$. Therefore, $AR(\{d^{JS}_{c_1}(b_n, g_n)\}, b^0) = w c_1^2 - 2n c_1 + k$. The first and second derivatives of the asymptotic risk with respect to c_1 are $2(w c_1 - n)$ and $2w$. Since $w > 0$, $AR(\{d^{JS}_{c_1}(b_n, g_n)\}, b^0)$ is strictly convex in c_1. By setting the first derivative to zero and solving for c_1, we have $c_1^* = n / w$. Since $w = 1/Z'M_1Z$ and $n = Z'M_2Z / Z'M_1Z$, a straightforward application of Lemma 2 in Ullah (1990) delivers the expressions for w and n in (i). Plugging c_1^* into the asymptotic risk, we have the minimum asymptotic risk $AR(\{d^{JS}_{c_1^*}(b_n, g_n)\}, b^0) = -n^2 / w + k$. Since $k \equiv AR(\{b_n\}, b^0)$, $AR(\{d^{JS}_{c_1^*}(b_n, g_n)\}, b^0) \leq AR(\{b_n\}, b^0)$, where the equality holds only when $n = 0$. The result in (iv) is obtained from the strict convexity of the asymptotic risk. The last result follows from an application of Lemma 2 in Ullah (1990). *QED.*

Proof of Theorem 3. It follows from Lemma 3 that $AR(\{d^{OW}_{l}(b_n, g_n)\}, b^0) = a l_1^2 - 2r l_1 + 2 l_1 l_2 + w l_2^2 - 2n l_2 + k$. The Hessian is given by $\begin{bmatrix} 2a & 0 \\ 0 & 2w \end{bmatrix}$, which is positive definite. Hence, $AR(\{d^{OW}_{l}(b_n, g_n)\}, b^0)$ is strictly convex in l. By setting the first derivative to zero and solving for l, we have $l_1^* = (aw - 1)^{-1}(rw - n)$ and $l_2^* = (aw - 1)^{-1}(an - r)$. Plugging l^* into the asymptotic risk, we have the minimum asymptotic risk $AR(\{d^{OW}_{l^*}(b_n, g_n)\}, b^0) = (aw - 1)^{-2}\{-ar^2w^2 - (2arn - a^2n^2 + r^2)w + (an^2 - 2rn)\} + k$. For the last result, we define $h(w) \equiv -\{-ar^2w^2 - (2arn - a^2n^2 + r^2)w + (an^2 - 2rn)\}$. We want to show $h(w) \geq 0$ for all w, which delivers the desired result. (Case 1) $r = 0$ and $n = 0$. Then $h(w) = 0$. (Case 2) $r = 0$ and $n \neq 0$. Then $h(w) = an^2(aw - 1) > 0$ because $a > 0$, $n \neq 0$ and $aw > 1$. (Case 3) $r \neq 0$ and $n = 0$. Then $h(w) = r^2w(aw - 1) > 0$ because $w > 0$, $r \neq 0$ and $aw > 1$. (Case 4) $r \neq 0$ and $n \neq 0$. Define $w^* \in \{w \mid h(w) = 0\}$. Suppose that $an - r = 0$. Then

$w^* = -(a^2n^2 - r^2 - 2arn)/2ar^2$. It can be shown that $w > w^*$, because $aw > 1$ and $r \neq 0$. This implies that $h(w) \geq 0$ for all w. Now consider the case that $an - r \neq 0$. Define $w_-^* \equiv n(2r - a)/r^2$ and $w_+^* \equiv 1/a$. It follows that $w_-^* < w_+^*$ and $w_+^* < w$ because $aw > 1$. This implies that $h(w) \geq 0$ for all w. Again, the last result follows from an application of Lemma 2 in Ullah (1990). *QED.*

Proof of Corollary 1. Because of strict convexity, $AR(\{d_{l^*}^{OW}(b_n, g_n)\}, b^0) \leq AR(\{d_l^{OW}(b_n, g_n)\}, b^0)$ for any l. Choose $l = (0, l_2^*)'$. Then, $AR(\{d_l^{OW}(b_n, g_n)\}, b^0) = -n^2/w + k$, which is $AR(\{d_{c_1^*}^{JS}(b_n, g_n)\}, b^0)$. Since l^* is the unique and global solution, the strict inequality holds if l_1^* is not equal to zero. For the second claim, choose $l = (l_1^*, 0)'$. Then, $AR(\{d_l^{OW}(b_n, g_n)\}, b^0) = -r^2/a + k$, which is equal to $AR(\{d_{c_2^*}^{NR}(b_n, g_n)\}, b^0)$. The same argument applies to the strict inequality. *QED.*

Proof of Lemma 4. It is trivial to show that $|a| < \infty$ and $|r| < \infty$ because these are obtained by adding variances and covariances of normal random variables. In order to show that $|w| < \infty$, we note that $w = \frac{1}{Z'Z}\frac{Z'Z}{Z'M_1Z}$. It can be shown that $\frac{1}{l_M} \leq \frac{Z'Z}{Z'M_1Z} \leq \frac{1}{l_m}$ where l_M and l_m are the largest and smallest eigenvalues of M_1. This implies that $\frac{1}{l_M}E\left[\frac{1}{Z'Z}\right] \leq w \leq \frac{1}{l_m}E\left[\frac{1}{Z'Z}\right]$. If $k > 2$, then $\frac{1}{l_M(k-2)} \leq w \leq \frac{1}{l_m(k-2)}$. For the last claim, note that $\nu^2 \leq E\left[(Z'M_2Z)^2\right]E\left[\frac{1}{(Z'M_1Z)^2}\right]$ by the Cauchy-Schwarz inequality. Since Z is a normal random variable, $E\left[(Z'M_2Z)^2\right] < \infty$. Hence, we have $\frac{1}{l_M^2}E\left[\frac{1}{(Z'Z)^2}\right] \leq E\left[\frac{1}{(Z'M_1Z)^2}\right] \leq \frac{1}{l_m^2}E\left[\frac{1}{(Z'Z)^2}\right]$. If $k > 4$, then $\frac{1}{l_M^2(k-2)(k-4)} \leq E\left[\frac{1}{(Z'M_1Z)^2}\right] \leq \frac{1}{l_m^2(k-2)(k-4)}$ by Theorem A.2.20 in Judge and Bock (1978). Therefore, $|\nu| < \infty$. *QED.*

Proof of Theorem 4. Since trace is a continuous function, $\hat{a}_n \equiv tr[(\hat{A}_n - \hat{\Delta}_n - \hat{\Delta}_n' + \hat{B}_n)\hat{Q}_n]$ is consistent. By the same reasoning, $\hat{r}_n \equiv tr[(\hat{A}_n - \hat{\Delta}_n')\hat{Q}_n]$ is consistent. The argument for $\hat{w}_n$ and $\hat{n}_n$ is more involved. We define $g(M_1, t) \equiv \det(I + 2tM_1)^{-1/2}$. We want to show that there exists a dominating function $d(t)$ such that (i) $|g(M_1, t)| \leq d(t)$ for all Σ_{22} and Q in a compact parameter space and (ii) $\int_0^\infty d(t)dt < \infty$. Using the relationship between determinant and eigenvalues of a matrix, we can express $g(\cdot)$ in terms of eigenvalues; $g(M_1, t) = \left\{\prod_{i=1}^{k} l_i\right\}^{1/2}$ where l_i is an eigenvalue of the inverse matrix of $I + 2tM_1$. Using some linear algebra, we can obtain an upper bound given by $g(M_1, t)^2 \leq \left\{\frac{1}{2|\bar{k}|t+1}\right\}^k$ where $\bar{k}$ is the minimum (in absolute value) eigenvalue of M_1 Hence the natural candidate for the dominating function is $d(t) = \left\{\frac{1}{2|\bar{k}|t+1}\right\}^{k/2}$. As long as $k > 2$, the dominating function $d(t)$ will satisfy the second condition, $\int_0^\infty d(t)dt < \infty$. Hence we have the desired result. The proof for the consistency of $\hat{n}_n$ is more complicated, but the key step is again to find a dominating function. Since the argument is similar, we just give the dominating function: $D(t) = 2k\,|\bar{x}|\left\{\frac{1}{2\,|\bar{k}|\,t+1}\right\}^{k+1}$ where $\bar{k}$ is the minimum eigenvalue (in absolute value) of M_1 and $\bar{x}$ is the maximum (in absolute value) eigenvalue of M_2^2. *QED.*

Proof of Corollary 2. Both $\hat{l}_{1n}$ and $\hat{l}_{2n}$ are continuous function of the consistent estimators. Since the limit of continuous function of consistent estimators is the value of function evaluated at the limit of the consistent estimators, we have the desired results: $\hat{l}_{1n} \xrightarrow{p} l_1^*$ and $\hat{l}_{2n} \xrightarrow{p} l_2^*$. The consistency of the

estimated weights together with the Slutsky Theorem delivers $n^{1/2}(d^{OW}_{\hat{l}_n}(b_n, g_n) - b^0) \xrightarrow{d} d^{OW}_{l^*}(U_1, U_2)$ which in turn implies that $AR(\{d^{OW}_{\hat{l}_n}(b_n, g_n)\}, b^0) = AR(\{d^{OW}_{l^*}(b_n, g_n)\}, b^0)$. *QED.*

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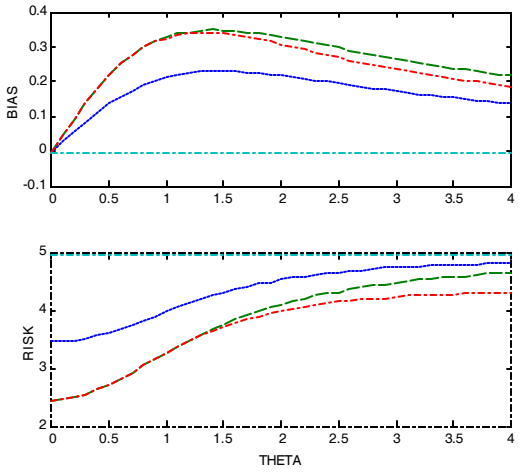
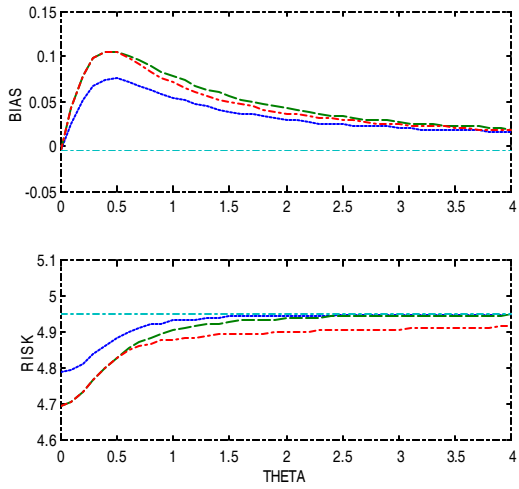
Figure1. Bias and Risk Comparison

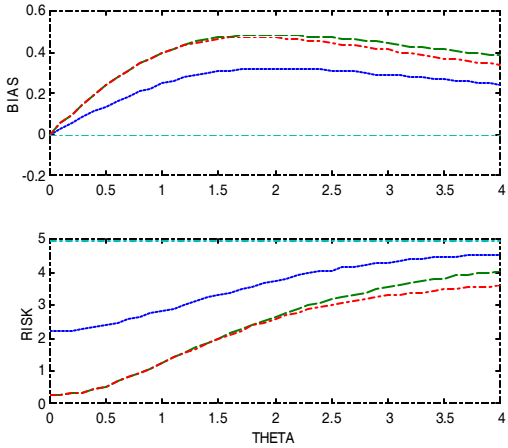
(k = 5 and d = 0)



(k = 5 and d = 0.9)



(k = 5 and d = -0.9)



Note: BASE (solid), JS(dotted), NR(dashed) and OW(dashdot)

Figure 2. Out-of-Sample Prediction Performance for ADC TeleCom.

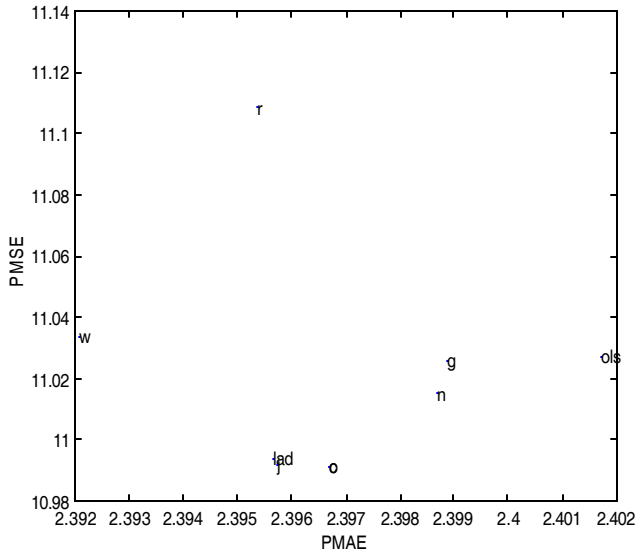


Figure 3. Out-of-Sample Prediction Performance for HomeStake.