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# The EPR Paradox: Roots and Ramifications

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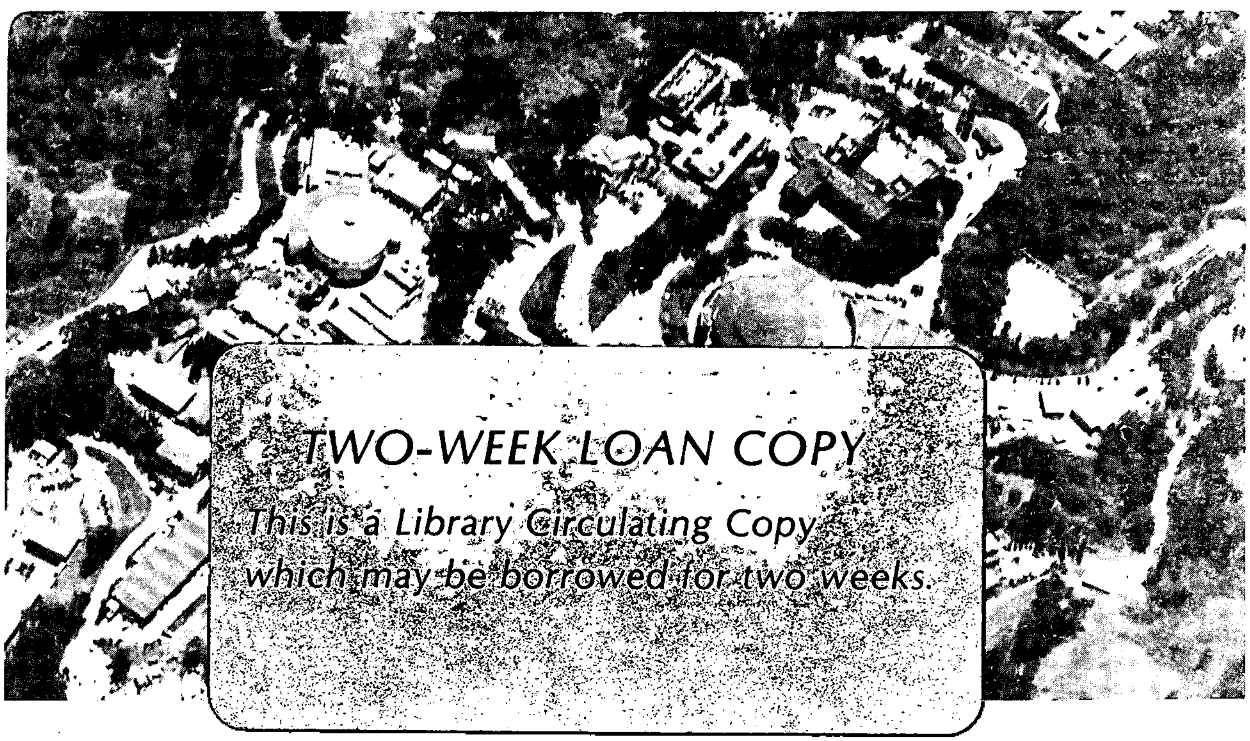





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# THE EPR PARADOX
# ROOTS AND RAMIFICATIONS†

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## ABSTRACT

Action at a distance at a speed greater than the speed of light was not an acceptable idea for Einstein, Podolsky, and Rosen (EPR) [1], but it was for Heisenberg [2]. Today, because of experimental results [3,4] and theoretical analysis [5,6], such action looks, in all likelihood, like a real effect. The argument leading to this conclusion can be explained in simple terms, as long as the word "real" is given a conventional meaning. However, the need for such action can also be shown with more general definitions of this word [7]. Several pictures of reality can still be drawn without contradicting the results of any experiment performed to date. The choice between the different possibilities depends partly on one's guess about the outcome of possible future experiments and partly on one's philosophical view of the world.

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# 1 A Debate Lasting More Than Fifty Years.

## 1.1 Are There Faster-Than-Light Effects in Quantum Phenomena?

The idea of physical effects propagating faster than the velocity of light may seem unacceptable to today's physicists, trained in an atmosphere of respect for the theory of relativity. For almost everyone, it also seems preposterous to give up the belief that there is a world out there, whether or not we make observations of it. However, there is evidence based on the results of experiments [3,4] of the "EPR-Bohm type" [8], that makes it very difficult to draw a picture of what this world might look like, with all the quantum phenomena accounted for, without some faster-than-light influences. This paper retraces the steps that, historically, led to this conclusion, analyzes the elements of the logical argument, and mentions a few possible solutions. In particular, the paper uses an example, [9], to show how it is still possible to find models of reality with a "rudimentary property of locality" that are compatible with *all* experimental results.

It is to explain the mechanisms behind the collapse of the wave function in quantum theory that faster-than-light actions are invoked. They were recognized already many years ago ( though long after the theory of relativity was well established ) by Werner Heisenberg [2]. It is well known that a measurement in quantum theory not only informs the observer but also modifies the quantum system under observation. When a linearly polarized photon impinges on a polarizer oriented at $90^o$ from the photon plane of polarization, there is a probability zero for the photon to pass the polarizer. If, on the other hand, in front of this polarizer, we place another polarizer tilted at $45^o$, there is a 25% chance that the photon will pass both polarizers. By testing the polarization at $45^o$, we transform some of these photons that could never pass the $90^o$ polarizer into photons that sometimes are able to do so. With the polarizer at $45^o$, we not only determine a property of the photons and make a selection among them, we also exert an influence on the selected ones as they are passing by. In quantum theory, this influence is described as an instantaneous effect but, in this particular example, it is exerted locally. For a quantum system extended in space, it is not so obvious that, by a similar mechanism, a measurement at some location in space can also influence the system's properties far from the measurement location. In particular, one may ask if such influence can also be exerted when it would have to propagate faster than light. Amazingly, from what we know today, as Heisenberg thought, the answer to this question is: *yes*, in all likelihood this influence has to be exerted at a distance and sometimes has to propagate faster than the speed of light. Some correlations between some results of measurements made at different locations in space under some experimental conditions seem explainable only by a change caused by the setting up of one measurement apparatus on the properties of the system itself (not simply of our information about it) near the other apparatus. This conclusion was reached after a long debate indicating the importance of the issues in question and recounted here in Sec.1. Necessary assumptions for this conclusion are analyzed in Sec.2. Some of the pictures of reality that still can be drawn are sketched in Sec.3.

It is in the following terms that Heisenberg was alluding to faster-than-light actions:

> "We imagine a photon which is represented by a wave packet built up out of Maxwell waves. It will thus have a certain spatial extension and also a certain range of frequency. By reflection at a semi-transparent mirror, it is possible to decompose it into two parts, a reflected and a transmitted packet. There is then a definite probability for finding the photon either in one part or in the other part of the divided wave packet. After a sufficient time the two parts will be separated by any distance desired; now if an experiment yields the result that the photon is, say, in the reflected part of the packet, then the probability of finding the photon in the other part of the packet immediately becomes zero. The experiment at the position of the reflected packet then exerts a kind of action (reduction of the wave packet) at the distant point occupied by the transmitted packet, and one sees that this action is propagated with a velocity greater than

> light. However, it is also obvious that this kind of action can never be utilized to transmit a signal so that it is not in conflict with the postulates of the theory of relativity." [2]

For Heisenberg, the postulates of relativity apply to results of observations and to what we can do with them. For an observable at a given point of space at a given time, consider the probability distribution one can determine not knowing the results of observations anywhere else. That distribution cannot depend on the action taken by anyone at a distance $\Delta x$ at another time, if the interval between the two times is less than that distance $\Delta x$ divided by the velocity of light $c$. However, the mathematical entities one uses to describe quantum systems between observations can depend on such action, because they are nothing more than convenient quantities to be used in calculations. Elsewhere in the same series of papers, Heisenberg expressed his opinion about the quantum theory calculations:

> "There exists a body of exact mathematical laws, but these cannot be interpreted as expressing simple relationships between objects existing in space and time." [10]

This point of view is consistent with the "Copenhagen interpretation" of quantum theory [11], according to which quantum theory is just a set of mathematical rules to predict future observations. There is no mention of a "reality" of the quantum system with features to be defined, described, or ruled by an equation of evolution between times at which we make observations. No attempt is made to describe that reality between measurements. Computations make use of a wave function $\psi$ (e.g., made of Maxwell waves), evolving according to the Schroedinger equation between observations, and collapsing instantaneously everywhere at the time every one of these observations is made. It expresses everything we can know about the system.[1] It is not an observable. It can be subjected to faster-than-light actions as long as these actions do not permit faster-than-light communication between observers.

[1] The wave function $\psi$ may depend partly on characteristics of the system and partly on our information about it. The two contributions are not sorted out in the Copenhagen interpretation.

Let us emphasize that Heisenberg talks about "exerting an action" at a distant point, not "finding out" about a feature (i.e., absence of a photon) in the transmitted packet when a photon has been seen in the reflected one. For him, obviously, there is more to the reduction of the wave function at a distance than just finding out about the property of a distant object.

## 1.2 Einstein's Point of View.

On the contrary, Albert Einstein was not satisfied with the Copenhagen interpretation of quantum theory. In 1934, for instance, he said:

> "I do not believe, however, that so elementary an ideal could do much to kindle the investigator's passion from which really great achievements have arisen. Behind the tireless efforts of the investigator there lurks a stronger, more mysterious drive: it is existence and reality that one wishes to comprehend." [12]

To comprehend reality, one has to write physical laws for the "real" features of the quantum systems, even between observations, not merely for the results of our observations when we make them.[2] To achieve such a goal may take considerable time and effort. One may question the wisdom and the usefulness of such an enterprise, but one cannot prove this approach to be wrong. However, Einstein made an additional assumption based on his view of relativity. Since human observation cannot identify a space-time restframe that would be more fundamental than others, then such a fundamental restframe does not exist. Then all laws ruling the evolution of the real features of a system should be written without reference to any such unobserved fundamental restframe. It follows that all kinds of faster-than-light effects are forbidden, whether or not they can be used to send a faster-than-light signal between observers. A measurement at some point in space can help us to find out

[2]For Einstein, reality was meant to be defined as usual, i.e., also in between times at which observations are made. Evidence for this can be found in the following account given by A. Pais of a discussion he had with Einstein during a walk with him:

> ". . . Einstein suddenly stopped, turned to me and asked whether I really believed that the moon exists only when I look at it." . . . [13]

about the real features existing at some distance $\Delta x$ but it cannot modify them before the time $\Delta x/c$ needed for light to cover that distance. This is an assumption, which we call "Einstein's locality postulate." This postulate is in contradiction with the point of view expressed by Heisenberg and quoted above [2]. It also turns out to have experimentally testable consequences.

If the constraint imposed by such a postulate is incorporated into our description of reality, the faster-than-light propagation of the wave-function collapse makes it impossible for this wave function $\psi$ to be identified with that reality. Another picture is necessary. In the 1930s, it was reasonable to hope that the laws of evolution of the real quantities could, without faster-than-light effects, be set up so that the predictions of quantum theory are reproduced. If this were indeed possible, then it can be demonstrated that this picture of reality would attribute more features to the quantum systems than can be described by a single wave function $\psi$. This demonstration was the subject of a paper written in 1935 by Einstein, Podolsky, and Rosen, [1], here abbreviated to EPR. We will give a simple version of their argument.

EPR describe a thought experiment involving a system of two particles, #1 and #2, which have interacted strongly in the past. In the basis of the positions $x_1$ and $x_2$ of the two particles, the system wave function is, at some instant,

$$\psi(x_1,\ x_2) \sim \delta(x_1 - x_2 + x_0) \tag{1}$$

where $\delta$ is the Dirac distribution and $x_0$ is a parameter.[3]

If position $x_1$ of particle #1 and then position $x_2$ of particle #2 are measured, the second measurement will, with absolute certainty, yield the value

$$x_2 = x_1 + x_0 \tag{2}$$

Just after the measurement of $x_1$, we know $x_1$, therefore we can know exactly the value $x_2$ that the second measurement would yield, if we decide to measure $x_2$ immediately

[3] Expression (1) has to be understood as a limit that can be approximated to any degree of accuracy by a wave function $\psi$ normalized to one.

afterward. EPR conclude that such a prediction is only possible if, before the measurement of $x_2$, there is an element of reality determining this unique value of $x_2$. According to EPR's own definition of reality,

> "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." [1]

This element exists immediately after the first measurement, i.e., before any signal emitted at the location and at the time of the first measurement has been able to reach the location of the second particle at a speed equal to or less than the speed of light. If, like EPR, we adopt Einstein's locality postulate which forbids any faster-than-light action, then, like they, we have to conclude that this element of reality determining the result of measurement #2 cannot possibly have been created or perturbed by measurement #1. Measurement #1 can only let us know the value $x_2$ to which this element corresponds. The element is present before, as well as after, measurement #1 is performed and therefore even if no measurement is made on particle #1.

The same argument can be repeated for the momenta $p_1$ and $p_2$ of the same particles #1 and #2. The wave function $\psi(x_1,\ x_2)$ of eq.(1) is an eigenvector of both operators $x_1 - x_2$ and $p_1 + p_2$, which commute. It also predicts that a measurement of $p_1$ will inform us exactly of the value $p_2$ that a measurement on particle #2 would yield, if we ever decide to measure its momentum $p_2$ anytime later.

$$p_2 = -p_1 \tag{3}$$

Therefore there is also an element of reality corresponding to that value of $p_2$. Applying once more Einstein's locality postulate as we did for $x_2$, we see that this element of reality corresponding to $p_2$ must also exist if we do not measure $p_1$. Then it follows that elements of reality determining a definite value for both the position $x_2$ and the momentum $p_2$ must be present, regardless of any measurement performed on particle #1. The uncertainty principle in quantum theory prevents a wave function $\psi$ from describing a particle with definite values of both position and momentum. EPR conclude:

> "We are thus forced to conclude that the quantum-mechanical description of physical reality given by wave functions is not complete." [1]

This result is called "the EPR paradox."

## 1.3 Dissenting Voices.

Niels Bohr believed in a principle of "complementarity" that was fundamentally incompatible with an idea of definite values of position and momentum at the same time. The uncertainty principle did not simply result from our lack of knowledge of the values of $x$ and $p$. It was an inherent property of the quantum system itself. In his answer to EPR, Bohr said:

> "Indeed we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way." [14]

According to Bohr, this impossibility of defining these quantities is also true in the context of the EPR experiment. To justify his point of view, Bohr did not challenge the EPR logic. He challenged the EPR assertion, based on Einstein's locality postulate forbidding faster-than-light effects, that measurement #1 did not disturb particle #2 in any way:

> "From our point of view we now see that the wording of the above-mentioned criterion of physical reality proposed by Einstein, Podolsky and Rosen contains an ambiguity as regards the meaning of the expression 'without in any way disturbing a system.' Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical state of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which*

*define the possible types of predictions regarding the future behavior of the system.* Since these conditions constitute an inherent element of the description of any phenomenon to which the term 'physical reality' can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete." [14]

Bohr attaches the term "physical reality" to conditions that define the possible types of predictions. Since measurement #1 affects predictions concerning measurement #2, there is an influence exerted by measurement #1 on the system's characteristics responsible for measurement #2, i.e., on those characteristics we like to associate with particle #2. It is an influence, not only an inference, i.e., not only a change in our information about particle #2. Of course, this influence must result from an action at a speed greater than the speed of light, but it does not matter. The disturbance is not a "mechanical disturbance." The word "influence" is used by Bohr instead of Heisenberg's word "action" quoted above, but the idea is the same. It is an effect we should assume to exist in order to explain our observations, but we cannot use it to send a signal and much of it is unknown.[4] Elsewhere in his answer to EPR, Bohr calls for "a radical revision of our attitude towards the problem of physical reality" but, on this point, he is not explicit as to how "physical reality" should be revised and how exactly his interpretation of this term is different from the definition given by EPR.

In any event, only if and when $x_1$ has been measured do we need to consider $x_2$ to have a definite value. Only if and when $p_1$ has been measured do we need to consider $p_2$ to have one either. Since we cannot measure both $x_1$ and $p_1$, the wave function $\psi$ is adequate to describe everything we need to describe. In this sense quantum theory is complete.

Bohr says: "From our point of view we now see . . ." He does not give a proof that this is the only possible point of view. Descriptions of a reality that could account for the predictions of orthodox quantum theory and satisfy Einstein's locality were still sought.

[4] In the case of the EPR experiment, one cannot even know if the correlation is due to an influence of the measurement of $x_1$ on the observable $x_2$ or to an influence of the measurement of $x_2$ on the observable $x_1$.

None was found. In the 1950s, David Bohm constructed a deterministic hidden variables model (i.e., a type of model of reality) which exactly reproduced the predictions of quantum theory [15]. For quantum systems made of only one particle, this model does not imply any faster-than-light action and therefore agrees with Einstein's locality. However, when more than one particle is involved in a quantum system, the model does involve such faster-than-light actions. Of course, for the Copenhagen interpretation, questions of this kind do not have to be settled. Therefore, for lack of a better alternative, it is understandable that quantum theory has widely been used as prescribed by that interpretation, i.e., just as a theory to predict observables.

The most significant progress toward a resolution of the controversy was accomplished by John Bell [5] much later, in 1964. Bell considered a thought experiment inspired by EPR but with an important modification suggested by Bohm [8]. In such an experiment, which we call an EPR-Bohm experiment, the two spatially separated particles have spins and, because of their past interaction, the spin-orientations are strongly correlated. Then one considers the measurement of the orientations of the spins of both particles with various settings of two spin-measurement apparatuses. Bell demonstrated two statements:

a) *Einstein's locality postulate imposes constraints on the predictions of spin correlations in the form of inequalities.* Assume, in accordance with that postulate, that the observations we can make on particle #2 are not affected by any faster-than-light action caused by the setting up of the apparatus for a measurement on particle #1. Assume also the converse that the observations on #1 are not affected by the apparatus setting on #2. Then the set of predictions for various configurations of the measurement apparatuses must satisfy some inequalities. Whatever picture of reality one can possibly draw, if it does not imply action of one apparatus setting on the result found in the other, these inequalities must hold. Since Bell's initial paper, these inequalities have been expressed in different forms by various authors [5,6,16,17]. We will refer to all these expressions by the generic name "Bell's inequalities."

b) *The predictions of quantum theory for the EPR-Bohm experiment violate these inequalities implied by Einstein's locality.* This is true no matter how small the time delay is between the two measurements, even if the two measurements are almost simultaneous.

The amount by which the inequalities are violated is finite, thus measurable.

In the literature, conclusions a) and b) are referred to as Bell's theorem. They are also demonstrated in Sec.2 of this paper. Together, they have far-reaching consequences. For an EPR-Bohm experiment, quantum theory implies an instantaneous action of one measurement apparatus on the result obtained in the other. Therefore either Einstein's locality postulate, or the quantum theory prediction, or both have to be violated. One cannot anymore hope, like EPR, to find a more complete description of quantum phenomena that would describe the reality (in a conventional sense) of quantum systems between measurements, reproduce all the predictions of quantum theory, and use only effects propagating at speed equal to or less than the velocity of light.

In papers following Bell's initial article, his conclusion was discussed and analyzed in the broadest possible perspective [7,16,17]. It was always upheld. If the predictions of quantum theory are correct, even approximately, for all time delays between measurements, then any picture of reality must contain these effects that Heisenberg, in the above quote [2], was infering from the instantaneous collapses of the wave function: the picture has to include instantaneous actions at distant locations, thus actions faster than the speed of light.

## 1.4 The Verdict of Experiment.

At that point in time, it was important to find out experimentally, in the circumstances of an EPR-Bohm experiment, whether it was quantum theory or Einstein's locality that actually was incorrect. Experiments have been performed to measure the probabilities involved in Bell's inequalities with nonzero-spin particles and with some of the features of the EPR-Bohm thought experiments incorporated in them, [3,4]. In 1981, Aspect, Dalibard, and

Roger performed such an experiment with the most important features all included in one experiment [4]. All experimental results ruled in favor of quantum theory. [18]

Not everyone of all the features of a real EPR-Bohm experiment have yet been included. Loopholes still exist, [19], and it is still possible, in principle, to believe in Einstein's locality, [20], therefore in violations of quantum theory and that would show up in a real EPR-Bohm experiment. However, the quantum theory predictions have been verified at high precision in all sorts of setups affected by different kinds of loopholes and it is difficult just to blame loopholes for all these apparent experimental violations of Einstein's locality. Then, if we do overlook those loopholes, we are forced to conclude against Einstein's locality postulate and accept the fact that any conventional picture of reality we can construct has to include some faster-than-light effects.

The EPR argument backfired. It was invented to demonstrate the shortcomings of the orthodox Copenhagen interpretation of quantum theory. It only ended up showing an additional difficulty that an alternative theory describing reality would have to face. It would have to include faster-than-light influences.

The above historical account is succinct and schematic. When reading the published work and the correspondence of the opponents in this debate, one becomes aware of subtleties and nuances, of the evolution of everyone's thinking on the subject, probably in reaction to what the others are saying, and sometimes of some difficulty in finding the right words to express their own thoughts. Sometimes the debate is obscured because words related to the concept of reality, such as "causality" and "locality," are not always used by everyone with the same significance, [17,21]. However, our goal is not to give a precise account of this historical debate. Our goal is only to give the general idea.

As of now, there essentially remain four possible options:

a) give up on any picture of reality between observations (as in the Copenhagen interpretation [11]) or overhaul our concept of reality. Quantum theory predictions for all present and future experiments can be absolutely correct. No violation of Lorentz invariance shows up anywhere.

b) capitalize on the existence of loopholes in the past experiments, [20]. Conventional reality is still a good concept. Einstein's locality is absolutely correct. There is a violation of quantum theory but it will be found experimentally only in the future, maybe only when a real EPR-Bohm experiment can be done.

c) accept the existence of effects propagating instantaneously in a fundamental space-time restframe. There are models of reality in which there are instantaneous actions at distances, [15,22]. They are not Lorentz invariant. The predictions of relativistic quantum theory are absolutely correct. Therefore it is and always will be impossible to find out what the fundamental Lorentz restframe actually is.

d) assume effects propagating faster than light but not instantaneously in a fundamental space-time restframe (i.e., "rudimentary locality"), [9]. Both Lorentz invariance and quantum theory are violated. For almost simultaneous measurements in the fundamental restframe, there are violations of the quantum theory predictions, but these violations can be found only in experiments involving measurements with time delays between them so small that these violations have gone unnoticed so far. The fundamental restframe, too, can be identified only in such experiments. Therefore, it has also yet been impossible to find out what it is.

Brief developments of these ideas a), b), c), and d) above are presented in Sec.3.

Unfortunately, Einstein died before Bell's theorem was published [5]. Had he known about it, what conclusion would he have drawn? Since he could not have both "plain reality" and equivalence of all space-time restframes, which would he have been willing to sacrifice? The following account given by Heisenberg of a conversation with him may give us a clue. They were discussing *unobservable* electron trajectories in an atom and Einstein was blaming quantum theory for not describing them. In this context, Heisenberg alluded to Einstein's own motivation for formulating special relativity the way he did, i.e., so that no *unobservable* privileged restframe be used in the theory.

". . . He (Einstein) thought that every theory in fact contains unobservable quantities. The principle of employing only observable quantities simply cannot be consistently carried out. And when I (Heisenberg) objected that in this I had merely been applying the type of philosophy that he, too, had made the basis of his special theory of relativity, he answered simply: 'Perhaps I did use such philosophy earlier, and also wrote it, but it is nonsense all the same.' . . ." [23]

# 2 A Far-Reaching Argument.

## 2.1 Example of an EPR-Bohm Experiment.

To help appreciate the constraints imposed by Bell's theorem on our pictures of reality, we will analyze the conditions under which the theorem is demonstrated. For this analysis, we will, in as concrete terms as possible, describe an "EPR-Bohm" experiment, where measurements are performed outside of the light cone of one another and where quantum theory predicts correlations that are adequate for the demonstration.[5] In Subsec.2.1, the experimental setup is described. In Subsec.2.2, the relevant quantum theoretical predictions are computed. In Subsec.2.3, the properties expected from a representation of reality in a conventional sense and from Einstein's locality are spelled out. In Subsec.2.4, it is shown that these properties cannot be satisfied by any model of reality reproducing those quantum theory predictions (Bell's theorem). In Subsec.2.5, an intuitive argument is given to show the generality of the demonstration. The reader who wishes to skip the details of both experimental lay out and quantum theoretical computation may want to read only the words written in italic in both this subsection and Subsec.2.2.

*Our EPR-Bohm experiment is a thought experiment involving three spaceships (#0, #1, and #2),* motionless with respect to one another, and lined up with respect to an axis which we call the $z$-axis, as shown in Fig.1. The distance between spaceship #0 in the middle and each of the other two (#1 and #2) is exactly the same and equal to the distance that light would cover in, let us say, one second. *Photons with correlated helicities are emitted from spaceship #0 at about the same time and their polarization is analyzed in the other two at*

[5]It has been shown that the original EPR experiment cannot be used to demonstrate the faster-than-light action, [24]. For this purpose, the kind of modification introduced by Bohm is necessary [8].

*the time of their arrival.* Arrival time for both is also about the same, one second later. The distance between spaceships is very large so that humans have about two seconds, i.e., enough time to make a choice of which experiment to perform in their own spaceship, change the setup of their equipment, and make their observations before any information about the choice of setup could be communicated between spaceships #1 and #2 at a speed equal to or less than the velocity of light.

*In spaceship #0, quantum systems made of two photons with correlated helicities and aimed at the other spaceships are being prepared.* The photon source is a gas of monoatomic molecules having a ground level and an excited level both of spin-parity $0^+$, as well as two excited levels of spin-parity $1^-$, as sketched in Fig.2. Light is shined on the gas to provoke the transition of molecules from their ground level to the higher of the two $1^-$ excited levels. Then, very often, one of the molecules returns to the ground level via the two other excited levels, as shown in Fig.2, in a cascade that generates three photons. The atom decays first from the higher $1^-$ level to the $0^+$ excited level while emitting photon #0 of wave length $\lambda_0$, then to the lower $1^-$ excited level emitting photon #1 of wave length $\lambda_1$, then to the $0^+$ ground level emitting photon #2 of wave length $\lambda_2$. In each spaceship, there is a photon detector. We are only interested in events where photon #0 is detected in the detector aboard spaceship #0 and the two photons #1 and #2 emerge in directions that intercept the photon detectors in spaceships #1 and #2, respectively. To recognize such events, an active collimator is attached to spaceship #0, as shown in Fig.1. Its efficiency is assumed to be 100%. It has three openings, two of them defining very small solid angles in the direction of the two spaceships #1 and #2. Through one opening, photons of wave length $\lambda_1$ can emerge toward the detector aboard spaceship #1 and, through another opening, photons of wave length $\lambda_2$ can emerge toward the detector aboard spaceship #2. Through the third opening, photons of wave length $\lambda_0$ can escape the active collimator and fall on the detector aboard spaceship #0. Any other photon configuration triggers the active collimator because, in the openings, there are optical devices made of prisms and lenses that deflect photons of the wrong wave length and make them fall on the collimator. Whenever, aboard spaceship #0, a photon of wave length $\lambda_0$ is detected but the active collimator does not detect any photon, the time of the event is recorded in spaceship #0 for future analysis of the data. Every one of these events corresponds to a quantum system made of two photons, #1 and #2, aimed at the detectors in spaceships #1 and #2, respectively. These events are the events of interest.

*In each one of the spaceships #1 and #2, there is a polarizer in front of the photon detector, as shown in Fig.1, and there is an observer. The polarization power of the polarizer and the detection efficiency of the detectors are assumed to be 100%.* In each spaceship, the observer can rotate the axis of his polarizer to form any angle he wants with respect to a direction called $x$-axis, common to both spaceships, normal to the $z$-axis, and shown in Fig.1. Let us call this angle $\phi_1$ and $\phi_2$ for the polarizers in spaceships #1 and #2, respectively. Every second, each observer freely chooses a value for the angle of his polarizer and sets his polarizer accordingly. He keeps a record of that polarizer angle as a function of time and, whenever he notices a photon detected in his apparatus, he also makes a record of the time of the event. Both photons reach their respective polarizers at about the same time, as can be seen from the space-time display of the event in Fig.3. Only if information travels faster than light can the angle of the polarizer for one photon in one spaceship be known in the other spaceship at the time of impact of the other photon on the polarizer located there.

A large amount of statistical data are collected under these conditions in the three spaceships. *Later, these data are gathered in one place and compared,* taking into account the delay of one second between the manifestation of the event in spaceship #0 and in the other two spaceships. We first discard all events but those called "events of interest," i.e., with a spaceship #0 record indicating a photon #0 in the appropriate detector but no other photon in the active collimator. Then, *we separate events registered with different sets of values $\phi_1$ and $\phi_2$ of the polarizer angles. In each sample of events corresponding to the same set of $\phi_1$ and $\phi_2$, we determine the fraction $F_1(\phi_1, \phi_2)$ of events for which there was a photon #1 detected in spaceship #1 but no photon detected in spaceship #2. Similarly, we determine the fraction $F_2(\phi_1, \phi_2)$ for which a photon #2 has been detected in spaceship*

*#2 but none in spaceship #1.* There are predictions for $F_1(\phi_1, \phi_2)$ and $F_2(\phi_1, \phi_2)$ from quantum theory and they will be shown to be incompatible with Einstein's locality.

## 2.2 How to Compute the Predictions in Quantum Theory.

*For the experiment described in Subsec.2.1, the quantum theoretical computation illustrates the purely mathematical role played by the wave function $\psi$.* As in any quantum theoretical computation, we first define a basis for the quantum states. We choose it to be made of states corresponding to the classical picture of a determined number of particles of each type, with each one of these particles having a determined one-particle wave function. If the wave functions of these particles do not overlap in space, they do not interact and one-particle quantum theory is applicable to each of them independently. All pure quantum states are coherent superpositions of such basis states. Every superposition corresponds to a set of complex numbers, one of them attached to each basis state.[6] Superpositions do not, in general, correspond to a classical picture. They are states where there are mathematical laws but, according to Heisenberg, they do not "express simple relationships between objects existing in space and time." [10]

As soon as photon #0 has been detected, it is possible to know that there is one of these monoatomic molecules in the $0^+$ excited state. Among other quantum numbers, this state has a zero total angular momentum component $S_z$ along the $z$-axis and is even under an operation $P_y$ of reflection about the $x$-$z$ plane

$$S_z = 0 \tag{4}$$

$$P_y = +1 \tag{5}$$

After a small time lapse, we expect the atom to return to its ground state while emitting the two photons, #1 and #2, of Fig.2. Then the quantum system is still in a state called "pure case." It is a coherent superposition of basis states, all corresponding to classical representations of two photons escaping in various directions and with positive and negative

[6] The mathematical space defined by all these superpositions, but with the symmetries and antisymmetries imposed by Bose and Fermi statistics, is called "Fock space".

helicities. The total spin-parity of the system is still $0^+$. Therefore $S_z$ and $P_y$ still satisfy eqs.(4) and (5).

A little later, the active collimator has had a chance to detect photons #1 and #2. In the cases of interest, it did not detect either of these two photons. The state has collapsed (reduction of the wave packet) into another coherent superposition of basis states. The basis states left with nonzero amplitudes correspond only to photons parallel to the $z$-axis, namely photon #1 in the direction of spaceship #1 and photon #2 in the direction of spaceship #2. Total spin and parity are not good quantum numbers anymore because they have been disrupted by the active collimator. However, the angular momentum component $S_z$ along the $z$-axis and the quantum number $P_y$ associated with the reflection about the $x$-$z$ plane are conserved. They still satisfy eqs.(4) and (5).

*At this time, the "preparation" of the two-photon system is complete. The quantum state is represented by a wave function $\psi_0$ which,* because of eq.(4), *has non-zero components for only two basis states, one with two positive and the other with two negative helicities. We call these two basis states $\chi^1_+ \otimes \chi^2_+$ and $\chi^1_- \otimes \chi^2_-$, respectively, where the symbols $\chi^1_\pm$ and $\chi^2_\pm$ refer to basis states of individual photons.*[7] Because of eq.(5), *the two components are equal.*

$$\psi_0 = \frac{1}{\sqrt{2}}(\chi^1_+ \otimes \chi^2_+ + \chi^1_- \otimes \chi^2_-) \tag{6}$$

When the wave packets of photons #1 and #2 are well separated in space, either basis state $\chi^1_+ \otimes \chi^2_+$ or $\chi^1_- \otimes \chi^2_-$ corresponds to a classical picture made of two objects separated in space. Single particle quantum theory can be applied for the predictions. However, this is not the case of the superposition $\psi_0$ of eq.(6), which is one of these examples of a situation where Heisenberg would not see "simple relationships between objects existing in space and time." Mathematics have to be used without the support of a classical picture.

As shown in Fig.3, in the Lorentz restframe at rest with respect to the spaceships, the time interval between the impacts of photons #1 and #2 on their respective polarizers is

[7] Bose symmetry has been explicitly omitted in the mathematical expressions of these states for reasons of simplicity and because it does not affect the final predictions. Bose symmetry can easily be reinstated by a reader who wants absolute rigor in all mathematical expressions.

very small, much less than the two seconds needed for light to travel from one to the other spaceship. It follows that there are Lorentz restframes where the impact of #1 appears to observers as occurring before the impact of #2 and there are restframes where it is the other way around. *Let us use a restframe like the $z_1$-$t_1$ restframe of Fig.3 where the impact of photon #1 precedes the impact of photon #2.*

*When photon #1 impinges on the polarizer in spaceship #1, the state of the two-photon system collapses* again. Let $\Pi_1(\phi_1)$ be the projection operator corresponding to the case where the photon passes the polarizer oriented at an angle $\phi_1$ with respect to the $x$-axis. First consider the effect of $\Pi_1$ on the basis states $\chi^1_+ \otimes \chi^2_+$ and $\chi^1_- \otimes \chi^2_-$ that correspond to classical pictures of photons separated in space and with the same helicity. If photon #1 passes the polarizer, it becomes a linearly polarized photon, i.e., a superposition of positive and negative helicities. Single particle quantum mechanics gives the rules to compute the complex coefficients

$$\Pi_1(\phi_1)\chi^1_+ = \frac{1}{2}(\chi^1_+ + e^{-2i\phi_1}\chi^1_-) \tag{7}$$

$$\Pi_1(\phi_1)\chi^1_- = \frac{1}{2}(e^{2i\phi_1}\chi^1_+ + \chi^1_-) \tag{8}$$

The operator $\Pi_1$ does not affect $\chi^2_+$ and $\chi^2_-$ since these symbols correspond to the classical image of a photon faraway from the measurement location. Therefore, for the two-photon system

$$\Pi_1(\phi_1)\chi^1_+ \otimes \chi^2_+ = [\Pi_1(\phi_1)\chi^1_+] \otimes \chi^2_+ \tag{9}$$

$$\Pi_1(\phi_1)\chi^1_- \otimes \chi^2_- = [\Pi_1(\phi_1)\chi^1_-] \otimes \chi^2_- \tag{10}$$

For the system of two photons corresponding to the superposition defined by eq.(6), using eqs.(7), (8), (9), and (10) as well as the linear property of the operator $\Pi_1$, *one can derive the expression of the state $\psi_1$ after photon #1 has passed its polarizer*

$$\psi_1 \sim \Pi_1(\phi_1)\psi_0 = \frac{1}{2\sqrt{2}}(e^{i\phi_1}\chi^1_+ + e^{-i\phi_1}\chi^1_-) \otimes (e^{-i\phi_1}\chi^2_+ + e^{+i\phi_1}\chi^2_-) \tag{11}$$

*The state $\psi_1$ now corresponds to a classical picture of two photons both linearly polarized in a plane containing the $z$-axis and tilted at an angle $\phi_1$ from the $x$-$z$ plane. The polarizer* *in spaceship #1 has made not only photon #1 but also the faraway photon #2 look like a linearly polarized photon at an angle $\phi_1$, though $\phi_1$ is the value of a polarizer angle set arbitrarily by an observer outside of the light cone of the photon #2 wave packet.* This is the kind of "influence" Bohr was referring to in his answer to EPR. [14]

From eqs.(6) and (11), one can compute the probability of photon #1 passing the polarizer; it is 50%. Since $\psi_1$ corresponds to classical objects separated in space, one-particle wave mechanics can be used to compute the probability that photon #2, now linearly polarized at the angle $\phi_1$, gets absorbed in the polarizer tilted at the angle $\phi_2$ in spaceship #2. It is $\sin^2(\phi_1 - \phi_2)$. Similar calculations yield the probabilities of photon #1 being absorbed in the polarizer of spaceship #1 and photon #2 passing the polarizer in spaceship #2. From this, *we derive the probabilities $F_1(\phi_1, \phi_2)$ to detect a photon only in spaceship #1 and $F_2(\phi_1, \phi_2)$ only in spaceship #2,*

$$F_1(\phi_1, \phi_2) = F_2(\phi_1, \phi_2) = \frac{1}{2}\sin^2(\phi_1 - \phi_2) \tag{12}$$

*These probabilities are the expectation values for the fractions $F_1(\phi_1, \phi_2)$ and $F_2(\phi_1, \phi_2)$, measured in the experiment described in Subsec.2.1.*[8]

### 2.3 Reality and Lorentz Invariance.

As the computation in Subsec.2.2 progresses, it would be interesting to have a picture of the quantum system itself at different stages of the event. If we accept instantaneous effects at distance in a model of reality, the wave function $\psi$ in one particular Lorentz restframe can be used to represent the real quantum system between observations.

[8] It is easy to see that, as Bohm wanted [8], an EPR-Bohm experiment such as the one of Subsec.2.1 can also be used to illustrate the EPR argument. For $\phi_1 = \phi_2$, the probabilities $F_1(\phi_1, \phi_2)$ and $F_2(\phi_1, \phi_2)$ of eq.(12) for counting a photon in one spaceship and not in the other are zero. It follows that one count in one spaceship #1 informs us with certainty that one count will be observed in spaceship #2, and no count in #1 allows us to predict that no count will be observed in #2. Measurement #1 determines exactly the result of measurement #2, as in the original EPR experiment described in Subsec.1.2. This is true regardless of the value of $\phi_1 = \phi_2$, though the operators associated with measurement #2 for different values of $\phi_2$ do not commute in general. Therefore, in this case, EPR would also conclude that, to avoid faster-than-light effects, a description of reality must include elements of reality corresponding to definite outcomes of measurements associated with non-commuting operators. Therefore they would also conclude that a description of reality has to be more complete than any description that can be given by a wave function.

Let us choose the Lorentz restframe $z_1$-$t_1$ of Fig.3, which we used for the computations above and where photon #1 is seen impinging on its polarizer *before* photon #2 does on its own. The probability of detection in spaceship #1 is a function $r_1(\psi_0, \phi_1)$ of the wave function $\psi_0$ before the measurement in spaceship #1 and of the polarizer angle $\phi_1$. It is not affected by the angle $\phi_2$. This is understandable because $\phi_2$ could be set in spaceship #2 after the impact of photon #1 on spaceship #1, according to the time order of events in the $z_1$-$t_1$ restframe. For the same reason, after the impact of photon #1, the state of the quantum system $\psi_1$, given by eq.(11), depends on $\psi_0$, on $\phi_1$, and on the result of the measurement in spaceship #1, but does not depend on $\phi_2$. The probability of detection in spaceship #2 depends on $\phi_2$ and on $\psi_1$, thus on $\psi_0$, on $\phi_1$, and the outcome of the event in spaceship #1. It is a function $r_2(\psi_0, \phi_1, \phi_2)$. As in the quantum theory computations using the $z_1$-$t_1$ restframe, the probability $F_1(\phi_1, \phi_2)$ is computed as the product

$$F_1(\phi_1, \phi_2) = r_1(\psi_0, \phi_1)[1 - r_2(\psi_0, \phi_1, \phi_2)] \tag{13}$$

which finally produces the expression given in eq.(12). In this description of reality, the correlation of eq.(12) is generated by the faster-than-light influence of a human action taken and an event occurring in spaceship #1 on an event in spaceship #2. In the $z_1$-$t_1$ restframe, the time of the cause comes *before* the time of the consequence, as it is natural to expect.

However, a problem arises because the times and distances involved (shown in Fig.3) imply that there are also Lorentz restframes where photon #1 is seen as impinging on spaceship #1 *after* photon #2 impinges on spaceship #2. In these other restframes, the setting up of $\phi_1$, which we just considered as the origin of an influence on measurement #2, may actually take place *after* measurement #2. Then cause and consequence do not appear in a natural time order. This makes these other restframes less suitable than $z_1$-$t_1$ to describe what happens in reality. All Lorentz restframes are not equivalent. Einstein's concept of relativity was incompatible with such an idea. To have consequences and causes appear always in a proper time order in all Lorentz restframes, it can be shown that it is necessary to have them in the light cones of one another. Therefore one has to forbid all faster-than-light effects.

Of course, in a restframe $z_2$-$t_2$ where measurement #1 follows measurement #2, quantum theory allows us to define another wave function which could also be used to describe reality. Then the probability of detection of photon #2 is independent of $\phi_1$ and the probability of detecting photon #1 depends on $\phi_2$ via an intermediate state $\psi_2$ completely different from $\psi_1$ of eq.(11). Then, $F_1(\phi_1, \phi_2)$ results from a computation of the form

$$F_1(\phi_1, \phi_2) = [1 - r_2'(\psi_0, \phi_2)]r_1'(\psi_0, \phi_1, \phi_2) \tag{14}$$

The probabilities of eqs.(13) and (14) end up being the same,[9] and equivalent to eq.(12). However, if the latter scenario involving the wave function $\psi_2$ in the $z_2$-$t_2$ restframe is interpreted as the real scenario, the correlation is due to an influence of an operation executed and an event taking place in spaceship #2 on an event in spaceship #1. When compared to the former scenario involving the wave function $\psi$ in the $z_1$-$t_1$ restframe, the roles of spaceships #1 and #2 are interchanged. The intermediate state $\psi_2$ is a function of $\phi_2$ instead of $\phi_1$. It is not possible to define a Lorentz transformation for wave functions that transform a function $\psi_1$ of only $\phi_1$ into a function $\psi_2$ of only $\phi_2$. If a wave function describes reality between measurements, it can only be the wave function in one restframe. Then there is a privileged space-time restframe.

The above considerations show why a picture of reality based on the wave function cannot abide with Lorentz invariance. They still do not rule out the possibility that other pictures of reality could exist, not using $\psi$ for the description but reproducing the probabilities computed using $\psi$ and satisfying Lorentz invariance. Correlations between measurement results can be generated by Lorentz covariant mechanisms if they involve a *common cause* in a space-time region common to the light cones in the past of both measurements. Then the reality of the quantum system is not necessarily represented by a wave function $\psi$ in any restframe. Each possible configuration of reality corresponds to a mathematical entity $\lambda$. For each wave function $\psi$, there may be several possible values $\lambda_k$ of $\lambda$ and $\lambda$ takes one of

[9]The equivalence between eqs.(13) and (14) is just an example of the Lorentz invariance of all the probability distributions computed in relativistic quantum theory.

these values $\lambda_k$ with a probability $w_k$. To avoid faster-than-light effects, this kind of representation of reality is what EPR concluded was necessary. However, Bell's theorem, which we will demonstrate in the next subsection, stipulates that even such a more "complete" description of reality cannot account for the predictions of eq.(12) without these unwanted faster-than-light effects.

At this stage, to avoid ambiguities, we need to give the word "reality" at least a partial definition and spell out the properties it should have to satisfy Einstein's concept of relativity. It will be a rather conventional but convenient definition of reality. Of course, it will be only a working definition, not a claim to any insight into the philosophical essence of "Reality." *For us, reality is defined as a means to explain what is observed. It includes all the quantities that an observer calls observables. It always corresponds to a mathematical entity $\lambda$, which we call the "representation of reality" and which can take different sets of values $\lambda_k$ corresponding to probabilities $w_k$ that $\lambda = \lambda_k$. The representation $\lambda$ is defined at all times $t$, i.e., $\lambda = \lambda(t)$, whether or not we make observations of it.*[10] Furthermore, $\lambda$ has to satisfy the following conditions. Consider a particular space-time restframe.

a) *The time evolution of $\lambda$ is ruled by an equation that is probabilistic or deterministic. It depends only on $\lambda$ itself, on the action of humans in the past* when, for instance, they set up a measuring apparatus, and possibly on some random numbers if the evolution is not deterministic. In particular, there are functions of $\lambda$ expressing the probability distributions for all the observables. They are functions of the value of $\lambda$ before the measurement and of the conditions of observation, which are set by human action. They allow us to express the observed probability distributions when, in addition, the uncertainties $w_k$ on $\lambda$ are folded in. The mathematical dependence of $\lambda$ on quantities determined by human actions in the past is what we define as a causal relationship.

   The quantity $\lambda$ does not determine human action, which is treated as an external parameter depending on the free will of an observer.

[10]Note that the existence of objects between times of observation is recognized as a crucial element of a definition of reality by psychologists studying human intelligence. [25]

Condition a) is satisfied by the wave function $\psi$.

b) *The representation $\lambda$ is the same for all observers.* As mentioned above, there are several possibilities $\lambda_k$ for $\lambda$ and, for each $\lambda_k$, a weight $w_k$ expressing the likelihood that reality $\lambda$ actually is $\lambda_k$. However, a given preparation of the quantum system corresponds to only one set of $\lambda_k$'s and $w_k$'s common to all observers (or maybe to different sets of $\lambda_k$'s that can be derived from each other using definite mathematical transformations, e.g., Lorentz transformations, which do not alter the causal relationships).

In our EPR-Bohm experiment, for different observers making calculations in different space-time restframes, we have seen that the wave function $\psi$ and the causal relationships in a model of reality based on that wave function $\psi$ are observer-dependent. Therefore, if $\psi$ is used for a description of reality in accordance with condition b), only the wave function $\psi$ in one privileged restframe can qualify.

To incorporate Einstein's relativity concept into this picture of reality, we need also the following conditions:

c) *The representation $\lambda$ consists of "objects existing in space and time,"* i.e., of "elements of reality" $Q(x,t)$ depending on a set of space coordinates $x$ and characterizing properties existing at the point of coordinates $x$ only. A short time after a human action has taken place at some location, this action has no influence on the evolution of $Q(x,t)$, except in the vicinity of that location. We call this property "locality in a rudimentary sense."[11]

[11] What is meant here by assumption c) or by the words "locality in a rudimentary sense" can be expressed in rigorous mathematical terms. Consider a given $\lambda$ at a time $t$. Consider the probability distributions of $Q(x,t+\epsilon)$ at a point of space coordinates $x$, a little time $\epsilon$ later. Consider not only the probability distribution one can determine not knowing what is observed elsewhere but also the conditional probability distributions whatever "condition" is set on any human action elsewhere. Assumption c) means that all these probability distributions for $Q(x,t+\epsilon)$ are functions of elements of reality existing and of human action taken in the vicinity of $x$, independent of the values of faraway elements of reality and independent of these faraway human actions. Therefore assumption c) implies "factorizability" of the probability distributions of the $Q(x,t)$'s for different $x$'s. This assumption is more restrictive than needed for the demonstration of Bell's theorem but is adequate for the purpose of this paper.

In any restframe, the wave function $\psi$ for more than one particle is, in general, not a function of a single space variable $x$. Therefore it does not have property c).

d) *A property of Lorentz covariance can be given to $Q(x,t)$ and the law of evolution of $\lambda$ is the same in all space-time restframes.* Then the particular restframe used above to define properties a), b), and c) is not different from any other. To satisfy property a) regarding the causal relationships, elements of reality have now to depend only on human actions taken at *previous times in every space-time restframe,* therefore in the light cone in the past. Then the maximum speed at which any influence can propagate is the velocity of light $c$.

This property d) is specific to "Einstein's locality."

Property d), namely Einstein's locality, implies property c), i.e., rudimentary locality. Since $\psi$ does not have property c), it does not have property d) either.

### 2.4 Bell's Theorem.

Now, considering all possible descriptions of reality, not only the ones relying on the wave function $\psi$, we will demonstrate that no representation of reality $\lambda$ satisfying the conditions above can reproduce the quantum theory predictions of eq.(12). For this demonstration, we will first show that these conditions imply some statistical properties (Bell's inequalities) for the results of our EPR-Bohm experiment and then we will show that eq.(12) predicts results that do not have these statistical properties.

Assume Einstein's locality. In the EPR-Bohm experiment of Subsec.2.1, measurements #1 and #2 occur at approximately the same time. Because of assumption a), the probability distributions of the observables depend on the mathematical representation $\lambda$ of the quantum system just before the measurements, and on the conditions $\phi_1$ and $\phi_2$ of observation. Because of assumption c), the probability for photon #1 to pass the polarizer oriented at the angle $\phi_1$ and be counted in spaceship #1 is a function of $\phi_1$ and of these elements $Q(x,t)$ of reality $\lambda$ that are located near spaceship #1, independent of $\phi_2$ and of any element of reality created near spaceship #2 by measurement #2. Because of the same assumption, it can be expressed as a function $r_1(\lambda, \phi_1)$ which is the same whether or not photon #2 passes the polarizer in spaceship #2.[11] Conversely, the same requirement can be applied to photon #2. Its probability of passing the polarizer in spaceship #2 is a function $r_2(\lambda, \phi_2)$ independent of $\phi_1$, whether or not photon #1 passes its polarizer and gets counted. One can define a joint probability for both photons to be detected as an expression of the form

$$r(\lambda, \phi_1, \phi_2) = r_1(\lambda, \phi_1) r_2(\lambda, \phi_2) \tag{15}$$

and the fractions of events $f_1(\lambda, \phi_1, \phi_2)$, where only photon #1, and $f_2(\lambda, \phi_1, \phi_2)$, where only photon #2 is detected, are expected to be

$$f_1(\lambda, \phi_1, \phi_2) = r_1(\lambda, \phi_1)[1 - r_2(\lambda, \phi_2)] \tag{16}$$

$$f_2(\lambda, \phi_1, \phi_2) = [1 - r_1(\lambda, \phi_2)] r_2(\lambda, \phi_2) \tag{17}$$

For a given $\lambda$, these fractions $f_1$ and $f_2$ correspond to uncorrelated joint probabilities, but the uncertainties about what $\lambda$ is have not yet been taken into account. One has to consider the possibility that $\lambda$ may change from event to event, even in the context of our EPR-Bohm experiment, even for the events of interest, i.e., when the active collimator has not fired and the quantum theoretical "preparation" has produced the same pure case represented by the same wave function $\psi_0$ for all events. As considered in Subsec.2.3, there are several possibilities $\lambda_k$ and there are probabilities $w_k$ that $\lambda = \lambda_k$. The observed statistical distribution is a weighted average over $k$ of expressions like eqs. (15), (16), and (17), but where $\lambda$ is replaced by $\lambda_k$. Then, the fractions $F_1(\phi_1, \phi_2)$ and $F_2(\phi_1, \phi_2)$ of events where one and only one of the photons #1 and #2 is counted are of the form

$$F_1(\phi_1, \phi_2) = \sum_k w_k f_1(\lambda_k, \phi_1, \phi_2) = \sum_k w_k r_1(\lambda_k, \phi_1)[1 - r_2(\lambda_k, \phi_2)] \tag{18}$$

$$F_2(\phi_1, \phi_2) = \sum_k w_k f_2(\lambda_k, \phi_1, \phi_2) = \sum_k w_k [1 - r_1(\lambda_k, \phi_1)] r_2(\lambda_k, \phi_2) \tag{19}$$

Now, expressions (18) and (19) correspond to joint probabilities with some correlations between the results of measurements #1 and #2. They are typical expressions for correlations generated by a "*common cause.*" However, they will be shown to be incapable of reproducing the predictions of eq.(12).

Consider any set of four numbers a, b, c, and d and the quantity

$$I_B(a,b,c,d) = F_1(a,b) + F_2(c,b) + F_1(c,d) - F_1(a,d) \tag{20}$$

If $F_1$ and $F_2$ can be expressed as in eqs.(18) and (19),

$$\begin{aligned} I_B(a,b,c,d) &= \textstyle\sum_k w_k[A_k(1-B_k) + B_k(1-C_k) + C_k(1-D_k) - A_k(1-D_k)] \\ &\equiv \textstyle\sum_k w_k\{A_k(1-B_k)C_k + (1-A_k)B_k(1-C_k) \\ &\qquad + A_k(1-C_k)D_k + (1-A_k)C_k(1-D_k)\} \\ &\geq 0 \end{aligned} \tag{21}$$

where

$$A_k = r_1(\lambda_k, a) \tag{22}$$

$$B_k = r_2(\lambda_k, b) \tag{23}$$

$$C_k = r_1(\lambda_k, c) \tag{24}$$

$$D_k = r_2(\lambda_k, d) \tag{25}$$

This quantity $I_B(a,b,c,d)$ is positive or zero because, in its second expression in eq.(21), it is the sum of only obviously positive or null terms, whenever the set of weights $w_k$, the function $r_1(\lambda,\phi_1)$, and the function $r_2(\lambda,\phi_2)$ are capable of expressing probabilities, i.e., if they have a range of values between zero and one. The constraint that $I_B(a,b,c,d)$ be positive or null, for any set of numbers a, b, c, and d, is one of the forms of Bell's inequalities.

On the other hand, for the events of interest in our EPR-Bohm experiment, we can compute the same quantity, $I_B(a,b,c,d)$, defined by eq.(20), using the fractions $F_1$ and $F_2$ of eq.(12) predicted by quantum theory. It is negative for large ranges of values of a, b, c, and d. For instance, it is easy to compute that

$$I_B(0^o, 22.5^o, 45^o, 67.5^o) = \frac{1}{2}(1-\sqrt{2}) = -.207... < 0 \tag{26}$$

It follows that one cannot find a set of weights $w_k$, a function $r_1(\lambda,\phi_1)$, and a function $r_2(\lambda,\phi_2)$, positive and smaller than one, such that the fractions $F_1(\phi_1,\phi_2)$ and $F_2(\phi_1,\phi_2)$ of eqs.(16) and (17) satisfy the predictions of eq.(12). However if, as in eq.(13) where we could equate $\psi_0$ with $\lambda$, the distribution defined as $r_2$ is not constant as a function of $\phi_1$ or if, as in eq.(14), $r_1$ is not constant as a function of $\phi_2$, one can satisfy eq.(12). Therefore, to reproduce the predictions of quantum theory, it is necessary that the $F$'s be not of the forms (16) and (17), but include a dependence of either $r_1$ or $r_2$ on both $\phi_1$ and $\phi_2$. This implies an influence of the setting up of a polarizer angle in one spaceship on the result of an observation in the other. Regardless of the time delay, quantum theory always predicts that $F_1$ and $F_2$ will satisfy eq.(12). Therefore, if quantum theory is right, the influence has to propagate not only faster than light, but instantaneously.

For the sake of completeness, it should be pointed out that, to reach that conclusion, we tacitly used condition b) of Subsec.2.3. The fractions $F$'s, which reveal the influence in question, are determined only in the last stage of the experiment of Subsec.2.1, when the data are gathered in one place and analyzed. They reveal a *faster-than-light* propagation of that influence only if the data used to compute these $F$'s are the same, unaltered, as the ones seen by observers #1 and #2 during the experiment. Therefore, in the demonstration, we have assumed that the results of observations available to all concerned are the same. This is a consequence of condition b) of Subsec.2.3.

So far we have considered that the observation of either photon #1 or #2 was ruled by a probabilistic law defined by the functions $r_1(\lambda, \phi_1)$ and $r_2(\lambda, \phi_2)$. The models assuming determinism of one or both observations for a given $\lambda$ are not excluded from this demonstration. To take into account these models, it is enough to assume that one or both of the functions $r_1(\lambda, \phi_1)$ and $r_2(\lambda, \phi_2)$ take only values equal to one or zero. The case of a unique value of $\lambda$ corresponding to the wave function $\psi_0$ is also covered by the possibility of making all weights $w_k$ equal to zero except one, which would be equal to unity.

### 2.5 Analogy with a Spy Story.

The demonstration of Subsec.2.4 assumes that reality can be represented at all times by a mathematical entity which we call $\lambda$. Is such a representation always possible? For quantum systems, "reality" has sometimes been alluded to without implying the existence of a mathematical representation, from which all probability distributions would be derived. Depending on which definition is given to the word "reality," is it possible that Bell's inequalities do not apply? Generalizations of Bell's theorem to as many concepts of reality as possible were made by Henry Stapp [7] and others [16,17]. They rely on properties of the experimental results themselves, on their definiteness, and/or on characteristics of their probability distribution. Here we will follow another approach.

Figuring out how nature operates may be compared to solving a difficult puzzle. This permits us to transpose our quantum-theory problem and discuss it in another context, where the concept of reality is not so ambiguous. Yet a satisfactory explanation of what happens in reality will not be easy to obtain. Bell's inequalities will have to be applied. The reader's intuition will then be needed to transpose the results from the subsequent analysis back into the realm of quantum theory and to get a feeling for how Bell's theorem is more general than just the case demonstrated above.

The transposed problem is a riddle of the kind used to amuse guests in social gatherings. There are two spies, #1 and #2, who play the same role in the story as nature in our scientific investigations. In this context, they are not to be considered as "observers" in the quantum theoretical sense. Their behavior is assumed to be determined by the information available to them. *We* are the observers observing them and trying to figure out how they operate.

These spies are sending coded messages to the same country abroad, a country interested in monitoring the mood of our population, aggressiveness or friendliness, in order to react to it appropriately. We are intercepting all their coded messages but we do not know the key. The coded messages are sets of zeros and ones. All probability tests performed on the distribution of these digits in either message have detected no deviation from the hypothesis of a purely random distribution. This is possible, of course, if the encoding consists of adding, modulo 2, a random number expressed in a binary form to each digit of the uncoded message, also expressed in binary form. Nonetheless, there are correlations between the messages, as there are between the results of measurements in an EPR-Bohm experiment.

When the messages are juxtaposed, one finds zeros or ones often at the same location. This is only possible if, for each message, the key used by one spy is not independent from the key used by the other. Do the spies also share information other than the key? The riddle consists of finding a proof that, in certain circumstances, they do. The proof is derived from the same argument as Bell's theorem.

The messages depend on the information available to the spies. We can change the information that either spy has by supplying him with biased misinformation. Let us call $\phi_1$ and $\phi_2$ that biased misinformation we supply to spies #1 and #2, respectively. It is a human action which we exert on the spies and which is analogous to the act of setting polarizer angles in the EPR-Bohm experiment of Subsec.2.1. We can either formulate threats against or say kind words about their country. We will then refer to either one of these attitudes of ours by the statement: "$\phi_1$ or $\phi_2$ = *threats* or *kind words*," whichever is relevant. If the correlation changes when we feed this biased information to the spies, we can conclude that the kind of information we feed them is relevant to the subject of the messages.

Let us now define the fraction $F_1(\phi_1, \phi_2)$ of digits equal to one in the message from spy #1, that correspond to a zero digit in the message from spy #2. Let $F_2(\phi_1, \phi_2)$ be the similar fraction of ones in the messages from #2 corresponding to zeros in the message from #1. After intercepting both messages we can determine $F_1(\phi_1, \phi_2)$ and $F_2(\phi_1, \phi_2)$, as in the data analysis of the EPR-Bohm experiment. After successively telling either spy "threats" and "kind words," we can determine the quantity

$$\begin{aligned} I_B = \quad & F_1(threats, threats) + F_2(kind\ words, threats) \\ & + F_1(kind\ words, kind\ words) - F_1(threats, kind\ words) \end{aligned} \tag{27}$$

The $F$'s are small if the two messages agree and large if they are very different. The quantity $I_B$ can be negative if the first three terms are small and the last one large. This can happen if, for instance, the following statements are both correct:

a) spy #1 believes the biased information given to him in any case,

b) spy #2 is an optimist. He does not believe threats he is hearing unless he knows that spy #1 has heard them too. Of course, this is possible only if spy #2 receives information from spy #1.

This solution implies some form of communication between the spies. If $I_B$ is negative, do all solutions imply such communication? It can be shown that they do. To prove that, let us assume the opposite, that each of the spies keeps all the information he has for himself, therefore the biased information we are giving him as well. Then the biased information $\phi_1$ is not known to spy #2, and $\phi_2$ is not known to spy #1 (like the polarizer angles in the EPR-Bohm experiment under the assumptions of Subsec.2.4: these angles were not supposed to influence the detection of the faraway photon ). Of course, there is more information available to both spies. There is the information they collect routinely which, to make it analogous to the case of quantum theory, we will assume to be constant in time, i.e., independent of the action we take to misinform the spies. This routine information as well as the key used to encode the messages will be represented by the symbol $\lambda$. From message to message, we expect $\lambda$ to change since the key changes. There are several values $\lambda_k$ and there is a statistical distribution represented by the weights $w_k$ expressing how frequently $\lambda = \lambda_k$.

If the spies do not communicate, the message from spy #1 depends on $\phi_1$, on $\lambda$, and on some randomness due to spy #1's way of writing messages, not on $\phi_2$. For each digit of his message, we can assume that there is a probability that this digit be one and that this probability is a function $r_1(\lambda, \phi_1)$ independent of $\phi_2$. Likewise, there is a probability $r_2(\lambda, \phi_2)$, independent of $\phi_1$, that the same digit be one in the message from spy #2. The probability that the digit is one in the first message and zero in the second is

$$F_1(\phi_1, \phi_2) = \sum_k w_k \, r_1(\lambda_k, \phi_1)[1 - r_2(\lambda_k, \phi_2)] \tag{28}$$

as in eq.(18). The probability that the digit is one in the second message and zero in the first is

$$F_2(\phi_1, \phi_2) = \sum_k w_k[1 - r_1(\lambda_k, \phi_1)]r_2(\lambda_k, \phi_2) \tag{29}$$

as in eq.(19).

As in Subsec.2.4, these forms for $F_1(\phi_1, \phi_2)$ and $F_2(\phi_1, \phi_2)$ require that the expected value of $I_B$ of eq.(27) be positive or zero. This is true for each digit of the messages, therefore for the average of all of them. Therefore, if the quantity $I_B$ is negative, any solution to our riddle implies that the probabilities do not have the form (28) and (29), which were derived from the assumption that the spies do not communicate. Therefore any solution must involve communication between the two spies.

To enhance the relevancy of our riddle to the problem of quantum theory at hand, let us assume that we have been able to switch from threatening to saying kind words and back so fast that no communication between the spies could have been established by mail. Let us assume that the correlations still exist with this fast switching of biased information and are exactly the same as when there is plenty of time for the spies to correspond with each other. If $I_B$ is negative, our conclusion should be that they communicate by faster-than-mail means. Now let us assume that we have tapped their telephone, have monitored radio waves around the spies' residences, and found that the communication is neither made by telephone or radio. Are we going to conclude that they have a way to communicate that we do not know about?

In the case of the spies as in the case of quantum theory, our demonstration depends on the existence of functions $r_1(\lambda, \phi_1)$ and $r_2(\lambda, \phi_2)$ to define the probabilities. It is conceivable, in the case of the spies as well as in the case of quantum theory, that such very well defined functions may not be easily justifiable. After all, the spies are humans and it may not be reasonable to expect human behavior to be represented accurately by a statistical process ruled by a well defined probability distribution. However, if the quantity $I_B$ of eq.(27) is negative, how is it possible to imagine a mechanism by which the spies generate their messages without communicating with one another? Intuitively, one sees that the theorem can be made more general than the conditions of definite probabilities $r_1(\lambda, \phi_1)$ and $r_2(\lambda, \phi_2)$

for which we have demonstrated it. It should matter for anyone asking the question: "How do the spies do it?" Similarly, in the realm of quantum phenomena, the theorem should concern anyone who is wondering: "How does nature do it?"

Whether dealing with an innocuous riddle about spies or a fundamental problem in physics, it is expected that, if it is difficult, most people will stop looking for a solution after a while. On the other hand, some others do not give up easily. If you are one of those, read on. A short description of some typical solutions is given in the next section.

# 3 A Sample of Possible Solutions.

## 3.1 Experimental Loopholes.

What has been demonstrated so far is a contradiction between the quantum theory predictions and a description of reality satisfying Einstein's locality postulate. Solutions to the paradox can be found if we give up either the quantum theory predictions, or the Lorentz invariance implied by Einstein's locality, or our concept of reality. Subsec.3.1 discusses the possibility of saving Einstein's locality by capitalizing on the loopholes of all experiments having confirmed quantum theory in the past. Subsec.3.2 gives examples of new attitudes toward the concept of reality. Subsec.3.3 contains a description of models of reality violating Einstein's locality, including one of them which is further elaborated on in Subsec.3.4. All this discussion is summarized in Subsec.3.5.

If an EPR-Bohm experiment of the type described in Subsec.2.1 had been done, proof would have been obtained either against quantum theory or against Einstein's locality. Unfortunately, such an experiment has not been done, [19]. However similar experiments incorporating many of its features have been carried out, [3,4]. They all have loopholes but they all vindicate quantum theory, [18]. They impose constraints on the models that can be constructed. Actually performed experiments differ from the EPR-Bohm experiment of Subsec.2.1 in several ways:

a) Actual experiments were performed in laboratories. The distances between the photon source and the polarizers were a few meters instead of the distance that light would cover in one second. During the flight time of the photons, either the polarizer angles were not changed [3], or they were changed automatically at the very high frequency of two independent oscillators (one for each polarizer), [4].

This loophole leaves the possibility open for some sort of a "conspiracy" between the two measurements. An element of reality depending on $\phi_1$ may have been set up in spaceship #2 before the impact of photon #2 as a result of effects propagating no faster than light. This element of reality could affect the probability of detecting photon #2, thus justifying eq.(13) instead of (18) and allowing $I_B$ of eq.(20) to be negative. However, because of the experiment performed with oscillators, ad-hoc phase shifts in the picture of reality have to be generated, in addition, to account for the fact that the results are the same with and without oscillators. In practice, it would be difficult to take advantage of such possible "conspiracy" to construct a model without faster-than-light effects.

b) The photons we called photons #0 were not detected. Rates instead of probabilities were measured. The relevant forms of Bell's inequalities are different from inequality (21). They involve rates.

Because of this difference alone, slightly different steps than those described in Subsec.2.2 are necessary to compute the quantum theory predictions in the experiments but, for Bell's theorem, it is essentially inconsequential.

c) There was no active collimator and the efficiencies of polarizers and detectors were not 100%. Both effects introduced a large background in the counting rate of one single photon. It follows that no form of Bell's inequalities per se are violated by the quantum theory predictions for these cases. Other inequalities are violated but these other inequalities can be justified only by adding other assumptions to the premises of Bell's theorem, [6]. These other assumptions concern properties of the efficiencies of detection as the polarizers are rotated and removed.

These other assumptions are reasonable assumptions and hard to avoid in a model of reality. However, it is possible to capitalize on this loophole and construct models free from faster-than-light effects. [20]

The most striking result of the experiments performed is the vindication of the predictions of quantum theory to the limit of experimental accuracy, [18]. Suppose that the strong violation of quantum theory predicted by Bell's theorem for a description of reality according to Einstein's locality exists and can be seen when the efficiencies are 100%. Why then are we not even observing a very small violation when efficiencies are only 10%? Such considerations are at the origin of the most commonly drawn conclusion that a picture of reality cannot satisfy Einstein's locality. However, if a future experiment with fewer loopholes shows a violation of quantum theory, this question will surely be open again.

## 3.2 Giving up on Conventional Concepts of Reality.

For those satisfied with the Copenhagen interpretation of quantum theory, Bell's theorem and the subsequent experimental partial confirmations of quantum theory are seen as an additional justification for their position. These faster-than-light effects complicate any picture of reality to the point that such picture could not be figured out or, if it could be figured out, that it would not fulfill a useful purpose. To further reinforce their position, eastern philosophies are sometimes invoked, [26]. Reality sometimes is alluded to, but between observations it is not represented by a mathematical quantity.

As suggested by Bohr [14], the concept of reality can be revised. Some models have been constructed with concepts of reality that violate either assumption a) or b) of Subsec.2.3, which we used to demonstrate the theorem in Subsec.2.4. These models can reproduce the predictions of quantum theory exactly and obey a strict law of Lorentz invariance. Of course, it is likely that Bohr meant something else and probably alluded to concepts of reality without a mathematical representation between times of observations. However we briefly describe two such models of revised reality to give examples of this approach.

a) The first model, [27], represents reality by a quantity $\lambda$ that, at any one time, does not depend only on $\lambda$ itself and on human actions in the past but on $\lambda$ and on human actions in the future as well, in contradiction with condition a) of Subsec.2.3. Causal relationships exist between space-time points in the light cones of each other but the action is exerted both forward and backward in time. Equations can be made Lorentz invariant because of the symmetry of most basic equations of physics with respect to a reflection of the direction of time. The symmetry alluded to here is the same as the one used to express electromagnetism in terms of both advanced and retarded potentials. For the EPR-Bohm experiment of Subsec.2.1, there are effects, caused by the act of setting up $\phi_1$ and $\phi_2$, that propagate backward in time from the points of impact of the photons on the polarizers to their common point of emission in spaceship #0 (see Fig.3). There, these backward-in-time effects influence elements of reality and make $\lambda$ dependent on the future settings of the polarizers $\phi_1$ and $\phi_2$. From this point of emission, the characteristics of $\lambda$ now evolve forward in time until the photons reach the polarizers. The probabilities at both points of impact can then be a function of both $\phi_1$ and $\phi_2$, as in eqs.(13) and (14), and escape the constraints of eqs. (15), (16) and (17). Thus $I_B$ of eq.(20) does not have to be positive.

Causal effects backward in time may create a problem because of causal loops. Tomorrow, we can set up a parameter that influences an observation made today and, if the theory allows information understandable to humans to flow backward in time, we can send ourselves a message from the future to the present. Then the theory would allow us to inform ourselves of the occurrence of some events tomorrow and we could take an action today to prevent their occurrence. Equations have to be carefully set up to avoid such nonsense. How a model avoiding such problems can be constructed may be easier to understand if the model is first transposed into the spy-story example of Subsec.2.5. In the transposed model, each of the two spies, #1 and #2, has the unusual ability to foresee what information, $\phi_1$ and $\phi_2$, respectively, is going to be given to him in the future. Because of that special gift, the spies have plenty of time to communicate with each other by mail and exchange that information before they send their messages out. However, for us who do not have the key, knowing only one coded message gives us no information. The correlation is the source of all intelligible information for us and it can be determined only when both messages are known, i.e., after both sets of biased information $\phi_1$ and $\phi_2$ have already been delivered. Thus there is no way for us to take advantage of the spies psychic powers to find out what decision, "*threats*" or "*kind words*," will be made by us in the future and, therefore, we cannot take an action to change it. We are bound to fulfill our destiny, not unlike the characters of the Greek legend of Oedipus. [28]

It may seem awkward to invoke backward in time influences. But this model preserves Lorentz invariance absolutely. Note that, by linking causal chains in the forward and the backward time directions in a row, one is actually producing causal effects propagating outside of the light cone. Therefore this kind of model, though Lorentz invariant, is not free from faster-than-light actions.

b) The second of our models uses a concept of reality that is not the same for all observers. Because of this, it does not have property b) of Subsec.2.3, which assumed the opposite. In the model, there are many universes. Observers who do not see the same reality do not belong to the same universe and cannot communicate with one another. Take the observer in spaceship #1. Upon impact of photon #1, he splits into two or more individuals who cannot communicate with each other. Some of these individuals detect the photon in their detector and, for the others, the photon gets absorbed in the polarizer. The observer in spaceship #2 similarly splits into different individuals upon impact of photon #2. To reproduce the statistical correlation of quantum theory, all that is needed is to match the right observer #1 with the right observer #2 in the same universe. This proper matching can be done much later, anytime before data analysis is performed in the EPR-Bohm experiment of Subsec.2.1. By that time, communication has been possible between spaceships at a speed not faster than the speed of light. The model is called the "Many Worlds Interpretation of Quantum Mechanics." [29]

When the "many worlds" model is transposed into the spy-story example of Subsec.2.5, one has to assume that each spy sends out all possible messages simultaneously. Before reading a message, each one of our counter-espionage agents is cloned at least as many times as there are such different possible messages. Each one of the cloned agents reads only one of these simultaneous messages and he is not aware of the other cloned agents' existence. This happens to the agents observing the message from spy #1 and to those observing the message from spy #2. There is communication by mail between one clone of any agent and one clone of any other. The mailmen know the content of the letters and which message each clone has been given. The mailmen select which one of the clones is receiving which letter in such a way that the observed correlations are in accordance with a prearranged scenario, (the equivalent of the quantum theory predictions). The mailmen are thus responsible for the correlations.

As observations continue to be made, the number of observers in the model increases exponentially. However, the model is logically viable.

## 3.3 Fundamental Space-Time Restframes.

For most of us, reality is a deep rooted concept and not everyone is willing to revise it. By accepting faster-than-light effects, it is possible to assume a more conventional description of reality, in particular one assuming causal effects propagating only forward in time and the same reality for all observers, i.e., with properties a) and b), though not property d) of Subsec.2.3. In such models, it is necessary to define a particular space-time restframe as being a fundamental one. In this restframe, a cause will always occur before its consequence. If cause and consequence occur outside of the light cones of one another, the time order of the two in other restframes may not be the same as in the fundamental restframe. Therefore, these other restframes cannot be considered as fundamental as this particular one, in which causes always precede consequences. However, we know that observations in accordance to the predictions of relativistic quantum theory do not permit identifying a fundamental restframe. In all experiments for which the model reproduces the predictions of quantum theory, observations predicted by the model will not permit identifying it either. We will also give examples of such models hereafter.

a) One possibility, already pointed out in Subsec.2.3, is to describe reality by that wave function $\psi$ that is used to make computations in one particular (fundamental) space-time restframe. In other restframes there is another wave function $\psi'$ but then, to make reality the same for all observers, that $\psi'$ cannot describe reality. That other wave function $\psi'$ can only be used to make calculations, as in the Copenhagen interpretation. A model of this type is being developed by Philip Pearle [22]. Long before him, Bohm also constructed a model [15], mentioned in Subsec.1.3, where the wave function $\psi$ is used in conjunction with a singularity following a trajectory determined by the function $\psi$. Within these models, finite durations for the collapse phenomenon have been suggested but the disagreement with quantum theory generated by this effect can be made small. Therefore one can say that these models are able to reproduce the predictions of quantum theory exactly.

   In both models, for more than one particle, reality is not expressed in general as a function of a single set of space-time coordinates. Reality does not consist of "objects existing in space and time." Furthermore, collapses correspond to effects propagating instantaneously. Consequently, these models do not have either Einstein's locality of paragraph d) of Subsec.2.3 or the rudimentary locality property of paragraph c).

When these models are transposed into the spy-story of Subsec.2.5, the communication between spies has to be instantaneous. That instantaneous interaction makes the description of reality as features distributed over space lose much of its interest. Therefore, instead of paying a great deal of attention to the spies' locations in space, one would be tempted to describe their instantaneous interaction as a connection that does not involve space. This kind of connection is suggestive of "telepathy" or any other nonspatial structure. In Bohm's model, a nonspatial structure is assumed to be fundamental and is called "implicate order." [15]

b) An even less earthshaking approach consists of constructing a picture of reality made of "objects existing in space and time," i.e., of elements $Q(x,t)$ distributed over space and time like a field in classical physics. One can write equations to have only effects propagating no faster than a finite velocity $V$, greater than the speed of light $c$. Such models have the "rudimentary property of locality" of paragraph c) of Subsec.2.3, though not Einstein's locality of paragraph d). In these models, there will be violations of some quantum theory predictions, since quantum theory implies instantaneous action at a distance and the models do not. However, if $V$ is very large, experimental situations where a violation due to this effect can show up are extremely rare. If $V$ is large enough, the discrepancy cannot have been detected by previously performed experiments.

Since relativistic quantum theory does not allow us to identify a fundamental space-time restframe, the only experimental circumstances in which such identification can be done are those rare cases where, due to the finite propagation $V$ in the fundamental restframe, a discrepancy with quantum theory could be observed. If $V$ is large enough, these cases are so rare that the fundamental restframe could not have been detected in experiments performed to date.

Such models require giving up our faith in the absolute validity of both the quantum theory predictions and Lorentz invariance. They imply the existence of a privileged restframe that could be identified in some rare experimental situations, not yet realized today. However, a conventional reality with elements distributed over space without instantaneous action at a distance is an appealing idea, because it has the features of a classical field theory before relativity was invented. It satisfies some of Einstein's most cherished principles. In a tribute to J.C. Maxwell, he wrote:

> "The latest and most successful creation of theoretical physics, namely Quantum Mechanics, is fundamentally different in its principles from the two programmes which we will briefly call Newton's and Maxwell's. . . Yet I incline to the belief that physicists will not be permanently satisfied with such an indirect description of Reality, even if the theory can be fitted successfully to the General Relativity postulates. They would then be brought back to the attempt to realize that

programme which may suitably be called Maxwell's: the description of physical reality by fields which satisfy without singularity a set of partial differential equations." [30]

When such a model with rudimentary locality is transposed into our spy story, it implies that, between the two spies, there is a means of communication that is faster than mail but not instantaneous. We realize that we may have been able to determine lower limits to the speed of their communication but also that we cannot test "instantaneousness." Then we probably want to investigate the possibilities of that means of communication, instead of ignoring it, as in the Copenhagen interpretation, or considering it as a manifestation of an exotic form of reality, as in some of the models above. In the spy story, this approach looks like the "down to earth" approach. In the case of quantum theory, we will leave the reader decide for himself whether or not these models, with a conventional concept of reality and elements distributed over space, also deserve the epithet "down to earth."

## 3.4 How a Model with Rudimentary Locality Can Work.

The elements of reality $Q(x,t)$, defined at every point of space and time, can be matrices, sets of tensors of one or several ranks, or any other mathematical quantity having a functional dependence on a single set of space coordinates $x$ and characterizing properties existing at the point of coordinates $x$. A convenient mathematical quantity for this purpose is a *positive definite operator in Fock space of trace equal to unity*, which we call $\underline{Q}(x,t)$. This operator has so many possible configurations that it can describe as complex a quantity as the density matrix of all the particles of the universe. Furthermore, equations can then be written that are very similar to the equations used in quantum field theory. Use of this operator has been the option taken in a model that is going to be published soon. [9]

Measurements are performed at given locations in space. Let $x_m$ be the coordinates of a point $m$ and $t_m$ the time at which a measurement $\mathcal{M}$ is performed. Let $\beta$ be the other conditions of observation, in addition to $x_m$ and $t_m$, that define measurement $\mathcal{M}$ (e.g., the orientation $\phi_1$ or $\phi_2$ of a polarizer). The probability $\wp_\mu$ of an outcome $\mu$ of measurement

$\mathcal{M}$ is a function of the parameters $\beta$ and of the elements of reality $\underline{Q}(x_m, t_m)$, which the model assumes to be the information available at point $m$.

$$\wp_\mu = \wp_\mu[\beta, \underline{Q}(x_m, t_m)] \tag{30}$$

where there is no other dependence of $\wp_\mu$ on $x_m$ than the explicit one via $\underline{Q}(x_m, t_m)$ in eq.(30).

At the time $t_m$ of the measurement, the elements of reality $\underline{Q}(x_m, t)$ attached to point $m$ change instantaneously, like the wave function $\psi$ in quantum theory. In the fundamental space-time restframe, at any point located at a distance $\Delta x$ from the measurement point $m$, a similar instantaneous change occurs but at a later time, $t_m + \Delta t$

$$\Delta t = \frac{\Delta x}{V} \tag{31}$$

This abrupt change is called "collapse" as in quantum theory. The time at which collapses occur in the model is shown in Fig.4 as a function of the distance $\Delta x$ from point $m$. It can be assumed that measurements occur frequently. Therefore space-time is traversed by many lines where $\underline{Q}(x, t)$ collapses, like the ones shown in Fig.4.

Outside of these lines of collapse, the operators $\underline{Q}(x, t)$ depend on $x$ and evolve with time $t$. Let $P$ be the momentum-operator and $H$ the Hamiltonian in Fock space. The model permits the definition of another operator in Fock space at any point of space-time.

$$\tilde{Q}(x, t) = e^{iHt} e^{-iPx} \underline{Q}(x, t) e^{iPx} e^{-iHt} \tag{32}$$

Equations of evolution in the model are set to make $\tilde{Q}(x, t)$ practically constant in $x$ and $t$ everywhere but across the lines of collapse. However, $\tilde{Q}(x, t)$ is dependent on the choice of origins of both the time and space coordinates, while $\underline{Q}(x, t)$ is not.

In quantum theory, predictions can be computed only for quantum systems not involving all the particles of the universe. These systems are essentially free of unitarity-violating interactions for time intervals during which they can be described as being independent from the rest of the universe. They interact with other systems only at some particular times $t_m$, when a "measurement" is said to be made. Quantum theory assumes that the effect of these interactions is, at least to a very good approximation, accounted for by the collapse of the wave function. To illustrate how the model works, we will consider a system $S$ in states called "pure cases of quantum theory," i.e., with a single wave function in the space of the variables of the system.

In the model, a system $S$ in a pure state corresponds to operators $\tilde{Q}(x,t)$ of the form

$$\tilde{Q}(x,t) = \tilde{Q}_{ext(S)}(x,t) \otimes \tilde{Q}_S(x,t) \tag{33}$$

where the index $S$ on an operator indicates that its range and domain are the variables of the system $S$ and where $ext(S)$ indicates range and domain to be all the variables of Fock space except the variables of $S$.[12] The operator $\tilde{Q}_S(x,t)$ is of rank one. Equation (33) can be valid over some time duration only under the condition that, between measurements on the variables of $S$, i.e., between these interactions called "measurements," the Hamiltonian operator $H$ for the system can be approximated as

$$H = H_{ext(S)} \otimes I_S + I_{ext(S)} \otimes H_S \tag{34}$$

where $I_S$ and $I_{ext(S)}$ are identity operators in the space defined by the variables $S$ and $ext(S)$, respectively.

The collapses due to the measurements on the variables $ext(S)$ not belonging to $S$ affect only $\tilde{Q}_{ext(S)}(x,t)$ and leave $\tilde{Q}_S(x,t)$ unchanged. The operators $\tilde{Q}_S(x,t)$ are practically constant in $x$ and $t$ except across the lines of collapse generated by the measurements on $S$. Any outcome $\mu$ of a measurement $\mathcal{M}$ on the variables $S$ made with an apparatus of characteristics $\beta$ at a point $m$ at time $t_m$ is associated, in the model, with a projection operator $\underline{\Pi}_\mu(\beta)$. This operator $\underline{\Pi}_\mu(\beta)$ is related to the projection operator $\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)$ used in quantum theory, in the Heisenberg representation, for the same measurement outcome of $\mathcal{M}$ , at the same point $m$, at the same time $t_m$, and in the same environment $\beta$.

$$\underline{\Pi}_\mu(\beta) = e^{iPx_m} e^{-iHt_m} [I_{ext(S)} \otimes \tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)] e^{iHt_m} e^{-iPx_m} \tag{35}$$

[12] Expression (33) does not involve symmetrized and antisymmetrized wave functions for the particles of the system $S$ and all other identical particles in the rest of the universe, as one would expect if Bose and Fermi statistics were properly taken into account. The cases considered here are cases where this symmetrization does not have any impact on the predictions of quantum theory. Then this effect does not have any impact in the model either.

which can be shown not to depend on $x_m$ or $t_m$.

In the model, the probability $\wp_\mu$ of the outcome $\mu$ of measurement $\mathcal{M}$ is given by

$$\wp_\mu = Tr[\underline{\Pi}_\mu(\beta)\underline{Q}(x_m, t_m - \epsilon)] \tag{36}$$

where $\epsilon$ is an infinitesimal time interval. Since $\underline{\Pi}_\mu(\beta)$ does not depend on $x_m$ or $t_m$, $\wp_\mu$ is of the form of eq.(30), as it was intended. Using eqs.(32), (33), (35) and (36), one can show that the expression (36) of $\wp_\mu$ reduces to

$$\wp_\mu = Tr[\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)\tilde{Q}_S(x_m, t_m - \epsilon)] \tag{37}$$

After measurement $\mathcal{M}$ , the collapse phenomenon propagates from point $m$ at the velocity $V$ and equations can be set up so that, at any point of coordinates $x$ and at the time of the collapse determined by eq.(31),

$$\tilde{Q}_S(x_m, t_m + \Delta t + \epsilon) = \frac{1}{\wp_\mu}\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)\tilde{Q}_S(x, t_m + \Delta t - \epsilon)\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m) \tag{38}$$

As long as no other measurement on the variables of $S$ induces any collapse of $\tilde{Q}_S(x,t)$ near point $m$ at about time $t_m$, one can define three regions of space-time as shown in Fig.4 around point $m$ and time $t_m$.

a) Region I, corresponding to times $t$ prior to $t_m$ for all $x$'s, where

$$\tilde{Q}_S(x, t) = \tilde{Q}_S(x_m, t_m - \epsilon) \tag{39}$$

b) Region II, corresponding to times $t$ after $t_m$ but, for each $x$, before the time of the collapse in the model as determined by eq.(31). In Region II, $\tilde{Q}_S(x,t)$ is still equal to the value given by eq.(39).

c) Region III, where, for each $x$, the time $t$ comes after the time of the collapse, and where

$$\tilde{Q}_S(x, t) = \frac{1}{\wp_\mu}\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)\tilde{Q}_S(x_m, t_m - \epsilon)\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m) \tag{40}$$

Let us assume that two measurements, $\mathcal{M}$ and $\mathcal{M}'$ , are performed on the system $S$ at points $m$ and $m'$, at times $t_m$ and $t'_m > t_m$, with apparatus settings defined by the parameters $\beta$ and $\beta'$, respectively. To make it easier to prepare the system in a pure case of quantum theory, let us assume that measurement $\mathcal{M}$ is not degenerate, i.e., that it determines the system $S$ entirely. (In quantum theory, it means that $\mathcal{M}$ completely defines the wave function $\psi$ after collapse except for an arbitrary overall phase.) Then the projection operator $\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)$ for the actual result $\mu$ yielded by $\mathcal{M}$ is of rank one. Therefore, in Region III of Fig.4, using eq.(40),

$$\tilde{Q}_S(x,t) = \tilde{\Pi}_{S,\mu}(\beta, x_m, t_m) \tag{41}$$

regardless of what $\tilde{Q}_S(x,t)$ is in Region I.[13]

If the second measurement, $\mathcal{M}'$ , is performed in Region III, the probability $\wp'_{\mu'}$ of its outcome being $\mu'$ is

$$\wp'_{\mu'} = Tr[\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)\tilde{\Pi}'_{S,\mu'}(\beta', x'_m, t'_m)] \tag{42}$$

where $\tilde{\Pi}'_{S,\mu'}(\beta', x'_m, t'_m)$ is the projection operator associated with the outcome $\mu'$ of measurement $\mathcal{M}'$ .

For the same sequence of measurements, in quantum theory, the wave function $\psi$ after measurement $\mathcal{M}$ is the only eigenvector of $\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)$ for its only nonzero eigenvalue. The probability $\wp'_{\mu'}$ of the outcome $\mu'$ of $\mathcal{M}'$ is

$$\wp'_{\mu'} = \psi^\dagger \tilde{\Pi}'_{S,\mu'}(\beta', x'_m, t'_m)\psi \tag{43}$$

Since $\psi$ is the only eigenvector of the projection operator $\tilde{\Pi}_{S,\mu}(\beta, x_m, t_m)$ for the eigenvalue one, it is easy to show that eq.(43) gives the same result as eq.(42). The model reproduces the predictions of quantum theory for the kind of pure cases considered if the second measurement occurs in Region III. This result can be generalized to pure cases obtained from the action of several incomplete measurements. It can also be generalized to

[13] In quantum theory, the arbitrary overall phase in the definition of $\psi$ affects its mathematical expression but does not affect the final predictions. In the model, even the mathematical expressions of $\tilde{Q}_S(x,t)$ thus of $\underline{Q}(x,t)$ are completely independent of this arbitrary phase.

"mixed cases," i.e., cases that need to be defined by a density matrix of rank greater than one. In any event, the predictions of quantum theory and from the model can be shown to be equivalent if the measurement $\mathcal{M}'$ occurs in Region III, for which the time interval $\Delta t$ between measurements $\mathcal{M}$ and $\mathcal{M}'$ is greater than than the distance $\Delta x$ between their locations divided by $V$. If $\Delta t$ is smaller than this limit, (i.e., if measurement $\mathcal{M}'$ occurred in Region II of Fig.4) there can be discrepancies. However, for very large $V$, Region II is very small and, as it was already mentioned in the previous subsection, such discrepancies are very rare. They could not have been noticed in experiments performed to date.

## 3.5 Conclusions.

Bell's theorem shows a contradiction between a conventional picture of reality, Einstein's locality, and quantum theory. It is possible to solve the paradox by sacrificing any one of the three ideas. These ideas are related to one's philosophical view of the world. Therefore the final choice of a solution may be affected by that view.

All experiments so far have confirmed the quantum theory predictions, [18]; therefore all solutions that reproduce these predictions exactly are not in contradiction with experiment. In addition, there are solutions based on pictures of reality of quantum systems with violations of the quantum theory predictions that could not have been detected in experiments performed so far, but may be detectable in other, not yet performed, experiments. In the category of solutions that can be made to reproduce the predictions of quantum theory exactly let us mention:

a) *The Copenhagen interpretation of quantum theory,* [11], *without any picture of reality between observations.* It is not always claimed that such picture of reality is impossible to draw but, if it were possible, that it would not be worth the effort to figure it out.

   Lorentz invariance is good for all observables when we observe them and for their probability distributions.

b) *The revised concepts of reality,* as in the models sketched in Subsec.3.2, relying on causal effects backward in time, [27], or assuming the existence of "many worlds,"

[29]. There are other examples, [31]. Such models can be made completely Lorentz invariant.

c) *Pictures of reality violating Lorentz invariance* as in the models mentioned in subsec. 3.3, paragraph a), [8,22]. There is a fundamental space-time restframe, which cannot be identified, and in that restframe there are *instantaneous actions at distances.* The interest of a description of reality as "objects existing in space and time," (i.e., with one of the properties characterizing "rudimentary locality") is much reduced. Except for this absence of "locality", these models use a conventional concept of reality.

The solutions above can all be made to predict the same results, all identical to the predictions of quantum theory. Therefore, choosing one solution instead of another depends essentially on one's philosophical preferences. In particular, it depends on the relative priorities that are assigned to Lorentz invariance and to the usefulness of a conventional description of reality.

In the second category of solutions, quantum theory is assumed to be only an approximate theory which is violated in some yet unexplored experimental conditions. In this category let us mention:

d) *Models capitalizing on the loopholes of previous experiments*, [20], as mentioned in Subsec.3.1. They imply pictures of reality with perfect Lorentz invariance and no faster-than-light effects. There are violations of the quantum theory predictions and they can be detected in the future, at least when a real EPR-Bohm experiment can be performed. It is generally assumed that models corresponding to such pictures would be complicated. If ever an experiment with more features resembling a real EPR-Bohm experiment reveals a violation of quantum theory, the question will surely be reopened.

e) *Models with a rudimentary property of locality* like the example [9] described in Subsec.3.4. These models have all the properties of a non relativistic classical field, therefore all properties expected from a conventional concept of reality, [25]. There is a fundamental space-time restframe, and in that restframe there is a maximum velocity $V$ of the propagation of any effect. However, $V$ is unknown as yet. Discrepancies with quantum theory predictions can show up only in experiments with measurements performed at time intervals $\Delta t$ smaller than the measurements' separation in space $\Delta x$ divided by $V$. Identification of the fundamental space-time restframe is also only possible in such experiments. For very large $V$, deviations from quantum theory will not be detectable. Therefore it is possible that future experiments show the failure of quantum theory but, if they agree with quantum theory, they cannot show a failure of the model. The situation is the same here as it was with the atomic theory soon after 1808, i.e., just after Dalton speculated that matter was discontinuous and made of atoms. It was possible experimentally to show that matter was discontinuous if one could measure an upper limit for Avogadro's number. However, as long as experiments revealed no discontinuity in the structure of matter, it was always possible to assume Avogadro's number large enough to explain the experimental results.

Future experiments should help clarify the situation. If quantum theory predictions are upheld in a real EPR-Bohm experiment, the models mentioned in paragraph d) will definitively be eliminated. If, instead, future experiments with more features of an EPR-Bohm experiment in their experimental setup than the ones performed so far show a violation of quantum theory, then models of the type considered in paragraphs a), b), and c) will be eliminated. It is likely that more than one solution will remain possible for quite a while. Then, the final choice will depend on one's philosophical inclination. Today, since all possible experiments have not yet been done, the choice is going to be based both on one's guess of the outcome of these experiments when they are performed in the future and on one's philosophical inclination.

There is no reason why everyone should agree about what that best choice is. Divergences of opinion must be expected that cannot be reconciled by a logical argument. Tolerance is in order.

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[14]In Subsec.2.5, both spies were incorrectly referred to by the pronoun "he." Actually spy #1 is a female. She met spy #2 once at a spy convention in their country of origin. They fell in love with one another, they married and lived happily ever after.

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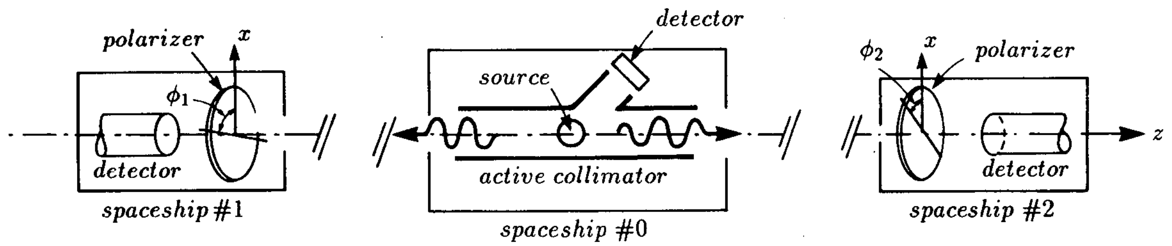


**FIG. 1.** Layout of an EPR-Bohm thought experiment: three spaceships and two photons (#1 and #2) with correlated helicities escaping spaceship #0. The events of interest are those events where a third photon (#0) has been detected in spaceship #0 but the active collimator did not detect any photon. Photons #1 and #2 will then fall on the polarizers in spaceships #1 and #2, respectively, and their plane of polarization will be analyzed by two observers posted there.

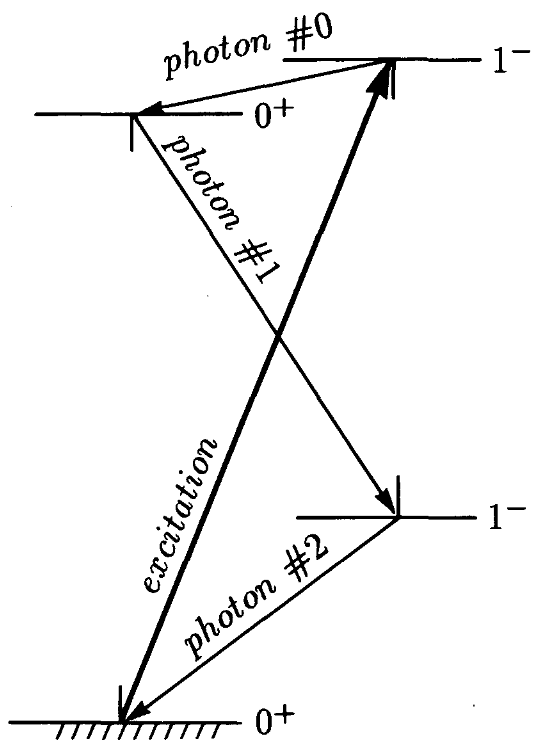


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**FIG. 2.** Excitation levels of atoms constituting the source of photons in the EPR-Bohm experiment. Incident light induces transitions of atoms from their ground level of spin-parity $0^+$ to the highest of the two $1^-$ excited levels. The events of interest are all events where the atom cascades down as shown by the arrows on the figure. In the text, it is shown that the polarization planes of photons #1 and #2 have, according to quantum theory, correlations appropriate to demonstrate the existence of faster-than-light influences.

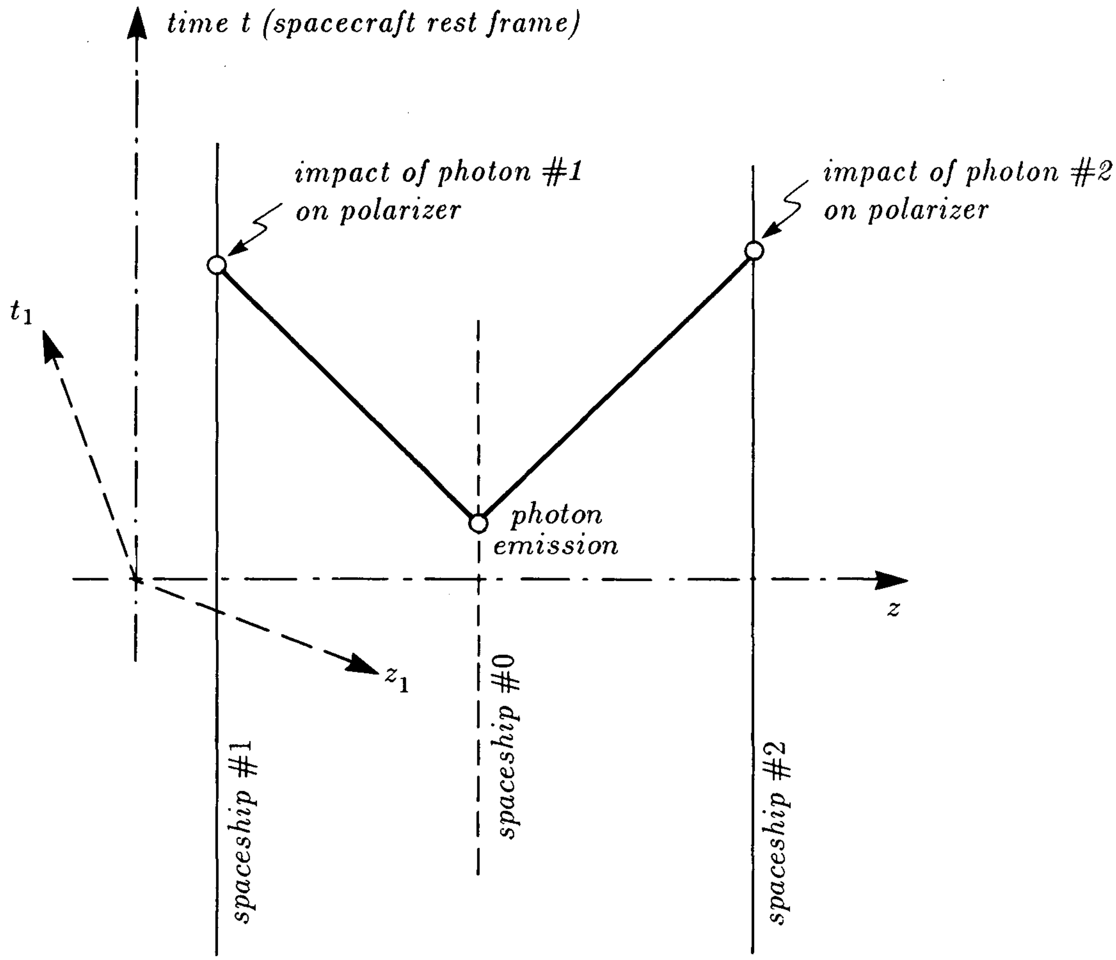


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**FIG. 3.** Space-time display of an event of interest in the spacecraft restframe. The two photon impacts are outside the light cones of one another. Therefore, there are Lorentz restframes, like the $z_1$-$t_1$ restframe, where photon #1 is seen impinging on the polarizer in spaceship #1 before photon #2 in spaceship #2 and there are restframes where it is the opposite.

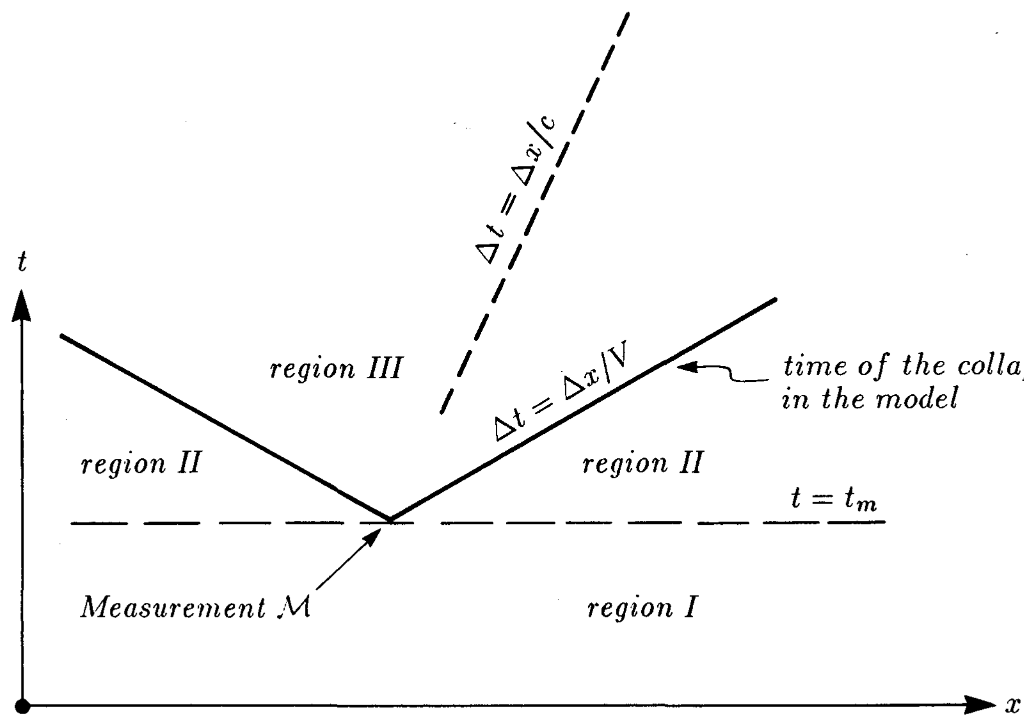


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**FIG. 4.** Space-time display of the collapse phenomenon in the model with "rudimentary locality" of Subsec.3.4. If the measurement $\mathcal{M}$ shown on the figure is followed by another measurement $\mathcal{M}'$ in Region III, the quantum theory predictions are reproduced. If $\mathcal{M}'$ occurs in Region II, they may not be. However, such cases can be made as rare as we wish by setting the parameter $V$ at a large enough value to make Region II very small. This way, the discrepancies between quantum theory and the model may have been invisible in experiments performed so far.