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LOWER BOUNDS OF LOCALIZATION UNCERTAINTY IN SENSOR NETWORKS

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ABSTRACT

Localization is a key application for sensor networks. We propose a Bayesian method to analyze the lower bound of localization uncertainty in sensor networks. Given the location and sensing uncertainty of individual sensors, the method computes the minimum-entropy target location distribution estimated by the network of sensors. We define the Bayesian bound (BB) as the covariance of such distribution, which is compared with the Cramér-Rao bound (CRB) through simulations. When the observation uncertainty is Gaussian, the BB equals the CRB. The BB is much simpler to derive than the CRB when sensing models are complex. We also characterize the localization uncertainty attributable to the sensor network topology and the sensor observation type through the analysis of the minimum entropy and the CRB. Given the sensor network topology and the sensor observation type, such characteristics can be used to approximately predict where the target can be relatively accurately located.

1. INTRODUCTION

The recent emergence of sensor network technology not only raises a new demand for locating internal nodes belonging to a sensor network but also provides a new means of locating external targets using a sensor network. Many localization technologies have recently been developed in the context of sensor networks, either for localization of cooperative nodes [1], or for localization of non-cooperative targets [2]. These localization technologies are mostly evaluated against the ground truth for a few randomly-generated sensor network topologies. The dependency of localization uncertainty on sensor network topologies has been largely neglected. This paper is devoted to the study of such dependency. In section 2, we propose a Bayesian method to characterize the lower bound of localization uncertainty in a sensor network. In section 3, we analyze the dependency of localization uncertainty on the sensor network topology and the sensor observation type in order to identify the region where the target is relatively accurately located. Section 4 concludes the paper.

2. BAYESIAN BOUND

In this section, we derive the BB of localization in a sensor network and compare it with the CRB through simulations.

2.1. Computing Bayesian Bound

Given the location and sensing uncertainty of individual sensors, for a target at $\mathbf{x}_t$, we first compute the target location distribution with the minimum entropy that could be achieved through the set of sensors. The covariance of such a distribution is the BB.

An observation can be associated with a single sensor, e.g. a range sensor that measures the distance from itself to the target. Let z_i denote such an observation associated with sensor i. An observation can also be associated with two sensors, e.g. two time-difference-of-arrival (TDOA) sensors that measure the difference of the arrival time of the target signal between them. Let z_{ij} denote such an observation associated with both sensor i and sensor j. A sensor observation depends on the true target location and the location(s) of the associated sensor(s). It also includes the perturbation caused by factors such as the ambient noise, the hardware imprecision, and the signal modeling inaccuracy. The optimal observation ζ_i or ζ_{ij} occurs in the situation where the perturbation happens to be zero. Hence, ζ_i or ζ_{ij} is determined only by the true target location and the associated sensor location(s),

$$\zeta_i = f(\mathbf{x_t}, \mathbf{x}_i) \quad \text{or} \quad \zeta_{ij} = f(\mathbf{x_t}, \mathbf{x}_i, \mathbf{x}_j) , \qquad (1)$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ are the location of sensor i and j respectively. When every sensor achieves its optimal observation, the posterior target location distribution is the optimal one that could be achieved through all sensors. Assuming independent sensor observations conditioned on the target location, and null prior information about the target location, the optimal posterior target location distribution using all sensors is constructed using sensing models for optimal observations,

$$p(\mathbf{x}|\zeta_i, 1 \le i \le N) = C \prod_{1 \le i \le N} p(z_i = \zeta_i|\mathbf{x}) , \quad \text{or}$$

$$p(\mathbf{x}|\zeta_{ij}, 1 \le i < j \le N) = C \prod_{1 \le i < j \le N} p(z_{ij} = \zeta_{ij}|\mathbf{x}) , \qquad (2)$$

where C is a normalization constant. The optimal target location distribution has the minimum entropy and its covariance is the BB,

$$BB = \int (\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^T p(\mathbf{x}|\zeta_i, 1 \le i \le N)d\mathbf{x} , \quad \text{or}$$

$$BB = \int (\mathbf{x} - \bar{\mathbf{x}})(\mathbf{x} - \bar{\mathbf{x}})^T p(\mathbf{x}|\zeta_{ij}, 1 \le i < j \le N)d\mathbf{x} , \qquad (3)$$

where T is the transpose operator, $\bar{\mathbf{x}}$ is the expectation of $\mathbf{x}$.
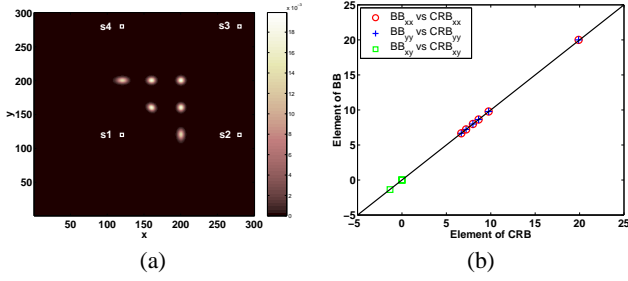
Fig. 1. Comparison of the BB with the CRB in localization using TDOA sensors. (a) The minimum-entropy target location distributions estimated by four TDOA sensors denoted by squares. The target signal propagation speed is assumed to be 1 distance unit per time unit. The Gaussian uncertainty with $\sigma = 6$ time units is assumed for TDOA observations. (b) Element-to-element comparison of the BB with the CRB of locating the same target.

2.2. Comparing Bayesian Bound with Cramér-Rao Bound

We compare the BB with the CRB through two simulations of two-dimensional localization using TDOA sensors and range sensors respectively. For simplicity, we assume the Gaussian distribution for all TDOA and range observations. In both simulations, the BB equals the CRB element to element. However, the BB is simpler to derive than the CRB when sensing uncertainty is complex.

The first simulation uses four TDOA sensors placed on a square as shown in Fig. 1(a). We assume that the target signal propagation speed is 1 distance unit per time unit. The Gaussian sensing model for TDOA observations is then

$$p(z_{ij}|\mathbf{x}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(z_{ij}-(\|\mathbf{x}-\mathbf{x}_i\|-\|\mathbf{x}-\mathbf{x}_j\|))^2/(2\sigma^2)} \ , \quad (4)$$

where $\|\cdot\|$ is the norm operator, σ is assumed to be 6 time units. The optimal TDOA observation is

$$\zeta_{ij} = \|\mathbf{x_t}-\mathbf{x}_i\| - \|\mathbf{x_t}-\mathbf{x}_j\| \ . \quad (5)$$

Using (2), (4) and (5), we compute the optimal target location distributions of six true target locations as shown in Fig. 1(a). The distribution covariance is the BB, which is computed using (3).

The CRB of the localization scenario in Fig. 1(a) is analyzed as follows. The parameter vector is the true target location $\mathbf{x_t} = (x_t, y_t)^T$. The observation vector $\mathbf{z}$ is formed by stacking all TDOA observations $z_{ij}, 1 \leq i < j \leq N$. The expected observation vector ζ is formed by stacking all optimal TDOA observations $\zeta_{ij}, 1 \leq i < j \leq N$ defined in (5). The observation vector $\mathbf{z}$ has a Gaussian distribution,

$$p(\mathbf{z}|\mathbf{x_t}) = \frac{1}{(\sigma\sqrt{2\pi})^{\frac{N(N-1)}{2}}} e^{-(\mathbf{z}-\zeta)^T(\mathbf{z}-\zeta)/(2\sigma^2)} \ . \quad (6)$$

The CRB is simply the inverse of the Fisher information matrix

$$\begin{aligned} J &= E\{[\nabla_{\mathbf{x_t}} \ln p(\mathbf{z}|\mathbf{x_t})][\nabla_{\mathbf{x_t}} \ln p(\mathbf{z}|\mathbf{x_t})]^T\} \ , \\ &= \frac{1}{\sigma^2}[\nabla_{\mathbf{x_t}} \zeta]^T[\nabla_{\mathbf{x_t}} \zeta] \ . \end{aligned} \quad (7)$$

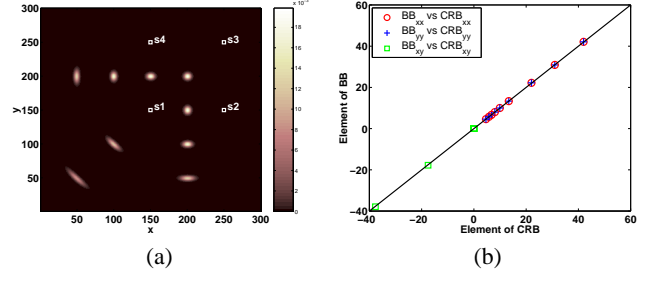The elements of the Fisher information matrix are

Fig. 2. Comparison of the BB with the CRB in localization using range sensors. (a) The the minimum-entropy target location distributions estimated by four range sensors denoted by squares. The Gaussian uncertainty with $\sigma = 4$ distance units is assumed for range observations. (b) Element-to-element comparison of the BB with the CRB of locating the same target.

$$\begin{aligned} J_{xx} &= \frac{1}{\sigma^2} \sum_{1 \leq i < j \leq N} (\frac{x_t - x_i}{\|\mathbf{x}_i\|} - \frac{x_t - x_j}{\|\mathbf{x}_j\|})^2 \ , \\ J_{yy} &= \frac{1}{\sigma^2} \sum_{1 \leq i < j \leq N} (\frac{y_t - y_i}{\|\mathbf{x}_i\|} - \frac{y_t - y_j}{\|\mathbf{x}_j\|})^2 \ , \\ J_{xy} &= \frac{1}{\sigma^2} \sum_{1 \leq i < j \leq N} (\frac{x_t - x_i}{\|\mathbf{x}_i\|} - \frac{x_t - x_j}{\|\mathbf{x}_j\|})(\frac{y_t - y_i}{\|\mathbf{x}_i\|} - \frac{y_t - y_j}{\|\mathbf{x}_j\|}) \ , \end{aligned}$$
$$(8)$$

where $\mathbf{x_t} = (x_t, y_t)$, $\mathbf{x}_i = (x_i, y_i)$, and $\mathbf{x}_j = (x_j, y_j)$.

We compute both the BB and the CRB for six true target locations depicted in Fig. 1(a), and compare the BB with the CRB element to element as shown in Fig. 1(b). Every element of the BB approximately equals to the corresponding element of the CRB.

The second simulation uses four range sensors placed on a square as shown in Fig. 2(a). We assume the Gaussian uncertainty with $\sigma = 4$ distance units for all range measurements. Similarly, we compute the minimum-entropy target location distribution for nine true target location as shown in Fig. 2(a). The CRB of localization using range sensors with Gaussian sensing uncertainty has been derived in [3]. We compute both the BB and the CRB of locating the same target and compare them component to component in Fig. 2(b). Every element of the BB approximately equals to the corresponding element of the CRB.

As revealed by both simulations, the BB equals the CRB element to element when the sensing uncertainty is Gaussian. When the minimum-entropy target location distribution derived through the Bayesian approach is not Gaussian, it provides more information than the CRB. It is simple to compute the BB of localization in a sensor network given arbitrary sensing models of individual sensors. In contrast, the CRB is difficult to derive when the sensing uncertainty are complex. Specifically, the partial derivative of the logarithm of the observation distribution, $\nabla_{\mathbf{x_t}} \ln p(\mathbf{z}|\mathbf{x_t})$, is difficult to derive when $p(\mathbf{z}|\mathbf{x_t})$ is complex.

3. SENSOR NETWORK COVERAGE

We define the coverage of a sensor network for localization as the region where the target can be relatively accurately located by the sensor network. In this section, we use the minimum entropy and the CRB to characterize the dependency of the localization uncer-
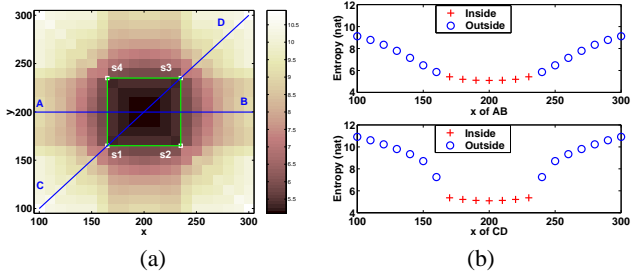
(a)   (b)

Fig. 3. Uncertainty characteristics of localization using TDOA sensors. (a) The minimum entropy of the target location distributions estimated by four TDOA sensors denoted by squares. The signal propagation speed is assumed to be 1 distance unit per time unit. The Gaussian uncertainty described in (4) with $\sigma = 4$ time units is assumed for all TDOA observations. (b) The minimum entropy of the target location distributions estimated for true target locations along profile AB and CD. The entropy corresponding to the true target locations inside the green square in (a) is denoted by red crosses. Otherwise, the entropy is denoted by blue circles.

(a)   (b)

Fig. 4. More scenarios of localization using TDOA information. (a) The minimum entropy of the target location distributions estimated by four TDOA sensors denoted by squares. The only difference from Fig. 3(a) is that four TDOA sensors are not evenly placed on a square. (b) The entropy converted from the CRB of the AML-based localization using eight sensors denoted by squares. The conversion is described by (10). We assume the white Gaussian noise with 20 db SNR 1 m from the target. The signal propagation speed is assumed to be 345 m/s.

tainty on the sensor network topology and the sensor observation type through simulations. The localization uncertainty characteristics obtained in this section provides guidance to approximately identify the coverage of a sensor network without detailed analysis. The localization uncertainty is measured in terms of Shannon entropy. We have considered three types of information provided by sensor observations, including TDOA, the range to the target, and the direction-of-arrival (DOA) of the target signal.

3.1. Characterizing Localization using TDOA

In this subsection, we study the uncertainty characteristics of localization using TDOA information. Our studies show that the coverage of a sensor network is the region inside the convex hull of all sensors if localization is essentially based on TDOA information among all sensors.

Fig. 3 shows the uncertainty characteristics analyzed through the Bayesian approach in terms of the minimum entropy of localization using four TDOA sensors placed on a square. The Gaussian uncertainty described in (4) with $\sigma = 4$ time units is assumed for all TDOA observations. The most significant feature is that the target inside the convex hull of TDOA sensors can be relatively accurately located. The localization uncertainty increases more abruptly when the target moves outside across a sensor at a convex hull vertex than when the target moves outside across a convex hull edge between sensors. As shown in Fig. 4(a), the coverage of the sensor network is still inside the convex hull of all sensors when four TDOA sensors are not placed evenly.

In contrast to the two-step localization approach in which an intermediate TDOA estimation is followed by a target location estimation, the approximate maximum-likelihood (AML) algorithm [4] directly estimates locations of near-field targets in a single-step. However, the most essential information utilized by AML is still TDOA at different sensors. The CRB for the near-field target localization using the AML algorithm has been derived in [5]. In this paper, we use the CRB to analyze the uncertainty characteristics of the AML-based single-target localization using eight sensors placed on a circle as shown in Fig. 4(b). The data collected
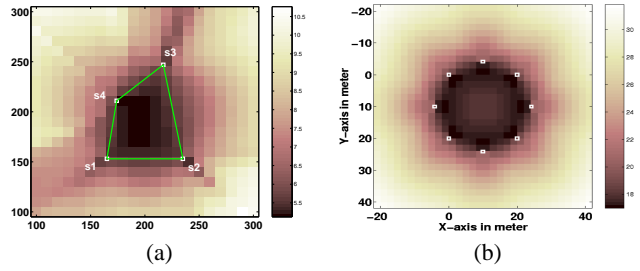
by the pth sensor at time n is assumed to be

$$x_p(n) = a_p s(n - t_p) + w_p(n) \ , \qquad (9)$$

where a_p is the signal gain at the pth sensor, s is the source signal, t_p is the time delay of the source signal at the pth sensor, w_p is the zero mean white Gaussian noise experienced by the pth sensor. In this case-study, we assume that the signal propagation speed is 345 m/s and that the signal-to-noise-ratio (SNR) at the distance of 1 m from the source is 20 db. In order for the result to be comparable with other localization uncertainty analysis in this paper, we treat the target location distribution approximately as Gaussian with the CRB as its covariance and compute the entropy H

$$H = 1 + \ln(2\pi\sigma_a\sigma_b) \ , \qquad (10)$$

where σ_a and σ_b are the square roots of the two eigen values of the CRB matrix. As shown in Fig. 4(b), the coverage of a sensor network using near-field AML algorithm for localization is also the region inside the convex hull of sensors.

3.2. Characterizing Localization using Range

Fig. 5 shows the uncertainty characteristics analyzed through the Bayesian approach in terms of the minimum entropy of localization using four range sensors placed on a square. The Gaussian sensing uncertainty with $\sigma = 4$ distance units is assumed for all range observations. In contrast to the coverage of a network of TDOA sensors, the coverage of a network of range sensors not only includes the area inside the convex hull of sensors, but also extends outside the convex hull to the area enclosed by arcs with convex hull edges as diameters. This result is consistent with the localization error characteristics of range sensors through the CRB analysis in [3]. When four range sensors are unevenly placed, our simulation indicates that the sensor network coverage is still enclosed by the arcs associated with the convex hull of sensors.

The cause of such extended coverage of a network of range sensors can be explained by the nature of range restrictions on the target location estimation. A range observation r with uncertainty $\pm\delta_r$ by a sensor S confines the target location estimate inside a
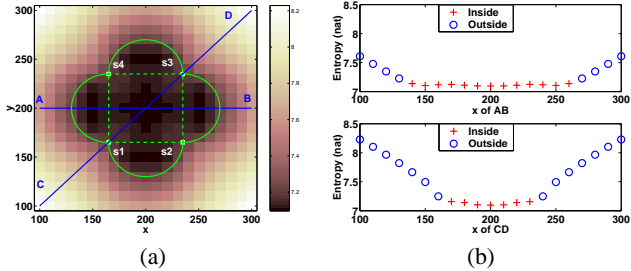
(a)   (b)

Fig. 5. Uncertainty characteristics of localization using range sensors. (a) The minimum entropy of the target location distributions estimated by four range sensors denoted by squares. The Gaussian uncertainty with $\sigma = 4$ distance units is assumed for all range observations. (b) The minimum entropy of the target location distributions estimated for true target locations along profile AB and CD. The entropy corresponding to the true target locations inside the solid green enclosure in (a) is denoted by red crosses. Otherwise, the entropy is denoted by blue circles.

circular belt with width $2\delta_r$. The center of the circle is S and the radius of the circle is r. Two range observations by different sensors together confine the target location estimate inside the intersection of two circular belts. When two circular belts cross each other perpendicularly, the intersection area is smallest and thus the target location estimate has the least uncertainty. When the target is actually at any point on the arch with a convex hull edge as the diameter (e.g. the arch between S_1 and S_2 in Fig. 5(a)), the circular belts of range measurements by the sensors at two ends of the convex hull edge cross each other perpendicularly. Therefore, the target can still be relatively accurately located around the arch although it is outside the convex hull of range sensors.

3.3. Characterizing Localization using DOA

Fig. 6 is the uncertainty characteristics analyzed through the Bayesian approach in terms of the minimum entropy of localization using four DOA sensors placed on a square. The observation uncertainty is modeled after the uncertainty characteristics of DOA estimation using the far-field AML algorithm [5]. We assume the DOA observation uncertainty is Gaussian and the standard deviation σ changes with the distance r between the sensor and the target,

$$\sigma = 180/r + 0.2r \quad \text{degrees} . \tag{11}$$

When the target is very far from the sensor, the SNR is low and thus σ should be high. When the target is very close to the sensor, the planar wave assumption of the far-field AML algorithm is violated and thus σ should also be high. The localization uncertainty characteristics using DOA information is very different from those using TDOA or range information. Although a target inside the convex hull of DOA sensors is still more accurately located than a target outside the convex hull and far from any sensor, the coverage of a network of DOA sensors is better described as the vicinity of individual DOA sensors. When σ does not change with r or σ changes with r differently from (11), simulations indicate that the coverage of a DOA sensor network is still the vicinity of individual sensors, similar to that shown in Fig. 6.
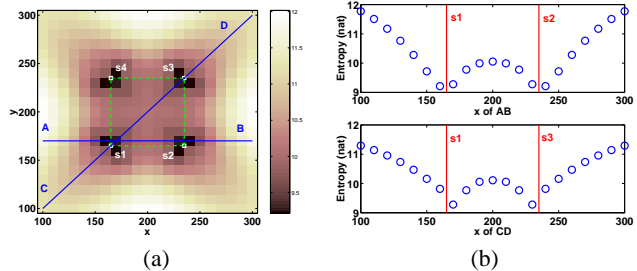

(a)   (b)

Fig. 6. Uncertainty characteristics of localization using DOA sensors. (a) The minimum entropy of the target location distributions estimated by four DOA sensors denoted by squares. The Gaussian uncertainty with σ described in (11) is assumed for all DOA observations. (b) The minimum entropy of the target location distributions estimated for true target locations along profile AB and CD. The red lines indicate the x coordinate of DOA sensors.

4. CONCLUSION AND FUTURE WORK

In this paper, we have proposed a Bayesian method to analyze the lower bound of localization uncertainty in sensor networks. We have compared the BB with the CRB through simulations. When the sensor observation uncertainty is Gaussian, the BB equals the CRB. Using the Bayesian method and the CRB, we have also analyzed the dependency of localization uncertainty on the sensor network topology and the sensor observation type. Such dependency can be used to approximately identify the region of relatively good localization accuracy without detailed analysis.

For simplicity, we have assumed the Gaussian sensing uncertainty throughout this paper. We plan to compare the BB with the CRB under non-Gaussian sensing models in the future. We have only analyzed the uncertainty characteristics of localization using the same type of sensors, future work is needed for localization using a mixture of multiple types of sensors.

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