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**DOUBLE DIFFRACTION COEFFICIENTS
FOR SOURCE AND OBSERVATION
AT FINITE DISTANCE FOR A PAIR OF WEDGES.**

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1. INTRODUCTION

A closed form, high-frequency solution is presented for describing double diffraction mechanism at a pair of parallel wedges, when they are illuminated by a spherical wave. The solution is obtained by using a spherical waves spectral representation of the first order diffracted field from each wedge. For the sake of simplicity, in this paper only the scalar case is considered when either hard or soft boundary condition (b.c.) may be imposed on the faces of the two wedges; this provides basic step for constructing the solution in the more general electromagnetic case. Although the procedure is applicable to any couple of parallel wedges, the case of two wedges sharing a common face is explicitly considered in the presented solution. The asymptotic evaluation of the double spectral integral leads to transition functions involving generalized Fresnel integrals. Numerical calculations show that the present high frequency formulation fails so gracefully that it gradually blends into the solution of a single wedge when the distance between the two edges vanishes. This is a desirable property for analyzing the shadowing effects of thick screen.

2. FORMULATION

Let us consider a pair of wedges with parallel edges, illuminated by a spherical source. It is useful to define a cylindrical (ρ_i, ϕ_i, z_i) and a spherical (r_i, β_i, ϕ_i) coordinate system at each edge ($i=1,2$). The position of the source is denoted by (ρ_i', ϕ_i', z_i') or $(r_i', \beta_i', \phi_i')$ ($i=1,2$). Our description of the double diffraction mechanism is constructed as the superposition of two analogous mechanisms; a field diffracted from edge 2 when it is illuminated by the field diffracted from edge 1 (12), and that from 1 when it is illuminated by 2 (21). For the sake of simplicity, only the contribution 12 is considered in herein after. The ray geometry for the field doubly diffracted at Q_1 and Q_2 is depicted in Fig. 1; the distance between the two points of diffraction Q_1 and Q_2 is denoted by $l=d/\sin \beta_1'$ where d is the distance between the two edges.

The singly diffracted field from the first wedge illuminated by a spherical source is conveniently expressed by

$$\psi_1^d = \frac{1}{2\pi j} \int_{\phi_1 - j\infty}^{\phi_1 + j\infty} \frac{e^{-jkR(\alpha_1 - \phi_1, \rho_1')}}{4\pi R(\alpha_1 - \phi_1, \rho_1')} G_1(\phi_1', \alpha_1) d\alpha_1' \quad (1)$$

where $G_1(\phi_1', \alpha_1)$ is the Sommerfeld spectral function [1] and

$$\tilde{R}(\alpha_1, \rho_1') = \sqrt{\rho_1'^2 + \rho_1^2 + 2\rho_1\rho_1' \cos \alpha_1 + |z_1 - z_1'|^2}$$

The integrand in (1) represents a spectral spherical source; this is used for illuminating the second wedge. Then, by analytic continuation of (1) the near field response of the second edge to a spherical spectral source is introduced and used to weight the spectrum of (1). Thus, a double spectral integral representation is obtained as

$$\psi_{12}^{dd} = -\frac{1}{8\pi^2} \int_{-j\infty}^{j\infty} \int_{-j\infty}^{j\infty} \frac{e^{-\mu R(\zeta_1, \zeta_2)}}{4\pi R(\zeta_1, \zeta_2)} G_1(\phi_1', \zeta_1) G_2(\zeta_2, \phi_2) d\zeta_1 d\zeta_2 \quad (2)$$

where $R(\zeta_1, \zeta_2) =$

$$= \sqrt{\rho_1'^2 + \rho_2^2 + d^2 + 2d\rho_1' \cos \zeta_1 + 2d\rho_2 \cos \zeta_2 + 2\rho_1\rho_2 \cos(\zeta_1 + \zeta_2) + |z_2 - z_2'|^2} \quad (3)$$

This representation which explicitly satisfies reciprocity, exhibits a two dimensional saddle point at $(\zeta_1, \zeta_2) = (0, 0)$; its asymptotic approximation is presented next.

2. ASYMPTOTIC EVALUATION

The integral in (2) is now asymptotically evaluated by stationary phase approximation. To this end, it should be noted that in the soft case the spectrum $G_1 G_2$ vanishes at the saddle point. Thus, this latter case requires a higher order term in the asymptotic evaluation. This is indeed expected since in the soft case double diffraction is essentially a slope contribution. A rigorous asymptotic analysis leads to the following expression for the doubly diffracted field

$$\psi_{12}^{dd}(r_1', r_2) = \psi^i(Q_1) \mathcal{A}(r_1', l, r_2) D_{12}^{s,h} e^{-jkr_2} \quad (4)$$

where

$$\psi^i(Q_1) = \frac{e^{-jkr_1'}}{4\pi r_1'} \quad ; \quad \mathcal{A}(r_1', l, r_2) = \frac{\sqrt{r_1'}}{\sqrt{l r_2} \sqrt{r_1' + l + r_2}} e^{-jkl} \quad (5)$$

are the incident field at Q_1 and the spreading factor for the doubly diffracted ray, respectively and $D_{12}^{s,h}$ is the diffraction coefficient for either soft (s)

$$D_{12}^s = \frac{1}{16\pi k^2 l \sin^4 \beta} \cdot \frac{1}{(n_1 n_2)^2} \sum_{pq=1}^2 (-1)^{p+q} \operatorname{cosec}^2\left(\frac{\Phi_1^p}{2n_1}\right) \operatorname{cosec}^2\left(\frac{\Phi_2^q}{2n_2}\right) \cdot T_{pq}^s \quad (6)$$

or hard (h) case

$$D_{12}^h = \frac{1}{4\pi k j \sin^2 \beta} \cdot \frac{1}{n_1 n_2} \sum_{pq=1}^2 (-1)^{p+q} \cot\left(\frac{\Phi_1^p}{2n_1}\right) \cot\left(\frac{\Phi_2^q}{2n_2}\right) \cdot T_{pq}^h \quad (7)$$

In (6) and (7) T_{pq} are the two dimension transition functions, that are defined herein after; they become unity for k large, away from the transition regions. It worth noting that at these aspects the hard coefficient $D_{12}^h \sim \frac{1}{k}$, while the soft coefficient $D_{12}^s \sim \frac{1}{k^2}$ as a slope contribution.

The transition functions are defined as

$$T_{pq}^s \triangleq j4k l \sin^2 \beta \sin \frac{\Phi_1^p}{2} \sin \frac{\Phi_2^q}{2} \frac{T(x_1, y_1^-, x_2, y_2^-) - T(x_1, y_1^+, x_2, y_2^+)}{2} \quad (8)$$

$$T_{pq}^h \triangleq \frac{T(x_1, y_1^-, x_2, y_2^-) + T(x_1, y_1^+, x_2, y_2^+)}{2}$$

The function T is the same as that defined in [2]; its expression is given below for convenience

$$T(x_1, y_1, x_2, y_2) \triangleq \frac{jk x_1 x_2}{x_1 y_2 + y_1 x_2} \left[\delta_1^2 G(\sqrt{k} x_1, \sqrt{k} y_1) + \delta_2^2 G(\sqrt{k} x_2, \sqrt{k} y_2) \right] \quad (9)$$

where $\delta_i^2 = x_i^2 + y_i^2$, G is the Generalized Fresnel Integral [2],

$$x_1 \simeq \sqrt{2} \sin \beta \sqrt{\frac{r_1' l}{r_1' + l}} \sin\left(\frac{\Phi_1^p}{2}\right) \quad x_2 \simeq \sqrt{2} \sin \beta \sqrt{\frac{r_2' l}{r_2' + l}} \sin\left(\frac{\Phi_2^q}{2}\right) \quad (10)$$

$$y_1^- \simeq \sqrt{2} \sin \beta \sqrt{\frac{r_1' r_1}{r_1' + r_1}} \sin\left(\frac{\Phi_1^p + \Phi_1^q}{2}\right) \quad y_2^- \simeq \sqrt{2} \sin \beta \sqrt{\frac{r_2' r_2}{r_2' + r_2}} \sin\left(\frac{\Phi_2^q + \Phi_2^p}{2}\right)$$

and Φ_1^q, Φ_2^p are given by $\Phi_1^q = \text{tg}^{-1}\left(\frac{\rho_2 \sin \Phi_2^q}{d + \rho_2 \cos \Phi_2^q}\right)$ $\Phi_2^p = \text{tg}^{-1}\left(\frac{\rho_1 \sin \Phi_1^p}{d + \rho_1 \cos \Phi_1^p}\right)$.

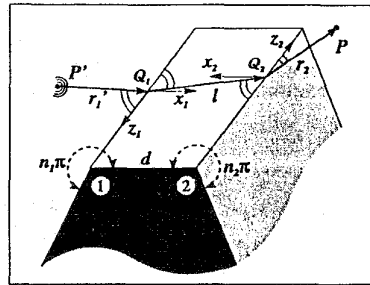


Fig. 1 Geometry

3. NUMERICAL RESULTS

The formulation presented in this paper has been applied to calculate double diffraction contributions from a strip illuminated close to grazing aspects by a TE field from a dipole. Calculations of the total field observed at a finite distance are presented in Fig. 2. The numerical results are compared with first order UTD calculations, that provide a discontinuous

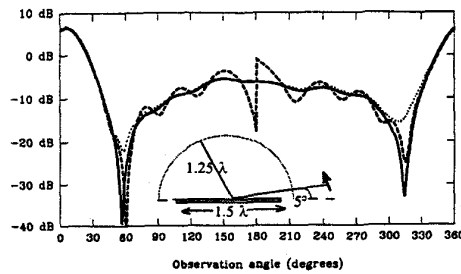


Fig. 2 Amplitude of the scattered field from a perfectly conducting strip; this solution (—) first order UTD (---), MoM (.....)

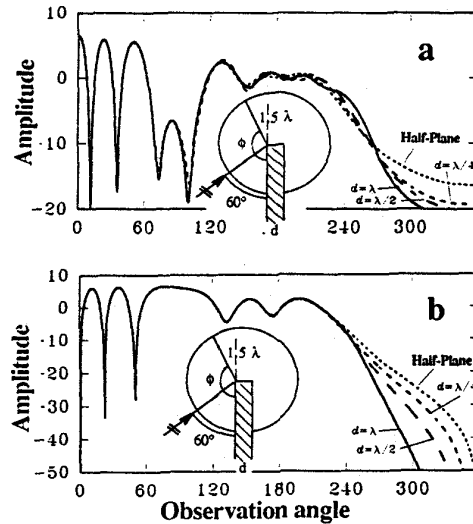


Fig. 3 Diffraction from a thick screen; a) TE polarisation, b) TM polarisation

pattern at near grazing observation aspects, and with results obtained from a Method of Moments (MoM) (from [3]). It is rather apparent that the discontinuity of the leading diffraction contribution is well compensated by the second order diffraction mechanism introduced here, and the total field is predicted with good accuracy. The field scattered from a thick screen with different thickness has been calculated for either a TE (Fig. 3a) or a TM (Fig. 3b) incident plane wave and an observation point at a finite distance, as shown in the inset. An increasing shadowing effects is predicted as the thickness increases, especially for TM polarisation, as expected. As the thickness decrease it is found that our solution fails so gracefully that it gradually blends into the calculations from the exact solution of the half-plane. Although this may not be expected, it is a rather desirable property for a h.f. solution.

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